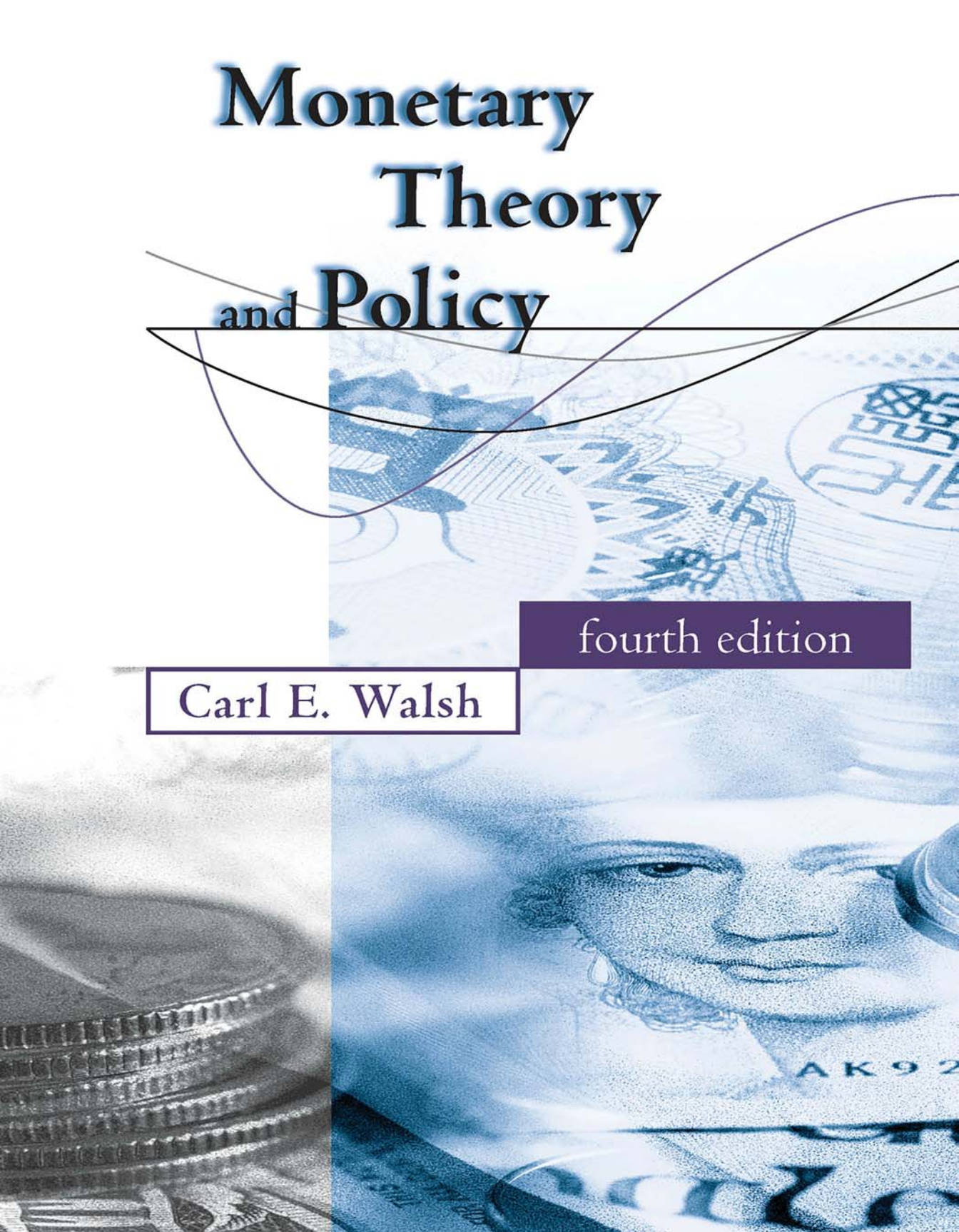


# Monetary Theory and Policy

The background of the cover is a collage of various currency notes and coins. On the right side, there are blue-toned banknotes, including a 100 Euro note and a 100 US Dollar note. On the left side, there are stacks of silver coins. Overlaid on the collage are several thin, black, curved lines that create a sense of movement and depth.

fourth edition

Carl E. Walsh



## **Monetary Theory and Policy**



# **Monetary Theory and Policy**

**Fourth Edition**

**Carl E. Walsh**

**The MIT Press  
Cambridge, Massachusetts  
London, England**

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# Contents

Preface	xiii
Introduction	xvii
<b>1 Evidence on Money, Prices, and Output</b>	<b>1</b>
1.1 Introduction	1
1.2 Some Basic Correlations	1
1.3 Estimating the Effect of Monetary Policy on Output and Inflation	8
1.3.1 The Evidence of Friedman and Schwartz	9
1.3.2 Granger Causality	13
1.3.3 Policy Uses	14
1.3.4 The VAR Approach	17
1.3.5 Structural Econometric Models	25
1.3.6 Alternative Approaches	27
1.4 Monetary Policy at Very Low Interest Rates	31
1.4.1 Measuring Policy at the Effective Lower Bound (ELB)	32
1.4.2 The Effects of Quantitative Easing (QE) Policies	33
1.5 Summary	40
<b>2 Money-in-the-Utility Function</b>	<b>41</b>
2.1 Introduction	41
2.2 The Basic MIU Model	43
2.2.1 Steady-State Equilibrium	48
2.2.2 Multiple Equilibria in Monetary Models	55
2.2.3 The Interest Elasticity of Money Demand	57
2.2.4 Limitations	61
2.3 The Welfare Cost of Inflation	61
2.4 Extensions	66

2.4.1	Interest on Money	66
2.4.2	Nonsuperneutrality	67
2.5	Dynamics in an MIU Model	69
2.5.1	The Decision Problem	70
2.5.2	The Steady State	73
2.5.3	The Linear Approximation	74
2.5.4	Calibration	78
2.5.5	Simulation Results	80
2.6	Summary	83
2.7	Appendix: Solving the MIU Model	83
2.7.1	The Linear Approximation	85
2.7.2	Collecting all Equations	90
2.7.3	Solving Linear Rational-Expectations Models with Forward-Looking Variables	91
2.8	Problems	94
<b>3</b>	<b>Money and Transactions</b>	<b>97</b>
3.1	Introduction	97
3.2	Resource Costs of Transacting	98
3.2.1	Shopping-Time Models	98
3.2.2	Real Resource Costs	102
3.3	Cash-in-Advance (CIA) Models	103
3.3.1	The Certainty Case	104
3.3.2	A Stochastic CIA Model	113
3.4	Search	120
3.4.1	Centralized and Decentralized Markets	122
3.4.2	The Welfare Costs of Inflation	127
3.5	Summary	129
3.6	Appendix: The CIA Approximation	130
3.6.1	The Steady State	130
3.6.2	The Linear Approximation	130
3.7	Problems	132
<b>4</b>	<b>Money and Public Finance</b>	<b>137</b>
4.1	Introduction	137
4.2	Budget Accounting	138
4.2.1	Intertemporal Budget Balance	143
4.3	Money and Fiscal Policy Frameworks	144
4.4	Deficits and Inflation	145



4.4.1	Ricardian and (Traditional) Non-Ricardian Fiscal Policies	148
4.4.2	The Government Budget Constraint and the Nominal Rate of Interest	151
4.4.3	Equilibrium Seigniorage	153
4.4.4	Cagan's Model	157
4.4.5	Rational Hyperinflation	159
4.5	The Fiscal Theory of the Price Level	162
4.5.1	Multiple Equilibria	163
4.5.2	The Basic Idea of the Fiscal Theory	164
4.5.3	Empirical Evidence on the Fiscal Theory	168
4.6	Optimal Taxation and Seigniorage	169
4.6.1	A Partial Equilibrium Model	170
4.6.2	Optimal Seigniorage and Temporary Shocks	173
4.6.3	Friedman's Rule Revisited	174
4.6.4	Nonindexed Tax Systems	186
4.7	Summary	188
4.8	Problems	189
<b>5</b>	<b>Informational and Portfolio Rigidities</b>	<b>193</b>
5.1	Introduction	193
5.2	Informational Frictions	194
5.2.1	Imperfect Information	194
5.2.2	The Lucas Model	195
5.2.3	Sticky Information	200
5.2.4	Learning	204
5.3	Limited Participation and Liquidity Effects	206
5.3.1	A Basic Limited-Participation Model	208
5.3.2	Endogenous Market Segmentation	212
5.3.3	Assessment	214
5.4	Summary	215
5.5	Appendix: An Imperfect-Information Model	215
5.6	Problems	219
<b>6</b>	<b>Discretionary Policy and Time Inconsistency</b>	<b>221</b>
6.1	Introduction	221
6.2	Inflation under Discretionary Policy	223
6.2.1	Policy Objectives	223
6.2.2	The Economy	225
6.2.3	Equilibrium Inflation	227

6.3	Solutions to the Inflation Bias	234
6.3.1	Reputation	235
6.3.2	Preferences	247
6.3.3	Contracts	251
6.3.4	Institutions	256
6.3.5	Targeting Rules	259
6.4	Is the Inflation Bias Important?	264
6.5	Summary	271
6.6	Problems	271
<b>7</b>	<b>Nominal Price and Wage Rigidities</b>	<b>277</b>
7.1	Introduction	277
7.2	Sticky Prices and Wages	277
7.2.1	An Example of Nominal Rigidities in General Equilibrium	278
7.2.2	Early Models of Intertemporal Nominal Adjustment	282
7.2.3	Imperfect Competition	285
7.2.4	Time-Dependent Pricing (TDP) Models	288
7.2.5	State-Dependent Pricing (SDP) Models	295
7.2.6	Frictions in the Timing of Price Adjustment or in the Adjustment of Prices?	300
7.3	Assessing Alternatives	301
7.3.1	Microeconomic Evidence	302
7.3.2	Evidence on the New Keynesian Phillips Curve	304
7.3.3	Sticky Prices versus Sticky Information	312
7.4	Summary	313
7.5	Appendix: A Sticky-Wage MIU Model	314
7.6	Problems	316
<b>8</b>	<b>New Keynesian Monetary Economics</b>	<b>319</b>
8.1	Introduction	319
8.2	The Basic Model	320
8.2.1	Households	321
8.2.2	Firms	323
8.2.3	Market Clearing	325
8.3	A Linearized New Keynesian Model	327
8.3.1	The Linearized Phillips Curve	327
8.3.2	The Linearized IS Curve	331
8.3.3	Local Uniqueness of the Equilibrium	332
8.3.4	The Monetary Transmission Mechanism	336
8.3.5	Adding Economic Disturbances	339

8.4	Monetary Policy Analysis in New Keynesian Models	341
8.4.1	Policy Objectives	342
8.4.2	Policy Trade-offs	344
8.4.3	Optimal Commitment and Discretion	346
8.4.4	Commitment to a Rule	353
8.4.5	Endogenous Persistence	354
8.4.6	Targeting Regimes and Instrument Rules	358
8.4.7	Model Uncertainty	363
8.5	Labor Market Frictions and Unemployment	365
8.5.1	Sticky Wages and Prices	366
8.5.2	Unemployment	369
8.6	Summary	377
8.7	Appendix	378
8.7.1	The New Keynesian Phillips Curve	378
8.7.2	Approximating Utility	381
8.8	Problems	388
<b>9</b>	<b>Monetary Policy in the Open Economy</b>	<b>397</b>
9.1	Introduction	397
9.2	A Two-Country Open-Economy Model	398
9.2.1	Households	398
9.2.2	International Consumption Risk Sharing	401
9.2.3	Firms	402
9.2.4	Equilibrium	404
9.2.5	Optimal Policy	414
9.3	A Model of the Small Open Economy	424
9.3.1	Households	424
9.3.2	International Risk Sharing and Uncovered Interest Parity	428
9.3.3	Domestic Firms	429
9.3.4	Equilibrium Conditions	430
9.3.5	Monetary Policy in the Linear Model	433
9.4	Additional Sources of Nominal Distortions	437
9.4.1	Imperfect Pass-Through	437
9.4.2	Local Currency Pricing	440
9.4.3	Sticky Tradeable and Nontradeable Goods Prices	442
9.5	Currency Unions	442
9.6	Summary	447
9.7	Appendix	448
9.8	Problems	449

<b>10</b>	<b>Financial Markets and Monetary Policy</b>	455
10.1	Introduction	455
10.2	Interest Rates and Monetary Policy	456
10.2.1	Interest Rate Rules and the Price Level	456
10.2.2	Interest Rate Policies in General Equilibrium	459
10.3	The Term Structure of Interest Rates	462
10.3.1	The Basic Expectations Theory	463
10.3.2	Expected Inflation and the Term Structure	465
10.4	Macrofinance	467
10.4.1	Affine Models of the Term Structure	467
10.4.2	A Preferred Habitat Term Structure Model	469
10.5	Policy and the Term Structure	473
10.5.1	A Simple Example	473
10.5.2	An Affine Example	476
10.6	Financial Frictions in Credit Markets	478
10.6.1	Adverse Selection	480
10.6.2	Moral Hazard	482
10.6.3	Monitoring Costs	484
10.6.4	Agency Costs	488
10.6.5	Intermediary-to-Intermediary Credit Flows	491
10.7	Macroeconomic Implications	495
10.7.1	General Equilibrium Models	496
10.7.2	Agency Costs and General Equilibrium	501
10.7.3	Agency Costs and Sticky Prices	504
10.8	Summary	505
10.9	Problems	505
<b>11</b>	<b>The Effective Lower Bound and Balance Sheet Policies</b>	509
11.1	Introduction	509
11.2	The Effective Lower Bound	510
11.3	Liquidity Traps	512
11.4	Conventional Policies at the ELB	515
11.4.1	Equilibria at the ELB	516
11.4.2	Analytics at the ELB	518
11.4.3	Commitment and Forward Guidance	522
11.4.4	Summary on the ELB	531
11.5	Balance Sheet Policies	532
11.5.1	Asset Pricing Wedges	534
11.5.2	Market Segmentation and Transaction Costs	538

11.5.3	Costly Intermediation	543
11.5.4	Moral Hazard in Banking	547
11.5.5	Resaleability Constraints	552
11.5.6	Summary on Balance Sheet Policies	556
11.6	Appendix: Derivation of the Asset Pricing Wedges	556
11.7	Problems	557
<b>12</b>	<b>Monetary Policy Operating Procedures</b>	<b>561</b>
12.1	Introduction	561
12.2	From Instruments to Goals	562
12.3	The Instrument Choice Problem	563
12.3.1	Poole’s Analysis	563
12.3.2	Policy Rules and Information	568
12.3.3	Intermediate Targets	570
12.3.4	Real Effects of Operating Procedures	578
12.4	Operating Procedures and Policy Measures	579
12.4.1	Money Multipliers	579
12.4.2	The Reserve Market	581
12.5	Interest on Reserves in a Channel System	590
12.6	A Brief History of Fed Operating Procedures	595
12.6.1	1972–1979	595
12.6.2	1979–1982	596
12.6.3	1982–1988	599
12.6.4	1988–2008	599
12.6.5	2009–2016	600
12.7	Other Countries	603
12.8	Summary	605
12.9	Problems	605
	References	609
	Name Index	645
	Subject Index	651



## Preface

This book covers the most important topics in monetary economics and models that economists have employed to understand the interactions between real and monetary factors. It deals with topics in both monetary theory and monetary policy and is designed for second-year graduate students specializing in monetary economics, for researchers in monetary economics wishing to have a systematic summary of recent developments, and for economists working in policy institutions such as central banks. It can also be used as a supplement for first-year graduate courses in macroeconomics, because it provides a more in-depth treatment of inflation and monetary policy topics than is customary in graduate macroeconomics textbooks. The chapters on monetary policy may be useful for advanced undergraduate courses.

For the fourth edition of *Monetary Theory and Policy*, every chapter has been revised to improve the exposition and to incorporate recent research contributions. When the first edition appeared in 1998, the use of models based on dynamic optimization and nominal rigidities in consistent general equilibrium frameworks was still relatively new. By the time of the second edition, these models had become the common workhorse for monetary policy analysis. They have continued to provide the theoretical framework for most monetary policy analysis, and they also provide the foundation for empirical models that have been estimated for a number of countries, with many central banks now employing or developing dynamic stochastic general equilibrium (DSGE) models that build on the new Keynesian model. The third edition incorporated expanded material on money in search equilibria, sticky information, adaptive learning, state-contingent pricing models, and channel systems of implementing monetary policy, among other topics. The fourth edition includes an entirely new chapter on the effective lower bound on nominal interest rates, forward guidance policies, and quantitative and credit-easing policies that have been at the forefront of monetary policy discussions since the global financial crisis of 2008–2009. In addition, the material on the basic new Keynesian model has been reorganized into a single chapter to provide a comprehensive analysis of this model and its policy implications. The chapter on the open economy has been completely rewritten to reflect the dominance of the new Keynesian approach.

In the introduction to the first edition, I cited three innovations of the book: the use of calibration and simulation techniques to evaluate the quantitative significance of the channels through which monetary policy and inflation affect the economy; a stress on the need to understand the incentives facing central banks and to model the strategic interactions between the central bank and the private sector; and a focus on interest rates in the discussion of monetary policy. All three aspects remain in the current edition, but each is now commonplace in monetary research. For example, it is rare today to see research that treats monetary policy in terms of money supply control, yet this was common well into the 1990s.

Monetary economics is a large field, and one must decide whether to provide broad coverage, giving students a brief introduction to many topics, or to focus more narrowly and in more depth. I have chosen to focus on particular models, models that monetary economists have employed to address topics in theory and policy. I discuss the major topics in monetary economics in order to provide sufficiently broad coverage of the field, but the focus within each topic is often on a small number of papers or models that I have found useful for gaining insight into a particular issue. As an aid to students, derivations of basic results are often quite detailed, but deeper technical issues of existence, multiple equilibria, and stability receive somewhat less attention. This choice was not made because the latter are unimportant. Instead, the relative emphasis reflects an assessment that to do these topics justice, while still providing enough emphasis on the core insights offered by monetary economics, would have required a much longer book. By reducing the dimensionality of problems and not treating them in full generality, I sought to achieve the right balance of insight, accessibility, and rigor. The many references guide students to the extensive treatments in the literature of all the topics touched on in this book.<sup>1</sup>

The organization of chapters 1–5 is similar to that of previous editions, although the appendix of chapter 2 gives more detail on Blanchard-Kahn conditions. The simulation results from all chapters are now done only using Dynare; the programs are available at <http://people.ucsc.edu/~walshc/mtp4e/>.

Chapters 6–12 have seen major revisions in content and organization. Chapter 6 now deals with issues of time inconsistency and the average inflation bias under discretion. The workhorse model employed in that literature can be motivated by the models based on informational frictions covered in chapter 5, so it seems natural to cover this model immediately after chapter 5. Chapter 7 then discusses nominal price rigidities and provides the background for the material on the new Keynesian model that begins in chapter 8.

Chapter 8 on the new Keynesian model includes a new section on labor market rigidities that contains new material on search and matching labor market frictions and unemployment. The derivation in the chapter appendix of the quadratic approximate to the welfare of the representative household has been rewritten to more closely parallel the model used

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1. A BibTeX file containing all references cited in the fourth edition is available at <http://people.ucsc.edu/~walshc/mtp4e/>.



in the chapter. Chapter 9 covers open-economy models and has been completely rewritten. Previous editions began with the Obstfeld-Rogoff model. This material has been cut but is available online. The chapter now begins with a two-country new Keynesian model, then moves to a model of a small open economy. These models assume sticky prices are the only nominal frictions. The implications of imperfect pass-through, local currency pricing, and sticky tradeable and nontradeable goods prices as additional sources of nominal frictions are discussed. The chapter ends with a model of a currency union. Chapter 10 has been expanded to discuss the role of intermediary-to-intermediary financial frictions based on moral hazard.

An important addition is a new chapter 11 that focuses on the effective lower bound (ELB) on nominal interest rates and on balance sheet policies. The chapter opens with a discussion of why standard models imply the nominal interest rate cannot be negative in equilibrium and discusses one modification to the basic model that could account for negative rates. The discussion then moves to the existence of a liquidity trap when policy follows a Taylor rule, material that was in chapter 10 of the third edition but which fits better now in a chapter devoted to ELB issues. Optimal interest rate policy at the ELB is discussed, as is forward guidance about the future path of interest rates. The existence of multiple equilibria at the ELB is emphasized, and it is shown that the economy may suffer deflation and depressed output while at the ELB or it may experience a boom, depending on the central bank's commitment to inflation in the post-ELB period. The chapter ends with a review of several models of balance sheet policies. Finally, chapter 12 provides a discussion of operating procedures and channel systems.

It is not possible to discuss here all the areas of monetary economics in which economists are pursuing active research, or to give adequate credit to all the interesting work that has been done. The topics covered and the space devoted to them reflect my own biases toward research motivated by policy questions or influential in affecting the conduct of monetary policy. The field has simply exploded with new and interesting research, much of it motivated by the financial crisis of 2008–2009, the Great Recession, the limits on policy due to the ELB, and the active use of balance sheet policies in ways not seen during the 40 years prior to 2008. At best, this edition, like the earlier ones, can only scratch the surface of many topics. To those whose research has been slighted, I offer my apologies.

I am grateful to all those who have read and commented on drafts of the various editions, and some deserve special mention. Lars Svensson and Berthold Herrendorf each made extensive comments on complete drafts of the first edition. Henning Bohn, Betty Daniel, Jordi Galí, Eric Leeper, Tim Fuerst, Ed Nelson, Federico Ravenna, and Kevin Salyer provided very helpful comments on early draft versions of some of the chapters of the second edition. Henrik Jensen provided a host of useful suggestions that helped improve the third edition in terms of substance and clarity. Federico Ravenna and Chris Limnios commented on material new to the fourth edition. Addressing the issues raised greatly improved each edition.

I have received many useful comments from users that have guided this and previous revisions. My thanks go to Jonathan Benchimol, Luigi Buttiglione, Julia Chiriaeva, Vasco Cúrdia, David Coble Fernández, Oliver Fries, William Gatt, Federico Guerrero, Basil Halperin, Marco Hoeberichts, Stefan Homburg, Michael Hutchison, Nancy Jianakoplos, Beka Lamazoshvili, Sendor Lczel, Jaewoo Lee, Haroan Lei, Francesco Lippi, Carlo Migliardo, Stephen Miller, Rasim Mutlu, Jim Nason, Mario Nigrinis, Doug Pearce, Xingyun Peng (who translated the third edition into Chinese), Gustavo Piga, Álvaro Pina, Glenn Rudebusch, Bo Sandemann, Stephen Sauer, Claudio Shikida, Teresa Simões, Paul Söderlind, Ulf Söderström, Robert Tchaidze, Oreste Tristani, Willem Verhagen, Yuichiro Waki, Chris Waller, Ken West, and Jizhong Zhou (who translated the second edition into Chinese). My apologies to anyone I have failed to mention.

Numerous graduate students at the University of California, Santa Cruz, have offered helpful comments and assistance on the various editions of this book. They include Alina Carare, Cesar Carrera, Wei Chen, David Florian-Hoyle, Peter Kriz, Sergio Lago Alves, Jamus Lim, Chris Limnios, Jerry McIntyre, David Munro, Akatsuki Sukeda, and Ethel Wang. Jules Leichter and Conglin Xu deserve special mention for providing excellent research assistance during the process of preparing earlier editions. The first edition was based on lecture notes developed when I taught in the first-year macroeconomics sequence at Stanford; feedback from students in that course and, in particular, Fabiano Schivardi, was most helpful.

Many of the changes in the book are the result of comments and suggestion from students and participants at intensive courses and lectures in monetary economics I have taught at the IMF Institute, the Bank of England, the Bank of Korea, the Bank of Portugal, the Bank of Spain, the Central Bank of Brazil, the Federal Reserve Bank of Philadelphia, the Finnish Post-Graduate Program in Economics, the Hong Kong Institute for Monetary Research, the Norges Bank Training Program for Economists, the Swiss National Bank Studienzentrums Gerzensee, the University of Oslo, the University of Rome “Tor Vergata,” and the ZEI Summer School. As always, remaining errors are my own.

I would also like to thank Jane MacDonald and Emily Taber, who have been my editors at MIT Press for the third and fourth editions; Nancy Lombardi, the production editor for the first and second editions; Deborah Cantor-Adams, production editor for the third edition; Virginia Crossman, production editor for the fourth edition; and Alice Cheyer, copy editor on the third and fourth editions, for their excellent assistance on the manuscript. Needless to say, all the remaining weaknesses and errors are my own responsibility. Terry Vaughan, my original editor at MIT Press, was instrumental in ensuring this project got off the ground initially, and Elizabeth Murry served ably as editor for the second edition.

I owe an enormous debt to my wife, Judy Walsh, for all her support, encouragement, and assistance on this fourth edition. Judy carefully read every chapter, editing my writing and improving the exposition.

## Introduction

Monetary economics investigates the relationship between real economic variables at the aggregate level (such as real output, real rates of interest, employment, and real exchange rates) and nominal variables (such as the inflation rate, nominal interest rates, nominal exchange rates, and the supply of money). So defined, monetary economics overlaps considerably with macroeconomics more generally, and these two fields have to a large degree shared a common history over most of the past 50 years. This statement was particularly true during the 1970s after the monetarist/Keynesian debates led to a reintegration of monetary economics with macroeconomics. The seminal work of Lucas (1972) provided theoretical foundations for models of economic fluctuations in which money was the fundamental driving factor behind movements in real output. The rise of real business cycle models during the 1980s and early 1990s, building on the contribution of Kydland and Prescott (1982) and focusing explicitly on nonmonetary factors as the driving forces behind business cycles, tended to separate monetary economics from macroeconomics. More recently, the real business cycle approach to aggregate modeling has been used to incorporate monetary factors into dynamic general equilibrium models. Today, macroeconomics and monetary economics share the common tools associated with dynamic stochastic approaches to modeling the aggregate economy.

Despite these close connections, a book on monetary economics is not a book on macroeconomics. The focus in monetary economics is distinct, emphasizing price level determination, inflation, and the role of monetary policy. Monetary economics is currently dominated by three alternative modeling strategies. The first two, representative agent models and overlapping-generations models, share a common methodological approach in building equilibrium relationships explicitly on the foundations of optimizing behavior by individual agents. The third approach is based on sets of equilibrium relationships that are often not derived directly from any decision problem. Instead, they are described as ad hoc by critics and as convenient approximations by proponents. The latter characterization is generally more appropriate, and these models have demonstrated great value in helping economists understand issues in monetary economics. This book deals with models in the representative agent class and with ad hoc models of the type more common in policy analysis.

There are several reasons for ignoring the overlapping-generations (OLG) approach. First, systematic expositions of monetary economics from the perspective of overlapping generations are already available. For example, Sargent (1987) and Champ, Freeman, and Haslag (2016) cover many topics in monetary economics using OLG models. Second, many of the issues studied in monetary economics require understanding the time series behavior of macroeconomic variables such as inflation or the relationship between money and business cycles. It is helpful if the theoretical framework can be mapped directly into implications for behavior that can be compared with actual data. This mapping is more easily done with infinite-horizon representative agent models than with OLG models. This advantage, in fact, is one reason for the popularity of real business cycle models that employ the representative agent approach, and so a third reason for limiting the coverage to representative agent models is that they provide a close link between monetary economics and other popular frameworks for studying business cycle phenomena. Fourth, monetary policy issues are generally related to the dynamic behavior of the economy over time periods associated with business cycle frequencies, and here again the OLG framework seems less directly applicable. Finally, OLG models emphasize the store-of-value role of money at the expense of the medium-of-exchange role that money plays in facilitating transactions. McCallum (1983b) has argued that some of the implications of OLG models that contrast most sharply with the implications of other approaches (the tenuousness of monetary equilibria, for example) are directly related to the lack of a medium-of-exchange role for money.

This book is about monetary theory and the theory of monetary policy. There are some references to empirical results, but no attempt is made to provide a systematic survey of the vast body of empirical research in monetary economics. Most of the debates in monetary economics, however, have at their root issues of fact that can only be resolved by empirical evidence. Empirical evidence is needed to choose among theoretical approaches, but theory is also needed to interpret empirical evidence. How one links the quantities in the theoretical model to measurable data is critical, for example, in developing measures of monetary policy actions that can be used to estimate the impact of policy on the economy. Because empirical evidence aids in discriminating between alternative theories, it is helpful to begin with a brief overview of some basic facts. Chapter 1 does so, focusing primarily on the estimated impact of monetary policy actions on real output. Here, as in the chapters that deal with institutional details of monetary policy, the evidence comes primarily from research on U.S. data. However, an attempt is made to cite cross-country studies and to focus on empirical regularities that seem to characterize most industrialized economies.

Chapters 2–4 emphasize the role of inflation as a tax, using models that provide the basic microeconomic foundations of monetary economics. These chapters cover topics of fundamental importance for understanding how monetary phenomena affect the general equilibrium behavior of the economy and how nominal prices, inflation, money, and interest rates are linked. Because the models studied in these chapters assume that prices are perfectly

flexible, they are most useful for understanding longer-run correlations between inflation, money, and output and cross-country differences in average inflation. However, they do have implications for short-run dynamics as real and nominal variables adjust in response to aggregate productivity disturbances and random shocks to money growth. These dynamics are examined by employing simulations based on linear approximations around the steady-state equilibrium.

Chapters 2 and 3 employ a neoclassical growth framework to study monetary phenomena. The neoclassical model is one in which growth is exogenous, and money either has no effect on the real economy's long-run steady state or has effects that are likely to be small empirically. However, because these models allow one to calculate the welfare implications of exogenous changes in the economic environment, they provide a natural framework for examining the welfare costs of alternative steady-state rates of inflation. Stochastic versions of the basic models are calibrated, and simulations are used to illustrate how monetary factors affect the behavior of the economy. Such simulations aid in assessing the ability of the models to capture correlations observed in actual data. Since policy can be expressed in terms of both exogenous shocks and endogenous feedbacks from real shocks, the models can be used to study how economic fluctuations depend on monetary policy.

In chapter 4 the focus turns to public finance issues associated with money, inflation, and monetary policy. The ability to create money provides governments with a means of generating revenue. As a source of revenue, money creation, along with the inflation that results, can be analyzed from the perspective of public finance as one among many tax tools available to governments.

The link between the dynamic general equilibrium models of chapters 2–4 and the models employed for short-run and policy analysis is developed in two stages. In the first stage, chapter 5 reviews some attempts to understand the short-run effects of monetary policy shocks while still maintaining the assumption of flexible prices. Lucas's misperceptions model provides an important example of one such attempt. Models of sticky information with flexible prices due to the work of Mankiw and Reis provide a modern approach that can be thought of as building on Lucas's original insight that imperfect information is important for understanding the short-run effects of monetary shocks.

Chapter 6 turns to the analysis of monetary policy using a model due to Barro and Gordon (1983a) that can be motivated by the information frictions discussed in chapter 5. The focus in chapter 6 is on monetary policy objectives and the ability of policy authorities to achieve these objectives. Understanding monetary policy requires an understanding of how policy actions affect macroeconomic variables, but it also requires models of policy behavior to understand why particular policies are undertaken. A large body of research over the past three decades has used game-theoretic concepts to model the monetary policymaker as a strategic agent. These models have provided new insights into the rules-versus-discretion debate, positive theories of inflation, and justification for many of the actual reforms of central banking legislation that have been implemented in recent years.

Despite the growing research on sticky information and on models with portfolio rigidities (see chapter 5), it remains the case that the most research in monetary economics in recent years has assumed that prices and/or wages adjust sluggishly in response to economic disturbances. Chapter 7 discusses some important models of price and inflation adjustment, and reviews some of the new microeconomic evidence on price adjustment by firms. This evidence is helping to guide research on nominal rigidities and has renewed interest in models of state-contingent pricing.

Models of sticky prices in dynamic stochastic general equilibrium form the foundation of the new Keynesian models that over the past two decades have become the standard models for monetary policy analysis. These models build on the joint foundations of optimizing behavior by economic agents and nominal rigidities, and they form the core material of chapter 8. The basic new Keynesian model is covered, and some of its policy implications are explored.

Chapter 9 extends the analysis to the open economy by focusing on two questions. First, what additional channels from monetary policy actions to the real economy are present in the open economy that were absent in the closed-economy analysis? Second, how do conclusions about monetary policy obtained in the context of a closed economy need to be modified when open-economy considerations are included?

There is a long tradition of treating the money stock or even the inflation rate as the direct instrument of monetary policy. In fact, major central banks have employed interest rates as their operational policy instrument, so chapter 10 emphasizes explicitly the role of the interest rate as the instrument of monetary policy and the term structure that link policy rates to long-term interest rates. While the channels of monetary policy emphasized in traditional models operate primarily through interest rates and exchange rates, an alternative view is that credit markets play an independent role in affecting the transmission of monetary policy actions to the real economy. The nature of credit markets and their role in the transmission process are affected by market imperfections arising from imperfect information, so chapter 10 also examines theories that stress the role of credit and credit market imperfections in the presence of moral hazard, adverse selection, and costly monitoring. Much of the literature has focused on financial frictions between firms (borrowing to finance capital projects) and lenders (financial intermediaries). A model of credit frictions that affect the flow of funds among financial intermediaries is also discussed.

Models that assume the central bank implements policy through its control over a short-term interest rate are useful in normal times when the policymaker can raise or lower the policy rate. However, when short-term rates are constrained by a lower bound, and rates have hit that bound and can no longer be reduced, new issues arise. These issues are the subject of chapter 11. Among the topics covered are liquidity traps, the role of future commitments when the policy rate is at its effective lower bound, and the importance of wedges in the standard asset-pricing formula if the composition of the central bank's balance sheet is to matter. A number of recent models of balance sheet policies are discussed. These

models are based on frictions due to transaction costs, moral hazard, and limitations on asset sales.

Chapter 12 focuses on monetary policy implementation. Here the discussion deals with the monetary instrument choice problem and monetary policy operating procedures. A long tradition in monetary economics has debated the usefulness of monetary aggregates versus interest rates in the design and implementation of monetary policy, and chapter 12 reviews the approach economists have used to address this issue. A simple model of the market for bank reserves is used to stress how the observed responses of short-term interest rates and reserve aggregates depend on the operating procedures used in the conduct of policy. New material on channel systems for interest rate control is added in this edition. A basic understanding of policy implementation is important for empirical studies that attempt to measure changes in monetary policy.





# **Monetary Theory and Policy**



# 1 Evidence on Money, Prices, and Output

## 1.1 Introduction

This chapter reviews some of the basic empirical evidence on money, nominal interest rates, inflation, and output. This review serves two purposes. First, these basic results about long-run and short-run relationships are benchmarks for judging theoretical models. Second, reviewing the empirical evidence provides an opportunity to discuss the approaches monetary economists have taken to estimate the effects of money and monetary policy on real economic activity. The discussion focuses heavily on evidence from vector autoregressions (VARs) because these have served as a primary tool for uncovering the impact of monetary phenomena on the real economy. The findings obtained from VARs have been criticized, and these criticisms as well as other methods that have been used to investigate the money-output relationship are also discussed.

## 1.2 Some Basic Correlations

What are the basic empirical regularities that monetary economics must explain? Monetary economics focuses on the behavior of prices, monetary aggregates, nominal and real interest rates, and output, so a useful starting point is to summarize briefly what macroeconomic data tell us about the relationships among these variables.

McCandless Jr. and Weber (1995) provided a summary of the long-run relationships based on data for inflation, the output gap, and the growth rate of various measures of money covering a 30-year period from 110 countries using several definitions of money. By examining data from many time periods and countries, they provided evidence on relationships that are unlikely to be dependent on unique, country-specific events (such as the particular means employed to implement monetary policy) that might influence the actual evolution of money, prices, and output in a particular country. The first of two primary conclusions that emerged from their analysis was that the correlation between inflation and the growth rate of the money supply is almost 1, varying between 0.92 and 0.96, depending on the definition of the money supply used. This strong positive relationship between

inflation and money growth is consistent with many other studies based on smaller samples of countries and different time periods.<sup>1</sup> This correlation is normally taken to support one of the basic tenets of the quantity theory of money: a change in the growth rate of money induces “an equal change in the rate of price inflation” (Lucas 1980b, 1005). Using U.S. data from 1955 to 1975, Lucas plotted annual inflation against the annual growth rate of money. While the scatter plot suggests only a loose but positive relationship between inflation and money growth, a much stronger relationship emerged when Lucas filtered the data to remove short-run volatility. Berentsen, Menzio, and Wright (2011) repeated Lucas’s exercise using data from 1955 to 2005, and like Lucas, they found a strong correlation between inflation and money growth as they removed more and more of the short-run fluctuations in the two variables.<sup>2</sup>

This high correlation between inflation and money growth does not, however, have any implication for causality. If countries followed policies under which money supply growth rates were exogenously determined, then the correlation could be taken as evidence that money growth causes inflation, with an almost one-to-one relationship between the two. An alternative possibility, equally consistent with the high correlation, is that other factors generate inflation, and central banks allow the growth rate of money to adjust. Most of the models examined in this book are consistent with a one-to-one long-run relationship between money growth and inflation.<sup>3</sup>

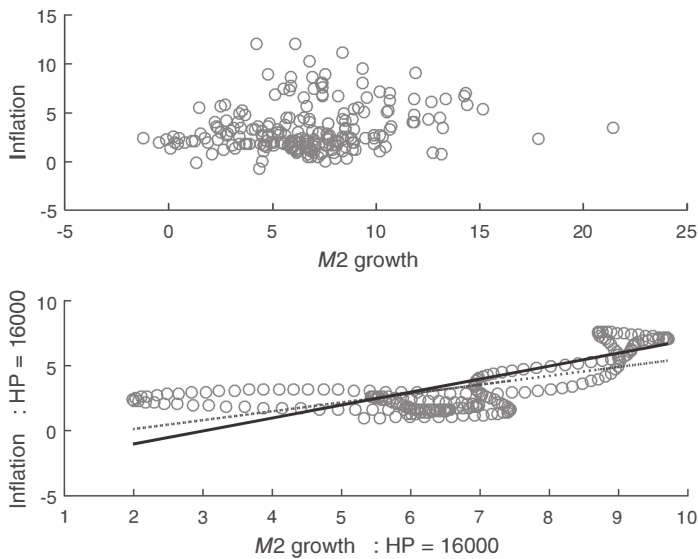
Money growth is not an exogenous variable; it depends on the actions of the central bank as well as the actions of the private sector. Inflation is also not exogenous. Because both money growth and inflation are endogenous variables, the correlation between the two depends on the types of disturbances affecting the economy as well as on changes in policy. Sargent and Surico (2011) emphasized that the strong relationship between money growth and inflation that Lucas found in the filtered data for 1955–1975 does not characterize other periods of U.S. history. They found that the regression of inflation on money growth yielded a coefficient of 1.01 for the 1960–1983 sample period when they first filtered out short-run volatility in the data. However, for 1984–2005, the regression coefficient was essentially equal to 0 (the point estimate was  $-0.03$ ). They attributed this changing relationship to shifts in U.S. monetary policy. Their interpretation is that the close association of money growth and inflation found by Lucas is likely to occur during periods in which the monetary authority has allowed persistent movements in money growth and fails to respond sufficiently to offset movements in inflation. The association breaks down when the monetary authority responds more strongly to inflation, leading to more stable inflation.

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1. Examples include Lucas (1980b), Geweke (1986), and Rolnick and Weber (1997), among others.

2. Berentsen, Menzio, and Wright (2011) employed an HP filter and progressively increased the smoothing parameter from 0 to 160,000.

3. Haldane (1997) found, however, that the money growth rate–inflation correlation is much less than 1 among low-inflation countries.

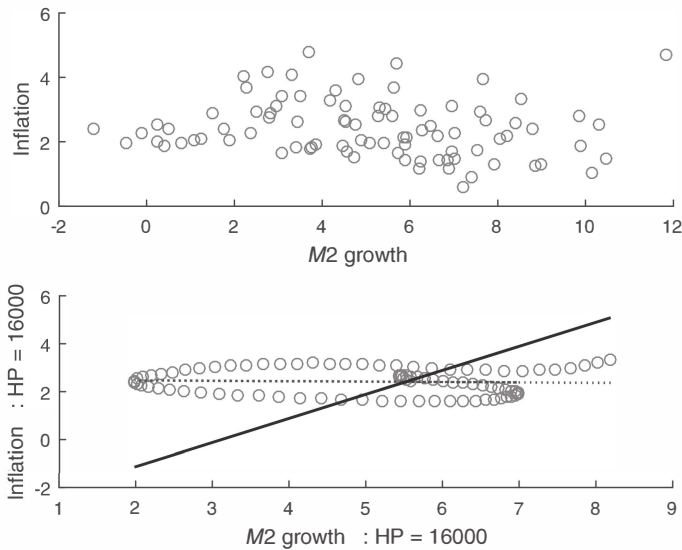


**Figure 1.1**

*Upper:* Quarterly inflation (GDP deflator) versus  $M2$  quarterly growth rate (both at annual rate). *Lower:* Filtered inflation and  $M2$  growth using HP smoothing parameter of 16,000. 45° line adjusted by mean annual rate of real GDP growth of 3.02 percent for 1960:1–2015:4. Dotted line in lower panel is fitted regression line.

The relationship between money growth and inflation in the United States for the period 1960:1–2015:4 is illustrated in figure 1.1. The upper panel is a scatter plot of money growth on the horizontal axis, measured by the quarterly growth rate of the  $M2$  measure of the money stock (expressed at an annual rate) and the quarterly inflation rate (at annual rates) measured by the GDP price deflator. The models investigated in chapters 2 and 3 imply inflation should equal the rate of money growth minus the rate of growth of real output. That is, on average the inflation points should lie along a 45 degree line with negative intercept equal to the average growth rate of real output (solid line in figure 1.1). The lower panel of the figure plots the same two variables after they have been filtered to remove much of the short-run volatility from each series. This is done using a Hodrick- Prescott (HP) filter with a smoothing weight of 16,000. The upper panel shows a weak relationship between money growth and inflation; the contemporaneous correlation between the two is 0.21. The lower panel, however, reveals a very positive relationship for the filtered data; the contemporaneous correlation is 0.66. The slope of the regression line in the lower panel is close to the adjusted 45 degree line, suggesting a one-to-one relationship between growth rate of money and inflation.

Figure 1.2 presents the same variables for the 1985:1–2006:4 period, an era sometimes referred to as the Great Moderation because macroeconomic volatility was much lower than it had been earlier (notice the differences in the inflation scales in figures 1.1 and 1.2).



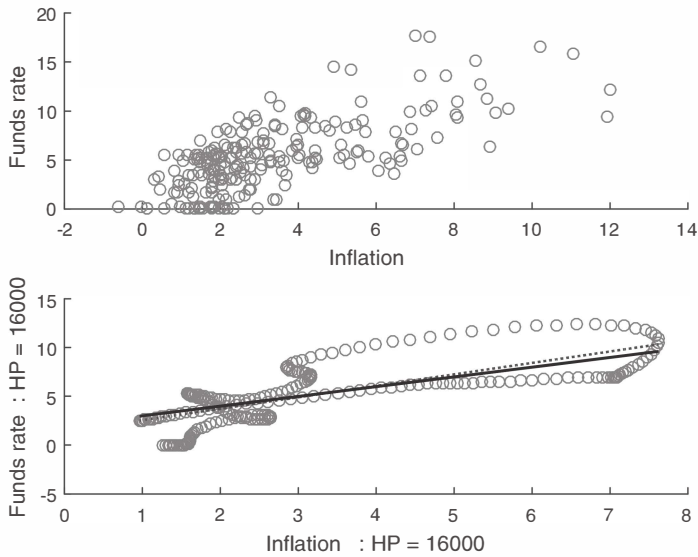
**Figure 1.2**

*Upper:* Quarterly inflation (GDP deflator) versus *M2* quarterly growth rate (both at annual rate). *Lower:* Filtered inflation and *M2* growth using HP smoothing parameter of 16,000. 45° line adjusted by mean annual rate of real GDP growth of 2.59 percent for 1985:1–2006:4. Dotted line in lower panel is fitted regression line.

The correlation between the data in the upper panel of figure 1.2 is actually negative ( $-0.15$ ); money growth varied significantly over the period while inflation remained within a narrow band. The lower panel shows the filtered data. There is little relationship to inflation. In fact, the correlation between money growth and inflation in the smoothed data is  $-0.05$ . Later chapters discuss the conduct of monetary policy. Changes in the conduct of policy over the two periods shown in these figures are important in accounting for the changing relationship between money growth and inflation, the point made by Sargent and Surico (2011).

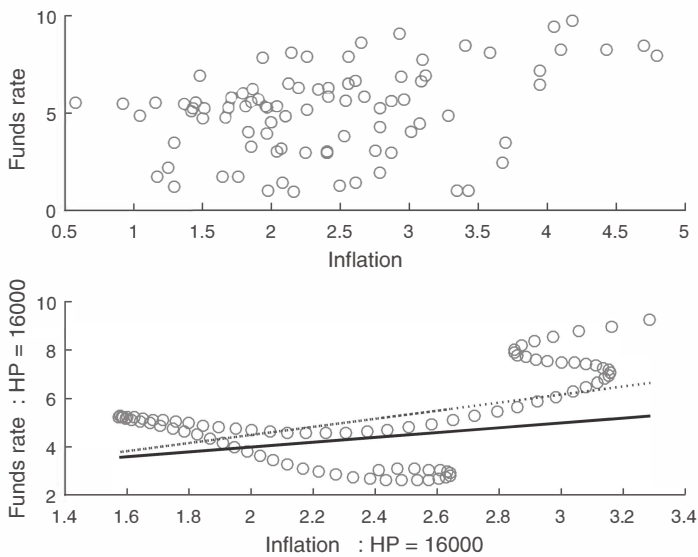
Models in monetary economics imply that nominal interest rates and inflation should tend to move together one-to-one. Figure 1.3 presents the data on the federal funds interest rate and inflation for 1960:1–2015:4. The funds rate plays an important role in monetary policy in the United States. A strong positive relationship between this interest rate and inflation shows up in both the quarterly and the smoothed data. The solid line in the lower panel is the 45° degree line with an intercept equal to 2 percent to adjust for the average real interest rate (see chapter 2). The regression line is essentially on top of this 45° degree line.

Figure 1.4 shows the funds rate and inflation data for 1985:1–2015:4, a period that includes the Great Moderation between 1985 and 2007 and the Great Recession of 2008–2009. The upper panel shows the correlation between the funds rate and inflation is much weaker than that seen in figure 1.3. In addition, the regression line in the lower



**Figure 1.3**

*Upper:* Federal funds interest rate versus inflation (GDP deflator). *Lower:* Filtered funds rate and inflation using HP smoothing parameter of 16,000. 45° line adjusted by 2.0 percent as an estimate of the real interest rate for 1960:1–2015:4. Dotted line in lower panel is fitted regression line.



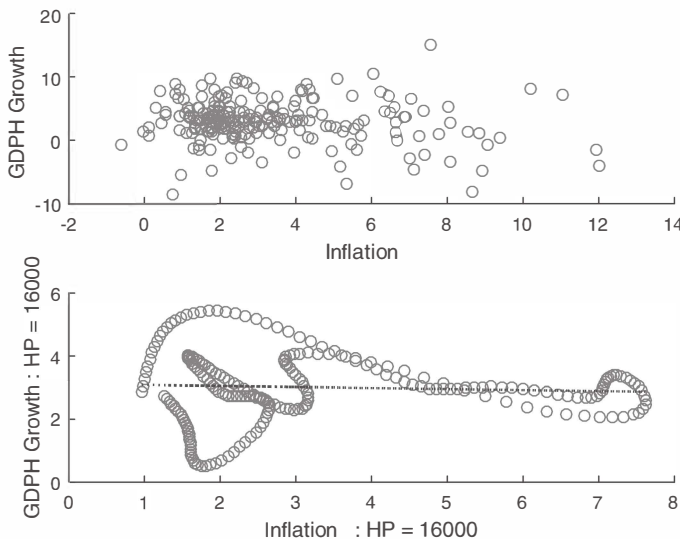
**Figure 1.4**

*Upper:* Federal funds interest rate versus inflation (GDP deflator). *Lower:* Filtered funds rate and inflation using HP smoothing parameter of 16,000. 45° line adjusted by 2.0 percent as an estimate of the real interest rate for 1985:1–2006:4. Dotted line in lower panel is fitted regression line.

panel now has a slope that is greater than one. If regression line is interpreted as reflecting the reaction of policy to inflation, a slope that is greater than one suggests the Fed increased the funds rate more than one-for-one in response to changes in inflation.

The appropriate interpretation of money-inflation correlations, both in terms of causality and in terms of tests of long-run relationships, also depends on the statistical properties of the underlying series. As Fischer and Seater (1993) noted, one cannot ask how a permanent change in the growth rate of money affects inflation unless actual money growth has exhibited permanent shifts. They showed how the order of integration of money and prices influences the testing of hypotheses about the long-run relationship between money growth and inflation. In a similar vein, McCallum (1984b) demonstrated that regression-based tests of long-run relationships in monetary economics may be misleading when expectational relationships are involved.

The second general conclusion that emerged from McCandless and Weber's (1995) work was that there is no correlation between either inflation or money growth and the growth rate of real output. Thus, there are countries with low output growth and low money growth and inflation, countries with low output growth and high money growth and inflation, and countries with every other combination as well. Figure 1.5 illustrates the lack of correlation between inflation and real GDP growth for the United States over the 1960:1–2015:4 period. This conclusion is not as robust as the money growth–inflation one; McCandless



**Figure 1.5**

*Upper:* Real GDP growth rate versus quarterly inflation rate (GDP deflator) (both at annual rate) for 1960:1–2015:4. *Lower:* Filtered real GDP growth rate and inflation using HP smoothing parameter of 16,000. Dotted line in lower panel is fitted regression line.

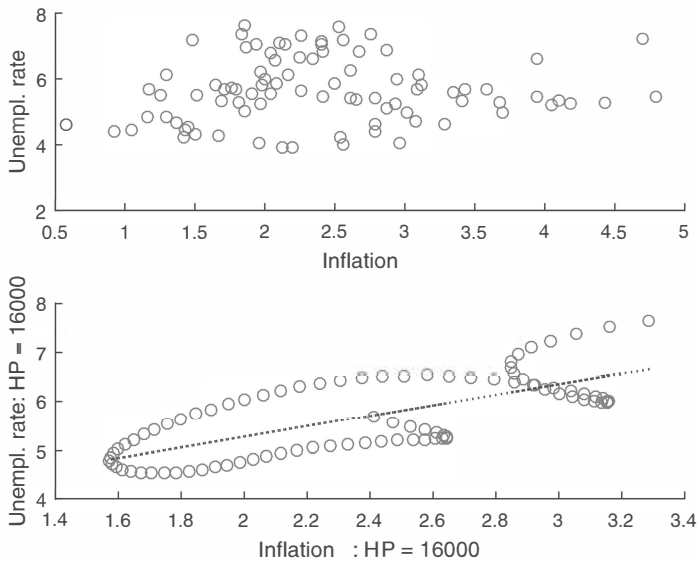


and Weber reported a positive correlation between real growth and money growth, but not inflation, for a subsample of OECD countries. Kormendi and Meguire (1984) for a sample of almost 50 countries and Geweke (1986) for the United States argued that the data reveal no long-run effect of money growth on real output growth. Barro (1995; 1996) reported a negative correlation between inflation and growth in a cross-country sample. Bullard and Keating (1995) examined post–World War II data from 58 countries, concluding for the sample as a whole that the evidence that permanent shifts in inflation produce permanent effects on the level of output is weak, with some evidence of positive effects of inflation on output among low-inflation countries and zero or negative effects for higher-inflation countries. Similarly, Boschen and Mills (1995a) concluded that permanent monetary shocks in the United States made no contribution to permanent shifts in GDP, a result consistent with the findings of King and Watson (1997).

Bullard (1999) surveyed much of the existing empirical work on the long-run relationship between money growth and real output, discussing both methodological issues associated with testing for such a relationship and the results of a large literature. Specifically, while shocks to the level of the money supply do not appear to have long-run effects on real output, this is not the case with respect to shocks to money growth. For example, the evidence based on postwar U.S. data reported in King and Watson (1997) is consistent with an effect of money growth on real output. Bullard and Keating (1995) did not find any real effects of permanent inflation shocks with a cross-country analysis, but Berentsen, Menzio, and Wright (2011), using the same filtering approach described earlier, argued that inflation and unemployment are positively related in the long run. A positive correlation between inflation and unemployment characterizes the Great Moderation period 1985:1–2006:4, as seen in figure 1.6.

However, despite this diversity of empirical findings concerning the long-run relationship between inflation and real growth, and other measures of real economic activity such as unemployment, the general consensus is summarized by the proposition, “about which there is now little disagreement . . . that there is no long-run trade-off between the rate of inflation and the rate of unemployment” (Taylor 1996, 186).

Monetary economics is also concerned with the relationship between interest rates, inflation, and money. According to the Fisher equation, the nominal interest rate equals the real return plus the expected rate of inflation. If real returns are independent of inflation, then nominal interest rates should be positively related to expected inflation. This relationship is an implication of the theoretical models discussed throughout this book. In terms of long-run correlations, it suggests that the level of nominal interest rates should be positively correlated with average rates of inflation. Because average rates of inflation are positively correlated with average money growth rates, nominal interest rates and money growth rates should also be positively correlated. Monnet and Weber (2001) examined annual average interest rates and money growth rates over the period 1961–1998 for a sample of 31 countries. They found a correlation of 0.87 between money growth and long-term



**Figure 1.6**

*Upper:* Unemployment rate versus quarterly inflation rate (GDP deflator) for 1985:1–2006:4. *Lower:* Filtered real GDP growth rate and inflation using HP smoothing parameter of 16,000. Dotted line in lower panel is fitted regression line.

interest rates. For the developed countries, the correlation is somewhat smaller (0.70); for the developing countries, it is 0.84, although this falls to 0.66 when Venezuela is excluded.<sup>4</sup> This evidence is consistent with the Fisher equation.<sup>5</sup>

### 1.3 Estimating the Effect of Monetary Policy on Output and Inflation

While long-run effects of money may fall entirely, or almost entirely, on prices and have little impact on real variables, most economists believe that monetary disturbances can, in the short run, have important effects on real variables such as output.<sup>6</sup> As Lucas (1996) put it in his Nobel lecture, “This tension between two incompatible ideas—that changes in money are neutral unit changes and that they induce movements in employment and production in the same direction—has been at the center of monetary theory at least since

4. Venezuela’s money growth rate averaged over 28 percent, the highest among the countries in Monnet and Weber’s sample.

5. Consistent evidence on the strong positive long-run relationship between inflation and interest rates was reported by Berentsen, Menzio, and Wright (2011).

6. For an exposition of the view that monetary factors have not played an important role in U.S. business cycles, see Kydland and Prescott (1982).

Hume wrote” (664).<sup>7</sup> The evidence of the short-run effects of money on real output comes from a variety of approaches.

The tools that have been employed to estimate the impact of monetary policy have evolved over time as the result of developments in time series econometrics and changes in the specific questions posed by theoretical models. This section reviews some of the empirical evidence on the relationship between monetary policy and U.S. macroeconomic behavior. One objective of this literature has been to determine whether monetary policy disturbances actually have played an important role in U.S. economic fluctuations. Equally important, the empirical evidence is useful in judging whether the predictions of different theories about the effects of monetary policy are consistent with the evidence. Among the excellent discussions of these issues are Leeper, Sims, and Zha (1996) and Christiano, Eichenbaum, and Evans (1999), where the focus is on the role of identified VARs in estimating the effects of monetary policy; King and Watson (1996), where the focus is on using empirical evidence to distinguish among competing business cycle models; and Boivin, Kiley, and Mishkin (2010), where the focus is on the channels through which monetary shocks affect the economy. Much of the empirical literature has focused on estimating the impact of a monetary shock such as an unpredicted change in policy on other macroeconomic variables. A discussion of the literature on estimating the effects of such shocks is provided by Ramey (2016).

### 1.3.1 The Evidence of Friedman and Schwartz

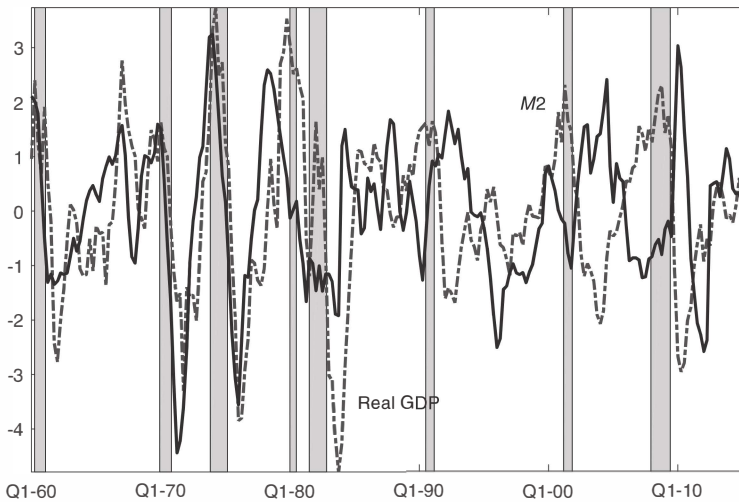
M. Friedman and Schwartz’s (1963) classic study of the relationship between money and business cycles still represents probably the most influential empirical evidence that money does matter for business cycle fluctuations. Their evidence, based on almost 100 years of U.S. data, relies heavily on patterns of timing; systematic evidence that money growth rate changes lead changes in real economic activity is taken to support a causal interpretation in which money causes output fluctuations.

Friedman and Schwartz concluded that the data “decisively support treating the rate of change series [of the money supply] as conforming to the reference cycle positively with a long lead” (36). That is, faster money growth tends to be followed by increases in output above trend, and slowdowns in money growth tend to be followed by declines in output. The inference Friedman and Schwartz drew was that variations in money growth rates cause, with long (and variable) lags, variations in real economic activity.

The nature of this evidence for the United States is apparent in figure 1.7, which shows the logs of the *M2* measure of the money supply and real GDP. Both variables are detrended using a Hodrick-Prescott filter. The sample is quarterly and spans 1960:1 to 2015:1, so this figure starts after the Friedman and Schwartz study ends. The figure reveals

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7. The reference is to David Hume’s 1752 essays *Of Money* and *Of Interest*.



**Figure 1.7**

Detrended log  $M2$  and real GDP. Shaded regions are NBER recession dates.

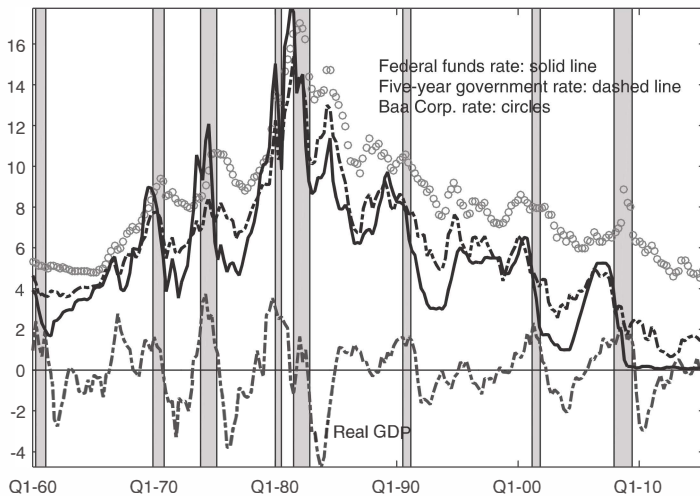
slowdowns in money leading most business cycle downturns through the early 1980s. However, the pattern is not so apparent after 1982. B. Friedman and Kuttner (1992) documented the seeming breakdown in the relationship between monetary aggregates and real output; this changing relationship between money and output has affected the manner in which monetary policy has been conducted, at least in the United States (see chapter 12).

While suggestive, evidence based on timing patterns and simple correlations may not indicate the true causal role of money. Since the Federal Reserve and the banking sector respond to economic developments, movements in the monetary aggregates are not exogenous, and the correlation patterns need not reflect any causal effect of monetary policy on economic activity. If, for example, the central bank is implementing monetary policy by controlling the value of some short-term market interest rate, the nominal stock of money will be affected both by policy actions that change interest rates and by developments in the economy that are not related to policy actions. An economic expansion may lead banks to expand lending in ways that produce an increase in the stock of money, even if the central bank has not changed its policy. If the money stock is used to measure monetary policy, the relationship observed in the data between money and output may reflect the impact of output on money, not the impact of money and monetary policy on output.

Tobin (1970) was the first to model formally the idea that the positive correlation between money and output, the correlation that M. Friedman and Schwartz interpreted as providing evidence that money caused output movements, could in fact reflect just the opposite—output might be causing money. This *reverse causation argument* was investigated by King and Plosser (1984). They showed that inside money—the component of

a monetary aggregate such as  $M1$  that represents the liabilities of the banking sector—is more highly correlated with output movements in the United States than is outside money, the liabilities of the Federal Reserve. King and Plosser interpreted this finding as evidence that much of the correlation between broad aggregates such as  $M1$  or  $M2$  and output arises from the endogenous response of the banking sector to economic disturbances that are not the result of monetary policy actions. Coleman (1996), in an estimated equilibrium model with endogenous money, found that the implied behavior of money in the model cannot match the lead-lag relationship in the data. Specifically, a money supply measure such as  $M2$  leads output, whereas Coleman found that his model implied money should be more highly correlated with lagged output than with future output.<sup>8</sup>

The endogeneity problem is likely to be particularly severe if the monetary authority has employed a short-term interest rate as its main policy instrument, and this has generally been the case in the United States. Changes in the money stock are then endogenous and cannot be interpreted as representing policy actions. Figure 1.8 shows the behavior of the federal funds rate and the 5-year U.S. government bond rate, together with detrended real GDP. The figure provides some support for the notion that monetary policy actions have contributed to U.S. business cycles. Interest rates have typically increased prior to economic downturns. But whether this is evidence that monetary policy has caused or



**Figure 1.8**

Nominal federal funds rate, 5-year U.S. bond rate, Baa corporate bond rate, and detrended real GDP, 1960:1–2015:4.

8. Lacker (1988) showed how the correlations between inside money and future output could also arise if movements in inside money reflected new information about future monetary policy.

contributed to cyclical fluctuations cannot be inferred from the figure; the movements in interest rates may simply reflect the Federal Reserve's response to the state of the economy.

Simple plots and correlations are suggestive, but they cannot be decisive. Other factors may be the cause of the joint movements of output, monetary aggregates, and interest rates. The comparison with business cycle reference points also ignores much of the information about the time series behavior of money, output, and interest rates that could be used to determine what impact, if any, monetary policy has on output. And the appropriate variable to use as a measure of monetary policy depends on how policy has been implemented.

One of the earliest time series econometric attempts to estimate the impact of money was due to M. Friedman and Meiselman (1963). Their objective was to test whether monetary or fiscal policy was more important for the determination of *nominal* income. To address this issue, they estimated the following equation:<sup>9</sup>

$$y_t^n \equiv y_t + p_t = y_0^n + \sum_{i=0} a_i A_{t-i} + \sum_{i=0} b_i m_{t-i} + \sum_{i=0} h_i z_{t-i} + u_t, \quad (1.1)$$

where  $y_t^n$  denotes the log of nominal income, equal to the sum of the logs of output and the price level,  $A$  is a measure of autonomous expenditures, and  $m$  is a monetary aggregate;  $z$  can be thought of as a vector of other variables relevant for explaining nominal income fluctuations. Friedman and Meiselman reported finding a much more stable and statistically significant relationship between output and money than between output and their measure of autonomous expenditures. In general, they could not reject the hypothesis that the  $a_i$  coefficients were zero, while the  $b_i$  coefficients were always statistically significant.

The use of equations such as (1.1) for policy analysis was promoted by a number of economists at the Federal Reserve Bank of St. Louis, so regressions of nominal income on money are often called *St. Louis equations* (see Andersen and Jordan 1968; B. Friedman 1977a; Carlson 1978). Because the dependent variable is nominal income, the St. Louis approach does not address directly the question of how a money-induced change in nominal spending is split between a change in real output and a change in the price level. The impact of money on nominal income was estimated to be quite strong, and Andersen and Jordan (1968, 22) concluded that this finding suggested monetary policy should be used to promote economic stabilization.<sup>10</sup>

The original Friedman-Meiselman result generated responses by Modigliani and Ando (1976) and De Prano and Mayer (1965), among others. This debate emphasized that an

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9. This is not exactly correct; because M. Friedman and Meiselman included autonomous expenditures as an explanatory variable, they also used consumption as the dependent variable (basically, output minus autonomous expenditures). They also reported results for real variables as well as nominal ones. Following modern practice, (1.1) is expressed in terms of logs; Friedman and Meiselman estimated their equation in levels.

10. B. Friedman (1977a) argued that updated estimates of the St. Louis equation did yield a role for fiscal policy, although the statistical reliability of this finding was questioned by Carlson (1978). Carlson also provided a bibliography listing many of the papers on the St. Louis equation (see his footnote 2, p. 13).

equation such as (1.1) is misspecified if  $m$  is endogenous. To illustrate the point with an extreme example, suppose that the central bank is able to manipulate the money supply to offset almost perfectly shocks that would otherwise generate fluctuations in nominal income. In this case,  $y^n$  would simply reflect the random control errors the central bank had failed to offset. As a result,  $m$  and  $y^n$  might be completely uncorrelated, and a regression of  $y^n$  on  $m$  would not reveal that money actually played an important role in affecting nominal income. If policy is able to respond to the factors generating the error term  $u_t$ , then  $m_t$ , and  $u_t$  will be correlated, ordinary least squares estimates of (1.1) will be inconsistent, and the resulting estimates will depend on the manner in which policy has induced a correlation between  $u$  and  $m$ . Changes in policy that altered this correlation will also alter the least squares regression estimates one would obtain in estimating (1.1).

Belongia and Ireland (2016) updated Friedman and Schwartz's evidence on money-output correlations by examining the 1967–2013 period and by employing a measure of the money supply that differentially weighs the various components added together in a standard measure such as  $M2$ . For example,  $M2$  consists of the sum of currency, checkable deposits, savings accounts, small time deposits, and retail money market mutual funds. If these components are not perfect substitutes, then it is incorrect to simply add them together dollar-for-dollar, as is done to obtain  $M2$ . Instead, Barnett (1980) advocated the use of Divisia measures of money that construct weighted averages rather than simply sums, with the weights a function of the user cost of each component of the monetary aggregate.<sup>11</sup> Belongia and Ireland (2016) found large and positive correlations between Divisia monetary aggregates and GDP, with money leading output, and they argued that the U.S. data since 1967 are consistent with the findings of M. Friedman and Schwartz for the prior 100 years. Belongia and Ireland found that the exact correlations and the lead of money over output varied over different subsamples, with some evidence that the lead time between changes in money and subsequent changes in real output has lengthened.

### 1.3.2 Granger Causality

The St. Louis equation related nominal output to the past behavior of money. Similar regressions employing *real* output have also been used to investigate the connection between real economic activity and money. In an important contribution, Sims (1972) introduced the notion of *Granger causality* into the debate over the real effects of money. A variable  $X$  is said to Granger-cause  $Y$  if and only if lagged values of  $X$  have marginal predictive content in a forecasting equation for  $Y$ . In practice, testing whether money Granger-causes output involves testing whether the  $a_i$  coefficients equal zero in a regression of the form

$$y_t = y_0 + \sum_{i=1} a_i m_{t-i} + \sum_{i=1} b_i y_{t-i} + \sum_{i=1} c_i z_{t-i} + e_t, \quad (1.2)$$

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11. See Barnett et al. (2013).

where key issues involve the treatment of trends in output and money, the choice of lag lengths, and the set of other variables (represented by  $z$ ) that are included in the equation.

Sims's original work used log levels of U.S. nominal GNP and money (both  $M1$  and the monetary base). He found evidence that money Granger-caused GNP. That is, the past behavior of money helped to predict future GNP. However, using the index of industrial production to measure real output, Sims (1980) found that the fraction of output variation explained by money was greatly reduced when a nominal interest rate was added to the equation (so that  $z$  consisted of the log price level and an interest rate). Thus, the conclusion seemed sensitive to the specification of  $z$ . Eichenbaum and Singleton (1986) found that money appeared to be less important if the regressions were specified in log first difference form rather than in log levels with a time trend. Stock and Watson (1989) provided a systematic treatment of the trend specification in testing whether money Granger-causes real output. They concluded that money does help to predict future output (they actually use industrial production) even when prices and an interest rate are included.

A large literature has examined the value of monetary indicators in forecasting output. One interpretation of Sims's finding was that including an interest rate reduced the apparent role of money because, at least in the United States, a short-term interest rate, rather than the money supply, provided a better measure of monetary policy actions (see chapter 12). B. Friedman and Kuttner (1992) and Bernanke and Blinder (1992), among others, looked at the role of alternative interest rate measures in forecasting real output. Friedman and Kuttner examined the effects of alternative definitions of money and different sample periods, concluding that the relationship in the United States is unstable and deteriorated in the 1990s. Bernanke and Blinder found that the federal funds rate "dominates both money and the bill and bond rates in forecasting real variables."

Regressions of real output on money were also popularized by Barro (1977; 1978; 1979b) as a way of testing whether only unanticipated money matters for real output. By dividing money into anticipated and unanticipated components, Barro obtained results suggesting that only the unanticipated part affected real variables (see also Barro and Rush 1980 and the critical comment by Small 1979). Subsequent work by Mishkin (1982) found a role for anticipated money as well. Cover (1992) employs a similar approach and finds differences in the impacts of positive and negative monetary shocks. Negative shocks were estimated to have significant effects on output, while the effect of positive shocks was usually small and statistically insignificant.

### 1.3.3 Policy Uses

Before reviewing other evidence on the effects of money on output, it is useful to ask whether equations such as (1.2) can be used for policy purposes. That is, can a regression of this form be used to design a policy rule for setting the central bank's policy instrument? If it can, then the discussions of theoretical models that form the bulk of this book would be unnecessary, at least from the perspective of conducting monetary policy.



Suppose that the estimated relationship between output and money takes the form

$$y_t = y_0 + a_0 m_t + a_1 m_{t-1} + c_1 z_t + c_2 z_{t-1} + u_t. \quad (1.3)$$

According to (1.3), systematic variations in the money supply affect output. Consider the problem of adjusting the money supply to reduce fluctuations in real output. If this objective is interpreted to mean that the money supply should be manipulated to minimize the variance of  $y_t$  around  $y_0$ , then  $m_t$  should be set equal to

$$\begin{aligned} m_t &= -\frac{a_1}{a_0} m_{t-1} - \frac{c_2}{a_0} z_{t-1} + v_t \\ &= \pi_1 m_{t-1} + \pi_2 z_{t-1} + v_t, \end{aligned} \quad (1.4)$$

where for simplicity it is assumed the monetary authority's forecast of  $z_t$  is equal to zero. The term  $v_t$  represents the control error experienced by the monetary authority in setting the money supply. Equation (1.4) represents a feedback rule for the money supply whose parameters are themselves determined by the estimated coefficients in the equation for  $y$ . A key assumption is that the coefficients in (1.3) are independent of the choice of the policy rule for  $m$ . Substituting (1.4) into (1.3), output under the policy rule given in (1.4) would be equal to  $y_t = y_0 + c_1 z_t + u_t + a_0 v_t$ .

Notice that a policy rule was derived using only knowledge of the policy objective (minimizing the expected variance of output) and knowledge of the estimated coefficients in (1.3). No theory of how monetary policy actually affects the economy was required. Sargent (1976) showed, however, that the use of (1.3) to derive a policy feedback rule may be inappropriate. To see why, suppose that real output actually depends only on unpredicted movements in the money supply; only surprises matter, with predicted changes in money simply being reflected in price-level movements with no impact on output.<sup>12</sup> From (1.4), the unpredicted movement in  $m_t$  is just  $v_t$ , so let the true model for output be

$$y_t = y_0 + d_0 v_t + d_1 z_t + d_2 z_{t-1} + u_t. \quad (1.5)$$

Now from (1.4),  $v_t = m_t - (\pi_1 m_{t-1} + \pi_2 z_{t-1})$ , so output can be expressed equivalently as

$$\begin{aligned} y_t &= y_0 + d_0 [m_t - (\pi_1 m_{t-1} + \pi_2 z_{t-1})] + d_1 z_t + d_2 z_{t-1} + u_t \\ &= y_0 + d_0 m_t - d_0 \pi_1 m_{t-1} + d_1 z_t + (d_2 - d_0 \pi_2) z_{t-1} + u_t, \end{aligned} \quad (1.6)$$

which has exactly the same form as (1.3). Equation (1.3), which was initially interpreted as consistent with a situation in which systematic feedback rules for monetary policy could affect output, is *observationally equivalent* to (1.6), which was derived under the assumption that systematic policy had no effect and only money surprises mattered. The two are observationally equivalent because the error term in both (1.3) and (1.6) is just  $u_t$ ; both equations fit the data equally well.

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12. The influential model of Lucas (1972) has this implication. See chapter 5.

A comparison of (1.3) and (1.6) reveals another important conclusion. The coefficients of (1.6) are functions of the parameters in the policy rule (1.4). Thus, changes in the conduct of policy, interpreted to mean changes in the feedback rule parameters, change the parameters estimated in an equation such as (1.6) (or in a St. Louis-type regression). This is an example of the Lucas (1976) critique: empirical relationships are unlikely to be invariant to changes in policy regimes.

Of course, as Sargent stressed, it may be that (1.3) is the true structure that remains invariant as policy changes. In this case, (1.5) will not be invariant to changes in policy. To demonstrate this point, note that (1.4) implies

$$m_t = (1 - \pi_1 L)^{-1} (\pi_2 z_{t-1} + v_t),$$

where  $L$  is the lag operator.<sup>13</sup> Hence, one can write (1.3) as

$$\begin{aligned} y_t &= y_0 + a_0 m_t + a_1 m_{t-1} + c_1 z_t + c_2 z_{t-1} + u_t \\ &= y_0 + a_0 (1 - \pi_1 L)^{-1} (\pi_2 z_{t-1} + v_t) \\ &\quad + a_1 (1 - \pi_1 L)^{-1} (\pi_2 z_{t-2} + v_{t-1}) + c_1 z_t + c_2 z_{t-1} + u_t \\ &= (1 - \pi_1) y_0 + \pi_1 y_{t-1} + a_0 v_t + a_1 v_{t-1} + c_1 z_t \\ &\quad + (c_2 + a_0 \pi_2 - c_1 \pi_1) z_{t-1} + (a_1 \pi_2 - c_2 \pi_1) z_{t-2} + u_t - \pi_1 u_{t-1}, \end{aligned} \tag{1.7}$$

where output is now expressed as a function of lagged output, the  $z$  variable, and money surprises (the  $v$  realizations). If this were interpreted as a policy-invariant expression, one would conclude that output was independent of any predictable or systematic feedback rule for monetary policy; only unpredicted money appears to matter. Yet, under the hypothesis that (1.3) is the true invariant structure, changes in the policy rule (the  $\pi_1$  coefficients) cause the coefficients in (1.7) to change.

Note that starting with (1.5) and (1.4), one derives an expression for output that is observationally equivalent to (1.3). But starting with (1.3) and (1.4), an expression for output is obtained that was not equivalent to (1.5); (1.7) contains lagged values of output,  $v$ , and  $u$ , and two lags of  $z$ , while (1.5) contains only the contemporaneous values of  $v$  and  $u$  and one lag of  $z$ . These differences would allow one to distinguish between the two, but they arise only because this example placed a priori restrictions on the lag lengths in (1.3) and (1.5). In general, one would not have the type of a priori information that would allow this.

The lesson from this simple example is that policy cannot be designed without a theory of how money affects the economy. A theory should identify whether the coefficients in a specification of the form (1.3) or the form (1.5) will remain invariant as policy changes. While output equations estimated over a single policy regime may not allow the true structure to be identified, information from several policy regimes might succeed in doing so.

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13. That is,  $L^i x_t = x_{t-i}$ .

If a policy regime change means that the coefficients in the policy rule (1.4) have changed, this would identify whether an expression of the form (1.3) or the form (1.5) was policy-invariant.

### 1.3.4 The VAR Approach

Much of our understanding of the empirical effects of monetary policy on real economic activity has come from the use of vector autoregression (VAR) frameworks. The use of VARs to estimate the impact of money on the economy was pioneered by Sims (1972; 1980). The development of the approach as it has moved from bivariate to trivariate to larger and larger systems, as well as the empirical findings the literature has produced, were summarized by Leeper, Sims, and Zha (1996). Christiano, Eichenbaum, and Evans (1999) provided a thorough discussion of the use of VARs to estimate the impact of money, and they provide an extensive list of references to work in this area.<sup>14</sup> More recent surveys on the effects of monetary policy that utilize VAR or related frameworks are Boivin, Kiley, and Mishkin (2010) and Ramey (2016) and the references they cite.

Suppose there is a bivariate system in which  $y_t$  is the natural log of real output at time  $t$ , and  $x_t$  is a candidate measure of monetary policy such as a measure of the money stock or a short-term market rate of interest.<sup>15</sup> The VAR system can be written as

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = A(L) \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}, \quad (1.8)$$

where  $A(L)$  is a  $2 \times 2$  matrix polynomial in the lag operator  $L$ , and  $u_{it}$  is a time  $t$  serially independent innovation to the  $i$ th variable. These innovations can be thought of as linear combinations of independently distributed shocks to output ( $e_{yt}$ ) and to policy ( $e_{xt}$ ):

$$\begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix} = \begin{bmatrix} e_{yt} + \theta e_{xt} \\ \phi e_{yt} + e_{xt} \end{bmatrix} = \begin{bmatrix} 1 & \theta \\ \phi & 1 \end{bmatrix} \begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix} = B \begin{bmatrix} e_{yt} \\ e_{xt} \end{bmatrix}. \quad (1.9)$$

The one-period-ahead error made in forecasting the policy variable  $x_t$  is equal to  $u_{xt}$ , and since from (1.9)  $u_{xt} = \phi e_{yt} + e_{xt}$ , these errors are caused by the exogenous output and policy disturbances  $e_{yt}$  and  $e_{xt}$ .

The random variable  $e_{xt}$  represents the exogenous shock to policy. To determine the role of policy in *causing* movements in output or other macroeconomic variables, one needs to estimate the effect of  $e_x$  on these variables. If  $\phi \neq 0$ , the innovation to the observed

14. Two references on the econometrics of VARs are Hamilton (1994) and Maddala (1992).

15. How one measures monetary policy is a critical issue in the empirical literature and is a topic of ongoing debate. See, for example, Romer and Romer (1989), Bernanke and Blinder (1992), Gordon and Leeper (1994), Christiano, Eichenbaum, and Evans (1996; 1999), Bernanke and Mihov (1998), Rudebusch and Svensson (1999), Leeper, Sims, and Zha (1996), and Leeper (1997). Zha (1997) and Ramey (2016) provided useful discussions of the general identification issues that arise in attempting to measure the impact of monetary policy. This issue is discussed further in chapter 12.

policy variable  $x_t$  depends both on the shock to policy  $e_{xt}$  and on the nonpolicy shock  $e_{yt}$ ; obtaining an estimate of  $u_{xt}$  does not provide a measure of the policy shock unless  $\phi = 0$ .

To make the example even more explicit, suppose the VAR system is

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}, \quad (1.10)$$

with  $0 < a_1 < 1$ . Then  $x_t = u_{xt}$  and  $y_t = a_1 y_{t-1} + u_{yt} + a_2 u_{xt-1}$ , and one can write  $y_t$  in moving average form as

$$y_t = \sum_{i=0}^{\infty} a_1^i u_{yt-i} + \sum_{i=0}^{\infty} a_1^i a_2 u_{xt-i-1}.$$

Estimating (1.8) yields estimates of  $A(L)$  and  $\Sigma_u$ , and from these the effects of  $u_{xt}$  on  $\{y_t, y_{t+1}, \dots\}$  can be calculated. If one interpreted  $u_{xt}$  as an exogenous policy disturbance, then the implied response of  $y_t, y_{t+1}, \dots$  to a policy shock would be<sup>16</sup>

$$0, \quad a_2, \quad a_1 a_2, \quad a_1^2 a_2, \dots$$

To estimate the impact of a policy shock on output, however, one needs to calculate the effect on  $\{y_t, y_{t+1}, \dots\}$  of a realization of the policy shock  $e_{xt}$ . In terms of the true underlying structural disturbances  $e_{yt}$  and  $e_{xt}$ , (1.9) implies

$$\begin{aligned} y_t &= \sum_{i=0}^{\infty} a_1^i (e_{yt-i} + \theta e_{xt-i}) + \sum_{i=0}^{\infty} a_1^i a_2 (e_{xt-i-1} + \phi e_{yt-i-1}) \\ &= e_{yt} + \sum_{i=0}^{\infty} a_1^i (a_1 + a_2 \phi) e_{yt-i-1} + \theta e_{xt} + \sum_{i=0}^{\infty} a_1^i (a_1 \theta + a_2) e_{xt-i-1}, \end{aligned} \quad (1.11)$$

so that the impulse response function giving the true response of  $y$  to the exogenous policy shock  $e_x$  is

$$\theta, \quad a_1 \theta + a_2, \quad a_1 (a_1 \theta + a_2), \quad a_1^2 (a_1 \theta + a_2), \dots$$

This response involves the elements of  $A(L)$  and the elements of  $B$ . And while  $A(L)$  can be estimated from (1.8),  $B$  and  $\Sigma_e$  are not identified without further restrictions.<sup>17</sup> Letting  $\Sigma_e$  denote the  $2 \times 2$  diagonal variance matrix of the  $e_{it}$ , and  $\Sigma_u$  denote the variance-covariance matrix of the VAR residues  $u_{it}$ , then  $\Sigma_u = B \Sigma_e B'$ . From estimates of the VAR, an estimate of the three elements of  $\Sigma_u$  can be obtained. But these are functions of six unknown

16. This represents the response to a nonorthogonalized innovation. The basic point, however, is that if  $\theta$  and  $\phi$  are nonzero, the underlying shocks are not identified, so the estimated response to  $u_x$  or to the component of  $u_x$  that is orthogonal to  $u_y$  will not identify the response to the policy shock  $e_x$ .

17. In this example, the three elements of  $\Sigma_u$ , the two variances and the covariance term, are functions of the four unknown parameters:  $\phi$ ,  $\theta$ , and the variances of  $e_y$  and  $e_x$ .

parameters (two elements of  $\Sigma_e$  and four elements of  $B$ ). In this example, the diagonal elements of  $B$  were normalized to equal 1, so the three elements of  $\Sigma_u$  (the two variances and the covariance term) are functions of four unknown parameters:  $\phi$ ,  $\theta$ , and the variances of  $e_y$  and  $e_x$ .

Several approaches have been taken to solving this identification problem. One approach imposes additional restrictions on the matrix  $B$  that links the observable VAR residuals to the underlying structural disturbances (see 1.9). This approach was used by Sims (1972; 1988), Bernanke (1986), Walsh (1987), Bernanke and Blinder (1992), Gordon and Leeper (1994), and Bernanke and Mihov (1998), among others. Sims (1972) treated the nominal money supply ( $M1$ ) as the measure of monetary policy (the  $x$  variable) and identified policy shocks by assuming that  $\phi = 0$ . This approach corresponds to the assumption that the money supply is predetermined and that policy innovations are exogenous with respect to the nonpolicy innovations (see 1.9). Alternatively, if policy shocks affect output with a lag, for example, the restriction that  $\theta = 0$  would allow the other parameters of the model to be identified. This type of restriction was imposed by Bernanke and Blinder (1992) and Bernanke and Mihov (1998).

A second approach achieves identification by imposing restrictions on the long-run effects of the disturbances on observed variables. For example, the assumption of long-run neutrality of money would imply that a monetary policy shock ( $e_x$ ) has no long-run permanent effect on output. In terms of the example that led to (1.11), long-run neutrality of the policy shock would imply that  $\theta + (a_1\theta + a_2) \sum a_1^i = 0$  or  $\theta = -a_2$ . Examples of this approach include Blanchard and Watson (1986), Blanchard (1989), Blanchard and Quah (1989), Judd and Trehan (1989), Hutchison and Walsh (1992), and Galí (1992). The use of long-run restrictions is criticized by Faust and Leeper (1997).

A third approach relies on sign restrictions. For example, Uhlig (2005) identifies a monetary shock by imposing restrictions on how such a shock should affect some of the variables in the VAR. He left unrestricted the effects of the shock on real GDP because estimating the effect of monetary policy shocks on GDP is his primary interest. Specifically, he assumes a contractionary monetary shock does not decrease the federal funds rate, increase prices, or increase nonborrowed reserves. He found that the contractionary policy shocks he identified had ambiguous effects on output.

Monetary policy shocks might be identified by using information from outside the VAR. For example, Romer and Romer (1989) developed a narrative measure of policy. Rather than identify a variable of interest like a monetary policy shock as directly observable, an alternative approach is to distinguish between the variables of interest, which might be unobservable, and the variables that are actually observed. In the factor-augmented VAR (FAVAR) approach of Bernanke (2005), the model structure takes the following form:

$$X_t = \Lambda F_t + e_t,$$

$$F_t = A(L)F_{t-1} + u_t,$$

where  $X_t$  is a vector of observable indicators that are a function of a vector  $F_t$  of potentially unobserved variables. The vector  $e_t$  consists of measurement errors that are variable specific.  $F_t$  follows a VAR, with innovations  $u_t$  that are functions of the underlying structural shocks. Monetary policy shocks are one of the structural disturbances that affect elements of  $u_t$ .<sup>18</sup> If the variables  $F_t$  that govern the evolution of  $X_t$  are identified with observable variables, then  $X_t = F_t$  and the system reduces to a standard VAR. However, the FAVAR framework is more general and allows one to deal with situations in which there may be data available on many macroeconomic variables that are functions of a smaller number of fundamental factors  $F_t$ . That is,  $X_t$  may contain many more variables than appear in a standard VAR. For example, many interest rates and monetary aggregates may be affected by monetary policy, but none of these variables is an exact measure of monetary policy. These variables can all be included in  $X_t$ , and their comovements help identify the evolution of monetary policy. For example, Boivin, Kiley, and Mishkin (2010) included almost 200 variables in  $X_t$  and assumed they were functions of five factors in  $F_t$ . To identify monetary policy shocks, they assumed monetary policy responds contemporaneously to real GDP, prices, and the unemployment rate, while there is at least a one-month lag in the response of these variables to monetary policy.

### Money and Output

Sims (1992) provided a useful summary of the VAR evidence on money and output from France, Germany, Japan, the United Kingdom, and the United States. He estimated separate VARs for each country, using a common specification that includes industrial production, consumer prices, a short-term interest rate as the measure of monetary policy, a measure of the money supply, an exchange rate index, and an index of commodity prices. Sims ordered the interest rate variable first. This corresponds to the assumption that  $\phi = 0$ ; innovations to the interest rate variable potentially affect the other variables contemporaneously (Sims used monthly data), while the interest rate is not affected contemporaneously by innovations in any of the other variables.<sup>19</sup>

The response of real output to an interest rate innovation was similar for all five of the countries Sims examined. In all cases, monetary shocks led to an output response that is usually described as following a hump-shaped pattern. The negative output effects of a contractionary shock, for example, build to a peak after several months and then gradually die out.

Eichenbaum (1992) compared the estimated effects of monetary policy in the United States using alternative measures of policy shocks and discussed how different choices can produce puzzling results, or at least puzzling relative to certain theoretical expectations.

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18. See Boivin and Giannoni (2006) and Boivin, Kiley, and Mishkin (2010).

19. Sims noted that the correlations among the VAR residuals, the  $u_{it}$ , are small, so the ordering has little impact on his results (i.e., sample estimates of  $\phi$  and  $\theta$  are small).

He based his discussion on the results obtained from a VAR containing four variables: the price level and output (these correspond to the elements of  $y$  in (1.8)),  $M1$  as a measure of the money supply, and the federal funds rate as a measure of short-term interest rates (these correspond to the elements of  $x$ ). He considered interpreting shocks to  $M1$  as policy shocks versus the alternative of interpreting funds rate shocks as policy shocks. He found that a positive innovation to  $M1$  is followed by an increase in the federal funds rate and a decline in output. This result is puzzling if  $M1$  shocks are interpreted as measuring the impact of monetary policy. An expansionary monetary policy shock would be expected to lead to increases in both  $M1$  and output. The interest rate was also found to rise after a positive  $M1$  shock, also a potentially puzzling result; a standard model in which money demand varies inversely with the nominal interest rate would suggest that an increase in the money supply would require a decline in the nominal rate to restore money market equilibrium. Gordon and Leeper (1994) showed that a similar puzzle emerges when total reserves are used to measure monetary policy shocks. Positive reserve innovations are found to be associated with increases in short-term interest rates and unemployment increases. The suggestion that a rise in reserves or the money supply might raise, not lower, market interest rates generated a large literature that attempted to search for a liquidity effect of changes in the money supply (e.g., Reichenstein 1987; Christiano and Eichenbaum 1992b; Leeper and Gordon 1992; Strongin 1995; Hamilton 1996).

When Eichenbaum used innovations in the short-term interest rate as a measure of monetary policy actions, a positive shock to the funds rate represented a contractionary policy shock. No output puzzle was found in this case; a positive interest rate shock was followed by a decline in the output measure. Instead, what has been called the *price puzzle* emerges: a contractionary policy shock is followed by a rise in the price level. The effect is small and temporary (and barely statistically significant) but still puzzling. The most commonly accepted explanation for the price puzzle is that it reflects the fact that the variables included in the VAR do not span the full information set available to the Federal Reserve. Suppose the Fed tends to raise the funds rate whenever it forecasts that inflation might rise in the future. To the extent that the Fed is unable to offset the factors that led it to forecast higher inflation, or it acts too late to prevent inflation from rising, the increase in the funds rate will be followed by a rise in prices. This interpretation would be consistent with the price puzzle. One solution is to include commodity prices or other asset prices in the VAR. Since these prices tend to be sensitive to changing forecasts of future inflation, they are a proxy for some of the Fed's additional information (Sims 1992; Chari, Christiano, and Eichenbaum 1995; Bernanke and Mihov 1998). Sims 1992 showed that the price puzzle is not confined to U.S. studies. He reported VAR estimates of monetary policy effects for France, Germany, Japan, and the United Kingdom as well as for the United States, and in all cases, a positive shock to the interest rate leads to a positive price response. These price responses tend to become smaller but do not in all cases disappear when a commodity price index and a nominal exchange rate are included in the VAR. In fact, Hansen (2004)

failed to find much relationship between an indicator's ability to forecast future prices and its ability to reduce the size of the price puzzle.

An alternative interpretation of the price puzzle was provided by Barth and Ramey (2002). They argued that contractionary monetary policy operates on aggregate supply as well as aggregate demand. For example, an increase in interest rates raises the cost of holding inventories and thus acts as a positive cost shock. This negative supply effect raises prices and lowers output. Such an effect is called the *cost channel* of monetary policy. In this interpretation, the price puzzle is simply evidence of the cost channel rather than evidence that the VAR is misspecified. Barth and Ramey combined industry-level data with aggregate data in a VAR and reported evidence supporting of the cost channel interpretation of the price puzzle (see also Ravenna and Walsh 2006 and Gaiotti and Secchi 2004).

One difficulty in measuring the impact of monetary policy shocks arises when operating procedures change over time. The best measure of policy during one period may no longer accurately reflect policy in another period if the implementation of policy has changed. Many of the earlier VAR papers employed measures of monetary aggregates as measures of monetary policy. However, during most of the past 50 years, the federal funds interest rate has been the key policy instrument in the United States, suggesting that unforecasted changes in this interest rate may provide good estimates of policy shocks. Bernanke and Blinder (1992) and Bernanke and Mihov (1998) argued for using the federal funds rate as the measure of monetary policy. While the Fed's operating procedures have varied over time, the funds rate is likely to be the best indicator of policy in the United States during the pre-1979 and post-1982–2008 periods. Policy during the period 1979–1982 is less adequately characterized by the funds rate.<sup>20</sup> The Fed's funds rate target remained fixed at 25 basis points between December 2008 and December 2015, while the Fed used other instruments to influence the economy.

Boivin, Kiley, and Mishkin (2010) summarized evidence on the impact of monetary policy on real GDP and the GDP price deflator. They found that the impact of monetary policy on real GDP was smaller in the 1984–2008 period than before 1980, evidence consistent with the findings of Boivin and Giannoni (2006) but not with those of Canova and Gambetti (2009) or Primiceri (2006), who used VAR approaches with time-varying coefficients. Thus, the issue of whether the effects of a monetary policy shock have changed over time is an open empirical issue.

While researchers disagree on the best means of identifying policy shocks, there is a surprising consensus on the general nature of the economic responses to monetary policy shocks. A variety of VARs estimated for a number of countries all indicate that in response

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20. During this period, nonborrowed reserves were set to achieve a level of interest rates consistent with the desired monetary growth targets. In this case, the funds rate may still provide a satisfactory policy indicator. Cook (1989) found that most changes in the funds rate during the 1979–1982 period reflected policy actions. See chapter 12 for a discussion of operating procedures and the reserve market.



to a policy shock, output follows a hump-shaped pattern in which the peak impact occurs several quarters after the initial shock. Monetary policy actions appear to be taken in anticipation of inflation, so a price puzzle emerges if forward-looking variables such as commodity prices are not included in the VAR.

If monetary policy shocks cause output movements, how important have these shocks been in accounting for actual business cycle fluctuations? Leeper, Sims, and Zha (1996) concluded that monetary policy shocks have been relatively unimportant. However, their assessment is based on monthly data for the period from the beginning of 1960 until early 1996. This sample contains several distinct periods, characterized by differences in how the Fed implemented monetary policy and differing contributions of monetary shocks over various subperiods. Christiano, Eichenbaum, and Evans (1999) concluded that estimates of the importance of monetary policy shocks for output fluctuations are sensitive to the way monetary policy is measured. When they used a funds rate measure of monetary policy, policy shocks accounted for 21 percent of the four-quarter-ahead forecast error variance for quarterly real GDP. This figure rose to 38 percent of the 12-quarter-ahead forecast error variance. Smaller effects were found using policy measures based on monetary aggregates. Christiano, Eichenbaum, and Evans found that very little of the forecast error variance for the price level could be attributed to monetary policy shocks. Romer and Romer (2004) found a larger role for monetary policy using their measure of policy shocks.

### **Criticisms of the VAR Approach**

Measures of monetary policy based on the estimation of VARs have been criticized on several grounds.<sup>21</sup> First, some of the impulse responses do not accord with most economists' priors. In particular, the price puzzle—the finding that a contractionary policy shock, as measured by a funds rate shock, tends to be followed by a rise in the price level—is troublesome. As noted earlier, the price puzzle can be solved by including oil prices or commodity prices in the VAR system, and the generally accepted interpretation is that lacking these inflation-sensitive prices, a standard VAR misses important information that is available to policymakers. A related but more general point is that many of the VAR models used to assess monetary policy fail to incorporate forward-looking variables. Central banks look at a lot of information in setting policy. Because policy is likely to respond to forecasts of future economic conditions, VARs may attribute the subsequent movements in output and inflation to the policy action. However, the argument that puzzling results indicate a misspecification implicitly imposes a prior belief about what the correct effects of monetary shocks should look like. Eichenbaum (1992), in fact, argued that short-term interest rate innovations have been used to represent policy shocks in VARs because they produce the types of impulse response functions for output that economists expect.

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21. These criticisms are detailed in Rudebusch (1998).

In addition, the residuals from the VAR regressions that are used to represent exogenous policy shocks often bear little resemblance to standard interpretations of the historical record of past policy actions and periods of contractionary and expansionary policy (Sheffrin 1995; Rudebusch and Svensson 1999). They also differ considerably depending on the particular specification of the VAR. Rudebusch reported low correlations between the residual policy shocks he obtained based on funds rate futures and those obtained from a VAR by Bernanke and Mihov. How important this finding is depends on the question of interest. If the objective is to determine whether a particular recession was caused by a policy shock, then it is important to know if and when the policy shock occurred. If alternative specifications provide differing and possibly inconsistent estimates of when policy shocks occurred, then their usefulness as a tool of economic history would be limited. If, however, the question of interest is how the economy responds when a policy shock occurs, then the discrepancies among the VAR residual estimates may be less important. Sims (1998a) argued that in a simple supply-demand model, different authors using different supply curve shifters may obtain quite similar estimates of the demand curve slope (since they all obtain consistent estimators of the true slope). At the same time, they may obtain quite different residuals for the estimated supply curve. If the true interest is in the parameters of the demand curve, the variations in the estimates of the supply shocks may not be important. Thus, the type of historical analysis based on a VAR, as in Walsh (1993), is likely to be more problematic than the use of a VAR to determine the way the economy responds to exogenous policy shocks.

While VARs focus on residuals that are interpreted as policy shocks, the systematic part of the estimated VAR equation for a variable such as the funds rate can be interpreted as a policy reaction function; it provides a description of how the policy instrument has been adjusted in response to lagged values of the other variables included in the VAR system. Rudebusch and Svensson (1999) argued that the implied policy reaction functions look quite different than results obtained from more direct attempts to estimate reaction functions or to model actual policy behavior.<sup>22</sup> A related point is that VARs are typically estimated using final, revised data and therefore do not capture accurately the historical behavior of the monetary policymaker who is reacting to preliminary and incomplete data. Woolley (1995) showed how the perception of the stance of monetary policy in the United States in 1972, and President Richard Nixon's attempts to pressure Fed Chairman Arthur F. Burns into adopting a more expansionary policy were based on initial data on the money supply that were subsequently very significantly revised.

At best the VAR approach identifies only the effects of monetary policy shocks, shifts in policy unrelated to the endogenous response of policy to developments in the economy.

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22. For example, Taylor (1993a) employed a simple interest rate rule that closely matches the actual behavior of the federal funds rate in recent years. Such a rule is now the standard way to model Fed behavior. Yet as Khoury (1990) noted in an earlier survey of many studies of the Fed's reaction function, few systematic conclusions had emerged from this empirical literature prior to Taylor's work.

Yet most, if not all, of what one thinks of in terms of policy and policy design represents the endogenous response of policy to the economy, and “most variation in monetary policy instruments is accounted for by responses of policy to the state of the economy, not by random disturbances to policy” (Sims 1998a, 933). This is also a major conclusion of Leeper, Sims, and Zha (1996). So it is unfortunate that VAR analysis, a primary empirical tool used to assess the impact of monetary policy, is uninformative about the role played by policy rules. If policy is completely characterized as a feedback rule on the economy, so that there are no exogenous policy shocks, then the VAR methodology would conclude that monetary policy doesn’t matter. Yet while monetary policy is not causing output movements in this example, it does not follow that policy is unimportant; the response of the economy to nonpolicy shocks may depend importantly on the way monetary policy endogenously adjusts.

Cochrane (1998) made a similar point that is related to the issues discussed in section 1.3.3. In that section, it was noted that one must know whether it is anticipated money with real effects (as in (1.3)) or unanticipated money (as in (1.5)) that matters. Cochrane argued that most of the VAR literature has focused on issues of lag length, detrending, ordering, and variable selection, and has largely ignored another fundamental identification issue: is it anticipated or unanticipated monetary policy that matters? If only unanticipated policy matters, then the subsequent systematic behavior of money after a policy shock is irrelevant. This means that the long hump-shaped response of real variables to a policy shock must be due to inherent lags of adjustment and the propagation mechanisms that characterize the structure of the economy. If anticipated policy matters, then subsequent systematic behavior of money after a policy shock is relevant. This means that the long hump-shaped response of real variables to a policy shock may only be present because policy shocks are followed by persistent, systematic policy actions. If this is the case, the direct impact of a policy shock, if it were not followed by persistent policy moves, would be small.

Attempts have been made to use VAR frameworks to assess the systematic effects of monetary policy. Sims (1998b), for example, estimated a VAR for the interwar years and used it to simulate the behavior of the economy if policy had been determined according to the feedback rule obtained from a VAR estimated using postwar data.

### 1.3.5 Structural Econometric Models

The empirical assessment of the effects of alternative feedback rules for monetary policy has traditionally been carried out using structural macroeconomic models. During the 1960s and early 1970s, the specification, estimation, use, and evaluation of large-scale econometric models for forecasting and policy analysis represented a major research agenda in macroeconomics. Important contributions to our understanding of investment, consumption, the term structure, and other aspects of the macroeconomy grew out of the need to develop structural equations for various sectors of the economy. An equation

describing the behavior of a policy instrument such as the federal funds rate was incorporated into these structural models, allowing model simulations of alternative policy rules to be conducted. These simulations would provide an estimate of the impact on the economy's dynamic behavior of changes in the way policy was conducted. For example, a policy under which the funds rate was adjusted rapidly in response to unemployment movements could be contrasted with one in which the response was more muted.

A key maintained hypothesis, one necessary to justify this type of analysis, was that the estimated parameters of the model would be invariant to the specification of the policy rule. If this were not the case, then one could no longer treat the model's parameters as unchanged when altering the monetary policy rule (as the example in section 1.3.3 shows). In a devastating critique of this assumption, Lucas (1976) argued that economic theory predicts that the decision rules for investment, consumption, and expectations formation will not be invariant to shifts in the systematic behavior of policy. The Lucas critique emphasized the problems inherent in the assumption, common in the structural econometric models of the time, that expectations adjust mechanically to past outcomes.

While large-scale econometric models of aggregate economies continued to play an important role in discussions of monetary policy, they fell out of favor among academic economists during the 1970s, in large part as a result of Lucas's critique, the increasing emphasis on the role of expectations in theoretical models, and the dissatisfaction with the empirical treatment of expectations in existing large-scale models.<sup>23</sup> The academic literature witnessed a continued interest in small-scale rational-expectations models, both single and multicountry versions (e.g., the work of Taylor 1993b) as well as the development of larger-scale models (Fair 1984), all of which incorporated rational expectations into some or all aspects of the model's behavioral relationships. However, recent empirical work investigating the impact of monetary policy has relied on estimated dynamic stochastic general equilibrium (DSGE) models. These models combine rational expectations with a microeconomic foundation in which households and firms are assumed to behave optimally, given their objectives (utility maximization, profit maximization) and the constraints they face. In general, these models are built on the theoretical foundations of the new Keynesian model. As discussed in chapter 8, this model is based on the assumption that prices and wages display rigidities and that this nominal stickiness accounts for the real effects of monetary policy. Early examples include the work of Yun (1996), Ireland (1997a), and Rotemberg and Woodford (1997). Among more recent examples are the DSGE models of Christiano, Eichenbaum, and Evans (2005), who estimated their model by matching VAR impulse responses, and Smets and Wouters (2003), who estimated their model using Bayesian techniques. The use of Bayesian estimation is now common; early examples include work by Smets and Wouters (2003; 2007); Levin et al. (2006), and Lubik

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23. For an example of a small-scale model in which expectations play no explicit role, see Rudebusch and Svensson (1999).

and Schorfheide (2005). Many central banks have built and estimated DSGE models to use for policy analysis, and many more central banks are in the process of doing so. A major advantage of these structural models is that they can be used to evaluate the effects of alternative, systematic rules for monetary policy rather than just the effects of policy shocks on macroeconomic variables.

The basic structure of these models can be expressed as

$$E_t Y_{t+1} = A_1 Y_t + A_2 X_t + B i_t + u_t, \quad (1.12)$$

where  $Y_t$  is a vector of endogenous variables,  $X_t$  is a vector of exogenous variables,  $i_t$  is the policy instrument, and  $u_t$  is an i.i.d. vector of mean zero shocks. The endogenous variables depend on expectations of their future values, on policy, and on the exogenous variables and shocks. The assumption in structural models is that the parameters in the coefficient matrices  $A_1$ ,  $A_2$ , and  $B$  are invariant to the particular policy rule followed by the central bank. Suppose the rule is

$$i_t = C_1 Y_t + C_2 X_t + v_t,$$

where  $v_t$  is an i.i.d. policy shock. Finally, assume the exogenous variables evolve according to  $X_t = \Gamma X_{t-1} + e_t$ , where  $\Gamma$  is also independent of the parameters of the policy rule.

Assuming rational expectations, the solution to this model takes the form

$$Y_t = M X_t + N (B v_t + u_t), \quad (1.13)$$

where  $M$  satisfies

$$M \Gamma = (A_1 + B C_1) M + A_2 + B C_2,$$

and

$$N = -(A_1 + B C_1)^{-1}.$$

The key implication is that the  $M$  and  $N$  matrices in (1.13) depend on the coefficients  $C_1$  and  $C_2$  in the policy rule. However, if the structural model (1.12) can be estimated, then one can investigate how  $M$  and  $N$  and the behavior of  $Y$  changes as the policy rule coefficients  $C_1$  and  $C_2$  are changed, because  $A_1$ ,  $A_2$ , and  $B$  remain constant as  $C_1$  and  $C_2$  are varied.

### 1.3.6 Alternative Approaches

The VAR approach is the most commonly used empirical methodology, and the accompanying results provide a fairly consistent view of the impact of monetary policy shocks. But other approaches have also influenced views on the role of policy. Two such approaches, one based on deriving policy directly from a reading of policy statements, the other based on case studies of disinflations, have influenced academic discussions of monetary policy.

### Announcement Effects

Monetary policy meetings of the Fed's Federal Open Market Committee (FOMC) are followed by announcements. The FOMC releases a statement describing any change in the target for the federal funds rate and guidance about the direction of future policy. Using data on asset prices from immediately before a policy announcement and immediately after the announcement can provide evidence on the impact of policy actions on financial markets and on how information about future policy affects those markets.

Measuring the impact of policy expectations by examining the reaction of financial markets to the release of new information has a long history. In the early 1980s, for example, attention focused on the weekly release of new data on the money supply. Because the Federal Reserve had established targets for money growth, if actual money growth was faster than expected, markets interpreted this as a sign that the Fed would tighten future policy to bring money growth back to target. Roley and Walsh (1985) described empirical work to investigate the impact of weekly money surprises on interest rates. Cook and Hahn (1989) focused on the effects of announced changes in the funds rate target on asset prices. Kuttner (2001) used data on Fed funds futures to distinguish between anticipated and unanticipated changes in the funds rate target and found significant effects on Treasury yields of the latter but not the former. Rigobon and Sack (2004) and Bernanke and Kuttner (2005) examined the stock market reaction to monetary policy.

Gürkaynak, Sack, and Swanson (2005) distinguished between the effects of announcements about policy changes and announcements providing information about future policy. For example, they pointed out that long-term interest rates jumped in response to a January 2004 post-policy meeting announcement even though there was no change in actual policy. Instead, the Fed changed its language about future conditions, which led market participants to anticipate a future rise in the policy rate. Using information on Fed announcements over a 15-year period, they showed that policy announcements affect asset prices through two factors: surprise changes in the actual funds rate target and surprise changes in the expected future path of the funds rate.

To assess the effects of Fed guidance about future policy on inflation and the real economy, Campbell et al. (2012) estimated the effects of policy surprises on professional forecasts of future inflation and unemployment. For the 1996–2007 period, they argued, the Fed was able to signal future policy actions that moved private sector forecasts in ways consistent with policy intentions.<sup>24</sup>

Event studies that estimate the effects of Federal Reserve policy announcements on asset prices have been used extensively to investigate the impact of Fed balance sheet policies undertaken between early 2009 and late 2015, when the Fed funds rate target was fixed at 0–25 basis points. These are discussed in section 1.4.

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24. Kool and Thornton (2012) provided a more skeptical assessment of the effectiveness of forward guidance by examining the experiences of the central banks of New Zealand, Norway, Sweden, and the United States.

### Narrative Measures of Monetary Policy

An alternative to the VAR statistical approach is to develop a measure of the stance of monetary policy from a direct examination of the policy record. This approach was taken by Romer and Romer (1989; 2004) and Boschen and Mills (1991), among others.<sup>25</sup>

Boschen and Mills developed an index of policy stance that takes on integer values from  $-2$  (strong emphasis on inflation reduction) to  $+2$  (strong emphasis on “promoting real growth”). Their monthly index is based on a reading of the FOMC policy directives and the records of the FOMC meetings. Boschen and Mills showed that innovations in their index corresponding to expansionary policy shifts are followed by subsequent increases in monetary aggregates and declines in the federal funds rate. They also concluded that all the narrative indices they examined yielded relatively similar conclusions about the impact of policy on monetary aggregates and the funds rates. And in support of the approach described in section 1.3.4, Boschen and Mills concluded that the funds rate is a good indicator of monetary policy. These findings were extended in Boschen and Mills (1995b), which compared several narrative-based measures of monetary policy, finding them to be associated with permanent changes in the level of  $M2$  and the monetary base and temporary changes in the funds rate.

Romer and Romer (1989) used the Fed’s “Record of Policy Actions” and, prior to 1976 when they were discontinued, minutes of FOMC meetings to identify episodes in which policy shifts occurred that were designed to reduce inflation. They found six different months during the postwar period that saw such contractionary shifts in Fed policy: October 1947, September 1955, December 1968, April 1974, August 1978, and October 1979. Leeper (1993) argued that the Romer-Romer index is equivalent to a dummy variable that picks up large interest rate innovations. Hoover and Perez (1994) provided a critical assessment of the Romers’ narrative approach, noting that the Romer dates are associated with oil price shocks, while Leeper (1997) found that the exogenous component of the Romers’ policy variable does not produce dynamic effects on output and prices that accord with general beliefs about the effects of monetary policy.

Romer and Romer (2004) used a narrative approach to identify changes in the Fed’s target for the federal funds rate and then took the component that was orthogonal to the Fed’s forecasts of macroeconomic variables. Using this measure of policy shocks, they found a much larger role for policy shocks both in affecting inflation and output and in accounting for historical fluctuations. Coibion (2012) reconciled these results with the smaller effects found in most VAR analyses by showing that the lag length structure assumed can play an important role, as does the treatment of the 1979–1982 period, during which Fed policy was better characterized as a nonborrowed reserve aggregates procedure (see chapter 12), implying movements of the funds rate were not the appropriate measures of policy.

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25. Boschen and Mills (1991) provided a discussion and comparison of some other indices of policy.

### Case Studies of Disinflations

Case studies of specific episodes of disinflation provide, in principle, an alternative means of assessing the real impact of monetary policy. Romer and Romer's approach to dating periods of contractionary monetary policy is one form of case study. However, the most influential example of this approach is that of Sargent (1986), who examined the ends of several hyperinflations. As discussed more fully in chapter 5, the distinction between anticipated and unanticipated changes in monetary policy played an important role in the 1980s in academic discussions of monetary policy, and a key hypothesis is that anticipated changes should affect prices and inflation, with little or no effect on real economic activity. This implies that a credible policy to reduce inflation should succeed in actually reducing inflation without causing a recession. This implication contrasts sharply with the view that any policy designed to reduce inflation would succeed only by inducing an economic slowdown and temporarily higher unemployment.

Sargent tested these competing hypotheses by examining the ends of the post-World War I hyperinflations in Austria, Germany, Hungary, and Poland. In each case, he found that the hyperinflations ended abruptly. In Austria, for example, prices rose by over a factor of 20 from December 1921 to August 1922, an annual inflation rate of over 8,800 percent. Prices then stopped rising in September 1922, actually declining by more than 10 percent during the remainder of 1922. While unemployment did rise during the price stabilizations, Sargent concluded that the output cost per percentage point reduction in inflation was much smaller than what some economists had estimated would be the costs of reducing U.S. inflation. Sargent's interpretation of the experiences in Germany, Poland, and Hungary is similar. In each case, the hyperinflation was ended by a regime shift that involved a credible change in monetary and fiscal policy designed to reduce government reliance on inflationary finance. Because the end of inflation reduced the opportunity cost of holding money, money demand grew and the actual stock of money continued to grow rapidly after prices had stabilized.

Sargent's conclusion that the output costs of these disinflations were small has been questioned, as have the lessons he drew for the moderate inflations experienced by the industrialized economies in the 1970s and early 1980s. As Sargent noted, the ends of the hyperinflations "were not isolated restrictive actions within a given set of rules of the game" but represented changes in the rules of the game, most importantly in the ability of the fiscal authority to finance expenditures by creating money. In contrast, the empirical evidence from VARs of the type discussed earlier in this chapter reflects the impact of policy changes within a given set of rules.

Schelde-Andersen (1992) and Ball (1993) provided other examples of the case study approach. In both cases, the authors examined disinflationary episodes in order to estimate the real output costs associated with reducing inflation.<sup>26</sup> Their cases, all involving OECD countries, represent evidence on the costs of ending moderate inflations. Ball calculated

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26. See also Gordon (1982) and Gordon and King (1982).



the deviation of output from trend during a period of disinflation and expressed this as a ratio to the change in trend inflation over the same period. The 65 disinflation periods he identifies in annual data yield an average sacrifice ratio of 0.77 percent; each percentage point reduction in inflation was associated with a 0.77 percent loss of output relative to trend. The estimate for the United States was among the largest, averaging 2.3 percent based on annual data. The sacrifice ratios are negatively related to nominal wage flexibility; countries with greater wage flexibility tend to have smaller sacrifice ratios. The costs of a disinflation also appear to be larger when inflation is brought down more gradually over a longer period of time.<sup>27</sup>

The case study approach can provide interesting evidence on the real effects of monetary policy. Unfortunately, as with the VAR and other approaches, the issue of identification needs to be addressed. To what extent have disinflations been exogenous, so that any resulting output or unemployment movements can be attributed to the decision to reduce inflation? If policy actions depend on whether they are anticipated or not, then estimates of the cost of disinflating obtained by averaging over episodes—episodes that are likely to have differed considerably in terms of whether the policy actions were expected or, if announced, credible—may yield little information about the costs of ending any specific inflation.

#### 1.4 Monetary Policy at Very Low Interest Rates

In December 2008 the Federal Reserve cut its federal funds rate target range to 0–25 basis points, and it remained there until December 2015. Bernanke and Reinhart (2004) argued that this was the effective lower bound for Fed’s target. Historically, zero was treated as the lower bound on nominal interest rates, but subsequently several central banks set negative interest rates (e.g., the central banks of Denmark, Japan, Sweden, and Switzerland as well as the European Central Bank).<sup>28</sup> How negative rates can go is uncertain, but it is certain that zero is not the lower bound for nominal interest rates. Thus, this book generally refers to the minimum possible level of the nominal interest rate as the effective lower bound (ELB) rather than the more common zero lower bound (ZLB). Regardless of what the value of the ELB is, with the funds rate effectively fixed for seven years in the United States, standard empirical strategies that used unforecastable movements in the funds rate to measure monetary policy shocks were no longer useful. While the funds rate target did not change, the Federal Reserve engaged in policy actions that expanded its balance sheet

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27. Brayton and Tinsley (1996) showed how the costs of disinflation can be estimated under alternative assumptions about expectations and credibility using the FRB/US structural model. Their estimates of the sacrifice ratio, expressed in terms of the cumulative annual unemployment rate increase per percentage point decrease in the inflation rate, range from 2.6 under imperfect credibility and VAR expectations to 1.3 under perfect credibility and VAR expectations. Under full-model expectations, the sacrifice ratio is 2.3 with imperfect credibility and 1.7 with full credibility.

28. Why zero was viewed as the lower bound is discussed in chapter 11.

from around \$850 billion in 2008 to \$4.5 trillion by 2015. It also altered the composition of the assets on its balance sheet by selling holdings of short-term government securities and purchasing long-term government securities and large quantities of mortgage-backed securities. These actions raise two questions. How can the stance of monetary policy be measured when the policy interest rate is at its lower bound? What have been the effects of balance sheet policies on financial markets and the macroeconomy?

#### 1.4.1 Measuring Policy at the Effective Lower Bound (ELB)

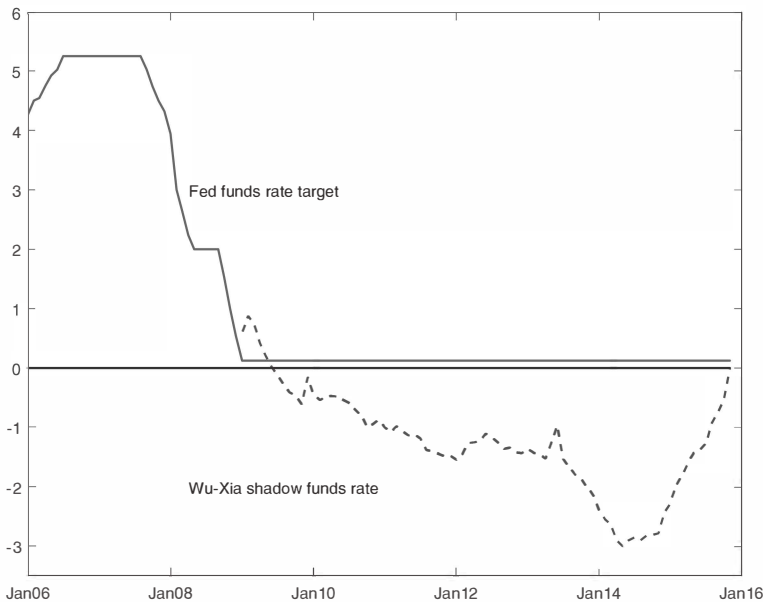
If the central bank is at its effective lower bound but is making announcements about the future path of the policy rate (forward guidance), expanding its balance sheet, and altering its asset holdings, it can be difficult to develop a summary measure of monetary policy. One approach is to employ data on various policy instruments and treat them as observable indicators of the unobservable policy stance. This is the approach, for example, in the FAVAR strategy see section 1.3.4. An alternative approach was developed by Wu and Xia (2016) to estimate an effective short-term rate when the policy rate is fixed at zero. They used a theory of the relationship between interest rates on government bonds of different maturities to estimate the value of the short-term rate that is consistent with the observed behavior of long-term rates. When the actual short-term rate is positive, their estimate corresponds to the actual short-term rate. When the actual short-term rate is fixed at its lower bound, they obtain an estimated shadow short-term rate. If nonstandard policies are effective at reducing long-term interest rates, even though the actual policy rate has not changed, the shadow rate will be below the policy rate. Its level can proxy for the impact of the nonstandard policies.

Figure 1.9 shows the Fed's target for the funds rate and the Wu-Xia shadow rate. The data are monthly from January 2006 to November 2015. The shadow rate has been negative since July 2009, suggesting that the unconventional balance sheet policies of the Fed succeeded in lowering long-term rates even though the funds rate target remained unchanged.

The models examined in this book generally imply both the current value of the short-term rate and its expected future path are important for households and firms making consumption and investment decisions. Long-term rates that affect spending decisions should respond directly to changes in expectations about future short-term rates. When the short-term rate is at the ELB, central bank announcements designed to affect expectations of future short-term rates may allow the central bank to influence economic activity, and a large literature has investigated the impact of central bank announcements on expectations and on long-term rates.<sup>29</sup> However, the effects of announcements can be difficult to interpret. Suppose the central bank announces it will keep interest rates lower for longer than it previously planned. If this is interpreted as indicating a more expansionary future

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29. See Kiley (2014) for evidence that aggregate demand is affected by both short-term and long-term interest rates.



**Figure 1.9**

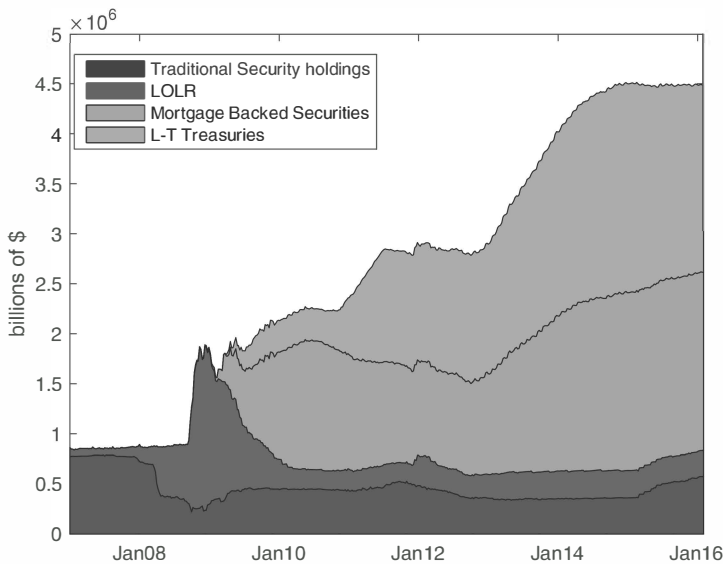
The Fed's target for the funds rate and the Wu-Xia shadow rate.

policy stance, the effect should be expansionary. Alternatively, if the public interprets the announcement of lower future rates as a signal that the central bank is pessimistic about future economic activity, the effect can be contractionary. Campbell et al. (2012) called the first effect Odyssean—the central bank is committing itself to keeping rates low in the future—and the second Delphic—the central bank is signaling a change in its outlook for the economy.

#### 1.4.2 The Effects of Quantitative Easing (QE) Policies

Between 2008 and 2015, the Fed employed balance sheet policies in an attempt to stimulate economic activity. These policies involved asset purchases and were collectively referred to as large-scale asset purchases (LSAP) programs or simply as quantitative easing (QE) policies. Many authors have described in detail the specific nature and timing of each of the Fed's QE policies. For example, see Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2013), and D'Amico et al. (2012) for the United States and Joyce et al. (2011) for the United Kingdom.<sup>30</sup> The net effects of these policies were to expand the Fed's balance sheet from \$850 billion to \$4.5 trillion and to extend the maturity and riskiness of the assets held by the Fed.

30. See also <http://projects.marketwatch.com/short-history-of-qe-and-the-market-timeline/#0>.



**Figure 1.10**

Asset holdings of the Federal Reserve.

Among the traditional tools a central bank has at its disposal in a financial crisis is the ability to provide short-term loans to solvent institutions. Providing liquidity in a crisis is part of the lender of last resort (LOLR) function of a central bank. Figure 1.10 shows the balance sheet assets of the Federal Reserve. The component labeled LOLR shows the rapid increase in the Federal Reserve's provision of loans to financial institutions and liquidity to key markets during the 2008–2009 financial crisis. These actions expanded the balance sheet from its precrisis level of roughly \$850 billion to a peak in December 2008 of just under \$1.9 trillion. As one would expect of LOLR activities, the crisis created a temporary expansion of the balance sheet, but the effects of these actions were quickly reversed as financial markets returned to more normal conditions.

As figure 1.10 clearly illustrates, however, the overall size of the Fed's balance sheet did not return to precrisis levels. The Fed also undertook QE policies. This section focuses on models designed to understand how expansions of the balance sheet and changes in the composition of the assets held by a central bank may affect asset prices and economic activity. The figure shows the consequences for the Fed's balance sheet of these QE policies, which involved the purchase of mortgage-backed securities and long-term Treasury securities. The balance sheet continued to grow after the end of the financial crisis, reaching \$4.5 trillion in early 2014, where it remained as of early 2016.

The first large expansion, under QE1 from November 2008 to March 2010, resulted in the purchase of \$300 billion in U.S. Treasuries, \$1.25 trillion of agency mortgage-backed

securities, and \$170 billion of agency debt. After a pause, the balance sheet expanded again under QE2, which began in November 2010 and lasted until June 2011. QE2 involved the purchases of long-term U.S. Treasuries. From July 2011 to December 2012, the total balance sheet remained relatively constant at about \$2.8–\$2.9 trillion. However, this period saw the Fed alter the composition of its balance sheet by purchasing long-term Treasuries financed by selling short-term Treasuries. This modern-day Operation Twist, or maturity extension program (MEP), began in September 2011 and continued through 2012.<sup>31</sup> September 2012 saw the start of QE3, under which the Fed shifted from announcing a fixed amount of purchases and instead committed to purchasing \$45 billion of U.S. Treasuries and \$40 billion of mortgage-backed securities per month with no end date. QE3 continued until late 2013, when the monthly amount purchased was reduced by \$10 billion in December 2013. A gradual tapering of purchases continued until QE3 ended in October 2014.

LSAP programs were designed to reduce long-term interest rates to stimulate spending. Normally, the Fed would lower its policy rate, a very short-term rate, if it wanted to lower longer-term rates. But if the policy rate is at its ELB, this option is not available. By purchasing long-term assets, the Fed reduced the supply of these assets available to the private sector. For example, when the Fed purchases long-term bonds, fewer are available for the private sector to hold, and this may increase their price, reducing long-term interest rates. Any assessment of balance sheet policies must address two separate questions. First, are such policies effective in altering yields? Second, if the answer to the first question is yes, are these changes effective in influencing real economic activity? Most of the empirical work on balance sheet policies has focused on the first question, but obtaining the answer to the second is clearly essential.

### **On Yields and Asset Prices**

The bulk of the empirical work designed to estimate the impact of balance sheet policies has focused on the effects of the announcements of balance sheet policies on asset prices and bond yields. In this context, an important issue is determining which asset prices and interest rates are most important for affecting the real economy. Consider a very simple economy with short-term and long-term government debt and a private security such as a corporate bond. There are three interest rates: the rate on short-term government debt, the rate on long-term government debt, and the rate on risky private debt. When the short-term rate is the policy instrument, increases or decreases in this rate are assumed to affect the other two rates. However, if the short-term rate is at its lower bound, are balance sheet policies more effective if they work by lowering long-term rates relative to the short-term rate or by lowering risk premiums so that the rate on private debt falls relative to riskless government debt?

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31. See Swanson (2011) for a comparison of MEP with the Operation Twist of the 1960s.

One way to address this question is to ask whether future real activity is better forecast by spreads between long-term and short-term rates on government securities or by credit spreads that reflect risk premiums. This forecasting exercise was conducted by Rudebusch, Sack, and Swanson (2007), who also summarized the earlier literature. Gilchrist and Zakrajšek (2012) undertook it, also using a number of new alternative measures of credit spreads. Rudebusch, Sack, and Swanson (2007) found that a rise in the long-rate relative to the short-rate predicts *higher* future real activity, a finding confirmed with more recent data by Walsh (2014). Rudebusch argued that if changes in spreads rather than levels are used, a rise in the long-term rate predicts slower future growth. However, Walsh found this was the case for industrial production but not for unemployment. Results for risk premiums as measured by spreads between the Aaa corporate bond rate and the 10-year government bond rate, or between Aaa and Baa bonds, were more robust, consistent with the findings of Gilchrist and Zakrajšek (2012) based on corporate credit risks. Increases in these risk spreads predicted weaker future industrial production and higher unemployment. Gilchrist and Zakrajšek (2013) found that the LSAP programs lowered overall credit risk as measured by the cost of default risk insurance outside the financial sector.

One of the first and most influential analyses of the Fed's LSAP policies is the work of Gagnon et al. (2011). They concluded "LSAPs cause economically meaningful and long-lasting reductions in longer-term interest rates on a range of securities, *including securities that were not included in the purchase programs*"<sup>(5)</sup> (italics added). They also concluded that the policies reduced risk premiums rather than expectations of future short-term rates. This suggested a low degree of substitutability between reserves and assets purchased (long-term Treasuries and mortgage-backed securities) and a high degree of substitutability between the assets purchased and corporate debt.

Krishnamurthy and Vissing-Jorgensen (2013) found, somewhat in contrast to Gagnon et al. (2011), that QE policies primarily affect the prices of the assets that the Fed purchases rather than broadly all long-term bonds.<sup>32</sup> This is an important finding, since it suggests that effects depend on particular assets and that QE policies are not good substitutes for general changes in the level of interest rates when the policy rate itself can be used. It may also suggest that the level of segmentation in financial markets is particularly high, limiting the arbitrage across broad categories that is implicitly assumed by arguments that lowering long-term rates on Treasuries have effects on a wide range of asset prices.<sup>33</sup>

A number of authors have used term structure factor models to investigate the effects of bond supply on interest rates. See, for example, Li and Wei (2013), Greenwood, Hanson, and Vayanos (2015), Hamilton and Wu (2012b), D'Amico et al. (2012), and

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32. See also Krishnamurthy and Vissing-Jorgensen (2011). Krishnamurthy and Vissing-Jorgensen (2013, table 1, p. 10), provided a summary of their findings for LSAP programs.

33. Attention is restricted to studies of the Fed's QE policies. Papers that focus on the Bank of England's policies include Joyce et al. (2011) and Kapetanios and Mumtaz (2012). As noted previously, Christensen and Rudebusch (2012) also estimated the effects of QE policies in the United Kingdom.

Swanson and Williams (2013). If financial assets are imperfect substitutes in investors' portfolios, then changes in the outstanding stocks of these assets should cause relative rates of return to adjust. Hamilton and Wu (2012b) echoed the earlier work by Bernanke, Reinhart, and Sack (2004) in stating, "Our conclusion is that although it appears to be possible for the Fed to influence the slope of the yield curve in normal times. . . very large operations are necessary to have an appreciable immediate impact. If there is no concern about a ZLB constraint, this potential tool should clearly be secondary to the traditional focus of open-market operations on the short end of the yield curve" (24). D'Amico et al. (2012) reached a more positive conclusion in arguing that changes in debt stocks affect yields independent of any signaling effects and that their results argued for the effectiveness of LSAPs as a useful tool of monetary policy.

Even in the absence of portfolio balance effects arising from investor heterogeneity or segmented markets, long-term yields could be affected by QE policies if these policies provide new information about the future path of short-term rates. This signaling channel is the only channel that operates in pure expectations models of the term structure. Christensen and Rudebusch (2012) and Bauer and Rudebusch (2013) argued that the commonly used Kim-Wright estimate of the term premium, the estimate used in several studies of QE policies, is based on a model in which the short-term rate's speed of reversion is overstated. Hence, they argued, work using the Kim-Wright model of the term premium, such as Gagnon et al. (2011), tend to overattribute movements in the long-term rate due to QE to movements in the term premium rather than to persistent movements in expected future short-term rates. Bauer and Rudebusch (2013) argued that the effects of QE policies on long-term rates in the United States and United Kingdom were similar but worked through the signaling channel in the United States and because of declines in term premium in the United Kingdom. They attributed these differences to a greater focus on providing forward guidance in the communications of the Fed.

While much of literature has focused on the effects of LSAPs on Treasury yields, Gilchrist and Zakrajšek (2013) focused on the default risk channel by looking at effects on measures of corporate credit risk. If LSAP programs help stimulate the economy, expected defaults should fall, reducing the default risk premium and increasing investor appetite for risk. They argued that event study estimates of LSAP policies are biased downward because of endogeneity of interest rate and credit risk responses to common shocks. They identified the credit risk response to QE policies using shifts in the variance of monetary policy shocks on announcement dates, based on the premise that a larger share of news is associated with monetary policy on these dates. Gilchrist and Zakrajšek (2013) concluded that declines in risk-free rates due to LSAP programs did succeed in reducing measures of risk for the corporate sector but not for the financial intermediary sector.

Gürkaynak, Sack, and Swanson (2005) identified two factors associated with the effects of Fed announcements on asset prices, with one factor associated with changes in the target for the funds rate and the other associated with information about the future path

of the target. Swanson (2015) applied this approach to the 2009–2015 period and found that the two factors can be identified as reflecting forward guidance and LSAP programs. He looked at the impact on 2-year, 5-year, and 10-year yields and found that LSAP policies significantly lowered long-term interest rates, with effects increasing as the maturity of the security increased.

Chen, Cúrdia, and Ferrero (2012, table 1) summarized the results of several papers that provided estimates of the impact of LSAP policies on the U.S. 10-year Treasury yield. A \$100 billion QE policy is estimated to reduce the 10-year rate from about 3 basis points (e.g., Hamilton and Wu 2012b) to 15 basis points (D’Amico et al. (2012)).

### **On the Macroeconomy**

While most studies of QE have focused on financial markets, understanding their impact on asset prices and yields provides at best a partial answer to the question of whether these policies have supported economic growth. Estimating such effects is inherently much more difficult than estimating the effects of announcements on asset yields.

Several authors have utilized DSGE models to simulate the effects of QE policies. For example, Chen, Cúrdia, and Ferrero (2012), building on the work of Andrés, López-Salido, and Nelson (2004), simulated the effects of a QE program in an estimated DSGE model with segmented financial markets and a transaction cost that limits arbitrage.<sup>34</sup> This transaction cost appears as a wedge between one-period returns on the short-term and long-term government bonds, and this wedge is assumed to depend on the maturity structure of publicly held government debt. Central bank balance sheet policies that alter the ratio of long-term to short-term debt held by the public affect the wedge between long-term rates and short-term rates. The resulting interest rate adjustments affect consumption behavior and real economic activity. The simulation results of Chen, Cúrdia, and Ferrero (2012) seem consistent with earlier findings that very large QE policies are necessary to move interest rate premiums significantly.<sup>35</sup> They conclude, “Asset purchase programmes are in principle effective at stimulating the economy because of limits to arbitrage and market segmentation between short-term and long-term government bonds. The data, however, provide little support for these frictions to be pervasive” (F313).

Another example of a DSGE model developed to investigate QE policies is that of Carlstrom, Fuerst, and Paustian (2014). Their model incorporated market segmentation and because of moral hazard issues, the net worth of financial intermediaries limits the ability of these institutions to arbitrage away the spread between long-term rates and deposit rates.

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34. The models of Andrés, López-Salido, and Nelson (2004) and Chen, Cúrdia, and Ferrero (2012) are discussed in chapter 11.

35. For example, they estimated that a commitment to keep the short-term rate at zero for four quarters combined with an LSAP of \$600 billion raises GDP growth by 0.13 percent at an annual rate and increases inflation by 3 basis points. The effects of LSAP are similar to a 25 basis point cut in the short-term rate, but (see their Figure 5, p. 313) it is interesting that the interest rate cut has a large impact on GDP growth but only a tiny impact on the 10-year rate, raising questions about the transmission channel of monetary policy in the model.



They also assumed that new investment is financed with long-term nominal debt, arguing that this leads to larger effects of QE policies because investment is more interest sensitive than is the consumption spending that is the focus of the segmented market's model of Chen, Cúrdia, and Ferrero (2012).<sup>36</sup> Financial intermediaries are the sole purchasers of long-term government bonds and investment bonds, but these are perfect substitutes from the perspective of the intermediaries, so they carry the same yield. Thus, QE policies that lower long-term rates on government debt automatically lower interest rates on private debt used to finance investment.

DelNegro et al. (2016) also developed a DSGE model to assess the Fed's policies. They found that the Fed's provision of liquidity during the financial crisis of 2008–2009 helped avert another Great Depression.

An alternative approach to specifying a DSGE model is provided by Baumeister and Benati (2013). They used a time-varying VAR that allows for stochastic volatility to estimate the impact of a decline in the long-term interest rate relative to the short-term policy rate. They then used estimates of the impact of QE policies in the United States and the United Kingdom in reducing long-term interest rates to obtain an estimate of the effects of these policies on inflation and output. Baumeister and Benati argued that for both countries the QE policies significantly reduce the risks of a major contraction.

Earlier, the shadow interest rate that Wu and Xia (2016) constructed from a term structure model as a measure of monetary policy was discussed. Wu and Xia found that the impact of their shadow interest rate on macroeconomic variables was similar to the estimated impact of the funds rate target in the prior zero interest rate period. They used their shadow rate term structure model in three ways to estimate the impact of unconventional monetary policies on the real economy. First, using their shadow rate in a VAR, they identified the estimated monetary shocks as reflecting unconventional policies. Setting the shocks to zero, they found the shadow rate would have been 0.4 percent higher during 2011–2013. They attributed this to unconventional policy generating expansionary shocks, leading the actual shadow rate to be below the counterfactual, no-shock path. However, the effects on the real economy were small. Without these shocks, unemployment in December 2013 would have been 6.83 percent rather than the actual 6.70 percent. The index of industrial production would have been 101.0 rather than the actual 101.8. Housing starts would have been 988,000 rather than the actual 999,000. They concluded that unconventional policy succeeded in stimulating the economy, but the effects seemed small. Second, they considered a counterfactual exercise in which the shadow rate never falls below a lower bound. In this case, they concluded the unemployment rate would have been 1 percentage point higher. Third, they estimated the impact of forward guidance by simulating expected lift-off dates (dates when the shadow rate is expected to exceed a lower bound). They found that a one-year increase in the expected time until lift-off leads to a 0.25 percent decrease in

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36. Since debt is issued in nominal terms, inflation has real effects even with flexible prices.

the unemployment rate (but the impulse response function was not statistically significant at the 10 percent level). Overall, they concluded this has roughly the same effect as a 35 basis point decline in the policy rate.

## 1.5 Summary

The consensus from the empirical literature on the long-run relationship between money, prices, and output is clear. Money growth and inflation essentially display a correlation of 1; the correlation between money growth or inflation and real output growth is probably close to 0, although it may be slightly positive at low inflation rates and negative at high rates.

The consensus from the empirical literature on the short-run effects of money is that exogenous monetary policy shocks produce hump-shaped movements in real economic activity. The peak effects occur after a lag of several quarters (as much as two or three years in some of the estimates) and then die out. The exact manner in which policy is measured makes a difference, and using an incorrect measure of monetary policy can significantly affect the empirical estimates obtained.

There is less consensus, however, on the role played by the systematic feedback responses of monetary policy. Structural econometric models have the potential to fill this gap, and they are widely used in policymaking settings. Disagreements over the “true” structure and the potential dependence of estimated relationships on the policy regime have, however, posed problems for the structural modeling approach. A major theme of the next 11 chapters is that the endogenous response of monetary policy to economic developments can have important implications for the empirical relationships observed among macroeconomic variables.

Finally, balance sheet policies that many central banks implemented during and after the 2008–2009 financial crisis appear to have been effective in lowering long-term interest rates. There is more uncertainty about the exact channels through which these policies affect the general level of economic activity. In addition, the conclusions of Bernanke, Reinhart, and Sack (2004) appear to have been supported by more recent work: large-scale balance sheet policies are required to have even modest effects on the real economy.

## 2 Money-in-the-Utility Function

### 2.1 Introduction

The neoclassical growth model due to Ramsey (1928) and Solow (1956) provides the basic framework for much of modern macroeconomics. Solow's growth model has just three key ingredients: a production function allowing for smooth substitutability between labor and capital in the production of output; a capital accumulation process in which a fixed fraction of output is devoted to investment each period; and a labor supply process in which the quantity of labor input grows at an exogenously given rate. Solow showed that such an economy would converge to a steady-state growth path along which output, the capital stock, and the effective supply of labor all grew at the same rate.

When the assumption of a fixed savings rate is replaced by a model of forward-looking households choosing savings and labor supply to maximize lifetime utility, the Solow model becomes the foundation for dynamic stochastic general equilibrium (DSGE) models of the business cycle. Productivity shocks or other real disturbances affect output and savings behavior, with the resultant effect on capital accumulation propagating the effects of the original shock over time in ways that can mimic some features of actual business cycles (see Cooley 1995).

The neoclassical growth model is a model of a nonmonetary economy, and while goods are exchanged and transactions must be taking place, there is no medium of exchange—that is, no “money”—used to facilitate these transactions. Nor is there an asset like money that has a zero nominal rate of return and is therefore dominated in rate of return by other interest-bearing assets. To employ the neoclassical framework to analyze monetary issues, a role for money must be specified so that the agents will wish to hold positive quantities of money. A positive demand for money is necessary if, in equilibrium, money is to have positive value.<sup>1</sup>

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1. This is just another way of saying that we would like the money price of goods to be bounded. If the price of goods in terms of money is denoted by  $P$ , then 1 unit of money will purchase  $1/P$  units of goods. If money has positive value,  $1/P > 0$ , and  $P$  is bounded ( $0 < P < \infty$ ). Bewley (1983) referred to the issue of why money has positive value as the Hahn problem (Hahn 1965).

Fundamental questions in monetary economics are the following: How should the demand for money be modeled? How do real economies differ from Arrow-Debreu economies in ways that give rise to a positive value for money? Three general approaches to incorporating money into general equilibrium models have been followed: (1) assume that money yields direct utility by incorporating money balances into the utility functions of the agents of the model (Sidrauski 1967); (2) impose transaction costs of some form that give rise to a demand for money, by making asset exchanges costly (Baumol 1952; Tobin 1956), requiring that money must be used for certain types of transactions (Clower 1967; Lagos and Wright 2005), assuming that time and money can be combined to produce transaction services that are necessary for obtaining consumption goods (Brock 1974; McCallum and Goodfriend 1987; Croushore 1993), or assuming that direct barter of commodities is costly (Kiyotaki and Wright 1989); or (3) treat money like any other asset used to transfer resources intertemporally (Samuelson 1958; Sims 2013).

All three approaches involve shortcuts; some aspects of the economic environment are simply specified exogenously to introduce a role for money. This can be a useful device, allowing one to focus on questions of primary interest without being unduly distracted by secondary issues. But confidence in the ability of a model to answer the questions brought to it is reduced if exogenously specified aspects appear to be critical to the primary issue. An important consideration in evaluating different approaches is to determine whether conclusions generalize beyond the specific model or depend on the exact manner in which a role for money has been introduced. Subsequent examples include results that are robust, such as the connection between money growth and inflation, and others that are sensitive to the specification of money's role, such as the impact of inflation on the steady-state capital stock.

This chapter develops the first of the three approaches by incorporating into the basic neoclassical model agents whose utility depends directly on their consumption of goods *and* their holdings of money.<sup>2</sup> Given suitable restrictions on the utility function, such an approach can guarantee that, in equilibrium, agents choose to hold positive amounts of money and money is positively valued. The money-in-the-utility function (MIU) model developed in this chapter is originally due to Sidrauski (1967), and it has been used widely.<sup>3</sup> It can be employed to examine some of the important issues in monetary economics: the relationship between money and prices, the effects of inflation on equilibrium, and the optimal rate of inflation. To better understand the role of money in such a framework, a linear approximation to the model is presented. This approximation can be used to derive

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2. The second approach, focusing on the transaction role of money, is discussed in chapter 3. The third approach has been developed primarily within the context of overlapping generation models; see Sargent (1987) or Champ, Freeman, and Haslag (2016).

3. See Patinkin (1965, ch. 5) for an earlier discussion of an MIU model, although he did not integrate capital accumulation into his model. However, the first-order condition for optimal money holdings that he presented (see his eq. 1, p. 89) is equivalent to the one derived in the next section.

some analytical implications and to study numerically the MIU model's implications for macrodynamics.

## 2.2 The Basic MIU Model

To develop the basic MIU approach, uncertainty and any labor-leisure choice are initially ignored to focus on the implications of the model for money demand, the value of money, and the costs of inflation.

Suppose the utility function of the representative household takes the form

$$U_t = u(c_t, z_t),$$

where  $z_t$  is the flow of services yielded by money holdings and  $c_t$  is time  $t$  per capita consumption. Utility is assumed to be increasing in both arguments, strictly concave, and continuously differentiable. The demand for monetary services is always positive if one assumes that  $\lim_{z \rightarrow 0} u_z(c, z) = \infty$  for all  $c$ , where  $u_z = \partial u(c, z) / \partial z$ .

What constitutes  $z_t$ ? To maintain the assumption of rational economic agents, what enters the utility function cannot just be the number of dollars (or euro or yen) that the individual holds. What should matter is the command over goods that are represented by those dollar holdings, or some measure of the transaction services, expressed in terms of goods, that money yields. In other words,  $z$  should be related to something like the number of dollars,  $M$ , times their price,  $1/P$ , in terms of goods:  $M(1/P) = M/P$ . If the service flow is proportional to the real value of the stock of money and  $N_t$  is the population, then  $z$  can be set equal to real per capita money holdings:

$$z_t = \frac{M_t}{P_t N_t} \equiv m_t.$$

To ensure that a monetary equilibrium exists, it is often assumed that for all  $c$ , there exists a finite  $\bar{m} > 0$  such that  $u_m(c, m) \leq 0$  for all  $m > \bar{m}$ . This means that the marginal utility of money eventually becomes nonpositive for sufficiently high money balances. The role of this assumption is made clear later, when the existence of a steady state is discussed. It is, however, not necessary for the existence of equilibrium, and some common functional forms employed for the utility function (which are used later in this chapter) do not satisfy this condition.<sup>4</sup>

The assumption that money enters the utility function is often criticized on the grounds that money itself is intrinsically useless (e.g., paper currency) and that it is only through its use in facilitating transactions that it yields valued services. Approaches that emphasize the transaction role of money are discussed in chapter 3, but models in which money helps

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4. For example,  $u(c, m) = \log c + b \log m$  does not exhibit this property, since  $u_m = b/m > 0$  for all finite  $m$ .

to reduce the time needed to purchase consumption goods can be represented by the MIU approach adopted in this chapter.<sup>5</sup>

The representative household is viewed as choosing time paths for consumption and real money balances subject to budget constraints specified later, with total utility given by

$$W = \sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \quad (2.1)$$

where  $0 < \beta < 1$  is a subjective rate of discount.

Equation (2.1) implies a much stronger notion of the utility provided by holding money than simply that the household would prefer having more money to less money. If the marginal utility of money is positive, then (2.1) implies that, *holding constant the path of real consumption for all  $t$* , the individual's utility is increased by an increase in money holdings. That is, even though the money holdings are never used to purchase consumption, they yield utility. This should seem strange; one usually thinks the demand for money is instrumental in that money is held to engage in transactions leading to the purchase of the goods and services that actually yield utility. All this is just a reminder that the money-in-the-utility function may be a useful shortcut for ensuring that there is a demand for money, but it is just a shortcut.

To complete the specification of the model, assume that households can hold money, bonds that pay a nominal interest rate  $i_t$ , and physical capital. Physical capital produces output according to a standard neoclassical production function. Given its current income, its assets, and any net transfers received from the government ( $\tau_t$ ), the household allocates its resources among consumption, gross investment in physical capital, and gross accumulation of real money balances and bonds.

If the rate of depreciation of physical capital is  $\delta$ , the aggregate economywide budget constraint of the household sector takes the form

$$Y_t + \tau_t N_t + (1 - \delta)K_{t-1} + \frac{(1 + i_{t-1})B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t}, \quad (2.2)$$

where  $Y_t$  is aggregate output,  $K_{t-1}$  is the aggregate stock of capital at the start of period  $t$ , and  $\tau_t N_t$  is the aggregate real value of any lump-sum transfers or taxes.

The timing implicit in this specification of the MIU model assumes that it is the household's real money holdings at the end of the period,  $M_t/P_t$ , *after* having purchased consumption goods, that yield utility. Carlstrom and Fuerst (2001) criticized this timing assumption, arguing that the appropriate way to model the utility from money is to assume it is money balances available *before* the purchase of consumption goods that yield utility. As they demonstrated, alternative timing assumptions can affect the correct definition of the opportunity cost of holding money and whether multiple real equilibria can be ruled

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5. Brock (1974), for example, developed two simple transaction stories that can be represented by putting money directly in the utility function. See also Feenstra (1986).

out. Because it is standard in the MIU approach to assume that it is end-of-period money holdings that yield utility, this assumption is maintained in the development of the model.<sup>6</sup>

The aggregate production function relates output  $Y_t$  to the available capital stock  $K_{t-1}$  and employment  $N_t$ :  $Y_t = F(K_{t-1}, N_t)$ .<sup>7</sup> Assuming this production function is linear homogeneous with constant returns to scale, output per capita  $y_t$  is a function of the per capita capital stock  $k_{t-1}$ :<sup>8</sup>

$$y_t = f\left(\frac{k_{t-1}}{1+n}\right), \quad (2.3)$$

where  $n$  is the population growth rate (assumed to be constant). Note that output is produced in period  $t$  using capital carried over from period  $t-1$ . The production function is assumed to be continuously differentiable and to satisfy the usual Inada conditions ( $f_k \geq 0$ ,  $f_{kk} \leq 0$ ,  $\lim_{k \rightarrow 0} f_k(k) = \infty$ ,  $\lim_{k \rightarrow \infty} f_k(k) = 0$ ).

Dividing both sides of the budget constraint (2.2) by population  $N_t$ , the per capita version becomes

$$\begin{aligned} \omega_t &\equiv f\left(\frac{k_{t-1}}{1+n}\right) + \tau_t + \left(\frac{1-\delta}{1+n}\right)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)(1+n)} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \quad (2.4)$$

where  $\pi_t$  is the rate of inflation,  $b_t = B_t/P_tN_t$ , and  $m_t = M_t/P_tN_t$ .

The representative household's problem is to choose paths for  $c_t$ ,  $k_t$ ,  $b_t$ , and  $m_t$  to maximize (2.1) subject to (2.4). This is a problem in dynamic optimization, and it is convenient to formulate the problem in terms of a value function. The value function gives the maximized present discounted value of utility that the household can achieve by optimally choosing consumption, capital holdings, bond holdings, and money balances, given its current state.<sup>9</sup> The state variable for the problem is the household's initial level of resources  $\omega_t$ , and the value function is defined by

$$V(\omega_t) = \max_{c_t, k_t, b_t, m_t} \{u(c_t, m_t) + \beta V(\omega_{t+1})\}, \quad (2.5)$$

where the maximization is subject to the budget constraint (2.4) and

$$\omega_{t+1} = f\left(\frac{k_t}{1+n}\right) + \tau_{t+1} + \left(\frac{1-\delta}{1+n}\right)k_t + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)}.$$

6. Problems 1 and 2 at the end of this chapter ask you to derive the first-order conditions for money holdings under an alternative timing assumption.

7. Since any labor-leisure choice is ignored in this section,  $N_t$  is used interchangeably for population and employment.

8. That is, if  $Y_t = F(K_{t-1}, N_t)$ , where  $Y$  is output,  $K$  is the capital stock, and  $N$  is labor input, and  $F(\lambda K, \lambda N) = \lambda F(K, N) = \lambda Y$ , we can write  $Y_t/N_t \equiv y_t = F(K_{t-1}, N_t)/N_t = F(K_{t-1}/N_t, 1) \equiv f(k_{t-1}/(1+n))$ , where  $n = (N_t - N_{t-1})/N_{t-1}$  is the constant labor force growth rate. In general, a lowercase letter denotes the per capita value of the corresponding uppercase variable.

9. Introductions to dynamic optimization designed for economists can be found in Sargent (1987), Lucas and Stokey (1989), Dixit (1990), Chiang (1992), Obstfeld and Rogoff (1996), Ljungquist and Sargent (2000), Wickens (2008), and Miao (2014).

Using (2.4) to express  $k_t$  as  $\omega_t - c_t - m_t - b_t$  and making use of the definition of  $\omega_{t+1}$ , (2.5) can be written as

$$V(\omega_t) = \max_{c_t, b_t, m_t} \left\{ u(c_t, m_t) + \beta V \left( f \left( \frac{\omega_t - c_t - m_t - b_t}{1+n} \right) + \tau_{t+1} + \left( \frac{1-\delta}{1+n} \right) (\omega_t - c_t - m_t - b_t) + \frac{(1+i_t)b_t + m_t}{(1+\pi_{t+1})(1+n)} \right) \right\},$$

with the maximization problem now an unconstrained one over  $c_t$ ,  $b_t$ , and  $m_t$ . The first-order necessary conditions for this problem are

$$u_c(c_t, m_t) - \frac{\beta}{1+n} \left[ f_k \left( \frac{k_t}{1+n} \right) + 1 - \delta \right] V_\omega(\omega_{t+1}) = 0, \quad (2.6)$$

$$\frac{1+i_t}{(1+\pi_{t+1})(1+n)} - \frac{1}{1+n} \left[ f_k \left( \frac{k_t}{1+n} \right) + 1 - \delta \right] = 0, \quad (2.7)$$

$$u_m(c_t, m_t) - \frac{\beta}{1+n} \left[ f_k \left( \frac{k_t}{1+n} \right) + 1 - \delta - \frac{1}{1+\pi_{t+1}} \right] V_\omega(\omega_{t+1}) = 0, \quad (2.8)$$

together with the transversality conditions

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t x_t = 0, \quad \text{for } x = k, b, m, \quad (2.9)$$

where  $\lambda_t$  is the marginal utility of period  $t$  consumption. The envelope theorem implies

$$V_\omega(\omega_t) = \frac{\beta}{1+n} \left[ f_k \left( \frac{k_t}{1+n} \right) + 1 - \delta \right] V_\omega(\omega_{t+1}),$$

which together with (2.6) yields

$$\lambda_t \equiv u_c(c_t, m_t) = V_\omega(\omega_t). \quad (2.10)$$

The first-order conditions have straightforward interpretations. Since initial resources  $\omega_t$  must be divided between consumption, capital, bonds, and money balances, each use must yield the same marginal benefit at an optimum allocation.<sup>10</sup> Using (2.6) and (2.10), (2.8) can be written as

$$u_m(c_t, m_t) + \frac{\beta u_c(c_{t+1}, m_{t+1})}{(1+\pi_{t+1})(1+n)} = u_c(c_t, m_t), \quad (2.11)$$

which states that the marginal benefit of adding to money holdings at time  $t$  must equal the marginal utility of consumption at time  $t$ . The marginal benefit of additional money holdings has two components. First, money directly yields utility  $u_m$ . Second, real money

10. For a general equilibrium analysis of asset prices in an MIU framework, see LeRoy (1984a; 1984b).



balances at time  $t$  add  $1/(1 + \pi_{t+1})(1 + n)$  to real per capita resources at time  $t + 1$ ; this addition to  $\omega_{t+1}$  is worth  $V_\omega(\omega_{t+1})$  at  $t + 1$ , or  $\beta V_\omega(\omega_{t+1})$  at time  $t$ . Thus, the total marginal benefit of money at time  $t$  is  $u_m(c_t, m_t) + \beta V_\omega(\omega_{t+1})/(1 + \pi_{t+1})(1 + n)$ . Equation (2.11) is then obtained by noting that  $V_\omega(\omega_{t+1}) = u_c(c_{t+1}, m_{t+1})$ .

From (2.6), (2.7), and (2.11),

$$\begin{aligned} \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} &= 1 - \left[ \frac{\beta}{(1 + \pi_{t+1})(1 + n)} \right] \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} \\ &= 1 - \frac{1}{(1 + r_t)(1 + \pi_{t+1})} \\ &= \frac{i_t}{1 + i_t} \equiv \Upsilon_t, \end{aligned} \quad (2.12)$$

where  $1 + r_t \equiv f_k(k_t/1 + n) + 1 - \delta$  is the real return on capital, and (2.6) implies  $\beta u_c(c_{t+1}, m_{t+1})/u_c(c_t, m_t) = (1 + n)/(1 + r_t)$ . Equation (2.12) also makes use of (2.7), which links the nominal return on bonds, inflation, and the real return on capital. This latter equation can be written as

$$1 + i_t = \left[ f_k \left( \frac{k_t}{1 + n} \right) + 1 - \delta \right] (1 + \pi_{t+1}) = (1 + r_t)(1 + \pi_{t+1}). \quad (2.13)$$

This relationship between real and nominal rates of interest is called the *Fisher relationship* after Irving Fisher (1896). It expresses the gross nominal rate of interest as equal to the gross real return on capital times 1 plus the expected rate of inflation. Note that  $(1 + x)(1 + y) \approx 1 + x + y$  when  $x$  and  $y$  are small, so (2.13) is often written as

$$i_t = r_t + \pi_{t+1}.$$

To interpret (2.12), consider a very simple choice problem in which the agent must pick  $x$  and  $z$  to maximize  $u(x, z)$  subject to a budget constraint of the form  $x + pz = y$ , where  $p$  is the relative price of  $z$ . The first-order conditions imply  $u_z/u_x = p$ ; in words, the marginal rate of substitution between  $z$  and  $x$  equals the relative price of  $z$  in terms of  $x$ . Comparing this to (2.12) shows that  $\Upsilon$  can be interpreted as the relative price of real money balances in terms of the consumption good. The marginal rate of substitution between money and consumption is set equal to the price, or opportunity cost, of holding money. The opportunity cost of holding money is directly related to the nominal rate of interest. The household could hold one unit less of money, purchasing instead a bond yielding a nominal return of  $i$ ; the real value of this payment is  $i/(1 + \pi)$ , and since it is received in period  $t + 1$ , its present value is  $i/[(1 + r)(1 + \pi)] = i/(1 + i)$ .<sup>11</sup> Since money is assumed to pay no interest, the opportunity cost of holding money is affected both by the real return on capital and

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11. Suppose households gain utility from the real money balances they have at the start of period  $t$  rather than the balances they hold at the end of the period, as has been assumed. Then the marginal rate of substitution between money and consumption will be set equal to  $i_t$  (see Lucas 1982; Carlstrom and Fuerst 2001). See also problem 1 at the end of this chapter.

the rate of inflation. If the price level is constant (so  $\pi = 0$ ), then the forgone earnings from holding money rather than capital are determined by the real return to capital. If the price level is rising ( $\pi > 0$ ), the real value of money in terms of consumption declines, and this adds to the opportunity cost of holding money.

In deriving the first-order conditions for the household's problem, it could have been equivalently assumed that the household rented its capital to firms, receiving a rental rate of  $r_k$ , and sold its labor services at a wage rate of  $w$ . Household income would then be  $r_k k + w$  (expressed on a per capita basis and ignoring population growth). With competitive firms hiring capital and labor in perfectly competitive factor markets under constant returns to scale,  $r_k = f'(k)$  and  $w = f(k) - kf'(k)$ , so household income would be  $r_k k + w = f_k(k)k + [f(k) - kf_k(k)] = f(k)$ , as in (2.4).<sup>12</sup>

While this system could be used to study analytically the dynamic behavior of the economy (e.g., Sidrauski 1967; Fischer 1979; Blanchard and Fischer 1989), the properties of the steady-state equilibrium are the initial focus. And because the main focus here is not on the exogenous growth generated by population growth, it provides some slight simplification to set  $n = 0$  in the following. After examining the steady state, we study the dynamic properties implied by a stochastic version of the model, a version that also includes uncertainty, a labor-leisure choice, and variable employment.

### 2.2.1 Steady-State Equilibrium

Consider the properties of this economy when it is in a steady-state equilibrium with  $n = 0$  and the nominal supply of money growing at the rate  $\theta$ . Let the superscript *ss* denote values evaluated at the steady state. The steady-state values of consumption, the capital stock, real money balances, inflation, and the nominal interest rate must satisfy the first-order necessary conditions for the household's decision problem given by (2.6)–(2.8), the economywide budget constraint, and the specification of the exogenous growth rate of  $M$ . Note that with real money balances constant in the steady state, it must be that the prices are growing at the same rate as the nominal stock of money, or  $\pi^{ss} = \theta$ .<sup>13</sup> Using (2.10) to eliminate  $V_\omega(\omega^{ss})$ , the equilibrium conditions can be written as

$$u_c(c^{ss}, m^{ss}) - \beta [f_k(k^{ss}) + 1 - \delta] u_c(c^{ss}, m^{ss}) = 0, \quad (2.14)$$

$$\frac{1 + i^{ss}}{1 + \theta} - [f_k(k^{ss}) + 1 - \delta] = 0, \quad (2.15)$$

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12. This follows from Euler's theorem: if the aggregate constant-returns-to-scale production function is  $F(N, K)$ , then  $F(N, K) = F_N N + F_K K$ . In per capita terms, this becomes  $f(k) = F_N + F_K k = w + r_k k$  if labor and capital are paid their marginal products.

13. If the population is growing at the rate  $n$ , then  $1 + \pi^{ss} = (1 + \theta)/(1 + n)$ .

$$u_m(c^{ss}, m^{ss}) - \beta [f_k(k^{ss}) + 1 - \delta] u_c(c^{ss}, m^{ss}) + \frac{\beta u_c(c^{ss}, m^{ss})}{1 + \theta} = 0, \quad (2.16)$$

$$f(k^{ss}) + \tau^{ss} + (1 - \delta)k^{ss} + \frac{m^{ss}}{1 + \theta} = c^{ss} + k^{ss} + m^{ss}, \quad (2.17)$$

where  $\omega^{ss} = f(k^{ss}) + \tau^{ss} + (1 - \delta)k^{ss} + m^{ss}/(1 + \pi)$ . In (2.14)–(2.17) use has been made of the fact that in this representative agent model, borrowing and lending must equal zero in equilibrium,  $b = 0$ . Equation (2.15) is the steady-state form of the Fisher relationship linking real and nominal interest rates. This can be seen by noting that the real return on capital (net of depreciation) is  $r^{ss} \equiv f_k(k^{ss}) - \delta$ , so (2.15) can be written as

$$1 + i^{ss} = (1 + r^{ss})(1 + \theta) = (1 + r^{ss})(1 + \pi^{ss}). \quad (2.18)$$

Notice that in (2.14)–(2.17) money appears only in the form of real money balances. Thus, any change in the nominal quantity of money that is matched by a proportional change in the price level, leaving  $m^{ss}$  unchanged, has no effect on the economy's real equilibrium. This is described by saying that the model exhibits *neutrality of money*. One-time changes in the level of the nominal quantity of money affect only the level of prices. If prices do not adjust immediately in response to a change in  $M$ , then a model might display non-neutrality in the short run but still exhibit monetary neutrality in the long run, once all prices have adjusted. In fact, this is the case with the models used in chapters 5–12 to examine issues related to short-run monetary policy.

Dividing (2.14) by  $u_c(c^{ss}, m^{ss})$  yields  $1 - \beta [f_k(k^{ss}) + 1 - \delta] = 0$ , or

$$f_k(k^{ss}) = \frac{1}{\beta} - 1 + \delta. \quad (2.19)$$

This equation defines the steady-state capital-labor ratio  $k^{ss}$  as a function of  $\beta$  and  $\delta$ . If the production function is Cobb-Douglas, say,  $f(k) = k^\alpha$  for  $0 < \alpha \leq 1$ , then  $f_k(k) = \alpha k^{\alpha-1}$  and

$$k^{ss} = \left[ \frac{\alpha \beta}{1 + \beta(\delta - 1)} \right]^{\frac{1}{1-\alpha}}. \quad (2.20)$$

What is particularly relevant for our purposes is the implication from (2.19) that the steady-state capital-labor ratio is independent of (1) all parameters of the utility function other than the subjective discount rate  $\beta$ , and (2) the steady-state rate of inflation  $\pi^{ss}$ . In fact,  $k^{ss}$  depends only on the production function, the depreciation rate, and the discount rate. It is independent of the rate of inflation and the growth rate of money.

Because changes in the nominal quantity of money are engineered in this model by making lump-sum transfers to the public, the real value of these transfers must equal  $(M_t - M_{t-1})/P_t = \theta M_{t-1}/P_t = \theta m_{t-1}/(1 + \pi_t)$ . Hence, steady-state transfers are given by

$\tau^{ss} = \theta m^{ss} / (1 + \pi^{ss}) = \theta m^{ss} / (1 + \theta)$ , and the budget constraint (2.17) reduces to the economy's resource constraint

$$c^{ss} = f(k^{ss}) - \delta k^{ss}. \quad (2.21)$$

The steady-state level of consumption per capita is equal to output minus replacement investment and is completely determined once the level of steady-state capital is known. Assuming  $f(k) = k^\alpha$ ,  $k^{ss}$  is given by (2.20) and

$$c^{ss} = \left[ \frac{\alpha\beta}{1 + \beta(\delta - 1)} \right]^{\frac{\alpha}{1-\alpha}} - \delta \left[ \frac{\alpha\beta}{1 + \beta(\delta - 1)} \right]^{\frac{1}{1-\alpha}}.$$

Steady-state consumption per capita depends on the parameters of the production function ( $\alpha$ ), the rate of depreciation ( $\delta$ ), and the subjective rate of time discount ( $\beta$ ).

The Sidrauski MIU model exhibits a property called the *superneutrality of money*; the steady-state values of the capital stock, consumption, and output are all independent of the *rate of growth* of the nominal money stock. That is, not only is money neutral, so that proportional changes in the *level* of nominal money balances and prices have no real effects, but changes in the *rate of growth* of nominal money also have no effect on the steady-state capital stock or therefore on output or per capita consumption. Because the real rate of interest is equal to the marginal product of capital, it also is invariant across steady states that differ only in their rates of money growth. Thus, the Sidrauski MIU model possesses the properties of both neutrality and superneutrality.

To understand why superneutrality holds, note from (2.10),  $u_c = V_\omega(\omega_t)$ , so using (2.6),

$$u_c(c_t, m_t) = \beta [f_k(k_t) + 1 - \delta] u_c(c_{t+1}, m_{t+1}), \quad \text{or}$$

$$\frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} = \frac{1/\beta}{f_k(k_t) + 1 - \delta}. \quad (2.22)$$

Recall from (2.19) that the right side of this expression is equal to 1 in the steady state. If  $k < k^{ss}$  so that  $f_k(k) > f_k(k^{ss})$ , then the right side is smaller than 1, and the marginal utility of consumption declines over time. It is optimal to postpone consumption to accumulate capital and have consumption grow over time (so  $u_c$  declines over time). As long as  $f_k + 1 - \delta > 1/\beta$ , this process continues, but as the capital stock grows, the marginal product of capital declines until eventually  $f_k(k) + 1 - \delta = 1/\beta$ . The converse holds if  $k > k^{ss}$ . Consumption remains constant only when  $f_k + 1 - \delta = 1/\beta$ . If an increase in the rate of money growth (and therefore an increase in the rate of inflation) were to induce households to accumulate more capital, this would lower the marginal product of capital, leading to a situation in which  $f_k + 1 - \delta < 1/\beta$ . Households would then want their consumption path to decline over time, so they would immediately attempt to increase current consumption and reduce their holdings of capital. The value of  $k^{ss}$  consistent with a steady state is independent of the rate of inflation.

What is affected by the rate of inflation? One thing to expect is that the interest rate on any asset that pays off in units of money at some future date will be affected; the real value of those future units of money will be affected by inflation, and this will be reflected in the interest rate required to induce individuals to hold the asset, as shown by (2.13). To understand this equation, consider the nominal interest rate that an asset must yield if it is to give a real return of  $r_t$  in terms of the consumption good. That is, consider an asset that costs 1 unit of consumption in period  $t$  and yields  $(1 + r_t)$  units of consumption at  $t + 1$ . In units of money, this asset costs  $P_t$  units of money at time  $t$ . Because the cost of each unit of consumption at  $t + 1$  is  $P_{t+1}$  in terms of money, the asset must pay an amount equal to  $(1 + r_t)P_{t+1}$ . Thus, the nominal return is  $[(1 + r_t)P_{t+1} - P_t]/P_t = (1 + r_t)(1 + \pi_{t+1}) - 1 \equiv i_t$ . In the steady state,  $1 + r^{ss} = 1/\beta$  and  $\pi^{ss} = \theta$ , so the steady-state nominal rate of interest is given by  $[(1 + \theta)/\beta] - 1$  and varies (approximately) one-to-one with inflation.<sup>14</sup>

### Existence of the Steady State

To ensure a steady-state monetary equilibrium, there must exist a positive but finite level of real money balances  $m^{ss}$  that satisfies (2.12), evaluated at the steady-state level of consumption. If utility is separable in consumption and money balances, say,  $u(c, m) = v(c) + \phi(m)$ , this condition can be written as  $\phi_m(m^{ss}) = \Upsilon^{ss} v_c(c^{ss})$ . The right side of this expression is a non-negative constant; the left side approaches  $\infty$  as  $m \rightarrow 0$ . If  $\phi_m(m) \leq 0$  for all  $m$  greater than some finite level, a steady-state equilibrium with positive real money balances is guaranteed to exist. This was the role of the earlier assumption that the marginal utility of money eventually becomes negative. Note that this assumption is not necessary;  $\phi(m) = \log m$  yields a positive solution to (2.12) as long as  $\Upsilon^{ss} v_c(c^{ss}) > 0$ .<sup>15</sup> When utility is not separable, one can still write (2.12) as  $u_m(c^{ss}, m^{ss}) = \Upsilon^{ss} u_c(c^{ss}, m^{ss})$ . If  $u_{cm} < 0$ , so that the marginal utility of consumption decreases with increased holdings of money, both  $u_m$  and  $u_c$  decrease with  $m$  and the solution to (2.12) may not be unique; multiple steady-state equilibria may exist.<sup>16</sup> However, it may be more plausible to assume money and consumption are complements in utility, an assumption that would imply  $u_{cm} \geq 0$ .

When  $u(c, m) = v(c) + \phi(m)$ , the dynamics of real balances around the steady state can be described easily by multiplying both sides of (2.11) by  $M_t$  and noting that  $M_{t+1} = (1 + \theta)M_t$ :

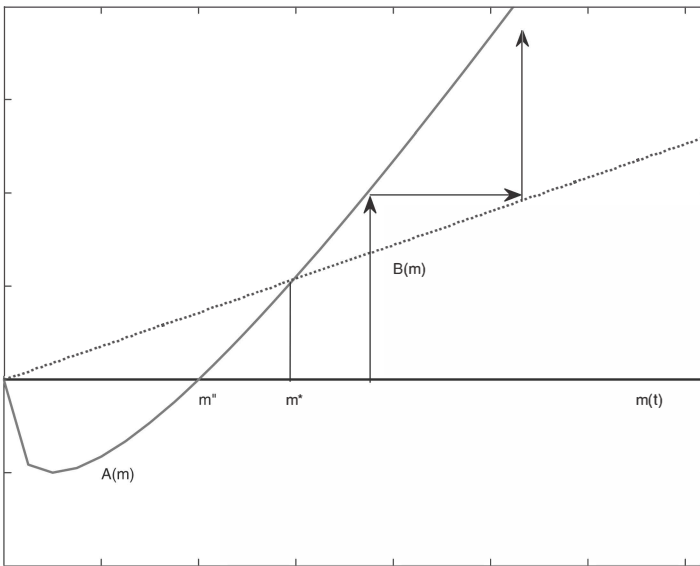
$$B(m_{t+1}) \equiv \frac{\beta}{1 + \theta} v_c(c^{ss}) m_{t+1} = [v_c(c^{ss}) - \phi_m(m_t)] m_t \equiv A(m_t), \quad (2.23)$$

14. Outside of the steady state, the nominal rate can still be written as the sum of the expected real rate plus the expected rate of inflation, but there is no longer any presumption that short-run variations in inflation will leave the real rate unaffected.

15.  $\Upsilon^{ss} > 0$  requires that  $i^{ss} > 0$ .

16. For more on the conditions necessary for the existence of monetary equilibria, see Brock (1974; 1975) and Bewley (1983).

which gives a difference equation in  $m$ . The properties of this equation have been examined by Brock (1974) and Obstfeld and Rogoff (1983; 1986). A steady-state value for  $m$  satisfies  $B(m^{ss}) = A(m^{ss})$ . The functions  $B(m)$  and  $A(m)$  are illustrated in figure 2.1.  $B(m)$  is a straight line with slope  $\beta v_c(c^{ss})/(1 + \theta)$ .  $A(m)$  has slope  $(v_c - \phi_m - \phi_{mm}m)$ . For the case drawn,  $\lim_{m \rightarrow 0} \phi_m m = 0$ , so there are two steady-state solutions to (2.23), one at  $m^*$  and one at 0. Only one of these involves positive real money balances (and a positive value for money). If  $\lim_{m \rightarrow 0} \phi_m m = \tilde{m} > 0$ , then  $\lim_{m \rightarrow 0} A(m) < 0$  and there is only one solution. Paths for  $m_t$  originating to the right of  $m^*$  involve  $m_{t+s} \rightarrow \infty$  as  $s \rightarrow \infty$ . When  $\theta \geq 0$  (non-negative money growth), such explosive paths for  $m$ , involving a price level going to zero, violate the transversality condition that the discounted value of asset holdings must go to zero.<sup>17</sup> More recently, Benhabib, Schmitt-Grohé, and Uribe (2001b; 2001a; 2002) noted that the existence of an effective lower bound on the nominal interest rate may not allow ruling out paths that begin to the right of  $m^*$ . Suppose the effective lower bound is at zero. As the rate of deflation rises along these deflationary paths, the nominal interest



**Figure 2.1**

Steady-state real balances (separable utility).

17. Obstfeld and Rogoff (1986) showed that any such equilibrium path with an implosive price level violates the transversality condition unless  $\lim_{m \rightarrow \infty} \phi(m) = \infty$ . This condition is implausible because it would require that the utility yielded by money be unbounded. See also Obstfeld and Rogoff (1983).

rate must fall. Once it reaches zero, the process cannot continue, so the economy may find itself in a zero interest rate equilibrium that does not violate any transversality condition.<sup>18</sup>

When  $\lim_{m \rightarrow 0} A(m) < 0$ , paths originating to the left of  $m^*$  converge to  $m < 0$ ; but this is clearly not possible because real balances cannot be negative. For the case drawn in figure 2.1, however, some paths originating to the left of  $m^*$  converge to zero without ever involving negative real balances. For example, a path that reaches  $m''$  at which  $A(m'') = 0$  then jumps to  $m = 0$ . Along such an equilibrium path, the price level is growing faster than the nominal money supply (so that  $m$  declines). Even if  $\theta = 0$ , so that the nominal money supply is constant, the equilibrium path would involve a speculative hyperinflation with the price level going to infinity.<sup>19</sup> Unfortunately, Obstfeld and Rogoff showed that the conditions needed to ensure  $\lim_{m \rightarrow 0} \phi_m m = \tilde{m} > 0$ , so that speculative hyperinflations can be ruled out, are restrictive. They showed that  $\lim_{m \rightarrow 0} \phi_m m > 0$  implies  $\lim_{m \rightarrow 0} \phi(m) = -\infty$ ; essentially, money must be so necessary that the utility of the representative agent goes to minus infinity if real balances fall to zero.<sup>20</sup>

When paths originating to the left of  $m^*$  cannot be ruled out, the model exhibits multiple equilibria. For example, suppose the nominal stock of money is held constant, with  $M_t = M_0$  for all  $t > 0$ . Then there is a rational-expectations equilibrium path for the price level and real money balances starting at any price level  $P_0$  as long as  $M_0/P_0 < m^*$ . Chapter 4 examines an approach called the *fiscal theory of the price level*, which argues that the initial price level may be determined by fiscal policy.

### Steady States with a Time-Varying Money Stock

The previous section considered the steady state associated with a constant growth rate of the nominal supply of money. Often, particularly when the focus is on the relationship between money and prices, one might be more interested in a steady state in which real quantities such as consumption and the capital stock are constant but the growth rate of money varies over time. Assume that  $c_t = c^*$  and  $k_t = k^*$  for all  $t$ . Setting population growth  $n$  to zero and using (2.10), the equilibrium conditions (2.6) and (2.7) can be written as

$$u_c(c^*, m_t) = \beta [f_k(k^*) + 1 - \delta] u_c(c^*, m_{t+1}), \quad (2.24)$$

$$\frac{1 + i_t}{(1 + \pi_{t+1})} = [f_k(k^*) + 1 - \delta], \quad (2.25)$$

18. See the discussion of the Taylor principle in chapter 8 and of liquidity traps in chapter 11.

19. The hyperinflation is labeled speculative, since it is not driven by fundamentals such as the growth rate of the nominal supply of money.

20. Speculative hyperinflations are shown by Obstfeld and Rogoff (1986) to be ruled out if the government holds real resources to back a fraction of the outstanding currency. This ensures a positive value below which the real value of money cannot fall.

while (2.12) implies

$$\frac{u_m(c^*, m_t)}{u_c(c^*, m_t)} = \frac{i_t}{1 + i_t}, \quad (2.26)$$

and the economy's resource constraint becomes

$$c^* = f(k^*) - \delta k^*.$$

The evolution of the real stock of money is given by

$$m_t = \left( \frac{1 + \theta_t}{1 + \pi_t} \right) m_{t-1}. \quad (2.27)$$

If  $\theta$  is constant, one has the situation previously studied. There is a steady state with inflation equal to the rate of growth of money ( $\pi = \theta$ ), and real money balances are constant. With  $m$  constant, (2.24) uniquely determines the capital stock such that  $\beta [f_k(k^{ss}) + 1 - \delta] = 1$ . The economy's resource constraint then determines  $c^*$ .

There may also be steady-state equilibria in the real variables in which  $m$  is changing over time. Reis (2007) investigated how monetary policies that allow the money stock to be time-varying can alter the steady-state values of consumption and capital. To understand intuitively how  $c^*$  and  $k^*$  could be affected by monetary policy, consider (2.24) for  $k^* > k^{ss}$ .<sup>21</sup> Because of diminishing marginal productivity,  $\beta [f_k(k^*) + 1 - \delta] < 1$ , so for (2.24) to hold requires the marginal utility of consumption to rise over time such that

$$\frac{u_c(c^*, m_{t+1})}{u_c(c^*, m_t)} = \frac{1}{\beta [f_k(k^*) + 1 - \delta]} > 1. \quad (2.28)$$

For example, suppose  $u_{cm} > 0$ , so that higher levels of real money balances increase the marginal utility of consumption. Then (2.28) can be satisfied if real money balances grow over time. For real money balances to grow over time, (2.12) implies that the nominal interest rate must be decreasing, reducing the opportunity cost of holding money. Of course, a steady state that satisfies (2.28) may not be feasible. If the marginal utility of money goes to zero for some  $\bar{m} > 0$ , then such a steady state does not exist. Note also that if utility is separable in consumption and real money balances, (2.24) becomes  $u_c(c^*) = \beta [f_k(k^*) + 1 - \delta] u_c(c^*)$ , which implies  $k^* = k^{ss}$ , and the steady state is independent of real money balances.

If, following Fischer (1979), the utility function takes the form

$$u(c, m) = \frac{(c^{1-\gamma} m^\gamma)^{1-\eta}}{1-\eta}, \quad (2.29)$$

---

21. Recall  $k^{ss}$  is such that  $\beta [f_k(k^{ss}) + 1 - \delta] = 1$ .



with  $\eta < 1$  and  $\gamma \in (0, 1)$ , then (2.28) requires that real money balances evolve according to

$$\frac{m_{t+1}}{m_t} = \left\{ \frac{1}{\beta [f_k(k^*) + 1 - \delta]} \right\}^{\frac{1}{\gamma(1-\eta)}}. \quad (2.30)$$

Rather than characterize the steady state in terms of the growth rate of the nominal stock of money, Reis (2007) examined the behavior of the nominal interest rate directly, since central banks today generally employ a nominal interest rate and not a nominal quantity as their policy instrument. The equilibrium condition (2.26) implicitly defines a money demand function of the form

$$m_t = \phi(i_t, c^*),$$

so (2.30) implies the path of the nominal rate must satisfy

$$\frac{\phi(i_{t+1}, c^*)}{\phi(i_t, c^*)} = \left\{ \frac{1}{\beta [f_k(k^*) + 1 - \delta]} \right\}^{\frac{1}{\gamma(1-\eta)}}.$$

With  $k$  constant, (2.25) implies the real interest rate, given by  $(1 + i_t)/(1 + \pi_{t+1})$ , is constant, so the required path for the nominal rate also pins down the path followed by the inflation rate. Advancing (2.27) one period then determines the growth rate of the nominal money stock consistent with the specified equilibrium path. Reis discussed how the monetary authority could, through a policy of declining nominal interest rates, sustain a steady state in which consumption and output remain above the levels that would be reached under a constant growth rate of money policy.<sup>22</sup>

## 2.2.2 Multiple Equilibria in Monetary Models

Section 2.2.1 considered the stationary, steady-state equilibrium of the MIU model, in which real money balances were constant. With  $M_t/P_t$  constant in such an equilibrium, inflation was pinned down by the growth rate of the nominal money supply (perhaps adjusted for income growth) and one-time permanent changes to the level of  $M_t$  would produce proportional changes in the price level. These conclusions are typically associated with the quantity theory of money. The discussion of figure 2.1 suggested the existence of a unique steady state with a constant level of real money balances could not be taken for granted. This section focuses on dynamic paths for the price level and examines whether, given a path for the nominal money supply, there exists a unique equilibrium path for the price level. Or, can there be multiple values of  $P_t$  all of which are consistent with the model's equilibrium conditions?

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22. Of course, an effective lower bound on the nominal interest rate (conventionally assumed to be zero) would halt the decline in the nominal interest rate when rates reached the effective lower bound. See chapter 11.

It is convenient to restrict attention to the case of separable utility in which superneutrality holds and focus on the case in which real consumption and the gross real interest rate are constant, with the latter equal to its steady-state value  $\beta^{-1}$ . The analysis can also be simplified by assuming the nominal money supply is fixed and equal to  $M_0$ . In this case, the key equilibrium condition in the MIU model, (2.12), can be written as

$$u_m\left(\frac{M_0}{P_t}\right) = \left[1 - \beta\left(\frac{P_t}{P_{t+1}}\right)\right] u_c(c). \quad (2.31)$$

This is a forward difference equation in the price level; does it uniquely determine  $P_t$ ?

One solution to (2.31) is  $P_{t+i} = P^*$  for all  $i \geq 0$ , where

$$u_m\left(\frac{M_0}{P^*}\right) = (1 - \beta) u_c(c),$$

or  $P^* = M_0/u_m^{-1}((1 - \beta) u_c(c))$ . In this equilibrium, the quantity theory holds, and the price level is proportional to the money supply. However, this may not be the only equilibrium for the price level. Rewriting (2.31) as

$$P_{t+1} = \left[ \frac{\beta u_c(c)}{u_c(c) - u_m\left(\frac{M_0}{P_t}\right)} \right] P_t \equiv \phi(P_t)$$

makes explicit that it defines a difference equation in the price level. Because  $u_c(c) - u_m\left(\frac{M_0}{P_t}\right) > 0$ , one solution is characterized by a constant price level  $P^* = \phi(P^*)$ .<sup>23</sup> Since  $u_{mm} \leq 0$ , it follows that  $\phi'(P_t) > 0$ . In figure 2.2,  $\phi(P_t)$  is shown as an increasing function of  $P_t$ . Also shown in the figure is the 45° line. Using the fact that  $P^* = \phi(P^*)$  implies

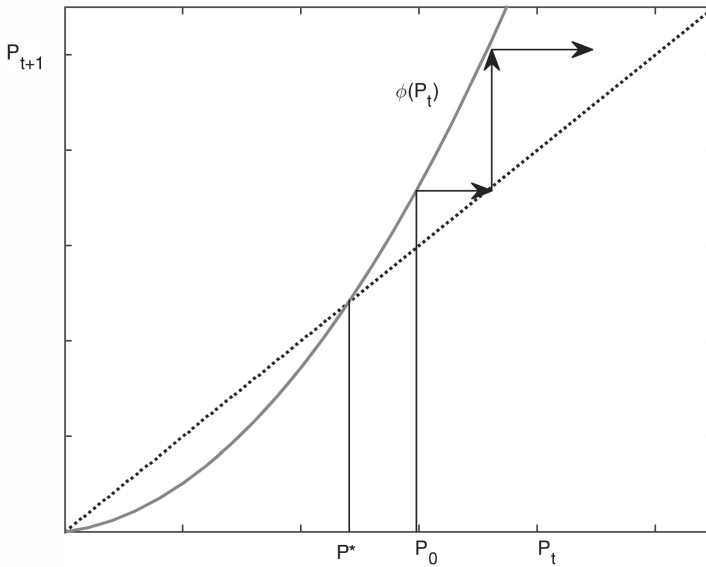
$$\left[ \frac{\beta u_c(c)}{u_c(c) - u_m\left(\frac{M_0}{P^*}\right)} \right] = 1,$$

the slope of  $\phi(P_t)$ , evaluated at  $P^*$ , is

$$\phi'(P^*) = \left[ \frac{u_c(c) - u_m\left(\frac{M_0}{P^*}\right) - u_{mm}\left(\frac{M_0}{P^*}\right)\left(\frac{M_0}{P^*}\right)}{u_c(c) - u_m\left(\frac{M_0}{P^*}\right)} \right] > 1.$$

Thus,  $\phi$  cuts the 45° line from below at  $P^*$ . Any price path starting at  $P_0 > P^*$  is consistent with (2.31) and involves a positive rate of inflation. As the figure illustrates,  $P \rightarrow \infty$ , but the equilibrium condition (2.31) is satisfied along this path. As the price level explodes, real money balances go to zero. But this is consistent with private agents' demand for money because inflation and therefore nominal interest rates are rising, lowering the real

23. From (2.31),  $u_c - u_m\left(\frac{M_0}{P_t}\right) = \beta\left(\frac{P_t}{P_{t+1}}\right) u_c > 0$ .



**Figure 2.2**  
Equilibrium inflationary path with a fixed nominal money supply.

demand for money. Any price level to the right of  $P^*$  is a valid equilibrium. These equilibria all involve speculative hyperinflations. Equilibria originating to the left of  $P^*$  eventually violate a transversality condition because  $M/P$  is exploding as  $P \rightarrow 0$ .<sup>24</sup> By itself, (2.31) is not sufficient to uniquely determine the equilibrium value of the initial price level, even though the nominal quantity of money is fixed.

Monetary models typically focus on stationary equilibria. In this case,  $P^*$  is the unique stationary equilibrium for the price level, and the focus is on the properties of this equilibrium.

### 2.2.3 The Interest Elasticity of Money Demand

Equation (2.12) characterizes the demand for real money balances as a function of the nominal rate of interest and real consumption. For example, suppose that the utility function in consumption and real balances is of the constant elasticity of substitution (CES) form:

$$u(c_t, m_t) = \left[ ac_t^{1-b} + (1-a)m_t^{1-b} \right]^{\frac{1}{1-b}}, \tag{2.32}$$

24. As  $P$  falls toward zero, the nominal interest rate will eventually be driven to zero, an issue ignored here but explored in chapter 11.

with  $0 < a < 1$  and  $b > 0, b \neq 1$ . Then

$$\frac{u_m}{u_c} = \left( \frac{1-a}{a} \right) \left( \frac{c_t}{m_t} \right)^b,$$

and (2.12) can be written<sup>25</sup> as

$$m_t = \left( \frac{1-a}{a} \right)^{\frac{1}{b}} \left( \frac{i}{1+i} \right)^{-\frac{1}{b}} c_t. \quad (2.33)$$

In terms of the more common log specification used to model empirical money demand equations,

$$\log \frac{M_t}{P_t N_t} = \frac{1}{b} \log \left( \frac{1-a}{a} \right) + \log c - \frac{1}{b} \log \frac{i}{1+i}, \quad (2.34)$$

which gives the real demand for money as a negative function of the nominal rate of interest and a positive function of consumption.<sup>26</sup> The consumption (income) elasticity of money demand is equal to 1 in this specification. The elasticity of money demand with respect to the opportunity cost variable  $\Upsilon_t = i_t/(1+i_t)$  is  $1/b$ . For simplicity, this is often referred to as the *interest elasticity of money demand*.<sup>27</sup>

As  $b$  approaches 1 in the limit, the CES specification yields a Cobb-Douglas utility function  $u(c_t, m_t) = c_t^a m_t^{1-a}$ . Note from (2.34) that in this case the consumption (income) elasticity of money demand and the elasticity with respect to the opportunity cost measure  $\Upsilon_t$  are both equal to 1.

While the parameter  $b$  governs the interest elasticity of demand, the steady-state level of money holdings depends on the value of  $a$ . From (2.33), the ratio of real money balances to consumption in the steady state is<sup>28</sup>

$$\frac{m^{ss}}{c^{ss}} = \left( \frac{1-a}{a} \right)^{\frac{1}{b}} \left( \frac{1+\pi^{ss}-\beta}{1+\pi^{ss}} \right)^{-\frac{1}{b}}.$$

25. In the limit, as  $b \rightarrow \infty$ , (2.33) implies that  $m = c$ . This is then equivalent to the cash-in-advance models examined in chapter 3.

26. The standard specification of money demand would use income in place of consumption; but see Mankiw and Summers (1986).

27. The elasticity of money demand with respect to the nominal interest rate is

$$-\frac{\partial m_t}{\partial i_t} \frac{i_t}{m_t} = \frac{1}{b} \frac{1}{1+i_t}.$$

Empirical work often estimates money demand equations in which the log of real money balances is a function of log income and the *level* of the nominal interest rate. The coefficient on the nominal interest rate is then equal to the semielasticity of money demand with respect to the nominal interest rate ( $m^{-1} \partial m / \partial i$ ), which for (2.34) is  $1/bi(1+i)$ . Note that an increase in the nominal interest rate reduces money demand, but the elasticity is expressed as a positive value.

28. This makes use of the fact that  $1+i^{ss} = (1+r^{ss})(1+\pi^{ss}) = (1+\pi^{ss})/\beta$  in the steady state.

The ratio of  $m^{SS}$  to  $c^{SS}$  is decreasing in  $a$ ; an increase in  $a$  reduces the weight given to real money balances in the utility function and results in smaller steady-state holdings of money (relative to consumption). Increases in inflation also reduce the ratio of money holdings to consumption by increasing the opportunity cost of holding money.

### Empirical Evidence on the Interest Elasticity of Money Demand

The empirical literature on money demand is vast. See, for example, the references in Judd and Scadding (1982), Laidler (1985), or Goldfeld and Sichel (1990) for earlier surveys. More recent contributions include Lucas (1988), Hoffman and Rasche (1991), Stock and Watson (1993), Ball (2001), Knell and Stix (2005), Teles and Zhou (2005), Bae and De Jong (2007), and Ireland (2009). Ball argued that in postwar samples ending prior to the late 1980s, the high degree of collinearity between output and interest rates made it difficult to obtain precise estimates of the income and interest elasticities of money demand. Based on data from 1946 to 1996, he found the income elasticity of the demand for the  $M1$  monetary aggregate to be about 0.5 and the interest semielasticity to be about 0.5. An income elasticity less than 1 (the value implied by equation 2.34) is consistent with the findings of Knell and Stix. Teles and Zhou argued that  $M1$  is not the relevant measure of money after 1980 because of the widespread changes in financial regulations. They focused on a monetary aggregate constructed by the Federal Reserve Bank of St. Louis, called money zero maturity (MZM), which measures balances available immediately for transactions at zero cost. Teles and Zhou also assumed an income elasticity of 1 and estimated the interest elasticity of money demand to be 0.24.

Holman (1998) directly estimated the parameters of the utility function under various alternative specifications of its functional form, including (2.32), using annual U.S. data from 1889 to 1991.<sup>29</sup> She obtained estimates of  $b$  of about 0.1 and  $a$  of about 0.95. This value of  $b$  implies an elasticity of money demand equal to 10. However, in shorter samples, the data fail to reject  $b = 1$ , the case of Cobb-Douglas preferences, indicating that the interest elasticity of money demand is estimated very imprecisely.

Using annual data, Lucas (2000) obtained an estimate of 0.5 for the interest elasticity of  $M1$  demand. Chari, Kehoe, and McGrattan (2000) estimated (2.34) using quarterly U.S. data and the  $M1$  definition of money. They obtained an estimate for  $a$  of about 0.94 and an estimate of the interest elasticity of money demand of 0.39, implying a value of  $b$  on the order of  $1/0.39 \approx 2.6$ . Christiano, Eichenbaum, and Evans (2005) reported an interest semielasticity of 0.96 (the partial of log real money holdings with respect to the gross nominal interest rate), obtained as part of the estimation of a dynamic stochastic general equilibrium (DSGE) model of the United States.

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29. Holman considered a variety of specifications for the utility function, including Cobb-Douglas ( $b = 1$ ) and nested CES functions of the form given in section 2.5.

Hoffman, Rasche, and Tieslau (1995) conducted a cross-country study of money demand and found a value of about 0.5 for the U.S. and Canadian money demand interest elasticities, with somewhat higher values for the United Kingdom and lower values for Japan and Germany. An elasticity of 0.5 implies a value of 2 for  $b$ . Ireland (2001) estimated the interest elasticity as part of a general equilibrium model and obtained a value of 0.19 for the pre-1979 period and 0.12 for the post-1979 period. These translate into values for  $b$  of 5.26 and 8.33, respectively.

The log-log specification for money demand given by (2.33) is consistent with the specification adopted by Lucas (2000) and is also used by Bae and De Jong (2007). Ireland (2009) focused on what recent data on interest rates and  $M1$  reveal about the appropriate functional form for the money demand equation. He contrasted two alternative functions. The first is a standard log-log specification, in which the log of real money balances relative to income is related to the log of the nominal interest rate. The second is a semilog specification linking the log of real money balances relative to income to the *level* of the nominal interest rate:

$$\log \frac{M_t}{P_t N_t} = \alpha_0 + \log c - \xi i.$$

Estimated elasticities for the log-log form were in the range of 0.05 to 0.09, corresponding to a value of  $b$  in (2.34) ranging from 11 to 20. The semilog form yielded a coefficient in the range of 1.5 to 1.9 on the level of the interest rate. Ireland found that the semilog specification fits the post-1980 data for the United States much better than the log-log specification. The form of the money demand equation and the sensitivity of money demand to the opportunity cost of holding money are important for assessing the welfare costs of inflation (see section 2.3).

Reynard (2004) found that an increase in financial market participation had increased the interest elasticity of U.S. money demand. He reported the interest rate elasticity rose from 0.065 for the 1949–1969 period to 0.134 for 1977–1999.

Obtaining estimates of the money demand equation is important when monetary policy is implemented through control of a monetary aggregate. The extent to which interest rates adjust in response to a change in the money supply, for example, depends on the interest elasticity of money demand. As many central banks switched during the 1990s to policies that focused directly on using a short-term market interest rate as the instrument of monetary policy, the money demand equation became less relevant for monetary policy, and interest in estimating money demand equations declined. However, as Ireland (2009) showed, estimates of the welfare cost of inflation can depend importantly on the value of the interest elasticity of demand that is used.

Most empirical estimates of the interest elasticity of money demand employ aggregate time series data. At the household level, many U.S. households hold no interest-earning assets, so the normal substitution between money and interest-earning assets as the nominal

interest rate changes is absent. As nominal interest rates rise, more households find it worthwhile to hold interest-earning assets. Changes in the nominal interest rate then affect both the extensive margin (the decision whether to hold interest-earning assets) and the intensive margin (the decision of how much to hold in interest-earning assets, given that the household already holds some wealth in this form). Mulligan and Sala-i-Martin (2000) focused on these two margins and used cross-sectional evidence on household holdings of financial assets to estimate the interest elasticity of money demand. They found that the elasticity increases with the level of nominal interest rates and is low at low nominal rates of interest.

#### 2.2.4 Limitations

Before moving on to use the MIU framework to analyze the welfare cost of inflation, one needs to consider the limitations of the money-in-the-utility approach. In the MIU model, there is a clearly defined reason for individuals to hold money: it provides utility. However, this essentially solves the problem of generating a positive demand for money by assumption; it doesn't address the reasons that money, particularly money in the form of unbacked pieces of paper, might yield utility. The money-in-the-utility function approach should be thought of as a shortcut for a fully specified model of the transaction technology faced by households that gives rise to a positive demand for a medium of exchange.

Shortcuts are often extremely useful. But one problem with such a shortcut is that it does not provide any real understanding of, or possible restrictions on, partial derivatives such as  $u_m$  or  $u_{cm}$  that play a role in determining equilibrium and the outcome of comparative static exercises. One possible scenario that can generate a rationale for money to appear in the utility function is based on the idea that money can reduce the time needed to purchase consumption goods. This shopping-time model is discussed in chapter 3.

### 2.3 The Welfare Cost of Inflation

Because money holdings yield direct utility and higher inflation reduces real money balances, inflation generates a welfare loss. This raises two questions: Is there an optimal rate of inflation that maximizes the steady-state welfare of the representative household? How large is the welfare cost of inflation? Some important results on these questions are illustrated here, and chapters 4 and 8 provide more discussion on the optimal rate of inflation.

The optimal rate of inflation was originally addressed by Bailey (1956) and M. Friedman (1969). Their basic intuition was that the private opportunity cost of holding money depends on the nominal rate of interest (see 2.12). The social marginal cost of producing money, that is, running the printing presses, is essentially zero. The wedge that arises between the private marginal cost and the social marginal cost when the nominal rate of interest is positive generates an inefficiency. This inefficiency would be eliminated if the private opportunity cost were also equal to zero, and this is the case if the nominal

rate of interest equals zero. But  $i = 0$  requires that  $\pi = -r/(1+r) \approx -r$ . So the optimal rate of inflation is a rate of *deflation* approximately equal to the real return on capital.<sup>30</sup>

In the steady state, real money balances are directly related to the inflation rate, so the optimal rate of inflation is also frequently discussed under the heading of the *optimal quantity of money* (M. Friedman 1969). With utility depending directly on  $m$ , one can think of the government choosing its policy instrument  $\theta$  (and therefore  $\pi$ ) to achieve the steady-state optimal value of  $m$ . Steady-state utility is maximized when  $u(c^{ss}, m^{ss})$  is maximized subject to the constraint that  $c^{ss} = f(k^{ss}) - \delta k^{ss}$ . But because  $c^{ss}$  is independent of  $\theta$ , the first-order condition for the optimal  $\theta$  is just  $u_m(\partial m/\partial \theta) = 0$ , or  $u_m = 0$ , and from (2.12), this occurs when  $i = 0$ .<sup>31</sup>

The major criticism of this result is due to Phelps (1973), who pointed out that money growth generates revenue for the government—the inflation tax. The implicit assumption so far has been that variations in money growth are engineered via lump-sum transfers. Any effects on government revenue can be offset by a suitable adjustment in these lump-sum transfers (taxes). But if governments only have distortionary taxes available for financing expenditures, then reducing inflation tax revenue to achieve the Friedman rule of a zero nominal interest rate requires that the lost revenue be replaced through increases in other distortionary taxes. Reducing the nominal rate of interest to zero would increase the inefficiencies generated by the higher level of other taxes needed to replace the lost inflation tax revenue. To minimize the total distortions associated with raising a given amount of revenue, it may be optimal to rely on the inflation tax to some degree. A number of authors have reexamined these results. See, for example, Chari, Christiano, and Kehoe (1991; 1996), Correia and Teles (1996; 1999), and Mulligan and Sala-i-Martin (1997)). The revenue implications of inflation and optimal inflation are major themes of chapter 4.

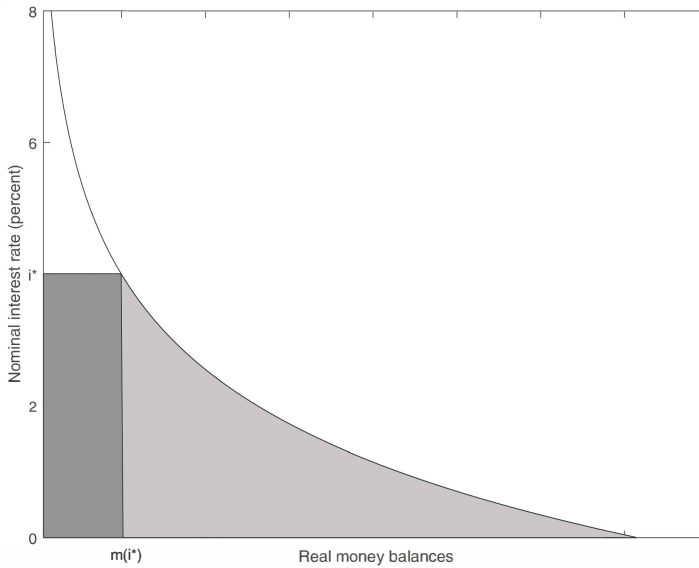
Now let's return to the question, what is the welfare cost of inflation? Beginning with Bailey (1956), this welfare cost has been calculated from the area under the money demand curve (showing money demand as a function of the nominal rate of interest) because this provides a measure of the consumer surplus lost as a result of having a positive nominal rate of interest. Figure 2.3 is based on a money demand function given by  $\ln(m) = B - \xi i$ . At a nominal interest rate of  $i^*$ , agents hold real money balances  $m(i^*)$ , and the shaded area measures the loss in consumer surplus relative to zero nominal interest rate. The darker shaded area represents the inflation tax revenue the government gains when the nominal interest rate is positive, so only the light shaded area represents a deadweight loss.<sup>32</sup> Consumer surplus is maximized when  $i = 0$ .

30. Since  $(1+i) = (1+r)(1+\pi)$ ,  $i = 0$  implies  $\pi = -r/(1+r) \approx -r$ .

31. Note that the earlier assumption that the marginal utility of money goes to zero at some finite level of real balances ensures that  $u_m = 0$  has a solution with  $m < \infty$ . The focus here is on the steady state, but a more appropriate perspective for addressing the optimal inflation question would not restrict attention solely to the steady state. The more general case is considered in chapter 4.

32. See chapter 4.





**Figure 2.3**  
Welfare costs of inflation as measured by area under money demand curve.

Nominal interest rates reflect *expected* inflation, so calculating the area under the money demand curve provides a measure of the costs of *anticipated* inflation and is therefore appropriate for evaluating the costs of alternative constant rates of inflation. However, not all the shaded area in the figure represents a deadweight loss to society. The rectangle equal to  $i^*$  times  $m(i^*)$  equals the seigniorage revenue the inflation tax generates for the government (see chapter 4). In addition to the loss of consumer surplus when agents economize on their holdings of money when the nominal interest rate is positive, there are costs of inflation associated with tax distortions and with variability in the rate of inflation; these are discussed in the survey on the costs of inflation by Driffill, Mizon, and Ulph (1990). In the presence of multiple economic distortions, it may not be optimal to completely eliminate the distortion generated by inflation; doing so may worsen the other distortions. The interactions of inflation with other distortions is discussed in connection with search models of money demand (chapter 3), the inflation tax when integrated into a model of optimal taxation (chapter 4), and the role of inflation in generating relative price distortions when prices are sticky (chapter 8).

Lucas (2000) provided estimates of the welfare costs of inflation, starting with the following specification of the instantaneous utility function:

$$u(c, m) = \frac{1}{1 - \sigma} \left\{ \left[ c\varphi \left( \frac{m}{c} \right) \right]^{1 - \sigma} - 1 \right\}. \tag{2.35}$$

With this utility function, (2.12) becomes

$$\frac{u_m}{u_c} = \frac{\varphi'(x)}{\varphi(x) - x\varphi'(x)} = \frac{i}{1+i} = \Upsilon, \quad (2.36)$$

where  $x \equiv m/c$ .<sup>33</sup> Normalize so that steady-state consumption equals 1; then  $u(1, m)$  is maximized when  $\Upsilon = 0$ , implying that the optimal  $x$  is defined by  $\varphi'(m^*) = 0$ . Lucas proposed to measure the costs of inflation by the percentage increase in steady-state consumption necessary to make the household indifferent between a nominal interest rate of  $i$  and a nominal rate of 0. If this cost is denoted  $w(\Upsilon)$ , it is defined by

$$u(1 + w(\Upsilon), m(\Upsilon)) \equiv u(1, m^*), \quad (2.37)$$

where  $m(\Upsilon)$  denotes the solution of (2.36) for real money balances evaluated at steady-state consumption  $c = 1$ .

Suppose, following Lucas, that  $\varphi(m) = (1 + Bm^{-1})^{-1}$ , where  $B$  is a positive constant. Solving (2.36), one obtains  $m(i) = B^{-5}\Upsilon^{-5}$ .<sup>34</sup> Note that  $\varphi' = 0$  requires that  $m^* = \infty$ . But  $\varphi(\infty) = 1$ , and  $u(1, \infty) = 0$ , so  $w(\Upsilon)$  is the solution to  $u(1 + w(\Upsilon), B^{-5}\Upsilon^{-5}) = u(1, \infty) = 0$ . Using the definition of the utility function, one obtains  $1 + w(\Upsilon) = 1 + \sqrt{B\Upsilon}$ , or

$$w(\Upsilon) = \sqrt{B\Upsilon}. \quad (2.38)$$

Based on U.S. annual data from 1900 to 1985, Lucas reported an estimate of 0.0018 for  $B$ . Hence, the welfare loss arising from a nominal interest rate of 10 percent would be  $\sqrt{(0.0018)(0.1/1.1)} = 0.013$ , or just over 1 percent of aggregate consumption.

Since U.S. government bond yields were about 10 percent in 1979 and 1980, one can use 1980 aggregate personal consumption expenditures of \$2,447.1 billion to get a rough estimate of the dollar welfare loss (although consumption expenditures include purchases of durables). In this example, 1.3 percent of \$2,447.1 billion is about \$32 billion. Because this is the annual cost in terms of steady-state consumption, one needs the present discounted value of \$32 billion. Using a real rate of return of 2 percent, one obtains to  $\$32(1.02)/0.02 = \$1.632$  billion; at 4 percent, the cost would be \$832 billion.

An annual welfare cost of \$32 billion seems a small number, especially when compared to the estimated costs of reducing inflation. For example, Ball (1993) reported a “sacrifice ratio” of 2.4 percent of output per 1 percentage point inflation reduction for the United States. Inflation was reduced from about 10 percent to about 3 percent in the early 1980s, so Ball’s estimate would put the cost of this disinflation at approximately 17 percent of GDP (2.4 percent times an inflation reduction of 7 percentage points). Based on a 1980 GDP of

33. In Lucas’s framework, the relevant expression is  $u_m/u_c = i$ ; problem 1 at the end of this chapter provides an example of the timing assumptions Lucas employed.

34. Lucas actually started with the assumption that money demand is equal to  $m = Ai^{-5}$  for  $A$  equal to a constant. He then derived  $\varphi(m)$  as the utility function necessary to generate such a demand function, where  $B = A^2$ .

\$3,776.3 billion (1987 prices), this would be \$642 billion. This looks large when compared to the \$32 billion annual welfare cost, but the trade-off starts looking more worthwhile if the costs of reducing inflation are compared to the present discounted value of the annual welfare cost; see also Feldstein (1979).

Gillman (1995) provided a useful survey of different estimates of the welfare cost of inflation. The estimates differ widely. One important reason for these differences arises from the choice of the base inflation rate. Some estimates compare the area under the money demand curve between an inflation rate of zero and, say, 10 percent. This is incorrect in that a zero rate of inflation still results in a positive nominal rate (equal to the real rate of return) and therefore a positive opportunity cost associated with holding money. Gillman concluded, based on surveying the empirical estimates, that a reasonable value of the welfare cost of inflation for the United States is in the range of 0.85 percent to 3 percent of real GNP per percentage rise in the nominal interest rate above zero, a loss in 2008 dollars of \$120 billion to \$426 billion per year.<sup>35</sup>

It should be clear from figure 2.3 that the size of the area under the demand curve depends importantly on both the shape and the position of the demand curve. For example, if money demand exhibits a constant elasticity with respect to the nominal interest rates, then at low levels of interest rates, further declines in the interest rate generate larger and larger increases in the absolute level of money demand, as illustrated in the figure. The area under the demand curve, and thus the welfare costs of inflation, will correspondingly be large.

Lucas (2000) calculated the welfare costs of inflation for two alternative specifications of money demand. The first takes the form

$$\ln(m) = \ln(A) - \eta \ln(i); \quad (2.39)$$

the second takes the form

$$\ln(m) = \ln(B) - \xi i. \quad (2.40)$$

Based on annual U.S. data from the period 1900–1994, Lucas obtained estimates of 0.5 for  $\eta$  and 7 for  $\xi$ . Ireland (2009) illustrated how these two functional forms have very different curvatures at low nominal interest rates. Real money demand becomes very large as  $i$  approaches zero under the log-log specification but approaches the finite limit  $\ln(B)$  with the semilog version. Equation (2.40) implies that a fall of interest rates from 3 percent to 2 percent produces the same increase in money demand as a fall from 10 percent to 9 percent, unlike the functional form in figure 2.3. If the welfare costs of positive nominal interest rates are measured from the area under the money demand function, these costs appear much larger when using (2.39) rather than (2.40). For example, at a real interest rate of 3 percent, an average inflation rate of 2 percent carries a welfare cost of just

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35. These estimates apply to the United States, which has experienced relatively low rates of inflation. They may not be relevant for high-inflation countries.

over 1 percent of income if (2.39) is the correct specification of money demand, but only 0.25 percent if (2.40) is correct.

Ireland (2009) argued that the support for the log-log specification comes primarily from two historical periods. The first is the late 1940s, when interest rates were very low and money demand very high (relative to income). The second is the period of the disinflation beginning in 1979 through the early 1980s, when interest rates were very high and money demand was unexpectedly low (often referred to as the period of missing money; see Goldfeld 1976). Ireland found, using a measure of the money stock that accounts for some of the changes due to financial market deregulation, that the data since 1980 provide much more support for the semilog specification with a small value of  $\xi$ . Rather than the value of 7 estimated by Lucas, Ireland found values below 2. His estimates imply the welfare cost of 2 percent inflation is less than 0.04 percent of income.

The Sidrauski model provides a convenient framework for calculating the steady-state welfare costs of inflation, both because the lower level of real money holdings that result at higher rates of inflation has a direct effect on welfare when money enters the utility function and because the superneutrality property of the model means that the other argument in the utility function, real consumption, is invariant across different rates of inflation. This latter property simplifies the calculation because it is not necessary to account for both variations in money holdings and variations in consumption when making the welfare cost calculation. However, the area under the demand curve is a partial equilibrium measure of the welfare costs of inflation if superneutrality does not hold, because steady-state consumption is no longer independent of the inflation rate. Gomme (1993) and Dotsey and Ireland (1996) examined the effects of inflation in general equilibrium frameworks that allow for the supply of labor and the average rate of economic growth to be affected (in models that do not display superneutrality; see section 2.4.2). Gomme found that even though inflation reduces the supply of labor and economic growth, the welfare costs are small because of the increased consumption of leisure that households enjoy.<sup>36</sup> Dotsey and Ireland found much larger welfare costs of inflation in a model that generates an interest elasticity of money demand that matches estimates for the United States. See also De Gregorio (1993) and Imrohorolu and Prescott (1991).

## 2.4 Extensions

### 2.4.1 Interest on Money

If the welfare costs of inflation are related to the positive private opportunity costs of holding money, paying explicit interest on money would be an alternative to deflation as

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36. The effect of money (and inflation) on labor supply is discussed in section 2.4.2.

a means of eliminating these costs.<sup>37</sup> There are obvious technical difficulties in paying interest on cash, but ignoring these, assume that the government pays a nominal interest rate of  $i^m$  on money balances. Assume further that these interest payments are financed by lump-sum taxes. The household's budget constraint, (2.4), now becomes (setting  $n = 0$ )

$$f(k_{t-1}) + \tau_t + (1 - \delta)k_{t-1} + (1 + r_{t-1})b_{t-1} + \frac{1 + i_{t-1}^m}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t + b_t, \quad (2.41)$$

where  $\tau_t$  represents transfers net of taxes. The first-order condition (2.8) becomes

$$-u_c(c_t, m_t) + u_m(c_t, m_t) + \frac{\beta(1 + i_t^m)V_\omega(\omega_{t+1})}{(1 + \pi_{t+1})} = 0, \quad (2.42)$$

while (2.12) is now

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t - i_t^m}{1 + i_t}.$$

The opportunity cost of money is related to the interest rate gap  $i - i^m$ , which represents the difference between the nominal return on bonds and the nominal return on money. Thus, the optimal quantity of money can be achieved as long as  $i - i^m = 0$ , regardless of the rate of inflation. The optimal quantity of money is obtained with a positive nominal interest rate as long as  $i^{ss} = i^m = r^{ss} > 0$ .

The assumption that interest payments are financed by revenue from lump-sum taxes is critical for this result. Problem 7 at the end of this chapter considers what happens if the government simply finances the interest payments on money by printing more money.

## 2.4.2 Nonsuperneutrality

Calculations of the steady-state welfare costs of inflation in the Sidrauski model are greatly simplified by the fact that the model exhibits superneutrality. But how robust is the result that money is superneutral? The empirical evidence of Barro (1995) suggests that inflation has a negative effect on growth, a finding inconsistent with superneutrality.<sup>38</sup> Berentsen, Menzio, and Wright (2011) also argued that there is evidence of a long-run positive relationship between inflation and unemployment. One channel through which inflation can have real effects in the steady state is introduced if households face a labor supply choice. That is, suppose utility depends on consumption, real money holdings, and leisure:

$$u = u(c, m, l). \quad (2.43)$$

37. Since 2008 the Federal Reserve has been paying interest on reserves. The implications this has for the implementation of monetary policy are discussed in chapter 12.

38. Of course, the empirical relationship may not be causal; both growth and inflation may be reacting to common factors. As noted in chapter 1, McCandless Jr. and Weber (1995) found no relationship between inflation and average real growth.

The economy's production function becomes

$$y = f(k, n), \quad (2.44)$$

where  $n$  is employment. If the total supply of time is normalized to equal 1, then  $n = 1 - l$ . The additional first-order condition implied by the optimal choice of leisure is

$$\frac{u_l(c, m, l)}{u_c(c, m, l)} = f_n(k, 1 - l). \quad (2.45)$$

Now, both steady-state labor supply and consumption may be affected by variations in the rate of inflation. Specifically, an increase in the rate of inflation reduces holdings of real money balances. If this affects the marginal rate of substitution between leisure and consumption  $u_l/u_c$ , then (2.45) implies the supply of labor will be affected, leading to a change in the steady-state per capita stock of capital, output, and consumption. But why would changes in money holdings affect  $u_l/u_c$ ? Because money has simply been assumed to yield utility, with real no explanation for the reason, it is difficult to answer this question. Chapter 3 examines a model in which money helps to reduce the time spent in carrying out the transactions necessary to purchase consumption goods; in this case, a rise in inflation would lead to more time spent engaged in transactions, and this would raise the marginal utility of leisure time. But one might expect that this channel is unlikely to be important empirically, so superneutrality may remain a reasonable first approximation to the effects of inflation on steady-state real magnitudes.

Equation (2.45) suggests that if  $u_l/u_c$  were independent of  $m$ , then superneutrality would hold. This is the case because the steady-state values of  $k$ ,  $c$ , and  $l$  could then be found from

$$\begin{aligned} \frac{u_l}{u_c} &= f_n(k^{ss}, 1 - l^{ss}), \\ f_k(k^{ss}, 1 - l^{ss}) &= \frac{1}{\beta} - 1 + \delta, \\ c^{ss} &= f(k^{ss}, 1 - l^{ss}) - \delta k^{ss}. \end{aligned}$$

Superneutrality reemerges when the utility function takes the general form  $u(c, m, l) = v(c, l)g(m)$ ; in this case  $u_l/u_c = v_l/v_c$  is independent of  $m$ . Variations in inflation affect the agent's holdings of money, but the consumption-leisure choice is not directly affected. As McCallum (1990a) noted, Cobb-Douglas specifications of utility, which are quite common in the literature, satisfy this condition. So with Cobb-Douglas utility, the ratio of the marginal utility of leisure to the marginal utility of consumption is independent of the level of real money balances, and superneutrality holds. Superneutrality also holds if utility is separable in money holdings.

Another channel through which inflation can affect the steady-state stock of capital is if money enters directly into the production function (Fischer 1974). Since steady states with different rates of inflation have different equilibrium levels of real money balances, they also lead to different marginal products of capital for given levels of the capital-labor ratio.

With the steady-state marginal product of capital determined by  $1/\beta - 1 + \delta$  (see 2.19), the two steady states can have the same marginal product of capital only if their capital-labor ratios differ. If  $\partial \text{MPK}/\partial m > 0$  (so that money and capital are complements), higher inflation, by leading to lower real money balances, also leads to a lower steady-state capital stock.<sup>39</sup> This is the opposite of the *Tobin effect*; Tobin (1965) argued that higher inflation would induce a portfolio substitution toward capital that would increase the steady-state capital-labor ratio (see also Stein 1969; Fischer 1972). For higher inflation to be associated with a higher steady-state capital-labor ratio requires that  $\partial \text{MPK}/\partial m < 0$  (that is, higher money balances reduce the marginal product of capital; money and capital are substitutes in production).

This discussion has, by ignoring taxes, excluded what is probably the most important reason that superneutrality may fail in actual economies. Taxes generally are not fully indexed to inflation and are levied on nominal capital gains instead of real capital gains. Effective tax rates depend on the inflation rate, generating real effects on capital accumulation and consumption as inflation varies (e.g., see Feldstein 1978; 1998; Summers 1981).

## 2.5 Dynamics in an MIU Model

The analysis of the MIU approach has, up to this point, focused on steady-state properties. It is also important to understand the model's implications for the dynamic behavior of the economy as it adjusts to exogenous disturbances. Even the basic Sidrauski model can exhibit nonsuperneutralities during the transition to the steady state. For example, Fischer (1979) showed that for the constant relative risk aversion class of utility functions, the rate of capital accumulation is positively related to the rate of money growth except for the case of log separable utility. Section 2.2.1 discussed how the steady state can be affected when money growth varies over time (Reis 2007).<sup>40</sup>

One way to study the model's dynamics is to employ numerical methods to carry out simulations using the model. The results can then be compared to actual data generated by real economies. This approach was popularized by the real business cycle literature (see Cooley 1995). Since the parameters of theoretical models can be varied in ways the characteristics of real economies cannot, simulation methods allow one to answer a variety of "what if" questions. For example, how does the dynamic response to a temporary

39. That is, in the steady state,  $f_k(k^{SS}, m^{SS}) = \beta^{-1} - 1 + \delta$ , where  $f(k, m)$  is the production function and  $f_i$  denotes the partial derivative with respect to argument  $i$ . It follows that  $dk^{SS}/dm^{SS} = -f_{km}/f_{kk}$ , so with  $f_{kk} \leq 0$ ,  $\text{sign}(dk^{SS}/dm^{SS}) = \text{sign}(f_{km})$ .

40. Superneutrality holds during the transition if  $u(c, m) = \ln(c) + b \ln(m)$ . The general class of utility functions Fischer considered is of the form  $u(c, m) = \frac{1}{1-\Phi} (c^a m^b)^{1-\Phi}$ ; log utility obtains when  $\Phi = 1$ . See also Asako (1983), who showed that faster money growth can lead to slower capital accumulation under certain conditions if  $c$  and  $m$  are perfect complements. These effects of inflation on capital accumulation apply during the transition from one steady-state equilibrium to another; they differ therefore from the Tobin (1965) effect of inflation on the steady-state capital-labor ratio.

change in the growth rate of the money supply depend on the degree of intertemporal substitution characterizing individual preferences or the persistence of money growth rate disturbances? Numerical solutions allow one to investigate whether simulation results are sensitive to parameter values. In addition, easily adaptable programs for solving linear dynamic stochastic general equilibrium rational-expectations models are now freely available.<sup>41</sup>

This section develops a linearized version of an MIU model that also incorporates a labor-leisure choice. This introduces a labor supply decision into the analysis, an important and necessary extension for studying business cycle fluctuations because employment variation is an important characteristic of cycles. It is also important to allow for uncertainty by adding exogenous shocks that disturb the system from its steady-state equilibrium. The two types of shocks considered are productivity shocks, the driving force in real business cycle models, and shocks to the growth rate of the nominal stock of money.

### 2.5.1 The Decision Problem

The household's decision problem is conveniently expressed using the value function. In studying a similar problem without a labor-leisure choice (see section 2.2), the state could be summarized by the resource variable  $\omega_t$ , which included current income. When the household chooses how much labor to supply, current income is no longer predetermined from the perspective of the household. Consequently, income (output)  $y_t$  cannot be part of the state vector for period  $t$ . Instead, let

$$a_t = \left( \frac{1 + i_{t-1}}{1 + \pi_t} \right) b_{t-1} + \left( \frac{1}{1 + \pi_t} \right) m_{t-1} + \tau_t$$

be the household's real financial wealth plus net transfer at the start of period  $t$ . If  $n_t$  denotes the fraction of time the household devotes to market employment (so that  $n_t = 1 - l_t$ , where  $l_t$  is the fraction of time spent in leisure activities), output per household  $y_t$  is given by

$$y_t = f(k_{t-1}, n_t, z_t),$$

where  $z_t$  is a stochastic productivity disturbance.

Define the value function  $V(a_t, k_{t-1})$  as the maximum present value of utility the household can achieve if the current state is  $(a_t, k_{t-1})$ . The value function for the household's decision problem satisfies

$$V(a_t, k_{t-1}) = \max \{ u(c_t, m_t, 1 - n_t) + \beta E_t V(a_{t+1}, k_t) \}, \quad (2.46)$$

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41. For example, MATLAB programs provided by Harald Uhlig can be obtained from <https://www.wiwi.hu-berlin.de/de/professuren/vwl/wipo/research/MATLAB.Toolkit>, and Paul Söderlind's Gauss and MATLAB programs are available at <https://sites.google.com/site/paulsoderlindecon/home/software>. Dynare for MATLAB is available at <http://www.dynare.org/>.



where the maximization is over  $(c_t, m_t, b_t, k_t, n_t)$  and is subject to

$$f(k_{t-1}, n_t, z_t) + (1 - \delta)k_{t-1} + a_t \geq c_t + k_t + b_t + m_t, \quad (2.47)$$

$$a_{t+1} = \tau_{t+1} + \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) b_t + \frac{m_t}{1 + \pi_{t+1}}. \quad (2.48)$$

Note that the presence of uncertainty arising from the stochastic productivity and money growth rate shocks means that the expected value of  $V(a_{t+1}, k_t)$  appears in the value function (2.46). The treatment of  $a_t$  as a state variable assumes that the money growth rate is known at the time the household decides on  $c_t$ ,  $k_t$ ,  $b_t$ , and  $m_t$  because it determines the current value of the transfer  $\tau_t$ . Assume also that the productivity disturbance  $z_t$  is known at the start of period  $t$ .

Equation (2.47) always holds with equality (as long as  $u_c > 0$ ); it can be used to eliminate  $k_t$ , and (2.48) can be used to substitute for  $a_{t+1}$ , allowing the value function to be rewritten<sup>42</sup> as

$$\begin{aligned} V(a_t, k_{t-1}) = \max_{c_t, n_t, b_t, m_t} & \left\{ u(c_t, m_t, 1 - n_t) \right. \\ & + \beta E_t V \left( \tau_{t+1} + \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) b_t + \frac{m_t}{1 + \pi_{t+1}}, f(k_{t-1}, n_t, z_t) \right. \\ & \left. \left. + (1 - \delta)k_{t-1} + a_t - c_t - b_t - m_t \right) \right\}, \end{aligned}$$

where this is now an unconstrained maximization problem. The first-order necessary conditions with respect to  $c_t$ ,  $n_t$ ,  $b_t$ , and  $m_t$  are

$$u_c(c_t, m_t, 1 - n_t) - \beta E_t V_k(a_{t+1}, k_t) = 0, \quad (2.49)$$

$$-u_l(c_t, m_t, 1 - n_t) + \beta E_t V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t, z_t) = 0, \quad (2.50)$$

$$\beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) V_a(a_{t+1}, k_t) - \beta E_t V_k(a_{t+1}, k_t) = 0, \quad (2.51)$$

$$u_m(c_t, m_t, 1 - n_t) + \beta E_t \left[ \frac{V_a(a_{t+1}, k_t)}{1 + \pi_{t+1}} \right] - \beta E_t V_k(a_{t+1}, k_t) = 0, \quad (2.52)$$

and the envelope theorem yields

$$V_a(a_t, k_{t-1}) = \beta E_t V_k(a_{t+1}, k_t), \quad (2.53)$$

$$V_k(a_t, k_{t-1}) = \beta E_t V_k(a_{t+1}, k_t) [f_k(k_{t-1}, n_t, z_t) + 1 - \delta]. \quad (2.54)$$

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42. Rather than introduce firms explicitly, households are assumed to directly operate the production technology.

Updating (2.54) one period and using (2.53), one obtains

$$V_k(a_{t+1}, k_t) = E_t [f_k(k_t, n_{t+1}, z_{t+1}) + 1 - \delta] V_a(a_{t+1}, k_t).$$

Now using this to substitute for  $V_k(a_{t+1}, k_t)$  in (2.49) yields

$$u_c(c_t, m_t, 1 - n_t) - \beta E_t [f_k(k_t, n_{t+1}, z_{t+1}) + 1 - \delta] V_a(a_{t+1}, k_t) = 0. \quad (2.55)$$

When it is recognized that  $u_c(c_t, m_t, 1 - n_t) = \beta E_t V_k(a_{t+1}, k_t)$ , (2.52), (2.55), and (2.53) take the same form as (2.8), (2.6), and (2.10), the first-order conditions for the basic Sidrauski model that did not include a labor-leisure choice. The only new condition is (2.50), which can be written, using (2.49), as

$$\frac{u_l(c_t, m_t, 1 - n_t)}{u_c(c_t, m_t, 1 - n_t)} = f_n(k_{t-1}, n_t, z_t).$$

This states that at an optimum, the marginal rate of substitution between leisure and consumption must equal the marginal product of labor.

Note that (2.49), (2.51), and (2.53) imply that

$$u_c(c_t, m_t, 1 - n_t) = \beta(1 + i_t) E_t \left[ \frac{u_c(c_{t+1}, m_{t+1}, 1 - n_{t+1})}{1 + \pi_{t+1}} \right].$$

Using this relationship and (2.49), one can now write (2.50), (2.52), and (2.55) as

$$u_l(c_t, m_t, 1 - n_t) = u_c(c_t, m_t, 1 - n_t) f_n(k_{t-1}, n_t, z_t), \quad (2.56)$$

$$u_m(c_t, m_t, 1 - n_t) = u_c(c_t, m_t, 1 - n_t) \left( \frac{i_t}{1 + i_t} \right), \quad (2.57)$$

$$u_c(c_t, m_t, 1 - n_t) = \beta E_t (1 + r_t) u_c(c_{t+1}, m_{t+1}, 1 - n_{t+1}), \quad (2.58)$$

where in (2.58)

$$r_t = f_k(k_t, n_{t+1}, z_{t+1}) - \delta \quad (2.59)$$

is the marginal product of capital net of depreciation. In addition, the economy's aggregate resource constraint, expressed in per capita terms, requires that

$$k_t = (1 - \delta)k_{t-1} + y_t - c_t, \quad (2.60)$$

while the production function is

$$y_t = f(k_{t-1}, n_t, z_t). \quad (2.61)$$

Finally, real money balances evolve according to

$$m_t = \left( \frac{1 + \theta_t}{1 + \pi_t} \right) m_{t-1}, \quad (2.62)$$

where  $\theta_t$  is the stochastic growth rate of the nominal stock of money.

Once processes for the exogenous disturbances  $z_t$  and  $\theta_t$  have been specified, equations (2.56)–(2.62) constitute a nonlinear system of equations to determine the equilibrium values of the model's seven endogenous variables:  $y_t, c_t, k_t, m_t, n_t, r_t, \pi_t$ .<sup>43</sup>

## 2.5.2 The Steady State

Consider a steady-state equilibrium of this model in which all real variables (including  $m$ ) are constant and shocks are set to zero. It follows immediately from (2.58) that  $1 + r^{ss} = \beta^{-1}$  and from (2.59) that

$$f_k(k^{ss}, n^{ss}, 0) = \beta^{-1} - 1 + \delta. \quad (2.63)$$

Thus, the marginal product of capital is a function only of  $\beta$  and  $\delta$ . If the production function exhibits constant returns to scale,  $f_k$  depends only on the capital-labor ratio  $k^{ss}/n^{ss}$ . In this case, (2.63) uniquely determines  $k/n$ . That is, the capital-labor ratio is independent of inflation or the real quantity of money.

With constant returns to scale,  $\phi(k/n) = f/n$  can be defined as the intensive production function. Then, from the economy's resource constraint,

$$c^{ss} = f(k^{ss}, n^{ss}, 0) - \delta k^{ss} = \left[ \phi\left(\frac{k^{ss}}{n^{ss}}\right) - \delta\left(\frac{k^{ss}}{n^{ss}}\right) \right] n^{ss} = \bar{\phi} n^{ss},$$

where  $\bar{\phi} \equiv \phi(k^{ss}/n^{ss}) - \delta(k^{ss}/n^{ss})$  does not depend on anything related to money. Now, (2.56) implies that

$$\frac{u_l(c^{ss}, m^{ss}, 1 - n^{ss})}{u_c(c^{ss}, m^{ss}, 1 - n^{ss})} = f_n(k^{ss}, n^{ss}, 0).$$

In the constant returns to scale case,  $f_n$  depends only on  $k^{ss}/n^{ss}$ , which is a function of  $\beta$  and  $\delta$ , so using the definition of  $\phi$ , one can rewrite this last equation as

$$\frac{u_l(\bar{\phi} n^{ss}, m^{ss}, 1 - n^{ss})}{u_c(\bar{\phi} n^{ss}, m^{ss}, 1 - n^{ss})} = \phi\left(\frac{k^{ss}}{n^{ss}}\right) - \left(\frac{k^{ss}}{n^{ss}}\right) \phi'\left(\frac{k^{ss}}{n^{ss}}\right). \quad (2.64)$$

This relationship provides the basic insight into how money can affect the real equilibrium. Suppose the utility function is separable in money so that neither the marginal utility of leisure nor the marginal utility of consumption depends on the household's holdings of real money balances. Then (2.64) becomes

$$\frac{u_l(\bar{\phi} n^{ss}, 1 - n^{ss})}{u_c(\bar{\phi} n^{ss}, 1 - n^{ss})} = \phi\left(\frac{k^{ss}}{n^{ss}}\right) - \left(\frac{k^{ss}}{n^{ss}}\right) \phi'\left(\frac{k^{ss}}{n^{ss}}\right),$$

which determines the steady-state supply of labor. Steady-state consumption is then given by  $\bar{\phi} n^{ss}$ . Thus, separable preferences imply superneutrality. Changes in the steady-state rate of inflation alter nominal interest rates and the demand for real money balances (see 2.57),

43. Since all households are identical,  $b_t = 0$  in equilibrium.

but different inflation rates have no effect on the steady-state values of the capital stock, labor supply, or consumption.

If utility is nonseparable, so that either  $u_l$  or  $u_c$  (or both) depend on  $m^{ss}$ , then money is not superneutral. Variations in average inflation that affect the opportunity cost of holding money affect  $m^{ss}$ . Different levels of  $m^{ss}$  change the value of  $n^{ss}$  that satisfies (2.64). Since  $1 + i^{ss} = (1 + r^{ss})(1 + \pi^{ss}) = \beta^{-1}(1 + \theta^{ss})$ , one can rewrite (2.57) as

$$\frac{u_m(\bar{\phi}n^{ss}, m^{ss}, 1 - n^{ss})}{u_c(\bar{\phi}n^{ss}, m^{ss}, 1 - n^{ss})} = \left( \frac{i^{ss}}{1 + i^{ss}} \right) = \frac{1 + \theta^{ss} - \beta}{1 + \theta^{ss}}.$$

This equation, together with (2.64) must be jointly solved for  $m^{ss}$  and  $n^{ss}$ . Even in this case, however, the ratios of output, consumption, and capital to labor are independent of the rate of money growth. The steady-state *levels* of the capital stock, output, and consumption depend on the money growth rate through the effects of inflation on labor supply, with inflation-induced changes in  $n^{ss}$  affecting  $y^{ss}$ ,  $c^{ss}$ , and  $k^{ss}$  equiproportionally.

The effect of faster money growth depends on how  $u_c$  and  $u_l$  are affected by  $m$ . For example, suppose money holdings do not affect the marginal utility of leisure ( $u_{lm} = 0$ ), but money and consumption are Edgeworth complements; higher inflation that reduces real money balances decreases the marginal utility of consumption ( $u_{cm} > 0$ ). In this case, faster money growth reduces  $m^{ss}$  and the marginal utility of consumption. Households substitute away from labor and toward leisure. Steady-state employment, output, and consumption fall. These effects go in the opposite direction if consumption and money are Edgeworth substitutes ( $u_{cm} < 0$ ).

### 2.5.3 The Linear Approximation

To further explore the effects of money outside the steady state, it is useful to approximate the model's equilibrium conditions around the steady state. The steps involved in obtaining the linear approximation around the steady state follow the approach of Campbell (1994) and Uhlig (1999). Details on the approach used to linearize (2.56)–(2.62) are discussed in the chapter appendix. With the exception of interest rates and inflation, variables are expressed as percentage deviations around the steady state. Percentage deviations of a variable  $q_t$  around its steady-state value are denoted by  $\hat{q}_t$ , where  $q_t \equiv q^{ss}(1 + \hat{q}_t)$ . For interest rates and inflation,  $\hat{r}_t$ ,  $\hat{i}_t$ , and  $\hat{\pi}_t$  denote  $r_t - r^{ss}$ ,  $i_t - i^{ss}$ , and  $\pi_t - \pi^{ss}$  respectively.<sup>44</sup> In what follows, uppercase letters denote economywide variables, lowercase letters denote random disturbances and variables expressed in per capita terms, and the superscript ss indicates the steady-state value of a variable. However,  $m$ ,  $m^{ss}$ , and  $\hat{m}$  refer to *real* money balances per capita, whereas  $M$  represents the aggregate *nominal* stock of money.

44. That is, if the interest rate is 0.0125 at a quarterly rate (i.e., 5 percent at an annual rate) and the steady-state value of the interest rate is 0.01, then  $\hat{r}_t = 0.0125 - 0.01 = 0.0025$ , i.e., 25 basis points, not  $(0.0125 - 0.01)/0.01 = 0.25$ , a 25 percent deviation.

As is standard, the production function is taken to be Cobb-Douglas with constant returns to scale, so

$$y_t = e^{z_t} k_{t-1}^\alpha n_t^{1-\alpha}, \quad (2.65)$$

with  $0 < \alpha < 1$ . Note the timing convention in (2.65): the capital carried over from period  $t - 1$ ,  $k_{t-1}$ , is available for use in producing output during period  $t$ . For the utility function, assume

$$u(c_t, m_t, 1 - n_t) = \frac{\left[ a c_t^{1-b} + (1-a) m_t^{1-b} \right]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta}. \quad (2.66)$$

King, Plosser, and Rebelo (1988) demonstrated that with the exception of the log case, utility must be multiplicatively separable in labor to be consistent with steady-state growth, in which the share of time devoted to work remains constant as real wages rise. Equation (2.66) does not have this property. However, abstracting from growth factors and assuming linear separability in leisure is common in the literature on business cycles. The problems at the end of this chapter present an example using a utility function consistent with growth.

The resulting linearized system consists of the exogenous processes for the productivity shock and the money growth rate plus the eight additional equilibrium conditions: the production function, the goods market clearing condition, the definition of the real return on capital, the Euler equation for optimal intertemporal consumption allocation, the first-order conditions for labor supply and money holdings, the Fisher equation linking nominal and real interest rates, and the money market equilibrium condition. These can be solved for the capital stock, money holdings, output, consumption, employment, the real rate of interest, the nominal interest rate, and the inflation rate.

To this system of eight endogenous variables, it is convenient to add investment,  $x_t$ , given by

$$x_t = k_t - (1 - \delta) k_{t-1},$$

and to define  $\lambda_t$  as the marginal utility of consumption. The linearized expression for  $\hat{\lambda}_t$  is

$$\hat{\lambda}_t = \Omega_1 \hat{c}_t + \Omega_2 \hat{m}_t, \quad (2.67)$$

where

$$\Omega_1 = [(b - \Phi) \gamma - b], \Omega_2 = (b - \Phi) (1 - \gamma),$$

$$\gamma = a(c^{ss})^{1-b} / \left[ a(c^{ss})^{1-b} + (1-a)(m^{ss})^{1-b} \right].$$

Then, in linearized form, the equilibrium conditions are (see the chapter appendix):

$$\hat{y}_t = \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t, \quad (2.68)$$

$$\left(\frac{y^{ss}}{k^{ss}}\right)\hat{y}_t = \left(\frac{c^{ss}}{k^{ss}}\right)\hat{c}_t + \delta\hat{x}_t, \quad (2.69)$$

$$\hat{k}_t = (1 - \delta)\hat{k}_{t-1} + \delta\hat{x}_t, \quad (2.70)$$

$$\hat{r}_t = \alpha \left(\frac{y^{ss}}{k^{ss}}\right) (E_t\hat{y}_{t+1} - \hat{k}_t), \quad (2.71)$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t, \quad (2.72)$$

$$-\hat{\lambda}_t + \eta \left(\frac{n^{ss}}{1 - n^{ss}}\right)\hat{n}_t = \hat{y}_t - \hat{n}_t, \quad (2.73)$$

$$\hat{i}_t = \hat{r}_t + E_t\hat{\pi}_{t+1}, \quad (2.74)$$

$$\hat{m}_t - \hat{c}_t = -\left(\frac{1}{b}\right) \left(\frac{1 - i^{ss}}{i^{ss}}\right)\hat{i}_t. \quad (2.75)$$

A process for the nominal stock of money needs to be specified. In previous sections,  $\theta$  denoted the growth rate of the nominal money supply. Assume that the average growth rate is  $\theta^{ss}$ , and let  $u_t \equiv \theta_t - \theta^{ss}$  be the deviation in period  $t$  of the growth rate from its unconditional average value. Then

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t, \quad (2.76)$$

and  $u_t$  will be treated as a stochastic process given by

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \varphi_t, \quad (2.77)$$

where  $\varphi_t$  is a white noise process and  $|\rho_u| < 1$ . This formulation allows the growth rate of the money stock to display persistence (if  $\rho_u > 0$ ), respond to the real productivity shock  $z$  (if  $\phi \neq 0$ ), and be subject to random disturbances through the realizations of  $\varphi_t$ .

Consistent with the real business cycle literature, a stochastic disturbance to total factor productivity that follows an AR(1) process is incorporated:

$$z_t = \rho_z z_{t-1} + e_t. \quad (2.78)$$

Assume  $e_t$  is a serially uncorrelated mean zero process and  $|\rho_z| < 1$ .

Equation (2.68) is the economy's production function in which output deviations from the steady state are a linear function of the percentage deviations of the capital stock and labor supply from the steady state plus the productivity shock. Equation (2.69) is the resource constraint derived from the condition that output equals consumption plus investment. Deviations of the marginal product of capital are tied to deviations of the real return by (2.71). Equations (2.72)–(2.75) are derived from the representative household's first-order conditions for consumption, leisure, and money holdings. Changes in the deviation from steady state of real money balances are related by (2.76) to the inflation rate and

the growth of the nominal money stock. Finally, the exogenous disturbances for nominal money growth and productivity are given by (2.78) and (2.77).

One conclusion follows immediately from inspecting this system. If  $\Phi = b$ , then  $\Omega_2 = 0$  and money no longer affects the marginal utility of consumption. Thus, money drops out of both (2.72) and (2.73) and (2.68)–(2.73) can be solved for  $\hat{y}$ ,  $\hat{c}$ ,  $\hat{r}$ ,  $\hat{k}$ , and  $\hat{n}$  independently of the money supply process and inflation. This implies that superneutrality characterizes dynamics around the steady state as well as the steady state itself.<sup>45</sup>

Separability allows the real equilibrium to be solved independently of money and inflation, but the assumption is more commonly used in monetary economics to allow the study of inflation and money growth to be conducted independently of the real equilibrium. When  $\Phi = b$ , (2.75) and (2.76) constitute a two-equation system in inflation and real money balances, with  $u$  representing an exogenous random disturbance and  $\hat{c}$  and  $\hat{r}$  determined by (2.68)–(2.73) and exogenous to the determination of inflation and real money balances. Using (2.74), equation (2.75) can then be written as

$$E_t \pi_{t+1} \equiv E_t \hat{p}_{t+1} - \hat{p}_t = - \left( \frac{b i^{ss}}{1 - i^{ss}} \right) \hat{m}_t + \chi_t = - \left( \frac{b i^{ss}}{1 - i^{ss}} \right) (\hat{M}_t - \hat{p}_t) + \chi_t,$$

where  $\hat{M}_t$  represents the nominal money stock (so  $\hat{m}_t = \hat{M}_t - \hat{p}_t$ ). This is an expectational difference equation that can be solved for the equilibrium path of  $\hat{p}$  for a given process for the nominal money supply and the exogenous variable  $\chi_t \equiv [(b i^{ss} / (1 - i^{ss})) \hat{c}_t - \hat{r}_t]$ . Models of this type have been widely employed in monetary economics (see chapter 4).

A second conclusion revealed by the dynamic system is that when money does matter (i.e., when  $b \neq \Phi$ ), it is only *anticipated* changes in money growth that matter. To see this, suppose  $\rho_u = \phi = 0$ , so that  $u_t = \varphi_t$  is a purely unanticipated change in the growth rate of money that has no effect on anticipated future values of money growth. Now consider a positive realization of  $\varphi_t$  (nominal money growth is faster than average). This increases the nominal stock of money. If  $\rho_u = \phi = 0$ , future money growth rates are unaffected by the value of  $\varphi_t$ . This means that future expected inflation,  $E_t \pi_{t+1}$ , is also unaffected. Therefore, a permanent jump in the price level that is proportional to the unexpected rise in the nominal money stock leaving  $m_t$  unaffected also leaves (2.68)–(2.75) unaffected. From (2.76), for  $\varphi_t$  to have no effect on  $\hat{m}_t$  requires that  $\pi_t = \varphi_t$ . So an unanticipated money growth rate disturbance has no real effects and simply leads to a one-period change in the inflation rate (and a permanent change in the price level). Unanticipated money doesn't matter.<sup>46</sup>

Now consider what happens when  $\phi = 0$  but  $\rho_u$  differs from zero. In the United States, money growth displays positive serial correlation, so assume that  $\rho_u > 0$ . A positive shock to money growth ( $\varphi_t > 0$ ) now has implications for the future growth rate of money. With

45. This result, for the preferences given by (2.66), generalizes the findings of Brock (1974) and Fischer (1979).

46. During the 1970s macroeconomics was heavily influenced by a model developed by Lucas (1972) in which only unanticipated changes in the money supply had real effects. See chapter 5.

$\rho_u > 0$ , future money growth will be above average, so expectations of future inflation will rise. From (2.75), however, for real consumption and the expected real interest rate to remain unchanged in response to a rise in expected future inflation, current real money balances must fall. This means that  $\hat{p}_t$  would need to rise more than in proportion to the rise in the nominal money stock. But when  $\Omega_2 \neq 0$ , the decline in  $\hat{m}_t$  affects the first-order conditions given by (2.73) and (2.75), so the real equilibrium does not remain unchanged. Monetary disturbances have real effects by affecting the expected rate of inflation.

A positive monetary shock *increases* the nominal rate of interest. Monetary policy actions that increase the growth rate of money are usually thought to reduce nominal interest rates, at least initially. The negative effect of money on nominal interest rates is usually called the *liquidity effect*, and it arises if an increase in the nominal quantity of money also increases the real quantity of money because nominal interest rates would need to fall to ensure that real money demand also increased. However, in the MIU model, prices have been assumed to be perfectly flexible; the main effect of money growth rate shocks when  $\rho_u > 0$  is to increase expected inflation and raise the nominal interest rate. Because prices are perfectly flexible, the monetary shock generates a jump in the price level immediately. The real quantity of money actually falls, consistent with the decline in real money demand that occurs as a result of the increase in the nominal interest rate.

To actually determine how the equilibrium responds to money growth rate shocks and how the response depends quantitatively on  $\rho_u$  and  $\phi$ , one must calibrate the parameters of the model and numerically solve for the rational-expectations equilibrium.

#### 2.5.4 Calibration

Thirteen parameters appear in the equations that characterize behavior around the steady state:  $\alpha, \delta, \rho_z, \sigma_e^2, \beta, a, b, \eta, \Phi, \theta^{ss}, \rho_u, \phi, \sigma_\varphi^2$ . Some of these parameters are common to standard real business cycle models; for example, Cooley and Prescott (1995) report values of, in our notation,  $\alpha$  (the share of capital income in total income),  $\delta$  (the rate of depreciation of physical capital),  $\rho_z$  (the autoregressive coefficient in the productivity process),  $\sigma_e$  (the standard deviation of productivity innovations), and  $\beta$  (the subjective rate of time discount in the utility function). These values are based on a time period equal to three months (one quarter). Cooley and Prescott's values are adopted except for the depreciation rate  $\delta$ ; Cooley and Prescott calibrate  $\delta = 0.012$  based on a model that explicitly incorporates growth. Here the somewhat higher value of 0.019 given in Cooley and Hansen (1995) is used. The value of  $\sigma_e$  is set to match the standard deviation of quarterly HP-filtered log U.S. real GDP for the 1985:1–2014:4 period of 1.10 percent. Over this same period, U.S. money growth as measured by  $M1$  averaged 5.54 percent. An annual rate of 5.54 percent would imply a quarterly value of 1.38 for  $1 + \theta^{ss}$ , so we set  $1 + \theta^{ss} = 1.0138$  to match  $M1$ . Estimating an  $AR(1)$  process for  $M1$  growth rate (expressed at quarterly rates to be consistent with the timing of the model) yields  $\rho_u = 0.69$  for 1985:1–2014:4 and an estimated standard error of the residual of 1.17 percent. Various alternative values for



the autoregression coefficient for money growth,  $\rho_u$ , and the coefficient on the productivity shock,  $\phi$ , are considered to see how the implications of the model are affected by the manner in which money growth evolves.

The remaining parameters are those in the utility function. The value of  $\Psi$  can be chosen so that the steady-state value of  $n^{ss}$  is equal to one-third, as in Cooley and Prescott. Ireland (2009) estimated a money demand equation for  $M1$  that is of the same form as (2.75), except that he uses GDP rather than consumption. For the 1980–2008 period, he finds the coefficient on the level of the interest rate to be around 1.85. The coefficient on  $\hat{i}_t$  in (2.75) is  $b^{-1}(1 - i^{ss})/i^{ss}$ , implying  $b = (1 - i^{ss}) / (1.85 \times i^{ss})$  to match Ireland’s estimate. The average for the 3-month Treasury bill rate over this period was 5.64. Taking this value for  $i^{ss}$ ,  $b = (1 - 0.0056) / (1.85 \times 0.056) \approx 9$ .<sup>47</sup>

The chapter appendix shows that the steady-state value of real money balances relative to consumption is equal to  $[a\Upsilon^{ss}/(1 - a)]^{-1/b}$ , where  $\Upsilon^{ss} = (1 + \theta^{ss} - \beta)/(1 + \theta^{ss}) = i^{ss}/(1 + i^{ss})$ . For real  $M1$ , this ratio in the data is just under 0.2 when consumption is expressed at annual rates, or about 0.78 at quarterly rates. If  $b = 9$ , this would imply  $a = 0.997$ .

The inverse of the intertemporal elasticity of substitution,  $\Phi$ , is set equal to 2 in the benchmark simulations. With  $b = 9$ , this means  $b - \Phi > 0$  and faster expected money growth decreases employment and output. Finally,  $\eta$  is set equal to 1. With  $n^{ss} = 1/3$ , a value of  $\eta = 1$  yields a labor supply elasticity of  $[\eta n^{ss}/(1 - n^{ss})]^{-1} = 2$ .

These parameter values are summarized in table 2.1. Using the information in this table, the steady-state values for the variables can be evaluated. These are given in table 2.2. The effect of money growth on the steady-state level of employment can be derived using (2.80). The elasticity of the steady-state labor supply with respect to the growth rate of

**Table 2.1**  
Baseline Parameter Values

$\alpha$	$\delta$	$\beta$	$\Phi$	$\eta$	$a$	$b$	$1 + \theta^{ss}$	$\rho_z$	$\sigma_e$	$\rho_u$	$\sigma_\varphi$
0.36	0.019	0.989	2	1	0.997	9	1.014	0.95	0.34	0.69	0.85

**Table 2.2**  
Steady-State Values at Baseline Parameter Values

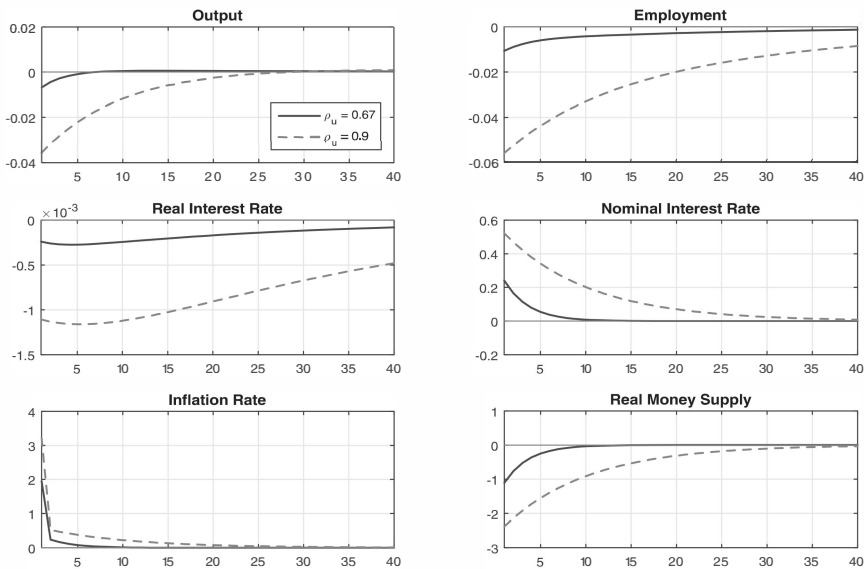
$1 + r^{ss}$	$\frac{y^{ss}}{k^{ss}}$	$\frac{c^{ss}}{k^{ss}}$	$\frac{m^{ss}}{k^{ss}}$	$\frac{n^{ss}}{k^{ss}}$
1.011	0.084	0.065	0.051	0.021

47. To match the estimates of Ireland (2009), this value of  $b$  is larger than was used in the third edition of this book.

the nominal money supply depends on the sign of  $u_{cm}$ ; this, in turn, depends on the sign of  $b - \Phi$ . For the benchmark parameter values, this is positive. With  $\Phi$  less than  $b$ , the marginal utility of consumption is increasing in real money balances. Hence, higher inflation decreases the marginal utility of consumption, increases the demand for leisure, and decreases the supply of labor (see 2.45). If  $b - \Phi$  is negative, higher inflation leads to a rise in labor supply and output. The dependence of the elasticity of labor with respect to inflation on the partial derivatives of the utility function in a general MIU model is discussed more fully by Wang and Yip (1992).

### 2.5.5 Simulation Results

Figure 2.4 shows the effect of a one standard deviation monetary shock on output, employment, real and nominal interest rates, inflation, and the real stock of money.<sup>48</sup> Because  $b > \Phi$ , a positive money growth rate shock reduces employment and output.<sup>49</sup> Notice that a positive monetary shock *increases* the nominal rate of interest. In the MIU model, prices are assumed to be perfectly flexible; when  $\rho_u > 0$ , money growth rate shocks increase



**Figure 2.4**

Responses to a positive money growth rate shock in the MIU model;  $\rho_u = 0.67$ ,  $\rho_u = 0.9$ .

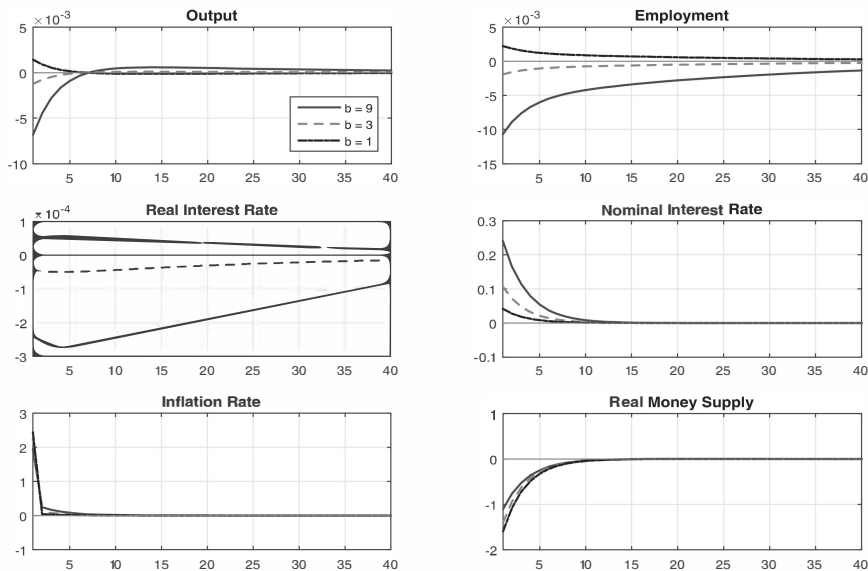
48. Simulation results are obtained using Dynare. See the chapter appendix and the programs available at <http://people.ucsc.edu/~walshc/mtp4e/> for details.

49. Recall that the transitional dynamics exhibit superneutrality when  $\Phi = b$ . In this case, neither output nor employment would be affected by the monetary shock.

expected inflation and raise the nominal interest rate. The price level jumps immediately and the real quantity of money actually falls, consistent with the decline in real money demand that occurs as a result of the increase in the nominal interest rate. The effects clearly depend on the degree of persistence in the money growth process. Higher values of  $\rho_u$  generate much larger effects on labor input and output.<sup>50</sup>

The value of  $b$  relative to  $\Phi$  is critical for determining the real effects of a money growth rate shock. The results in figure 2.4 are for the baseline calibration in which  $b = 9 > \Phi$ . The effects when  $b = 3$  (i.e., smaller than the baseline value but still greater than  $\Phi$ ) and  $b = 1 < \Phi$  are shown in figure 2.5.<sup>51</sup> When  $b < \Phi$ , higher expected inflation (and therefore lower real money balances) raise the marginal utility of consumption and lead to a decrease in leisure demand; labor supply and output rise in this case, as shown in the figure. In all cases, inflation jumps immediately and then quickly returns to its steady-state value.

How do the properties of the model vary if money growth responds to productivity shocks. Figure 2.6 illustrates the effects of varying  $\phi$ , the response of money growth to

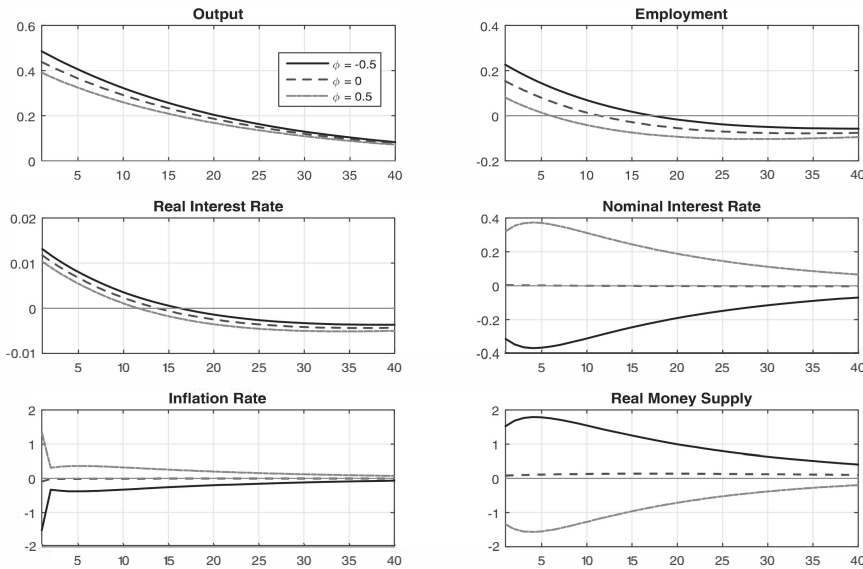


**Figure 2.5**

Responses to a positive money growth rate shock;  $b = 9 > \Phi$ ,  $b = 3 > \Phi$ ,  $b = 1 < \Phi$ .

50. Effects would also be larger if the model were calibrated to match a broader monetary aggregate by reducing  $a$ , increasing  $b$ , and increasing  $\sigma_\varphi$ .

51. The value  $b = 3$  was used as the baseline in the third edition of this book.



**Figure 2.6**

Responses in the MIU model for different values of  $\phi$  in the money growth process (2.77).

the productivity shock.<sup>52</sup> The major effect of  $\phi$  is on the behavior of inflation and the nominal rate of interest. When money growth does not respond to a productivity shock or when it decreases in response (i.e., when  $\phi \leq 0$ ), output and inflation are negatively correlated, as the positive shock to productivity increases output and reduces prices. When  $\phi < 0$ , a positive technology shock leads to lower expected money growth and inflation. Lower expected inflation raises real money balances, increases the marginal utility of consumption, and increases the labor supply when, as in the case here,  $b > \Phi$ . Hence, employment and output are slightly higher after a technology shock when  $\phi < 0$  than when  $\phi = 0$ . Conversely, when  $\phi > 0$ , a positive technology shock leads to higher expected inflation, and the output-inflation correlation becomes positive. Employment and output respond less than in the base case. Changes in the money growth process have their main effect on the behavior of the nominal interest rate and inflation. Both the sign and the magnitude of the correlation between these variables and output depend on the money growth process. Consistent with the earlier discussion, the monetary shock  $\varphi_t$  affects the labor-leisure choice only when the nominal money growth rate process exhibits serial correlation ( $\rho_u \neq 0$ ) or responds to the technology shock ( $\phi \neq 0$ ).

52. When  $\phi \neq 0$ , the variance of the innovation to  $u$  is adjusted to keep the standard deviation of nominal money growth equal to its value in the baseline case.

## 2.6 Summary

Assuming that holdings of real money balances yield direct utility is a means of ensuring a positive demand for money so that, in equilibrium, money is held and has value. This assumption is clearly a shortcut; it does not address the issue of why money yields utility or why certain pieces of paper that we call money yield utility but other pieces of paper presumably do not.

The Sidrauski model, because it assumes that agents act systematically to maximize utility, allows the welfare effects of alternative inflation rates to be assessed. The model illustrates the logic behind Friedman's conclusion that the optimal inflation rate is the rate that produces a zero nominal rate of interest, a result that also appears in the models discussed in chapters 3 and 4. Finally, by developing a linear approximation to the basic money in the utility function model (augmented to include a labor supply choice), it was shown how the effects of variations in the growth rate of the money supply on the short-run dynamic adjustment of the economy depend on effects of money holdings on the marginal utility of consumption and leisure.

## 2.7 Appendix: Solving the MIU Model

The basic MIU model is linearized around the nonstochastic steady state, so the first task is to derive the steady-state equilibrium. Setting all shocks to zero and all endogenous variables equal to constants, and using the functional forms assumed for production and utility, the Euler condition, the definition of the real return, the production function, the capital accumulation equation, and the goods market clearing condition imply

$$\begin{aligned}\lambda^{ss} &= \beta(1 + r^{ss})\lambda^{ss} \Rightarrow 1 + r^{ss} = \frac{1}{\beta}, \\ r^{ss} &= \alpha \left( \frac{y^{ss}}{k^{ss}} \right) - \delta \Rightarrow \left( \frac{y^{ss}}{k^{ss}} \right) = \left( \frac{1}{\alpha} \right) \left( \frac{1}{\beta} - 1 + \delta \right), \\ y^{ss} &= (k^{ss})^\alpha (n^{ss})^{1-\alpha} \Rightarrow \frac{n^{ss}}{k^{ss}} = \left( \frac{y^{ss}}{k^{ss}} \right)^{\frac{1}{1-\alpha}} = \left( \frac{\frac{1}{\beta} - 1 + \delta}{\alpha} \right)^{\frac{1}{1-\alpha}}, \\ x^{ss} &= k^{ss} - (1 - \delta)k^{ss} = \delta k^{ss} \Rightarrow \frac{x^{ss}}{k^{ss}} = \delta, \\ c^{ss} &= y^{ss} - x^{ss} \Rightarrow \frac{c^{ss}}{k^{ss}} = \left( \frac{y^{ss}}{k^{ss}} \right) - \delta = \left( \frac{1}{\alpha} \right) \left[ \frac{1}{\beta} - 1 + (1 - \alpha)\delta \right].\end{aligned}$$

These five equations pin down the steady-state values of the real return as well as the steady-state ratios of output, employment, investment, and consumption to the capital stock.

In the text, the intensive production function was defined as

$$\phi\left(\frac{k^{ss}}{n^{ss}}\right) = \left(\frac{k^{ss}}{n^{ss}}\right)^\alpha.$$

Then  $y^{ss}/n^{ss} = \phi(k^{ss}/n^{ss})$ ,  $c^{ss} = y^{ss} - \delta k^{ss} = [\phi - \delta(k^{ss}/n^{ss})]n^{ss} = \bar{\phi}n^{ss}$ . Section 2.5.2 also made use of the fact that because  $y = f = \phi n$ ,  $f_n = \phi - (k^{ss}/n^{ss})\phi'$ .

From

$$m^{ss} = \left(\frac{1 + \theta^{ss}}{1 + \pi^{ss}}\right) m^{ss},$$

one obtains  $\pi^{ss} = \theta^{ss}$ , and this means

$$1 + r^{ss} = \frac{1 + i^{ss}}{1 + \pi^{ss}} \Rightarrow 1 + i^{ss} = (1 + r^{ss})(1 + \pi^{ss}) = \frac{1 + \theta^{ss}}{\beta},$$

or  $i^{ss} = (1 + \theta^{ss} - \beta) / \beta$ . The first-order condition for money holdings then becomes

$$\frac{u_m(c^{ss}, m^{ss}, 1 - n^{ss})}{\lambda^{ss}} = \left(\frac{m^{ss}}{c^{ss}}\right)^{-b} = \left(\frac{i^{ss}}{1 + i^{ss}}\right) = \left(\frac{a}{1 - a}\right) \left(\frac{1 + \theta^{ss} - \beta}{1 + \theta^{ss}}\right), \text{ or}$$

$$\left(\frac{m^{ss}}{c^{ss}}\right) = \left[\left(\frac{a}{1 - a}\right) \left(\frac{1 + \theta^{ss} - \beta}{1 + \theta^{ss}}\right)\right]^{-\frac{1}{b}}. \quad (2.79)$$

From the first-order condition for the household's choice of hours and the definition of the marginal utility of consumption,

$$\frac{u_l(c^{ss}, m^{ss}, 1 - n^{ss})}{\lambda^{ss}} = \frac{\Psi(1 - n^{ss})^{-\eta}}{aQ^{\frac{b-\Phi}{1-b}}(c^{ss})^{-b}} = (1 - \alpha) \left(\frac{y^{ss}}{n^{ss}}\right),$$

where  $Q \equiv [a(c^{ss})^{1-b} + (1 - a)(m^{ss})^{1-b}]$ . This can be rewritten as

$$\frac{\Psi(1 - n^{ss})^{-\eta}}{a \left[ a + (1 - a) \left(\frac{m^{ss}}{c^{ss}}\right)^{1-b} \right]^{\frac{b-\Phi}{1-b}} (c^{ss})^{-\Phi}} = (1 - \alpha) \left(\frac{y^{ss}}{n^{ss}}\right).$$

Rearranging, and using the earlier results,  $n^{ss}$  satisfies

$$(1 - n^{ss})^{-\eta} (n^{ss})^\Phi = \frac{H}{\Psi}, \quad (2.80)$$

where

$$H = (1 - \alpha) \left(\frac{y^{ss}}{k^{ss}}\right) a \left[ a + (1 - a) \left(\frac{m^{ss}}{c^{ss}}\right)^{1-b} \right]^{\frac{b-\Phi}{1-b}} \left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi} \left(\frac{k^{ss}}{n^{ss}}\right)^{1-\Phi}$$

$$= (1 - \alpha) a \left[ a + (1 - a) \left(\frac{m^{ss}}{c^{ss}}\right)^{1-b} \right]^{\frac{b-\Phi}{1-b}} \left(\frac{c^{ss}}{k^{ss}}\right)^{-\Phi} \left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{\Phi-\alpha}{1-\alpha}}.$$

Using  $\frac{n^{ss}}{k^{ss}} = \left(\frac{y^{ss}}{k^{ss}}\right)^{\frac{1}{1-\alpha}}$ ,  $\left(\frac{y^{ss}}{k^{ss}}\right) = \left(\frac{1}{\alpha}\right) \left(\frac{1}{\beta} - 1 + \delta\right)$ ,  $\frac{c^{ss}}{k^{ss}} = \left(\frac{1}{\alpha}\right) \left[\frac{1}{\beta} - 1 + (1-\alpha)\delta\right]$ , and (2.79),

$$H = a(1-\alpha) \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{\Phi-\alpha}{1-\alpha}} \left[ a + (1-a) \left(\frac{a}{1-a} \frac{1 + \theta^{ss} - \beta}{1 + \theta^{ss}}\right)^{-\frac{1-b}{b}} \right]^{\frac{b-\Phi}{1-b}} \times \left[\frac{\frac{1}{\beta} - 1 + (1-\alpha)\delta}{\alpha}\right]^{-\Phi}.$$

Only  $H$  depends on the rate of money growth (and thus on the steady-state rate of inflation), and if  $b = \Phi$ , then  $H$ , too, is independent of  $1 + \theta^{ss}$ . In this case,  $n^{ss}$  and all other real variables (except  $m^{ss}$ ) are independent of the rate of money growth.

The next step is to obtain the linear approximation for each equilibrium condition of the model so that the dynamic behavior as the economy fluctuates around the steady state can be studied.

### 2.7.1 The Linear Approximation

Three basic rules are employed in deriving the linear approximations (see Uhlig 1999). First, for two variables  $u$  and  $w$ ,

$$uw = u^{ss}(1 + \hat{u})w^{ss}(1 + \hat{w}) \approx u^{ss}w^{ss}(1 + \hat{u} + \hat{w}). \quad (2.81)$$

That is, assume that product terms like  $\hat{u}\hat{w}$  are approximately equal to zero. Second,

$$u^a = (u^{ss})^a(1 + \hat{u})^a \approx (u^{ss})^a(1 + a\hat{u}), \quad (2.82)$$

which can be obtained as a repeated application of the first rule. Furthermore,

$$\ln u = \ln u^{ss}(1 + \hat{u}) = \ln u^{ss} + \ln(1 + \hat{u}) \approx \ln u^{ss} + \hat{u}. \quad (2.83)$$

Finally, because variables such as interest rates and inflation rates are already expressed in percentages, it is natural to write them as absolute deviations from steady state. So, for example,  $\hat{r}_t \equiv r_t - r^{ss}$ . Assuming interest rates and inflation rates are small, (2.83) permits approximating the log deviation of  $(1 + r_t)$  around the steady state by  $\ln(1 + r_t) - \ln(1 + r^{ss}) \approx r_t - r^{ss} = \hat{r}_t$ , and similarly for  $i_t$  and  $\pi_t$ . This also means  $(1 + r_t)/(1 + r^{ss})$  is approximately equal to  $1 - r_t - r^{ss} = 1 + \hat{r}_t$ .<sup>53</sup> By applying these rules, one obtains a system of linear equations that characterizes the dynamic behavior of the MIU model for small deviations around its steady state.

53. This requires that terms such as  $r_t$  be small. Otherwise, one should use the exact Taylor series expansion. For example, in the case of  $(1 + r_t)/(1 + r^{ss})$ , this would be

$$\frac{1 + r_t}{1 + r^{ss}} \approx 1 + \left(\frac{1}{1 + r^{ss}}\right)(r_t - r^{ss}) = 1 + \left(\frac{1}{1 + r^{ss}}\right)\hat{r}_t.$$

With the calibration employed,  $r^{ss} = 0.011$ , so  $1/(1 + r^{ss}) = \beta = 0.989$ .

### The Production Function

First, rewrite the production relationship (2.65) by replacing each variable with its steady-state value times one plus the percent deviation of its time  $t$  value from the steady state, noting that  $e^{z_t}$  can be approximated by  $1 + z_t$  for small  $z_t$ :

$$y^{\text{ss}}(1 + \hat{y}_t) = (1 + z_t) (k^{\text{ss}})^{\alpha} (1 + \hat{k}_{t-1})^{\alpha} (n^{\text{ss}})^{1-\alpha} (1 + \hat{n}_t)^{1-\alpha}.$$

Because

$$y^{\text{ss}} = (k^{\text{ss}})^{\alpha} (n^{\text{ss}})^{1-\alpha},$$

both sides can be divided by  $y^{\text{ss}}$  to obtain

$$\begin{aligned} (1 + \hat{y}_t) &= (1 + z_t) (1 + \hat{k}_{t-1})^{\alpha} (1 + \hat{n}_t)^{1-\alpha} \\ &\approx 1 + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t, \text{ or} \\ \hat{y}_t &= \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t. \end{aligned} \tag{2.84}$$

### Goods Market Clearing

Goods market clearing requires that  $y_t = c_t + x_t$ , where  $x_t$  is investment. Write this as

$$y^{\text{ss}}(1 + \hat{y}_t) = c^{\text{ss}}(1 + \hat{c}_t) + x^{\text{ss}}(1 + \hat{x}_t).$$

Because  $y^{\text{ss}} = c^{\text{ss}} + x^{\text{ss}}$ , it follows that

$$y^{\text{ss}} \hat{y}_t = c^{\text{ss}} \hat{c}_t + x^{\text{ss}} \hat{x}_t.$$

Dividing both sides by  $k^{\text{ss}}$  and noting that  $x^{\text{ss}}/k^{\text{ss}} = \delta$  gives

$$\left(\frac{y^{\text{ss}}}{k^{\text{ss}}}\right) \hat{y}_t = \left(\frac{c^{\text{ss}}}{k^{\text{ss}}}\right) \hat{c}_t + \delta \hat{x}_t. \tag{2.85}$$

### Capital Accumulation

The capital stock evolves according to  $k_t = (1 - \delta)k_{t-1} + x_t$ , or

$$k^{\text{ss}}(1 + \hat{k}_t) = (1 - \delta)k^{\text{ss}}(1 + \hat{k}_{t-1}) + x^{\text{ss}}(1 + \hat{x}_t),$$

which implies

$$\hat{k}_t = (1 - \delta)\hat{k}_{t-1} + \left(\frac{x^{\text{ss}}}{k^{\text{ss}}}\right)\hat{x}_t,$$

but  $x^{\text{ss}}/k^{\text{ss}} = \delta$ , so

$$\hat{k}_t = (1 - \delta)\hat{k}_{t-1} + \delta \hat{x}_t. \tag{2.86}$$

### Labor Hours

The first-order condition for the choice of labor hours is

$$u_l(c_t, m_t, 1 - n_t) = \lambda_t f_n(k_{t-1}, n_t, z_t),$$



where  $\lambda_t$  is the marginal utility of consumption. Using the production and utility functions, this becomes

$$\frac{u_t}{\lambda_t} = \frac{\Psi(1-n_t)^{-\eta}}{\lambda_t} = \frac{\Psi l_t^{-\eta}}{\lambda_t} = (1-\alpha) \left( \frac{y_t}{n_t} \right).$$

Written in terms of deviation, this is

$$\frac{\Psi (l^{ss})^{-\eta} (1 + \hat{l}_t)^{-\eta}}{\lambda^{ss} (1 + \hat{\lambda}_t)} = (1-\alpha) \left( \frac{y^{ss}}{n^{ss}} \right) \left( \frac{1 + \hat{y}_t}{1 + \hat{n}_t} \right).$$

But in the steady state,

$$\frac{\Psi (l^{ss})^{-\eta}}{\lambda^{ss}} = (1-\alpha) \left( \frac{y^{ss}}{n^{ss}} \right),$$

so

$$\frac{(1 + \hat{l}_t)^{-\eta}}{(1 + \hat{\lambda}_t)} = \left( \frac{1 + \hat{y}_t}{1 + \hat{n}_t} \right), \text{ or}$$

$$(1 - \eta \hat{l}_t) (1 - \hat{\lambda}_t) \approx 1 - \eta \hat{l}_t - \hat{\lambda}_t \approx 1 + \hat{y}_t - \hat{n}_t.$$

From  $l_t = 1 - n_t$ ,

$$l^{ss} (1 + \hat{l}_t) = 1 - n^{ss} (1 + \hat{n}_t) \Rightarrow \hat{l}_t = - \left( \frac{n^{ss}}{l^{ss}} \right) \hat{n}_t.$$

Hence,

$$-\eta \hat{l}_t - \hat{\lambda}_t = \eta \left( \frac{n^{ss}}{l^{ss}} \right) \hat{n}_t - \hat{\lambda}_t \approx \hat{y}_t - \hat{n}_t,$$

which can be written as

$$\left[ 1 + \eta \left( \frac{n^{ss}}{l^{ss}} \right) \right] \hat{n}_t = \hat{y}_t + \hat{\lambda}_t. \quad (2.87)$$

### The Marginal Utility of Consumption

The marginal utility of consumption is

$$\lambda_t = a \left[ a c_t^{1-b} + (1-a) m_t^{1-b} \right]^{\frac{b-\Phi}{1-b}} c_t^{-b}.$$

Define

$$Q_t = a c_t^{1-b} + (1-a) m_t^{1-b}.$$

Then

$$\lambda_t = a Q_t^{\frac{b-\Phi}{1-b}} c_t^{-b}, \text{ or}$$

$$\lambda^{\text{ss}}(1 + \hat{\lambda}_t) = a(Q^{\text{ss}})^{\frac{b-\Phi}{1-b}} (c^{\text{ss}})^{-b} \left[ 1 + \left( \frac{b-\Phi}{1-b} \right) \hat{q}_t \right] (1 - b\hat{c}_t).$$

Since

$$\lambda^{\text{ss}} = a(Q^{\text{ss}})^{\frac{b-\Phi}{1-b}} (c^{\text{ss}})^{-b},$$

the right side of the previous equation can be approximated by

$$\lambda^{\text{ss}} \left[ 1 + \left( \frac{b-\Phi}{1-b} \right) \hat{q}_t - b\hat{c}_t \right],$$

so

$$\hat{\lambda}_t = \left( \frac{b-\Phi}{1-b} \right) \hat{q}_t - b\hat{c}_t.$$

To obtain an expression for  $\hat{q}_t$ , note that from the definition of  $Q_t$ ,

$$\begin{aligned} (1 + \hat{q}_t) &= \frac{a(c^{\text{ss}})^{1-b}}{Q^{\text{ss}}} (1 + \hat{c}_t)^{1-b} + \frac{(1-a)(m^{\text{ss}})^{1-b}}{Q^{\text{ss}}} (1 + \hat{m}_t)^{1-b} \\ &= \gamma [1 + (1-b)\hat{c}_t] + (1-\gamma) [1 + (1-b)\hat{m}_t], \end{aligned}$$

where

$$\gamma \equiv \frac{a(c^{\text{ss}})^{1-b}}{Q^{\text{ss}}}.$$

Hence,

$$\hat{q}_t = \gamma(1-b)\hat{c}_t + (1-\gamma)(1-b)\hat{m}_t.$$

Combining these results,

$$\hat{\lambda}_t = \Omega_1 \hat{c}_t + \Omega_2 \hat{m}_t, \tag{2.88}$$

where  $\Omega_1 = b(\gamma - 1) - \gamma\Phi$  and  $\Omega_2 = (b - \Phi)(1 - \gamma)$ . Note that if  $b = \Phi$ ,  $\hat{\lambda}_t = -b\hat{c}_t$ .

### The Euler Condition

The Euler condition is

$$\lambda_t = \beta E_t(1 + r_t)\lambda_{t+1},$$

which, because  $\beta = (1 + r^{\text{ss}})^{-1}$ , can be written as

$$\lambda^{\text{ss}} (1 + \hat{\lambda}_t) = \beta \lambda^{\text{ss}} (1 + r_t) E_t (1 + \hat{\lambda}_{t+1}) = \lambda^{\text{ss}} \left( \frac{1 + r_t}{1 + r^{\text{ss}}} \right) E_t (1 + \hat{\lambda}_{t+1}).$$

Dividing both sides by  $\lambda^{\text{ss}}$ , recalling that  $\hat{r}_t \equiv r_t - r^{\text{ss}}$ , and using (2.81),

$$(1 + \hat{\lambda}_t) \approx (1 + \hat{r}_t + E_t \hat{\lambda}_{t+1});$$

then

$$\hat{\lambda}_t = \hat{r}_t + E_t \hat{\lambda}_{t+1}.$$

**Marginal Product, Real Return Condition**

Start with

$$1 + r_t = 1 - \delta + \alpha E_t \left( \frac{y_{t+1}}{k_t} \right).$$

Using the same general approach as applied to the other equations,

$$1 + r_t \approx 1 - \delta + \alpha \left( \frac{y^{ss}}{k^{ss}} \right) E_t \left( 1 + \hat{y}_{t+1} - \hat{k}_t \right).$$

Since  $r^{ss} = \alpha(y^{ss}/k^{ss}) - \delta$ ,

$$\hat{r}_t = r_t - r^{ss} = \alpha \left( \frac{y^{ss}}{k^{ss}} \right) E_t \left( \hat{y}_{t+1} - \hat{k}_t \right). \quad (2.89)$$

**Money Holdings**

The first-order condition for money holdings is

$$\frac{u_m(c_t, m_t, 1 - n_t)}{u_c(c_t, m_t, 1 - n_t)} = \left( \frac{i_t}{1 + i_t} \right).$$

From the specification of the utility function, the left side can be approximated as

$$\begin{aligned} \frac{u_m}{u_c} &= \frac{(1-a)m_t^{-b}}{ac_t^{-b}} \approx \left( \frac{1-a}{a} \right) \left( \frac{m^{ss}}{c^{ss}} \right)^{-b} (1 - b\hat{m}_t + b\hat{c}_t) \\ &= \left( \frac{i^{ss}}{1 + i^{ss}} \right) (1 - b\hat{m}_t + b\hat{c}_t). \end{aligned}$$

Therefore,

$$\left( \frac{i^{ss}}{1 + i^{ss}} \right) (1 - b\hat{m}_t + b\hat{c}_t) \approx \left( \frac{i_t}{1 + i_t} \right).$$

Multiplying both sides by  $1 + i_t$  and approximating  $(1 + i_t) / (1 + i^{ss})$  by  $1 + i_t - i^{ss}$  yields

$$i^{ss} (1 + i_t - i^{ss}) (1 - b\hat{m}_t + b\hat{c}_t) \approx i_t, \text{ or}$$

$$i^{ss} (i_t - i^{ss} - b\hat{m}_t + b\hat{c}_t) = i_t - i^{ss}.$$

Therefore, the money demand equation is given by

$$\hat{m}_t = \hat{c}_t - \left( \frac{1}{b} \right) \left( \frac{1 - i^{ss}}{i^{ss}} \right) (i_t - i^{ss}). \quad (2.90)$$

**Real Money Growth**

Because  $\theta^{ss} = \pi^{ss}$ , one can approximate

$$m_t = \left( \frac{1 + \theta_t}{1 + \pi_t} \right) m_{t-1}$$

by

$$m_t = \left( \frac{1 + \theta_t}{1 + \pi_t} \right) \left( \frac{1 + \pi^{ss}}{1 + \theta^{ss}} \right) m_{t-1} \approx \left( \frac{1 + \hat{\theta}_t}{1 + \hat{\pi}_t} \right) m_{t-1},$$

or

$$m^{ss} (1 + \hat{m}_t) \approx (1 + \hat{\theta}_t - \hat{\pi}_t) m^{ss} (1 + \hat{m}_{t-1}),$$

where  $\hat{\theta}_t \equiv \theta_t - \theta^{ss}$ . Dividing both sides by  $m^{ss}$  and using (2.81) yields

$$\hat{m}_t \approx \hat{\theta}_t - \hat{\pi}_t + \hat{m}_{t-1} = u_t - \hat{\pi}_t + \hat{m}_{t-1}.$$

### The Fisher Equation

The relationship between the nominal interest rate, the real interest rate, and expected inflation is

$$1 + r_t = E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right), \text{ or}$$

$$r_t \approx i_t - E_t \pi_{t+1}.$$

Subtracting steady state values from both sides,

$$\hat{r}_t = \hat{i}_t - E_t \hat{\pi}_{t+1}.$$

### 2.7.2 Collecting all Equations

The linearized model consists of twelve equations to determine the exogenous disturbances  $\hat{z}_t$  and  $\hat{u}_t$  and the ten endogenous variables  $\hat{y}_t, \hat{k}_t, \hat{n}_t, \hat{x}_t, \hat{c}_t, \hat{\lambda}_t, \hat{r}_t, \hat{i}_t, \hat{\pi}_t, \hat{m}_t$ . These twelve equations are

$$z_t = \rho_z z_{t-1} + e_t,$$

$$\hat{y}_t = z_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t,$$

$$\left( \frac{y^{ss}}{k^{ss}} \right) \hat{y}_t = \left( \frac{c^{ss}}{k^{ss}} \right) \hat{c}_t + \delta \hat{x}_t,$$

$$\hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \delta \hat{x}_t,$$

$$\left[ 1 + \eta \left( \frac{n^{ss}}{l^{ss}} \right) \right] \hat{n}_t = \hat{y}_t + \hat{\lambda}_t,$$

$$\hat{r}_t = \alpha \left( \frac{y^{ss}}{k^{ss}} \right) (E_t \hat{y}_{t+1} - \hat{k}_t),$$

$$\hat{\lambda}_t = \hat{r}_t + E_t \hat{\lambda}_{t+1},$$

$$\hat{\lambda}_t = \Omega_1 \hat{c}_t + \Omega_2 \hat{m}_t,$$

$$\hat{m}_t = \hat{c}_t - \left(\frac{1}{b}\right) \left(\frac{1 - i^{ss}}{i^{ss}}\right) \hat{l}_t,$$

$$\hat{m}_t = u_t - \hat{\pi}_t + \hat{m}_{t-1},$$

$$\hat{l}_t = \hat{r}_t + E_t \hat{\pi}_{t+1},$$

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \varphi_t,$$

where  $\Omega_1 = b(\gamma - 1) - \gamma\Phi$  and  $\Omega_2 = (b - \Phi)(1 - \gamma)$ . Note that if  $b = \Phi$  so that  $\Omega_2 = 0$ , the first eight equations can be solved for the behavior of the real variables  $z_t$ ,  $\hat{y}_t$ ,  $\hat{k}_t$ ,  $\hat{n}_t$ ,  $\hat{x}_t$ ,  $\hat{c}_t$ ,  $\hat{l}_t$ , and  $\hat{r}_t$ , while the last four then determine  $u_t$ ,  $\hat{l}_t$ ,  $\hat{\pi}_t$ , and  $\hat{m}_t$ .

### 2.7.3 Solving Linear Rational-Expectations Models with Forward-Looking Variables

This section provides a brief overview of the approach used to solve linear rational expectations models numerically. The basic reference is Blanchard and Kahn (1980). General discussions can be found in Wickens (2008), DeJong and Dave (2011), and Miao (2014).

Following the solutions methods of Blanchard and Kahn (1980), a linear rational expectations model can be written in the form

$$A_1 \begin{bmatrix} X_{t+1} \\ E_t x_{t+1} \end{bmatrix} = A_2 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A_3 G_t + \begin{bmatrix} \psi_{t+1} \\ 0 \end{bmatrix},$$

where  $X$  are predetermined variables ( $n_1$  in number) and  $x$  are non-predetermined (forward-looking) variables ( $n_2$  in number). Predetermined means that  $X_t$  is known at time  $t$  and not jointly determined with  $x_t$ , while  $x_t$  is endogenously determined at time  $t$ .  $G_t$  consists of a vector of exogenous variables. Premultiplying both sides by  $A_1$  inverse, we obtain

$$\begin{bmatrix} X_{t+1} \\ E_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B G_t + A_1^{-1} \begin{bmatrix} \psi_{t+1} \\ 0 \end{bmatrix}, \quad (2.91)$$

where  $A = A_1^{-1} A_2$  and  $B = A_1^{-1} A_3$ . King and Watson (1998) consider the case in which  $A_1$  is singular.

Blanchard and Kahn showed that the number of eigenvalues of  $A$  that are outside the unit circle must equal the number of forward-looking variables. Decompose  $A$  as  $Q^{-1} \Lambda Q$ , where  $\Lambda$  is a diagonal matrix of the eigenvalues of  $A$ , and  $Q$  is the corresponding matrix of eigenvectors. Next, order  $\Lambda$  so that  $\lambda_1$  is the smallest and  $\lambda_n$  is the largest eigenvalue, where  $n \equiv n_1 + n_2$ . Then, the Blanchard and Kahn conditions require that the first  $n_1$  eigenvalues must be inside the unit circle and the last  $n_2$  must be outside the unit circle if the system is to have a unique stationary rational-expectations equilibrium. If fewer than  $n_2$  eigenvalues are outside the unit circle, multiple equilibria exist and the system is said to be characterized by indeterminacy. If too many eigenvalues are outside the unit circle, no solution exists.

To understand the role these conditions play, it is convenient to write (2.91) as

$$Z_{t+1} = A Z_t + B G_t + (Z_{t+1} - E_t Z_{t+1}), \quad (2.92)$$

where  $Z_t \equiv [X_t \ x_t]'$  is a  $n \times 1$  vector of predetermined and non-predetermined variables and  $G_t$  is a stationary (possibly stochastic) vector of exogenous variables. Under the assumption of rational expectations,  $E_t(Z_{t+1} - E_t Z_{t+1}) = 0$ . Writing  $A = Q^{-1} \Lambda Q$ , both sides of (2.92) can be multiplied by  $Q$ , yielding

$$z_{t+1} = \Lambda z_t + g_t + \phi_{t+1}, \quad (2.93)$$

where  $z_t \equiv QZ_t$ ,  $g_t \equiv QBG_t$ , and  $\phi_{t+1} = Q(Z_{t+1} - E_t Z_{t+1})$ . Since  $\Lambda$  is a diagonal matrix, (2.93) consists of  $n$  independent equations of the form  $z_{i,t+1} = \lambda_i z_{i,t} + \hat{g}_{i,t}$ , where  $z_{i,t}$  is the  $i$ th element of  $z_t$  and similarly for  $g_{i,t}$ , while  $\lambda_i$  is the  $i$ th diagonal element of  $\Lambda$ .

Suppose  $\Lambda$  has  $\tilde{n}_1$  elements within the unit circle and  $\tilde{n}_2 = n - \tilde{n}_1$  on or outside the unit circle. The system can be written as

$$z_{t+1} = \begin{bmatrix} Y_{t+1} \\ P_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} Y_t \\ P_t \end{bmatrix} + \begin{bmatrix} g_{1,t} \\ g_{2,t} \end{bmatrix} + \begin{bmatrix} \phi_{1,t+1} \\ \phi_{2,t+1} \end{bmatrix}.$$

$Y_t$  consists of the first  $\tilde{n}_1$  elements of  $z_t$ , i.e., those corresponding to the eigenvalues in  $\Lambda$  less than 1 in absolute value and contained in the diagonal matrix  $\Lambda_1$ , and  $P_t$  consists of the  $\tilde{n}_2$  remaining elements of  $z_t$  associated with the eigenvalues equal to or greater than 1 in absolute value and contained in  $\Lambda_2$ . Writing this system out explicitly, one has

$$Y_{t+1} = \Lambda_1 Y_t + g_{1,t} + \phi_{1,t+1},$$

$$P_{t+1} = \Lambda_2 P_t + g_{2,t} + \phi_{2,t+1}.$$

This second set of equations is explosive (the elements of  $\Lambda_2$  are outside the unit circle). Hence, because  $E_t \phi_{i,t+1} = 0$ , it must be the case in any nonexplosive equilibrium that

$$P_t = \Lambda_2^{-1} (E_t P_{t+1} - g_{2,t}) = - \sum_{i=0}^{\infty} \Lambda_2^{-i-1} g_{2,t+i}.$$

This uniquely determines  $P_t$  as the only value of  $P_t$  consistent with a stationary equilibrium. Any other value of  $P_t$  leads to explosive behavior. For example, for  $P_t$  scalar and  $g_{2,t} = \rho g_{2,t-1} + e_t$ , where  $e$  is white noise and  $\rho$  less than 1 in absolute value,

$$P_t = - \sum_{i=0}^{\infty} \lambda_2^{-i-1} \rho^i g_{2,t} = - \left( \frac{1}{\lambda_2 - \rho} \right) g_{2,t},$$

where  $\lambda_2 - \rho > 0$  because  $|\lambda_2| \geq 1$  and  $|\rho| < 1$ .

The  $\tilde{n}_1$  equations of the form

$$Y_{t+1} = \Lambda_1 Y_t + g_{1,t}$$

can be solved backward because  $|\lambda_i| < 1$  for all  $i \leq \tilde{n}_1$ . That is, for these  $\tilde{n}_1$  equations, recursive substitution leads to

$$\begin{aligned} y_{i,t+1} &= \lambda_i y_{i,t} + g_{i,t} = \lambda_i [\lambda_i y_{i,t-1} + g_{i,t-1}] + g_{i,t} \\ &= \sum_{j=0}^{\infty} \lambda_i^j g_{i,t-j}. \end{aligned}$$

Recall that  $Y_t$  and  $P_t$  were linear combinations of the variables of interest  $X_t$  and  $x_t$ . Having obtained unique stationary solutions for  $Y_t$  and  $P_t$ , when can unique solutions for the variables of interest be obtained? Let  $W \equiv Q^{-1}$ . Then, since  $z_t = [Y_t \ P_t]' = Q[X_t \ x_t]'$ , one can write

$$\begin{bmatrix} X_t \\ x_t \end{bmatrix} = W \begin{bmatrix} Y_t \\ P_t \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ P_t \end{bmatrix},$$

where  $W$  has been partitioned to conform with the dimensions of the different vectors. This system yields

$$X_t = W_{11}Y_t + W_{12}P_t,$$

which gives  $n_1$  equations. Since  $X_t$  is predetermined, let  $X_0$  denote the initial conditions on the system. A unique value for  $P_0$  was obtained earlier. Thus,  $Y_0$  must satisfy

$$X_0 = W_{11}Y_0 + W_{12}P_0. \tag{2.94}$$

If the number of predetermined variables  $n_1 > \tilde{n}_1$ , then (2.94) consists of  $n_1$  equations in the  $\tilde{n}_1 < n_1$  unknown elements of  $Y_0$ . This imposes  $n_1 > \tilde{n}_1$  conditions on  $Y_0$ , and so there will generally be no solution. If  $n_1 < \tilde{n}_1$ , then there are  $n_1$  initial conditions  $X_0$  but  $\tilde{n}_1 > n_1$  unknowns in  $Y_0$ , a situation of too few equations, and generally multiple solution will exist. Thus, the Blanchard-Kahn condition for a unique stationary rational-expectations equilibrium is  $n_1 = \tilde{n}_1$ , or, as it is more commonly expressed,  $n_2 = n - n_1 = n - \tilde{n}_1 = \tilde{n}_2$ , that is, the number of forward-looking variables  $n_2$  must equal the number of eigenvalues outside the unit circle  $\tilde{n}_2$ . Assuming this condition is met, the unique solution of the original model (2.91) takes the form

$$X_{t+1} = MX_t + N\psi_{t+1},$$

$$x_t = CX_t.$$

The MATLAB code used to solve the MIU model is available at <http://people.ucsc.edu/~walshc/mtp4e/>. The programs use Dynare, available at <http://www.dynare.org/>.

## 2.8 Problems

1. The MIU model of section 2.2 implied that the marginal rate of substitution between money and consumption was set equal to  $i_t/(1 + i_t)$  (see (2.12)). That model assumed that agents entered period  $t$  with resources  $\omega_t$  and used those to purchase capital, consumption, nominal bonds, and money. The real value of these money holdings yielded utility in period  $t$ . Assume instead that money holdings chosen in period  $t$  do not yield utility until period  $t + 1$ . Utility is  $\sum \beta^i U(c_{t+i}, M_{t+i}/P_{t+i})$  as before, but the budget constraint takes the form

$$\omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t,$$

and the household chooses  $c_t$ ,  $k_t$ ,  $b_t$ , and  $M_{t+1}$  in period  $t$ . The household's real wealth  $\omega_t$  is given by

$$\omega_t = f(k_{t-1}) + (1 - \delta)k_{t-1} + (1 + r_{t-1})b_{t-1} + m_t.$$

Derive the first-order condition for the household's choice of  $M_{t+1}$  and show that

$$\frac{U_m(c_{t+1}, m_{t+1})}{U_c(c_{t+1}, m_{t+1})} = i_t.$$

(Suggested by Kevin Salyer.)

2. Carlstrom and Fuerst (2001). Assume that the representative household's utility depends on consumption and the level of real money balances available for spending on consumption. Let  $A_t/P_t$  be the real stock of money that enters the utility function. If capital is ignored, the household's objective is to maximize  $\sum \beta^i U(c_{t+i}, A_{t+i}/P_{t+i})$  subject to the budget constraint

$$Y_t + \frac{M_{t-1}}{P_t} + \tau_t + \frac{(1 + i_{t-1})B_{t-1}}{P_t} = C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t},$$

where income  $Y_t$  is treated as an exogenous process. Assume that the stock of money that yields utility is the real value of money holdings after bonds have been purchased but before income has been received or consumption goods have been purchased:

$$\frac{A_t}{P_t} = \frac{M_{t-1}}{P_t} + \tau_t + \frac{(1 + i_{t-1})B_{t-1}}{P_t} - \frac{B_t}{P_t}.$$

- a. Derive the first-order conditions for  $B_t$  and for  $A_t$ .
  - b. How do these conditions differ from those obtained in the text?
3. Assume the representative household's utility function is given by (2.29). Show that (2.24) implies (2.30). Now suppose  $u(c, m) = (1 - \gamma) \ln c + \gamma \ln m$ . Show that if consumption is constant in the steady state, there is a unique steady-state capital stock



$k^{ss}$  such that  $\beta [f_k(k^{ss}) + 1 - \delta] = 1$ . Explain why variations in the growth rate of the money supply do not affect the steady-state  $k$  in this case but do when the utility function is (2.29).

4. Suppose  $W = \sum \beta^t (\ln c_t + m_t e^{-\gamma m_t})$ ,  $\gamma > 0$ , and  $\beta = 0.95$ . Assume that the production function is  $f(k_t) = k_t^\delta$  and  $\delta = 0.02$ . What rate of inflation maximizes steady-state welfare? How do real money balances at the welfare-maximizing rate of inflation depend on  $\gamma$ ?
5. Suppose that the utility function (2.66) is replaced by

$$u(c_t, m_t, l_t) = \left( \frac{1}{1 - \Phi} \right) \left\{ \left[ a c_t^{1-b} + (1-a) m_t^{1-b} \right]^{\frac{1}{1-b}} l_t^{1-\eta} \right\}^{1-\Phi}.$$

- a. Derive the first-order conditions for the household's optimal money holdings.
  - b. Show how (2.72) and (2.73) are altered with this specification of the utility function.
6. Suppose the utility function (2.66) is replaced by

$$u(c_t, m_t, 1 - n_t) = \frac{\left[ a c_t^{1-b} + (1-a) m_t^{1-b} \right]^{\frac{1-\Phi}{1-b}}}{1 - \Phi} \left[ \frac{(1 - n_t)^{1-\eta}}{1 - \eta} \right].$$

- a. Derive the first-order conditions for the household's optimal money holdings.
  - b. Show how (2.72) and (2.73) are altered with this specification of the utility function.
7. Suppose a nominal interest rate of  $i^m$  is paid on money balances. These payments are financed by a combination of lump-sum taxes and printing money. Let  $a$  be the fraction financed by lump-sum taxes. The government's budget identity is  $\tau_t + v_t = i^m m_t$ , with  $\tau_t = a i^m m_t$  and  $v = \theta m_t$ . Using Sidrauski's model, do the following:
    - a. Show that the ratio of the marginal utility of money to the marginal utility of consumption equals  $r + \pi - i^m = i - i^m$ . Explain why.
    - b. Show how  $i - i^m$  is affected by the method used to finance the interest payments on money. Explain the economics behind your result.
  8. Suppose agents do not treat  $\tau_t$  as a lump-sum transfer but instead assume their transfers will be proportional to their own holdings of money (because in equilibrium,  $\tau = \theta m$ ). Solve for an agent's demand for money. What is the welfare cost of inflation?
  9. Suppose money is a productive input into production, so that the aggregate production function becomes  $y = f(k, m)$ . Incorporate this modification into the model of section 2.2. Is money still superneutral? Explain.

10. Consider the following two alternative specifications for the demand for money given by (2.39) and (2.40).
- Using (2.39), calculate the welfare cost as a function of  $\eta$ .
  - Using (2.40), calculate the welfare cost as a function of  $\xi$ .

11. In Sidrauski's MIU model augmented to include a variable labor supply, money is superneutral if the representative agent's preferences are given by

$$\sum \beta^i u(c_{t+i}, m_{t+i}, l_{t+i}) = \sum \beta^i (c_{t+i} m_{t+i})^b l_{t+i}^d$$

but not if they are given by

$$\sum \beta^i u(c_{t+i}, m_{t+i}, l_{t+i}) = \sum \beta^i (c_{t+i} + k m_{t+i})^b l_{t+i}^d.$$

Discuss. (Assume output depends on capital and labor, and the aggregate production function is Cobb-Douglas.)

12. Suppose preferences over consumption, money holdings, and leisure are given by ( $u = a \ln c_t + (1 - a) \ln m_t + \Psi l_t^{1-\eta} / (1 - \eta)$ ). Fischer (1979) showed that the transition paths are independent of the money supply in this case because the marginal rate of substitution between leisure and consumption is independent of real money balances. Write the equilibrium conditions for this case, and show that the model dichotomizes into a real sector that determines output, consumption, and investment, and a monetary sector that determines the price level and the nominal interest rate.
13. For the model of section 2.5, is the response of output and employment to a money growth rate shock increasing or decreasing in  $b$ ? Explain. (See <http://people.ucsc.edu/~walshc/mtp4e/> for programs to answer this question.)
14. For the model of section 2.5, is the response of output and employment to a money growth rate shock increasing or decreasing in  $a$ ? Explain. (See <http://people.ucsc.edu/~walshc/MTP4e/> for programs to answer this question.)

# 3 Money and Transactions

## 3.1 Introduction

The previous chapter introduced a role for money by assuming that individuals derive utility directly from holding real money balances. Therefore, real money balances appeared in the utility function along side consumption and leisure. Yet one usually thinks of money as yielding utility indirectly through use; it is valued because it is useful in facilitating transactions to obtain the consumption goods that do directly provide utility. As described by Clower (1967), goods buy money, and money buys goods, but goods don't buy goods. And because goods don't buy goods, a monetary medium of exchange that aids the process of transacting will have value.

A medium of exchange that facilitates transactions yields utility indirectly by allowing certain transactions to be made that would not otherwise occur or by reducing the costs associated with transactions. The demand for money is then determined by the nature of the economy's transaction technology. The first formal models of money demand that emphasized the role of transaction costs are due to Baumol (1952) and Tobin (1956).<sup>1</sup> Niehans (1978) developed a systematic treatment of the theory of money in which transaction costs play a critical role. These models are partial equilibrium models, focusing on the demand for money as a function of the nominal interest rate and income. In keeping with the approach used in examining money-in-the-utility function (MIU) models, the focus in this chapter is on *general* equilibrium models in which the demand for money arises from its use in carrying out transactions.

In the first models examined in this chapter, real resources and money are used to produce transaction services, which are required to purchase consumption goods. These real resources can take the form of either time or goods. Most of this chapter, however, is devoted to the study of models that impose a rigid restriction on the nature of transactions. Rather than allowing substitutability between time and money in carrying out transactions,

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1. Jovanovic (1982) and Romer (1986) embedded the Baumol-Tobin model in general equilibrium frameworks.

*cash-in-advance* (CIA) models simply require that money balances be held to finance certain types of purchases; without money, these purchases cannot be made. CIA models, like the MIU models of chapter 2, assume that money is special; unlike other financial assets, it either yields direct utility and therefore belongs in the utility function, or it has unique properties that allow it to be used to facilitate transactions. This chapter concludes with a look at some recent work based on search theory to explain how the nature of transactions gives rise to money.

## 3.2 Resource Costs of Transacting

A direct approach to modeling the role of money in facilitating transactions is to assume that the purchase of goods requires the input of transaction services. First a model is considered in which these services are produced using inputs of money and time. Then an alternative approach is studied in which there are real resource costs in terms of goods that are incurred in purchasing consumption goods. Larger holdings of money allow the household to reduce the resource costs of producing transaction services.

### 3.2.1 Shopping-Time Models

When transaction services are produced by time and money, the consumer must balance the opportunity cost of holding money against the value of leisure in deciding how to combine time and money to purchase consumption goods. The production technology used to produce transaction services determines how much time must be spent “shopping” for given levels of consumption and money holdings. Higher levels of money holdings reduce the time needed for shopping, thereby increasing the individual agent’s leisure. When leisure enters the utility function of the representative agent, shopping-time models provide a link between the MIU approach of chapter 2 and models of money that focus more explicitly on transaction services and money as a medium of exchange.<sup>2</sup>

Suppose that purchasing consumption requires transaction services  $\psi$ , with units chosen so that consumption of  $c$  requires transaction services  $\psi = c$ . These transaction services are produced with inputs of real cash balances  $m \equiv M/P$  and shopping time  $n^s$ :

$$\psi = \psi(m, n^s) = c, \tag{3.1}$$

where  $\psi_m \geq 0$ ,  $\psi_{n^s} \geq 0$ , and  $\psi_{mm} \leq 0$ ,  $\psi_{n^s n^s} \leq 0$ . This specification assumes that it is the agent’s holdings of *real* money balances that produce transaction services; a change in the price level requires a proportional change in nominal money holdings to generate the same

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2. See Brock (1974) for an earlier use of a shopping-time model to motivate an MIU approach. The use of a shopping-time approach to the study of the demand for money is presented in McCallum and Goodfriend (1987) and Croushore (1993).

level of real consumption purchases, holding shopping time  $n^s$  constant. Rewriting (3.1) in terms of the shopping time required for given levels of consumption and money holdings gives

$$n^s = g(c, m); \quad g_c > 0, \quad g_m \leq 0.$$

Household utility is assumed to depend on consumption and leisure:  $v(c, l)$ . Leisure is equal to  $l = 1 - n - n^s$ , where  $n$  is time spent in market employment and  $n^s$  is time spent shopping. Total time available is normalized to equal 1. With shopping time  $n^s$  an increasing function of consumption and a decreasing function of real money holdings, time available for leisure is  $1 - n - g(c, m)$ . Now define a function

$$u(c, m, n) \equiv v[c, 1 - n - g(c, m)]$$

that gives utility as a function of consumption, labor supply, and money holdings. Thus, a simple shopping-time model can motivate the appearance of an MIU function and, more important, can help determine the properties of the partial derivatives of the function  $u$  with respect to  $m$ . By placing restrictions on the partial derivatives of the shopping-time production function  $g(c, m)$ , one can potentially determine what restrictions might be placed on the utility function  $u(c, m, n)$ . For example, if the marginal productivity of money goes to zero for some finite level of real money balances  $\bar{m}$ , that is,  $\lim_{m \rightarrow \bar{m}} g_m = 0$ , then this property will carry over to  $u_m$ .

In the MIU model, higher expected inflation lowers money holdings, but the effect on leisure and consumption depends on the signs of  $u_{lm}$  and  $u_{cm}$ .<sup>3</sup> The shopping time model implies that  $u_m = -v_l g_m \geq 0$ , so

$$u_{cm} = (v_{ll} g_c - v_{cl}) g_m - v_l g_{cm}. \quad (3.2)$$

The sign of  $u_{cm}$  depends on such factors as the effect of variations in leisure time on the marginal utility of consumption ( $v_{cl}$ ) and the effect of variations in consumption on the marginal productivity of money in reducing shopping time ( $g_{cm}$ ). In the benchmark MIU model of chapter 2,  $u_{cm}$  was taken to be positive.<sup>4</sup> Relating  $u_{cm}$  to the partials of the underlying utility function  $v$  and the transaction production function  $g$  can suggest whether this assumption was reasonable. From (3.2), the assumption of diminishing marginal utility of leisure ( $v_{ll} \leq 0$ ) and  $g_m \leq 0$  implies that  $v_{ll} g_c g_m \geq 0$ . If greater consumption raises the marginal productivity of money in reducing shopping time ( $g_{cm} \leq 0$ ), then  $-v_l g_{cm} \geq 0$  as well. Wang and Yip (1992) characterized the situation in which these two dominate, so that  $u_{cm} \geq 0$ , as the transaction services version of the MIU model. In this case, the MIU model implies that a rise in expected inflation would lower  $m$  and  $u_c$ , and this would lower

3. This is a statement about the partial equilibrium effect of inflation on the representative agent's decision. In general equilibrium, consumption and leisure are independent of inflation in models that display superneutrality.

4. This corresponded to  $b > \Phi$  in the benchmark utility function used in chapter 2.

consumption, labor supply, and output (see section 2.5.2). The reduction in labor supply is reinforced by the fact that  $u_{lm} = -v_{ll}g_m < 0$ , so the reduction in  $m$  raises the marginal utility of leisure.<sup>5</sup> If consumption and leisure are strong substitutes so that  $v_{cl} \leq 0$ , then  $u_{cm}$  could be negative, a situation Wang and Yip describe as corresponding to an asset substitution model. With  $u_{cm} < 0$ , a monetary injection that raises expected inflation increases consumption, labor supply, and output.

The household's intertemporal problem analyzed in chapter 2 for the MIU model can be easily modified to incorporate a shopping-time role for money. The household's objective is to maximize

$$\sum_{i=0}^{\infty} \beta^i v[c_{t+i}, 1 - n_{t+i} - g(c_{t+i}, m_{t+i})], \quad 0 < \beta < 1,$$

subject to

$$f(k_{t-1}, n_t) + \tau_t + (1 - \delta)k_{t-1} + \frac{(1 + i_{t-1})b_{t-1} + m_{t-1}}{1 + \pi_t} = c_t + k_t + b_t + m_t, \quad (3.3)$$

where  $f$  is a standard neoclassical production function,  $k$  is the capital stock,  $\delta$  is the depreciation rate,  $b$  and  $m$  are real bond and money holdings, and  $\tau$  is a real lump-sum transfer from the government.<sup>6</sup> Defining  $a_t = \tau_t + [(1 + i_{t-1})b_{t-1} + m_{t-1}]/(1 + \pi_t)$ , the household's decision problem can be written in terms of the value function  $V(a_t, k_{t-1})$ :

$$V(a_t, k_{t-1}) = \max \{v[c_t, 1 - n_t - g(c_t, m_t)] + \beta V(a_{t+1}, k_t)\},$$

where the maximization is subject to the constraints  $f(k_{t-1}, n_t) + (1 - \delta)k_{t-1} + a_t = c_t + k_t + b_t + m_t$  and  $a_{t+1} = \tau_{t+1} + [(1 + i_t)b_t + m_t]/(1 + \pi_{t+1})$ . Proceeding as in chapter 2 by using these two constraints to eliminate  $k_t$  and  $a_{t+1}$  from the expression for the value function, the necessary first-order conditions for consumption, real money holdings, real bond holdings, and labor supply are

$$v_c - v_{lg}c - \beta V_k(a_{t+1}, k_t) = 0, \quad (3.4)$$

$$-v_{lg}m + \beta \frac{V_a(a_{t+1}, k_t)}{1 + \pi_{t+1}} - \beta V_k(a_{t+1}, k_t) = 0, \quad (3.5)$$

$$-v_l + \beta V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t) = 0, \quad (3.6)$$

$$\beta \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) V_a(a_{t+1}, k_t) - \beta V_k(a_{t+1}, k_t) = 0, \quad (3.7)$$

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5. I thank Henrik Jensen for pointing this out.

6. It is assumed that transaction services are needed only for the purchase of consumption goods, not for the purchase of capital goods. In the next section, alternative treatments of investment and the transaction technology are shown to have implications for the steady state.

and the envelope theorem yields

$$V_a(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t), \quad (3.8)$$

$$V_k(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t) [f_k(k_{t-1}, n_t) + 1 - \delta]. \quad (3.9)$$

Letting  $w_t$  denote the marginal product of labor, that is,  $w_t = f_n(k_{t-1}, n_t)$ , (3.6) and (3.8) yield  $v_l = w_t V_a(a_t, k_{t-1})$ . This implies that (3.4) can be written as

$$v_c(c_t, l_t) = V_a(a_t, k_{t-1}) [1 + w_t g_c(c_t, m_t)]. \quad (3.10)$$

The marginal utility of consumption is set equal to the marginal utility of wealth,  $V_a(a_t, k_{t-1})$ , plus the cost, in utility units, of the marginal time needed to purchase consumption. Thus, the total cost of consumption includes the value of the shopping time involved. A marginal increase in consumption requires an additional  $g_c$  in shopping time. The value of this time in terms of goods is obtained by multiplying  $g_c$  by the real wage  $w$ , and its value in terms of utility is  $V_a(a, k)w g_c$ .

With  $g_m \leq 0$ ,  $v_l g_m = V_a w g_m$  is the value in utility terms of the shopping time savings that result from additional holdings of real money balances. Equations (3.5) and (3.8) imply that money will be held to the point where the marginal net benefit, equal to the value of shopping time savings plus the discounted value of money's wealth value in the next period, or  $-v_l g_m + \beta V_a(a_{t+1}, k_t)/(1 + \pi_{t+1})$ , just equals the net marginal utility of wealth. The first-order condition for optimal money holdings, together with (3.7) and (3.8), implies

$$\begin{aligned} -v_l g_m &= \beta V_k(a_{t+1}, k_t) - \beta \frac{V_a(a_{t+1}, k_t)}{1 + \pi_{t+1}} \\ &= V_a(a_t, k_{t-1}) \left[ 1 - \beta \frac{V_a(a_{t+1}, k_t)/V_a(a_t, k_{t-1})}{1 + \pi_{t+1}} \right] \\ &= V_a(a_t, k_{t-1}) \left[ 1 - \left( \frac{1}{1 + i_t} \right) \right] \\ &= V_a(a_t, k_{t-1}) \left( \frac{i_t}{1 + i_t} \right), \end{aligned} \quad (3.11)$$

where  $i_t$  is the nominal rate of interest and, using (3.7) and (3.8),  $(V_a(a_{t+1}, k_t)/V_a(a_t, k_{t-1})) = (1 + \pi_{t+1})/(1 + i_t)$ .<sup>7</sup>

Further insight can be gained by using (3.6) and (3.8) to note that (3.11) can also be written as

$$-w_t g_m = \frac{i_t}{1 + i_t}. \quad (3.12)$$

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7. Note that (3.11) implies  $-v_l g_m/V_a = i/(1 + i)$ . The left side is the value of the shopping time savings from holding additional real money balances relative to the marginal utility of income. The right side is the opportunity cost of holding money. This expression can be compared to the result from the MIU model, which showed that the marginal utility of real balances relative to the marginal utility of income would equal  $i/(1 + i)$ . In the MIU model, however, the marginal utility of income and the marginal utility of consumption were equal.

The left side of this equation is the value of the transaction time saved by holding additional real money balances. At the optimal level of money holdings, this is just equal to the opportunity cost of holding money,  $i/(1+i)$ .

Since no social cost of producing money has been introduced, optimality would require that the private marginal product of money,  $g_m$ , be driven to zero. Equation (3.12) implies that  $g_m = 0$  if and only if  $i = 0$ ; one thus obtains the standard result for the optimal rate of inflation, as seen earlier in the MIU model.

The chief advantage of the shopping time approach as a means of motivating the presence of money in the utility function is its use in tying the partials of the utility function with respect to money to the specification of the production function relating money, shopping time, and consumption. But this representation of the medium-of-exchange role of money is also clearly a shortcut. The transaction services production function  $\psi(m, n^s)$  is simply postulated; this approach does not help to determine what constitutes money. Why, for example, do certain types of green paper facilitate transactions (at least in the United States), while yellow pieces of paper don't? Section 3.4 reviews models based on search theory that attempt to derive money demand from a more primitive specification of the transaction process.

### 3.2.2 Real Resource Costs

An alternative approach to the CIA and shopping-time models is to assume that transaction costs take the form of real resources that are used up in the process of exchange (Brock 1974; 1990). An increase in the volume of goods exchanged leads to a rise in transaction costs, while higher average real money balances for a given volume of transactions lower costs. In a shopping time model, these costs are time costs and so enter the utility function indirectly by affecting the time available for leisure.

If goods must be used up in transacting, the household's budget constraint must be modified, for example, by adding a transaction costs term  $\Upsilon(c, m)$  that depends on the volume of transactions (represented by  $c$ ) and the level of money holdings. The budget constraint (3.3) then becomes

$$f(k_{t-1}) + (1 - \delta)k_{t-1} + \tau_t + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} \geq c_t + m_t + b_t + k_t + \Upsilon(c_t, m_t).$$

Feenstra (1986) considered a variety of transaction cost formulations and showed that they all lead to the presence of a function involving  $c$  and  $m$  appearing on the right side of the budget constraint. He also showed that transaction costs satisfy the following condition for all  $c, m \geq 0$ :  $\Upsilon$  is twice continuously differentiable and  $\Upsilon \geq 0$ ;  $\Upsilon(0, m) = 0$ ;  $\Upsilon_c \geq 0$ ;  $\Upsilon_m \leq 0$ ;  $\Upsilon_{cc}, \Upsilon_{mm} \geq 0$ ;  $\Upsilon_{cm} \leq 0$ ; and  $c + \Upsilon(c, m)$  is quasi-convex, with expansion paths having a non-negative slope. These conditions all have intuitive meaning:  $\Upsilon(0, m) = 0$  means the consumer bears no transaction costs if consumption is zero. The sign restrictions on the partial derivatives reflect the assumptions that transaction costs rise at an increasing



rate as consumption increases and money has positive but diminishing marginal productivity in reducing transaction costs. The assumption  $\Upsilon_{cm} \leq 0$  means the marginal transaction costs of additional consumption do not increase with money holdings. Expansion paths with non-negative slopes imply  $c + \Upsilon$  increases with income. Positive money holdings can be ensured by the additional assumption that  $\lim_{m \rightarrow 0} \Upsilon_m(c, m) = -\infty$ ; that is, money is essential.

Now consider how the MIU approach compares to a transaction cost approach. Suppose a function  $W(x, m)$  has the following properties: for all  $x, m \geq 0$ ,  $W$  is twice continuously differentiable and satisfies  $W \geq 0$ ;  $W(0, m) = 0$ ;  $W(x, m) \rightarrow \infty$  as  $x \rightarrow \infty$  for fixed  $m$ ;  $W_m \geq 0$ ;  $0 \leq W_x \leq 1$ ;  $W_{xx} \leq 0$ ;  $W_{mm} \leq 0$ ;  $W_{xm} \geq 0$ ;  $W$  is quasi-concave with Engel curves with a non-negative slope.

Now simplify by dropping capital and consider the following two static problems representing simple transaction cost and MIU approaches:

$$\max U(c) \text{ subject to } c + \Upsilon(c, m) + b + m = y \quad (3.13)$$

$$\max V(x, m) \text{ subject to } x + b + m = y, \quad (3.14)$$

where  $V(x, m) = U[W(x, m)]$ . These two problems are equivalent if  $(c^*, b^*, m^*)$  solves (3.13) if and only if  $(x^*, b^*, m^*)$  solves (3.14) with  $x^* = c^* + \Upsilon(c^*, m^*)$ . Feenstra (1986) showed that equivalence holds if the functions  $\Upsilon(c, m)$  and  $W(x, m)$  satisfy the stated conditions.

This “functional equivalence” (Wang and Yip 1992) between the transaction cost and MIU approaches suggests that conclusions derived within one framework also hold under the alternative approach. However, this equivalence is obtained by redefining variables. So, for example, the consumption variable  $x$  in the utility function is equal to consumption *inclusive* of transaction costs ( $x = c + \Upsilon(c, m)$ ) and is therefore not independent of money holdings. At the very least, the appropriate definition of the consumption variable needs to be considered if one attempts to use either framework to draw implications for actual macroeconomic time series.<sup>8</sup>

### 3.3 Cash-in-Advance (CIA) Models

A direct approach to generating a role for money, proposed by Clower (1967) and developed formally by Grandmont and Younes (1972) and Lucas (1980a), captures the role of money as a medium of exchange by requiring explicitly that money be used to purchase goods. Such a requirement can also be viewed as replacing the substitution possibilities between time and money highlighted in the shopping-time model with a transaction technology in which shopping time is zero if  $M/P \geq c$  and infinite otherwise

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8. When distortionary taxes are introduced, Mulligan and Sala-i-Martin (1997) showed the functional equivalence between the two approaches can depend on whether money is required to pay taxes.

(McCallum 1990a). This specification can be represented by assuming that the individual faces, in addition to a standard budget constraint, a cash-in-advance (CIA) constraint.<sup>9</sup>

The exact form of the CIA constraint depends on which transactions or purchases are subject to the CIA requirements. For example, both consumption goods and investment goods might be subject to the requirement. Or only consumption might be subject to the constraint. Or only a subset of all consumption goods might require cash for their purchase. The constraint will also depend on what constitutes cash. Can bank deposits that earn interest, for example, also be used to carry out transactions? The exact specification of the transactions subject to the CIA constraint can be important.

Timing assumptions also are important in CIA models. Lucas (1982) allows agents to allocate their portfolio between cash and other assets at the start of each period, after observing any current shocks but prior to purchasing goods. This timing is often described by saying that the asset market opens first and then the goods market opens. If there is a positive opportunity cost of holding money and the asset market opens first, agents will only hold an amount of money that is just sufficient to finance their desired level of consumption. Svensson (1985) has the goods market open first. This implies that agents have available for spending only the cash carried over from the previous period, and so cash balances must be chosen before agents know how much spending they will wish to undertake. For example, if uncertainty is resolved *after* money balances are chosen, agents may find they are holding cash balances that are too low to finance their desired spending level. Or they may be left with more cash than they need, thereby forgoing interest income.

To elucidate the structure of CIA models, section 3.3.1 reviews a simplified version of a model due to Svensson (1985). The simplification involves eliminating uncertainty. Once the basic framework has been reviewed, a stochastic CIA model is considered as a means of studying the role of money in a stochastic dynamic general equilibrium model (DSGE) in which business cycles are generated by both real productivity shocks and shocks to the growth rate of money. Developing a linearized version of the model illustrates how the CIA approach differs from the MIU approach.

### 3.3.1 The Certainty Case

This section develops a simple cash-in-advance model. Issues arising in the presence of uncertainty or the presence of labor-leisure choices are postponed. The timing of transactions and markets follows Svensson (1985), although the alternative timing used by Lucas (1982) is also discussed. After the model and its equilibrium conditions are set out, the steady state is examined and the welfare costs of inflation in a CIA model are discussed.

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9. Boianovsky (2002) discussed the early use in the 1960s of a CIA constraint by the Brazilian economist Mario Simonsen.

### The Model

Consider the following representative agent model. The agent's objective is to choose a path for consumption and asset holdings to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \quad (3.15)$$

for  $0 < \beta < 1$ , where  $u(\cdot)$  is bounded, continuously differentiable, strictly increasing, and strictly concave, and the maximization is subject to a sequence of CIA and budget constraints. The agent enters the period with money holdings  $M_{t-1}$  and receives a lump-sum transfer  $T_t$  (in nominal terms). If the goods market opens first, the CIA constraint takes the form

$$P_t c_t \leq M_{t-1} + T_t,$$

where  $c$  is real consumption,  $P$  is the aggregate price level, and  $T$  represents lump-sum transfers. In real terms,

$$c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{1 + \pi_t} + \tau_t, \quad (3.16)$$

where  $m_{t-1} = M_{t-1}/P_{t-1}$ ,  $\pi_t = (P_t/P_{t-1}) - 1$  is the inflation rate, and  $\tau_t = T_t/P_t$ . Note the timing:  $M_{t-1}$  refers to nominal money balances chosen by the agent in period  $t - 1$  and carried into period  $t$ . The real value of these balances is determined by the period  $t$  price level  $P_t$ . Since certainty is assumed, the agent knows  $P_t$  at the time  $M_{t-1}$  is chosen. This specification of the CIA constraint assumes that income from production during period  $t$  is not be available for consumption purchases until period  $t + 1$ .

The budget constraint, in nominal terms, is

$$P_t \omega_t \equiv P_t f(k_{t-1}) + (1 - \delta)P_t k_{t-1} + M_{t-1} + T_t + (1 + i_{t-1})B_{t-1} \geq P_t c_t + P_t k_t + M_t + B_t, \quad (3.17)$$

where  $\omega_t$  is the agent's time  $t$  real resources, consisting of income generated during period  $t$ ,  $f(k_{t-1})$ ; the undepreciated capital stock,  $(1 - \delta)k_{t-1}$ ; money holdings,  $m$ ; the transfer from the government,  $\tau$ ; and gross nominal interest earnings on the agent's  $t - 1$  holdings of nominal one-period bonds,  $(1 + i_{t-1})B_{t-1}$ . Physical capital depreciates at the rate  $\delta$ . These resources are used to purchase consumption, capital, bonds, and nominal money holdings, which are then carried into period  $t + 1$ . Dividing through by the time  $t$  price level, the budget constraint can be rewritten in real terms as

$$\omega_t \equiv f(k_{t-1}) + (1 - \delta)k_{t-1} + \tau_t + \frac{m_{t-1} + (1 + i_{t-1})b_{t-1}}{1 + \pi_t} \geq c_t + m_t + b_t + k_t, \quad (3.18)$$

where  $m$  and  $b$  are real cash and bond holdings. Note that real resources available to the representative agent in period  $t + 1$  are given by

$$\omega_{t+1} = f(k_t) + (1 - \delta)k_t + \tau_{t+1} + \frac{m_t + (1 + i_t)b_t}{1 + \pi_{t+1}}. \quad (3.19)$$

The period  $t$  gross nominal interest rate  $1 + i_t$  divided by  $1 + \pi_{t+1}$  is the gross real rate of return from period  $t$  to  $t + 1$  and can be denoted by  $1 + r_t = (1 + i_t)/(1 + \pi_{t+1})$ . With this notation, (3.19) can be written as

$$\omega_{t+1} = f(k_t) + (1 - \delta)k_t + \tau_{t+1} + (1 + r_t)a_t - \left(\frac{i_t}{1 + \pi_{t+1}}\right)m_t,$$

where  $a_t \equiv m_t + b_t$  is the agent's holding of nominal financial assets (money and bonds). This form highlights that there is a cost to holding money when the nominal interest rate is positive. This cost is  $i_t/(1 + \pi_{t+1})$ . Since this is the cost in terms of period  $t + 1$  real resources, the discounted cost at time  $t$  of holding an additional unit of money is  $i_t/(1 + r_t)(1 + \pi_{t+1}) = i_t/(1 + i_t)$ . This is the same expression for the opportunity cost of money obtained in chapter 2 in an MIU model.

Equation (3.16) is based on the timing convention that the goods market opens before the asset market. The model of Lucas (1982) assumed the reverse, and individuals can engage in asset transactions at the start of each period before the goods market has opened. In the present model, this would mean that the agent enters period  $t$  with financial wealth that can be used to purchase nominal bonds  $B_t$  or carried as cash into the goods market to purchase consumption goods. The CIA constraint would then take the form

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t - b_t. \quad (3.20)$$

In this case, the household is able to adjust its portfolio between money and bonds before entering the goods market to purchase consumption goods.

To understand the implications of this alternative timing, suppose there is a positive opportunity cost of holding money. Then, if the asset market opens first, the agent will only hold an amount of money that is just sufficient to finance the desired level of consumption. Since the opportunity cost of holding  $m$  is positive whenever the nominal interest rate is greater than zero, (3.20) will always hold with equality as long as the nominal rate of interest is positive. When uncertainty is introduced, the CIA constraint may not bind when (3.16) is used and the goods market opens before the asset market. For example, if period  $t$ 's income is uncertain and is realized after  $M_{t-1}$  has been chosen, a bad income realization may cause the agent to reduce consumption to a point where the CIA constraint is no longer binding. Or a disturbance that causes an unexpected price decline might, by increasing the real value of the agent's money holdings, result in a nonbinding constraint.<sup>10</sup> Since

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10. While uncertainty may cause the CIA constraint not to bind, it does not follow that the nominal interest rate will be zero. If money is held, the constraint must be binding in some states of nature. The nominal interest rate will equal the discounted expected value of money; see problem 4 at the end of this chapter.

a nonstochastic environment holds in this section, the CIA constraint binds under either timing assumption if the opportunity cost of holding money is positive. For a complete discussion and comparison of alternative assumptions about the timing of the asset and goods markets, see Salyer (1991). The remainder of this chapter follows Svensson (1985) in using (3.16) and assuming that consumption in period  $t$  is limited by the cash carried over from period  $t - 1$  plus any net transfer.

The choice variables at time  $t$  are  $c_t, m_t, b_t$ , and  $k_t$ . An individual agent's state at time  $t$  can be characterized by resources  $\omega_t$  and real cash holdings  $m_{t-1}$ ; both are relevant since consumption choice is constrained by the agent's resources and by cash holdings. To analyze the agent's decision problem, one can define the value function

$$V(\omega_t, m_{t-1}) = \max_{c_t, k_t, b_t, m_t} \{u(c_t) + \beta V(\omega_{t+1}, m_t)\}, \quad (3.21)$$

where the maximization is subject to the budget constraint (from 3.18)

$$\omega_t \geq c_t + m_t + b_t + k_t, \quad (3.22)$$

the CIA constraint (3.16), and the definition of  $\omega_{t+1}$  given by (3.19). Using this expression for  $\omega_{t+1}$  in (3.21) and letting  $\lambda_t$  ( $\mu_t$ ) denote the Lagrangian multiplier associated with the budget constraint (the CIA constraint), the first-order necessary conditions for the agent's choice of consumption, capital, bond, and money holdings take the form<sup>11</sup>

$$u_c(c_t) - \lambda_t - \mu_t = 0, \quad (3.23)$$

$$\beta [f_k(k_t) + 1 - \delta] V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0, \quad (3.24)$$

$$\beta(1 + r_t) V_\omega(\omega_{t+1}, m_t) - \lambda_t = 0, \quad (3.25)$$

$$\beta \left[ 1 + r_t - \frac{i_t}{1 + \pi_{t+1}} \right] V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \lambda_t = 0. \quad (3.26)$$

From the envelope theorem,

$$V_\omega(\omega_t, m_{t-1}) = \lambda_t, \quad (3.27)$$

$$V_m(\omega_t, m_{t-1}) = \left( \frac{1}{1 + \pi_t} \right) \mu_t. \quad (3.28)$$

From (3.27),  $\lambda_t$  is equal to the marginal utility of wealth. According to (3.23), the marginal utility of consumption exceeds the marginal utility of wealth by the value of liquidity services,  $\mu_t$ . The individual must hold money in order to purchase consumption, so the "cost," to which the marginal utility of consumption is set equal, is the marginal utility of wealth plus the cost of the liquidity services needed to finance the transaction.<sup>12</sup>

11. The first-order necessary conditions also include the transversality conditions.

12. Equation (3.23) can be compared to (3.10) from the shopping-time model.

In terms of  $\lambda$ , (3.25) becomes

$$\lambda_t = \beta(1 + r_t)\lambda_{t+1}, \quad (3.29)$$

which is a standard asset pricing equation and is a familiar condition from problems involving intertemporal optimization. Along the optimal path, the marginal cost (in terms of today's utility) from reducing wealth slightly,  $\lambda_t$ , must equal the utility value of carrying that wealth forward one period, earning a gross real return  $1 + r_t$ , where tomorrow's utility is discounted back to today at the rate  $\beta$ ; that is,  $\lambda_t = \beta(1 + r_t)\lambda_{t+1}$  along the optimal path.

Using (3.27) and (3.28), the first-order condition (3.26) can be expressed as

$$\lambda_t = \beta \left( \frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right). \quad (3.30)$$

Equation (3.30) can also be interpreted as an asset pricing equation for money. The price of a unit of money in terms of goods is just  $1/P_t$  at time  $t$ ; its value in utility terms is  $\lambda_t/P_t$ . By dividing (3.30) through by  $P_t$ , it can be rewritten as  $\lambda_t/P_t = \beta(\lambda_{t+1}/P_{t+1} + \mu_{t+1}/P_{t+1})$ . Solving this equation forward<sup>13</sup> implies that

$$\frac{\lambda_t}{P_t} = \sum_{i=1}^{\infty} \beta^i \left( \frac{\mu_{t+i}}{P_{t+i}} \right). \quad (3.31)$$

From (3.28),  $\mu_{t+i}/P_{t+i}$  is equal to  $V_m(\omega_{t+i}, m_{t+i-1})/P_{t+i-1}$ . This last expression, though, is just the partial of the value function with respect to time  $t+i-1$  *nominal* money balances:

$$\begin{aligned} \frac{\partial V(\omega_{t+i}, m_{t+i-1})}{\partial M_{t+i-1}} &= V_m(\omega_{t+i}, m_{t+i-1}) \left( \frac{\partial m_{t+i-1}}{\partial M_{t+i-1}} \right) \\ &= \frac{V_m(\omega_{t+i}, m_{t+i-1})}{P_{t+i-1}} \\ &= \left( \frac{\mu_{t+i}}{P_{t+i}} \right). \end{aligned}$$

This means one can rewrite (3.31) as

$$\frac{\lambda_t}{P_t} = \sum_{i=1}^{\infty} \beta^i \frac{\partial V(\omega_{t+i}, m_{t+i-1})}{\partial M_{t+i-1}}.$$

In other words, the current value of money in terms of utility is equal to the present value of the marginal utility of money in all future periods. Equation (3.31) is an interesting result; it says that money is just like any other asset in the sense that its value (i.e., its price today) is equal to the present discounted value of the stream of returns generated by the asset. In

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13. For references on solving difference equations forward in the context of rational-expectations models, see Blanchard and Kahn (1980) or McCallum (1989).

the case of money, these returns take the form of liquidity services.<sup>14</sup> If the CIA constraint were not binding, these liquidity services would not have value ( $\mu = V_m = 0$ ) and neither would money. But if the constraint is binding, then money has value because it yields valued liquidity services.<sup>15</sup>

The result that the value of money,  $\lambda/P$ , satisfies an asset pricing relationship is not unique to the CIA approach. For example, a similar relationship is implied by the MIU approach. The model employed in analyzing the MIU approach (see chapter 2) implied that

$$\frac{\lambda_t}{P_t} = \beta \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) + \frac{u_m(c_t, m_t)}{P_t},$$

which can be solved forward to yield

$$\frac{\lambda_t}{P_t} = \sum_{i=0}^{\infty} \beta^i \left[ \frac{u_m(c_{t+i}, m_{t+i})}{P_{t+i}} \right].$$

Here, the marginal utility of money  $u_m$  plays a role exactly analogous to that played by the Lagrangian on the CIA constraint  $\mu$ . The one difference is that in the MIU approach,  $m_t$  yields utility at time  $t$ , whereas in the CIA approach, the value of money accumulated at time  $t$  is measured by  $\mu_{t+1}$ , since the cash cannot be used to purchase consumption goods until period  $t + 1$ .<sup>16</sup>

An expression for the nominal rate of interest can be obtained by using (3.29) and (3.30) to get  $\lambda_t = \beta(1 + r_t)\lambda_{t+1} = \beta(\lambda_{t+1} + \mu_{t+1})/(1 + \pi_{t+1})$ , or  $(1 + r_t)(1 + \pi_{t+1})\lambda_{t+1} = (\lambda_{t+1} + \mu_{t+1})$ . Since  $1 + i_t = (1 + r_t)(1 + \pi_{t+1})$ , the nominal interest rate is given by

$$i_t = \left( \frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}} \right) - 1 = \frac{\mu_{t+1}}{\lambda_{t+1}}. \quad (3.32)$$

Thus, the nominal rate of interest is positive if and only if money yields liquidity services ( $\mu_{t+1} > 0$ ). In particular, if the nominal interest rate is positive, the CIA constraint is binding ( $\mu > 0$ ).

One can use the relationship between the nominal rate of interest and the Lagrangian multipliers to rewrite the expression for the marginal utility of consumption, given in (3.23), as

$$u_c = \lambda(1 + \mu/\lambda) = \lambda(1 + i) \geq \lambda. \quad (3.33)$$

14. The parallel expression for the shopping-time model can be obtained from (3.5) and (3.8). See problem 2 at the end of this chapter.

15. Bohn (1991b) analyzed the asset pricing implications of a CIA model. See also Salyer (1991).

16. Carlstrom and Fuerst (2001) argued that utility at time  $t$  should depend on money balances available for spending during period  $t$ , or  $M_{t-1}/P_t$ . This would make the timing more consistent with CIA models. With this timing,  $m_t$  is chosen at time  $t$  but yields utility at  $t + 1$ . In this case,  $\lambda_t/P_t = \sum_{i=1}^{\infty} \beta^i [u_m(c_{t+i}, m_{t+i})/P_{t+i}]$ , and the timing is the same as in the CIA model.

Since  $\lambda$  represents the marginal value of income, the marginal utility of consumption exceeds that of income whenever the nominal interest rate is positive. Even though the economy's technology allows output to be directly transformed into consumption, the "price" of consumption is not equal to 1; it is  $1 + i$ , since the household must hold money to finance consumption. Thus, in this CIA model, a positive nominal interest rate acts as a tax on consumption; it raises the price of consumption above its production cost.<sup>17</sup>

The CIA constraint holds with equality when the nominal rate of interest is positive, so  $c_t = M_{t-1}/P_t + \tau_t$ . Since the lump-sum monetary transfer  $\tau_t$  is equal to  $(M_t - M_{t-1})/P_t$ , this implies that  $c_t = M_t/P_t = m_t$ . Consequently, the consumption velocity of money is identically equal to 1 (velocity  $= P_t c_t / M_t = 1$ ). Since actual velocity varies over time, CIA models have been modified in ways that break this tight link between  $c$  and  $m$ . One way to avoid this is to introduce uncertainty (see Svensson 1985). If money balances have to be chosen prior to the resolution of uncertainty, it may turn out after the realization of shocks that the desired level of consumption is less than the amount of real money balances being held. In this case, some money balances will be unspent, and velocity can be less than 1. Velocity may also vary if the CIA constraint only applies to a subset of consumption goods. Then variations in the rate of inflation can lead to substitution between goods whose purchase requires cash and those whose purchase does not (see problem 6 at the end of this chapter).

### The Steady State

If consideration is restricted to the steady state, (3.29) implies that  $(1 + r^{ss}) = 1/\beta$ , and  $i = (1 + \pi^{ss})/\beta - 1 \approx 1/\beta - 1 + \pi^{ss}$ . In addition, (3.24) gives the steady-state capital stock as the solution to

$$f_k(k^{ss}) = r^{ss} + \delta = \frac{1}{\beta} - 1 + \delta.$$

So this CIA model, like the Sidrauski MIU model, exhibits superneutrality. The steady-state capital stock depends only on the time preference parameter  $\beta$ , the rate of depreciation  $\delta$ , and the production function. It is independent of the rate of inflation. Since steady-state consumption is equal to  $f(k^{ss}) - \delta k^{ss}$ , it, too, is independent of the rate of inflation.<sup>18</sup>

It has been shown that the marginal utility of consumption could be written as the marginal utility of wealth ( $\lambda$ ) times 1 plus the nominal rate of interest, reflecting the opportunity cost of holding the money required to purchase goods for consumption. Using (3.32), the ratio of the liquidity value of money, measured by the Lagrangian multiplier  $\mu$ , to the

17. In the shopping-time model, consumption is also taxed. See problem 3 at the end of this chapter.

18. The expression for steady-state consumption can be obtained from (3.18) by noting that  $m_t = \tau_t + m_{t-1}/\Pi_t$  and, with all households identical,  $b = 0$  in equilibrium. Then (3.18) reduces to  $c^{ss} + k^{ss} = f(k^{ss}) + (1 - \delta)k^{ss}$ , or  $c^{ss} = f(k^{ss}) - \delta k^{ss}$ .



marginal utility of consumption is

$$\frac{\mu}{u_c} = \frac{\mu}{\lambda(1+i)} = \frac{i}{1+i}.$$

This expression is exactly parallel to the result in the MIU framework, where the ratio of the marginal utility of money to the marginal utility of consumption was equal to the nominal interest rate divided by 1 plus the nominal rate, that is, the relative price of money in terms of consumption.

With the CIA constraint binding, real consumption is equal to real money balances. In the steady state, constant consumption implies that the stock of nominal money balances and the price level must be changing at the same rate. Define  $\theta$  as the growth rate of the nominal quantity of money (so that  $T_t = \theta M_{t-1}$ ); then

$$\pi^{ss} = \theta^{ss}.$$

The steady-state inflation rate is, as usual, determined by the rate of growth of the nominal money stock.

One difference between the CIA model and the MIU model is that with  $c^{ss}$  independent of inflation and the cash-in-advance constraint binding, the fact that  $c^{ss} = m^{ss}$  in the CIA model implies that steady-state money holdings are also independent of inflation.

### The Welfare Costs of Inflation

The CIA model, because it is based explicitly on behavioral relationships consistent with utility maximization, can be used to assess the welfare costs of inflation and to determine the optimal rate of inflation. The MIU approach had very strong implications for the optimal inflation rate. Steady-state utility of the representative household was maximized when the nominal rate of interest equaled zero. It has already been suggested that this conclusion continues to hold when money produces transaction services.

In the basic CIA model, however, there is no optimal rate of inflation that maximizes the steady-state welfare of the representative household. The reason follows directly from the specification of utility as a function only of consumption and the result that consumption is independent of the rate of inflation (superneutrality). Steady-state welfare is equal to

$$\sum_{t=0}^{\infty} \beta^t u(c^{ss}) = \frac{u(c^{ss})}{1-\beta}$$

and is invariant to the inflation rate. Comparing across steady states, any inflation rate is as good as any other.<sup>19</sup>

This finding is not robust to modifications in the basic CIA model. In particular, once the model is extended to incorporate a labor-leisure choice, consumption will no longer be

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19. By contrast, the optimal rate of inflation was well defined even in the basic Sidrauski model that exhibited superneutrality, since real money balances vary with inflation and directly affect utility in an MIU model.

independent of the inflation rate, and there will be a well-defined optimal rate of inflation. Because leisure can be “purchased” without the use of money (i.e., leisure is not subject to the CIA constraint), variations in the rate of inflation affect the marginal rate of substitution between consumption and leisure (see section 3.3.2). With different inflation rates leading to different levels of steady-state consumption and leisure, steady-state utility is a function of inflation. This type of substitution plays an important role in the model of Cooley and Hansen (1989) (see section 3.3.2). In their model, inflation leads to an increased demand for leisure and a reduction in labor supply. But before including a labor-leisure choice, it is useful to review briefly some other modifications of the basic CIA model, modifications that will, in general, generate a unique optimal rate of inflation.

**Cash and Credit Goods** Lucas and Stokey (1983; 1987) introduced the idea that the CIA constraint may only apply to a subset of consumption goods. They modeled this by assuming that the representative agent’s utility function is defined over consumption of two types of goods: cash goods and credit goods. In this case, paralleling (3.23), the marginal utility of cash goods is equated to  $\lambda + \mu \geq \lambda$ , while the marginal utility of credit goods is equated to  $\lambda$ . Hence, the CIA requirement for cash goods drives a wedge between the marginal utilities of the two types of goods. It is exactly as if the consumer faces a tax of  $\mu/\lambda = i$  on purchases of cash goods. Higher inflation, by raising the opportunity cost of holding cash, raises the tax on cash goods and generates a substitution away from the cash good and toward the credit good (see also Hartley 1988).

The obvious difficulty with this approach is that the classifications of goods into cash and credit goods is exogenous. And it is common to assume a one-good technology so that the goods are not differentiated by any technological considerations. The advantage of these models is that they can produce time variation in velocity. Recall that in the basic CIA model, any equilibrium with a positive nominal rate of interest is characterized by a binding CIA constraint, and this means that  $c = m$ . With both cash and credit goods,  $m$  will equal the consumption of cash goods, allowing the ratio of total consumption to money holdings to vary with expected inflation.<sup>20</sup>

**CIA and Investment Goods** A second modification to the basic model involves extending the CIA constraint to cover investment goods. In this case, the inflation tax applies to both consumption and investment goods. Higher rates of inflation tend to discourage capital accumulation, and Stockman (1981) showed that higher inflation would lower the steady-state capital-labor ratio (see also Abel 1985 and problem 9 at the end of this chapter).<sup>21</sup>

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20. Woodford (1998) studied a model with a continuum of goods indexed by  $i \in [0, 1]$ . A fraction  $s$ ,  $0 \leq s \leq 1$ , are cash goods. He then approximated a cashless economy by letting  $s \rightarrow 0$ .

21. Abel (1985) studied the dynamics of adjustment in a model in which the CIA constraint applies to both consumption and investment.

**Implications for Optimal Inflation** In CIA models, inflation acts as a tax on goods or activities whose purchase requires cash. This tax then introduces a distortion by creating a wedge between the marginal rates of transformation implied by the economy's technology and the marginal rates of substitution faced by consumers. Since the CIA model, like the MIU model, offers no reason for such a distortion to be introduced (there is no inefficiency that calls for Pigovian taxes or subsidies on particular activities, and the government's revenue needs can be met through lump-sum taxation), optimality calls for setting the inflation tax equal to zero. The inflation tax is directly related to the nominal rate of interest; a zero inflation tax is achieved when the nominal rate of interest is equal to zero.

### 3.3.2 A Stochastic CIA Model

While the models of Lucas (1982), Svensson (1985), and Lucas and Stokey (1987) provide theoretical frameworks for assessing the role of inflation, they do not provide any guide to the empirical magnitude of inflation effects or to the welfare costs of inflation. What one would like is a dynamic equilibrium model that could be simulated under alternative monetary policies—for example, for alternative steady-state rates of inflation or alternative policy responses to shocks—in order to assess quantitatively the effects of inflation and monetary policy. Such an exercise was conducted by Cooley and Hansen (1989; 1991), who were the first to add money and a cash-in-advance constraint to a calibrated real business cycle model. They followed the basic framework of Lucas and Stokey (1987). However, important aspects of their specification include (1) introduction of capital, and consequently an investment decision; (2) the introduction of a labor-leisure choice; and (3) the identification of consumption as the cash good and investment and leisure as credit goods.

Inflation represents a tax on the purchases of the cash good, and therefore higher rates of inflation shift household demand away from the cash good and toward the credit good. In Cooley and Hansen's formulation, this implies that higher inflation increases the demand for leisure. One effect of higher inflation, then, is to reduce the supply of labor. This then reduces output, consumption, investment, and the steady-state capital stock.

Cooley and Hansen expressed welfare losses across steady states in terms of the consumption increase (as a percentage of output) required to yield the same utility as would arise if the CIA constraint were nonbinding.<sup>22</sup> For a 10 percent inflation rate, they reported a welfare cost of inflation of 0.387 percent of output if the CIA constraint is assumed to apply at a quarterly time interval. Not surprisingly, if the constraint binds only at a monthly time interval, the cost falls to 0.112 percent of output. These costs are small. For much higher rates of inflation, they start to look significant. For example, with a monthly time

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22. Refer to Cooley and Hansen (1989, sec. II) or Hansen and Prescott (1994) for discussions of the computational aspects of this exercise.

period for the CIA constraint, a 400 percent annual rate of inflation generates a welfare loss equal to 2.137 percent of output. The welfare costs of inflation are discussed further in section 3.4.2 and in chapter 4.

### The Basic Model

To model the behavior of the representative agent faced with uncertainty and a CIA constraint, assume the agent's objective is to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i}) = E_0 \sum_{i=0}^{\infty} \beta^i \left[ \frac{c_{t+i}^{1-\Phi}}{1-\Phi} + \Psi \frac{(1 - n_{t+i})^{1-\eta}}{1-\eta} \right], \quad (3.34)$$

with  $0 < \beta < 1$ . Here  $c_t$  is real consumption, and  $n_t$  is labor supplied to market activities, expressed as a fraction of the total time available, so that  $1 - n_t$  is equal to leisure time.<sup>23</sup> The parameters  $\Phi$ ,  $\Psi$ , and  $\eta$  are restricted to be positive.

Households supply labor and rent capital to firms that produce goods. The household enters each period with nominal money balances  $M_{t-1}$  and receives a nominal lump-sum transfer equal to  $T_t$ . In the aggregate, this transfer is related to the growth rate of the nominal supply of money. Letting the stochastic variable  $\theta_t$  denote the rate of money growth ( $M_t = (1 + \theta_t)M_{t-1}$ ), the per capita transfer equals  $\theta_t M_{t-1}$ . At the start of period  $t$ ,  $\theta_t$  is known to all households. Households purchase bonds  $B_t$ , and their remaining cash is available for purchasing consumption goods. Thus, the timing has asset markets opening first, and the CIA constraint, which is taken to apply only to the purchase of consumption goods, takes the form

$$P_t c_t \leq M_{t-1} + T_t - B_t,$$

where  $P_t$  is the time  $t$  price level. Note that time  $t$  transfers are available to be spent in period  $t$ . In real terms, the CIA constraint becomes

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t - b_t. \quad (3.35)$$

Here  $1 + \pi_t$  is equal to 1 plus the rate of inflation. The CIA constraint will always be binding if the nominal interest rate is positive.

In addition to the CIA constraint, the household faces a flow budget constraint in nominal terms of the form

$$M_t = P_t [Y_t + (1 - \delta)K_{t-1} - K_t] + (1 + i_t)B_t + (M_{t-1} + T_t - P_t c_t - B_t).$$

23. In order to allow for comparison between the MIU model developed earlier and a CIA model, the preference function used earlier, (2.66) in chapter 2, is modified by setting  $a = 1$  and  $b = 0$  so that real balances do not yield direct utility. The resulting utility function given in (3.34) differs from Cooley and Hansen's specification; they assume that the preferences of the identical (ex ante) households are log separable in consumption and leisure, a case obtained when  $\Phi = \eta = 1$ .

In real terms, this becomes

$$m_t = y_t + (1 - \delta)k_{t-1} + i_t b_t - k_t + \frac{m_{t-1}}{1 + \pi_t} + \tau_t - c_t, \quad (3.36)$$

where  $0 \leq \delta \leq 1$  is the depreciation rate.

The household is assumed to own the economy's technology, given by a Cobb-Douglas constant returns to scale production function, which can be expressed in per capita terms as

$$y_t = e^{z_t} k_{t-1}^\alpha n_t^{1-\alpha}, \quad (3.37)$$

where  $0 \leq \alpha \leq 1$ . The exogenous productivity shock  $z_t$  is assumed to follow an AR(1) process:

$$z_t = \rho_z z_{t-1} + e_t,$$

with  $0 \leq \rho_z \leq 1$ . The innovation  $e_t$  has mean zero and variance  $\sigma_e^2$ .

The individual's decision problem can be characterized by the value function

$$V(k_{t-1}, b_{t-1}, m_{t-1}) = \max_{c_t, n_t, k_t, b_t, m_t} \left[ \frac{c_t^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} + \beta E_t V(k_t, b_t, m_t) \right],$$

where the maximization is subject to the constraints (3.35) and (3.36).

If  $\lambda_t$  is the Lagrangian multiplier on the budget constraint and  $\mu_t$  is the multiplier on the cash-in-advance constraint, these first-order conditions take the form

$$c_t^{-\Phi} = \lambda_t + \mu_t, \quad (3.38)$$

$$\Psi(1-n)^{-\eta} = (1-\alpha) \left( \frac{y_t}{n_t} \right) \lambda_t, \quad (3.39)$$

$$\lambda_t = \beta E_t (1+r_t) \lambda_{t+1}, \quad (3.40)$$

$$i_t \lambda_t - \mu_t = 0, \quad (3.41)$$

$$\lambda_t = \beta E_t \left[ \frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right], \quad (3.42)$$

where  $r_t = \alpha (y_{t+1}/k_t) - \delta$ .

Finally, let  $u_t = \theta_t - \theta^{\text{SS}}$  be the deviation of money growth from its steady-state average rate and assume

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \varphi_t,$$

where  $\varphi_t$  is a white noise innovation with variance  $\sigma_\varphi^2$ . This is the same process for the nominal growth rate of money that was used in chapter 2.

### The Steady State

With the same parameter calibrations as those reported in section 2.5.4 for the MIU model, the steady-state values of the ratios that were reported for the MIU model are also the steady-state values for the CIA model (see the chapter appendix). The Euler condition ensures  $1 + r^{ss} = 1/\beta$ , which then implies  $y^{ss}/k^{ss} = (r^{ss} + \delta)/\alpha$  and, with investment in the steady state equal to  $\delta k^{ss}$ ,  $c^{ss}/k^{ss} = (y^{ss}/k^{ss}) - \delta$ . Even though the method used to generate a demand for money has changed in moving from the MIU model to the CIA model, the steady-state values of the output-capital and consumption-capital ratios are unchanged. Note that none of these steady-state ratios depends on the growth rate of the nominal money supply. The level of real money balances in the steady state is then determined by the cash-in-advance constraint, which is binding as long as the nominal rate of interest is positive. Hence,  $c^{ss} = m^{ss}/(1 + \pi^{ss}) + \tau^{ss} = m^{ss}$ , so  $m^{ss}/k^{ss} = c^{ss}/k^{ss}$ .

The steady-state labor supply depends on the money growth rate and therefore on the rate of inflation. The chapter appendix shows that  $n^{ss}$  satisfies

$$(1 - n^{ss})^{-\eta} (n^{ss})^{\Phi} = \left( \frac{1 - \alpha}{\Psi} \right) \left( \frac{\beta}{1 + \theta^{ss}} \right) \left( \frac{y^{ss}}{k^{ss}} \right)^{\frac{\Phi - \alpha}{1 - \alpha}} \left( \frac{c^{ss}}{k^{ss}} \right)^{-\Phi}, \quad (3.43)$$

where  $\theta$  is the steady-state rate of money growth. Since the left side of this expression is increasing in  $n^{ss}$ , a rise in  $\theta^{ss}$ , which implies a rise in the inflation rate, lowers the steady-state labor supply. Higher inflation taxes consumption and causes households to substitute toward more leisure. This is the source of the welfare cost of inflation in this CIA model. The elasticity of labor supply with respect to the growth rate of money is negative.

It is useful to note the similarity between the expression for steady-state labor supply in the CIA model and the corresponding expression (see (2.80) in chapter 2) that was obtained in the MIU model. With the MIU specification, faster money growth had an ambiguous effect on the supply of labor. With the calibrated values of the parameters of the utility function used in chapter 2, money and consumption were complements, so higher inflation, by reducing real money holdings, lowered the marginal utility of consumption and also reduced the supply of labor.

### Dynamics

The dynamic implications of the CIA model can be explored by obtaining a first-order linear approximation around the steady state of the model's equilibrium conditions. The derivation of the approximation is contained in the chapter appendix. As in chapter 2, a variable  $\hat{x}$  denotes the percentage deviation of  $x$  around the steady state.<sup>24</sup> The CIA model can be approximated around the steady state by the following ten linear equations:

$$\hat{y}_t = \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t, \quad (3.44)$$

24. The exceptions again being that  $\hat{r}$  and  $\hat{\tau}$  are expressed in percentage terms (e.g.,  $\hat{r}_t = r_t - r^{ss}$ ).

$$\left(\frac{y^{ss}}{k^{ss}}\right) \hat{y}_t = \left(\frac{c^{ss}}{k^{ss}}\right) \hat{c}_t + \delta \hat{x}_t, \quad (3.45)$$

$$\hat{k}_t = (1 - \delta) k_{t-1} + \delta \hat{x}_t, \quad (3.46)$$

$$\hat{r}_t = \alpha \left(\frac{y^{ss}}{k^{ss}}\right) \left(E_t \hat{y}_{t+1} - \hat{k}_t\right), \quad (3.47)$$

$$\hat{\lambda}_t = \hat{r}_t + E_t \hat{\lambda}_{t+1}, \quad (3.48)$$

$$\eta \left(\frac{n^{ss}}{1 - n^{ss}}\right) \hat{n}_t - \hat{\lambda}_t = \hat{y}_t - \hat{n}_t, \quad (3.49)$$

$$-\Phi \hat{c}_t = \hat{\lambda}_t + \hat{i}_t, \quad (3.50)$$

$$\hat{\lambda}_t = -\Phi E_t \hat{c}_{t+1} - E_t \hat{\pi}_{t+1}, \quad (3.51)$$

$$\hat{c}_t = \hat{m}_t, \quad (3.52)$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t. \quad (3.53)$$

The first six equations (production function, resource constraint, capital accumulation equation, marginal product of capital equation, Euler condition, and labor-leisure condition) are identical to those found with the MIU approach. The critical differences between the two approaches appear in a comparison of (3.50), (3.51), and (3.52) with (2.74) and (2.75) of chapter 2. In the MIU model, utility depended directly on money holdings, so (2.75) expressed the marginal utility of consumption in terms of  $\hat{c}_t$  and  $\hat{m}_t$ . In the CIA model, the marginal utility of income can differ from the marginal utility of consumption; (3.51) reflects the fact that an extra dollar of income received in period  $t$  cannot be spent on consumption until  $t + 1$ . Equation (3.42) gives  $\lambda_t = \beta E_t (\lambda_{t+1} + \mu_{t+1}) / (1 + \pi_{t+1})$ .<sup>25</sup> Since the marginal utility of consumption  $c_t^{-\Phi}$  is equated to  $\lambda_t + \mu_t$ , this becomes  $\lambda_t = \beta E_t c_{t+1}^{-\Phi} / (1 + \pi_{t+1}) = \beta E_t m_{t+1}^{-\Phi} / (1 + \pi_{t+1})$ . Linearizing this result produces (3.51). Equation (2.75) was the MIU money demand condition derived from the first-order condition for the household's holdings of real money balances. In the CIA model, (3.50) and (3.52) reflect the presence of the nominal interest rate as a tax on consumption and the binding cash-in-advance constraint in the CIA model. Finally, note that (3.48), (3.50), and (3.51) can be combined to yield the Fisher equation:  $\hat{r}_t = E_t (\hat{i}_{t+1} - \hat{\pi}_{t+1})$ .

### Calibration and Simulations

To assess the effects of money in this CIA model, values must be assigned to the specific parameters; that is, the model must be calibrated. The steady state depends on the values of  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\eta$ ,  $\Psi$ , and  $\Phi$ . The baseline values reported in section 2.5.4 for the MIU model can be employed for the CIA model as well.

25. Equation (3.30) is the corresponding equation for the nonstochastic CIA model of section 3.3.1.

Recall that the MIU model displayed short-run dynamics in which the real variables such as output, consumption, the capital stock, and employment were independent of the nominal money supply process when utility was log-linear in consumption and money balances.<sup>26</sup> While  $\hat{m}$  does not directly enter the utility function in the CIA model, note that in the case of log utility in consumption (that is, when  $\Phi = 1$ ), the short-run real dynamics in the CIA model are not independent of the process followed by  $\hat{m}$ , as they were in the MIU model. Equations (3.49), (3.51), and (3.53) imply, when  $\Phi = 1$ , that

$$\hat{\lambda}_t = -E_t(\hat{m}_{t+1} + \hat{\pi}_{t+1}) = -(\hat{m}_t + E_t u_{t+1}) = \left(1 + \eta \frac{n^{ss}}{l^{ss}}\right) \hat{n}_t - \hat{y}_t.$$

Thus, variations in the expected future growth rate of money,  $E_t u_{t+1}$ , force adjustment to  $\hat{y}$ ,  $\hat{c}$  ( $\hat{m}$ ), or  $\hat{n}$  (or all three). In particular, for given output and consumption, higher expected money growth (and therefore higher expected inflation) produces a fall in  $\hat{n}_t$ . This is the effect, discussed earlier, by which higher inflation reduces labor supply and output.

The current growth rate of the nominal money stock,  $u_t$ , and the current rate of inflation,  $\pi_t$ , only appear in the form  $u_t - \pi_t$  (see 3.53). Hence, as seen in the MIU model, *unanticipated* monetary shocks affect only current inflation and have no real effects unless they alter expectations of future money growth (i.e., unless  $E_t u_{t+1}$  is affected).

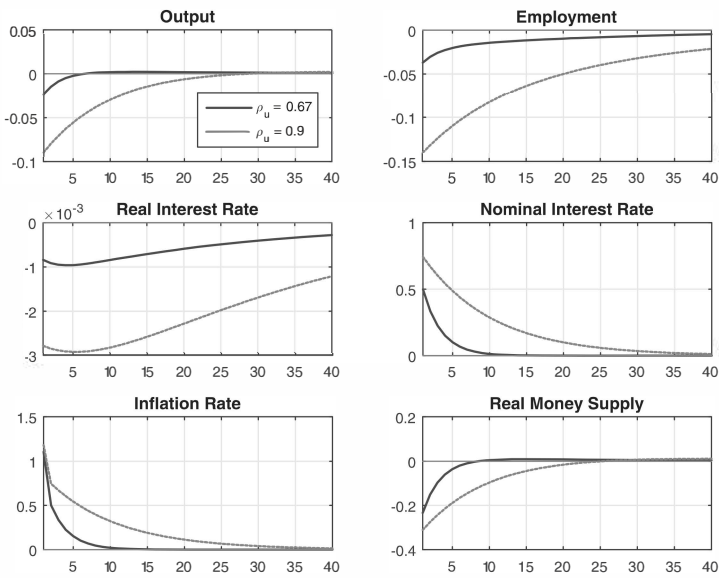
The responses of output, employment, and other variables to a positive money growth rate shock are illustrated in figure 3.1. As in the MIU model under the baseline calibration, a positive money growth rate shock reduces output and employment, and the impact is larger the more highly positively serially correlated the shock is. The rise in money growth immediately raises expected inflation when  $\rho_u > 0$  and the nominal interest rate. Greater persistence of the money growth rate process leads to larger movements in expected inflation in response to a monetary shock. By raising the expected rate of inflation and thereby increasing the inflation tax on consumption, the money growth rate increase induces a substitution toward leisure that lowers labor supply and output. These effects are larger the more persistent the rise in expected inflation.<sup>27</sup>

The economy's response to a productivity shock depends on the money growth rate process when  $\phi$  differs from zero. This is illustrated in figure 3.2. For example, when  $\phi$  is negative, a positive productivity shock implies that money growth will decline in the future. Consequently, expected inflation also declines. The resulting reduction in the nominal interest rate lowers the effective inflation tax on consumption and increases labor supply. In contrast, when  $\phi$  is positive, a positive productivity shock increases expected inflation and reduces labor supply. This tends to partially offset the effect of the productivity shock on

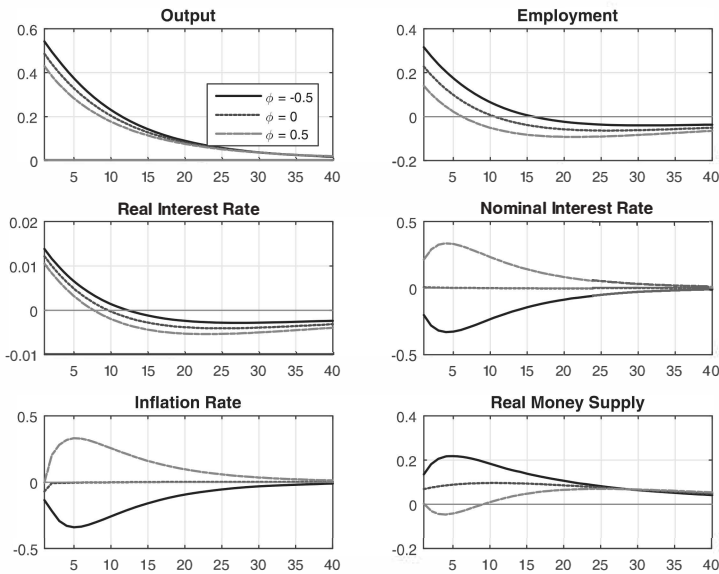
26. This was the case in which  $\Phi = b = 1$ .

27. Comparing figure 3.1 with figure 2.4 reveals that a money growth rate shock has a larger real impact in the CIA model than in the MIU model of chapter 2; this difference would be larger if a smaller value of the money demand parameter  $b$  had been used in the MIU model.





**Figure 3.1**  
Responses to a positive money growth rate shock in the CIA model;  $\rho_u = 0.67, \rho_u = 0.9$ .



**Figure 3.2**  
Effects of  $\phi$  on responses to a productivity shock in the CIA model.

output. Thus, output variability is less when  $\phi$  is positive than when it is zero or negative. However, the effects are small; as  $\phi$  goes from  $-0.5$  to  $0$  to  $0.5$ , the standard deviation of output falls from  $1.31$  to  $1.16$  to  $1.02$ .

### 3.4 Search

Both the MIU and the CIA approaches are useful alternatives for introducing money into a general equilibrium framework. However, neither approach is very specific about the exact role played by money. MIU models assume the direct utility yielded by money proxies for the services money produces in facilitating transactions. However, the nature of these transactions and, more important, the resource costs they might involve, and how these costs might be reduced by holding money, are not specified. Use of the CIA model is motivated by appealing to the idea that some form of nominal asset is required to facilitate transactions. Yet the constraint used is extreme, implying that there are no alternative means of carrying out certain transactions. The CIA constraint is meant to capture the essential role of money as a medium of exchange, but in this case one might wish to start from a specification of the transaction technology to understand why some commodities and assets serve as money and others do not.

A number of papers have employed search theory to motivate the development of media of exchange; this has been one of the most active areas of monetary theory. Examples include Jones (1976), Diamond (1984), Kiyotaki and Wright (1989; 1993), Oh (1989), Trejos and Wright (1993; 1995), Ritter (1995), Shi (1995), Rupert, Schindler, and Wright (2001), Lagos and Wright (2005), Rocheteau and Wright (2005), and the papers in the May 2005 issue of the *International Economic Review*. Williamson and Wright (2011) and Nosal and Rocheteau (2011) provided excellent surveys of the literature. In these models, individual agents must exchange the goods they produce (or with which they are endowed) for the goods they consume. During each period, individuals randomly meet other agents; exchange takes place if it is mutually beneficial. In a barter economy, exchange is possible only if an agent holding good  $i$  and wishing to consume good  $j$  (call this an  $ij$  agent) meets an individual holding good  $j$  who wishes to consume good  $i$  ( $ji$  agent). This requirement is known as the *double coincident of wants* and limits the feasibility of direct barter exchange when production is highly specialized. Trade could occur if agent  $ij$  meets a  $ki$  agent for  $k \neq j$  as long as exchange of goods is costless and the probability of meeting a  $jk$  agent is the same as meeting a  $ji$  agent. In this case, agent  $ij$  would be willing to exchange  $i$  for  $k$  (thereby becoming an  $kj$  agent).

In the basic Kiyotaki-Wright model, direct exchange of commodities is assumed to be costly, but there exists a fiat money that can be traded costlessly for commodities. The assumption that there exists money with certain exchange properties (costless trade with commodities) serves a role similar to that of putting money directly into the utility function

in the MIU approach or specifying that money must be used in certain types of transactions in the CIA approach.<sup>28</sup> More recent work on search and exchange assumes trading is anonymous, so credit is precluded; one would not accept an IOU from a trading partner if one were unable to identify or locate that person when wanting to collect.<sup>29</sup> However, whether an agent will accept money in exchange for goods will depend on the probability the agent places on being able later to exchange money for a consumption good.

Suppose agents are endowed with a new good according to a Poisson process with arrival rate  $a$ .<sup>30</sup> Trading opportunities arrive at rate  $b$ . A successful trade can occur if there is a double coincidence of wants. If  $x$  is the probability that another agent chosen at random is willing to accept the trader's commodity, the probability of a double coincidence of wants is  $x^2$ . A successful trade can also take place if there is a single coincidence of wants (i.e., one of the agents has a good the other wants), if one agent has money and the other agent is willing to accept it. That is, a trade can take place when an  $ij$  agent meets a  $jk$  agent if the  $ij$  agent has money and the  $jk$  agent is willing to accept it.

In this simple framework, agents can be in one of three states: an agent can be waiting for a new endowment to arrive (state 0), can have a good to trade and be waiting to find a trading partner (state 1), or have money and be waiting for a trading opportunity (state  $m$ ).

Three equilibria are possible. Suppose the probability of making a trade holding money is less than the probability of making a trade holding a commodity. In this case, individuals will prefer to hold on to their good when they meet another trader (absent a double coincidence) rather than trade for money. With no one willing to trade for money, money will be valueless in equilibrium. A second equilibrium arises when holding money makes a successful trade more likely than continuing to hold a commodity. So every agent will be willing to hold money, and in equilibrium all agents will be willing to accept money in exchange for goods. A mixed monetary equilibrium can also exist: agents accept money with some probability as long as they believe other agents will accept it with the same probability.

The Kiyotaki-Wright model emphasized the exchange process and the possibility for an intrinsically valueless money to be accepted in trade. It does so, however, by assuming a fixed rate of exchange—one unit of money is exchanged for one unit of goods whenever a trade takes place. The value of money in terms of goods is either 0 (in a nonmonetary

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28. In an early analysis, Alchian (1977) attempted to explain why there might exist a commodity with the types of exchange properties assumed in the search literature. He stressed the role of information and the costs of assessing quality. Any commodity whose quality can be assessed at low cost can facilitate the acquisition of information about other goods by serving as a medium of exchange. Models assuming an absence of a technology for record keeping rule out credit. For an analysis of "money as memory," see Kocherlakota (1998).

29. Anonymity is treated as given, and the role of third parties, such as credit card companies and banks, that solve this problem in monetary economies is precluded by assumption.

30. Kiyotaki and Wright (1993) interpreted this as a production technology.

equilibrium) or 1. In the subsequent literature, however, the goods price of money is determined endogenously as part of the equilibrium. For example, Trejos and Wright (1995) made price the outcome of a bargaining process between buyers and sellers who meet through a process similar to that in the Kiyotaki-Wright model. However, Trejos and Wright assumed money is indivisible, while goods are infinitely divisible (i.e., all trades involve one dollar, but the quantity of goods exchanged for that dollar may vary). Shi (1997) extended the Kiyotaki-Wright search model to include divisible goods *and* divisible money, and Shi (1999) also analyzed inflation and its effects on growth in a search model.

### 3.4.1 Centralized and Decentralized Markets

Lagos and Wright (2005) provided the core example of a monetary search model and the insights about the costs of inflation that this literature has provided. Money is assumed to be perfectly divisible and is the only storable good available to agents. Each period is divided into subperiods, called day and night. Agents consume and supply labor (produce) in both subperiods. The subperiods differ in terms of their market structure. Night markets are centralized and competitive; day markets are decentralized, and prices (and quantities) are set via bargaining between individual agents in bilateral meetings.

The preferences of agents are identical and given by

$$U = U(x, h, X, H) = u(x) - c(h) + U(X) - H,$$

where  $x$  ( $X$ ) is consumption during the day (night), and  $h$  ( $H$ ) labor supply during the day (night). The utility functions  $u$ ,  $c$ , and  $U$  have standard properties, and it is assumed that there exist  $q^*$  and  $X^*$  such that  $u'(q^*) = c'(q^*)$  and  $U'(X^*) = 1$ . Utility is linear in night labor supply  $H$ . The technology allows one unit of  $H$  to be transformed into one unit of  $X$ . Hence  $X^*$  is the quantity of the night good such that marginal utility equals marginal cost.

During the night, trading takes place in a centralized Walrasian market. Consider the decision problem of an agent who enters the night market with nominal money balances  $m$ . Let  $\phi_t$  denote the price of money in terms of goods (i.e., the price level, the price of goods in terms of money, is  $1/\phi_t$ ). Let  $W_t(m)$  be the value function for an agent at the start of the night market with money holdings  $m$ , and let  $V_{t+1}(m')$  be the value function for the agent entering the day market with money holdings  $m'$  (described later). Then  $W_t(m)$  is defined as

$$W_t(m) = \max_{X, H, m'} [U(X) - H + \beta V_{t+1}(m')],$$

where the maximization is subject to a budget constraint of the form

$$\phi_t m + H = X + \phi_t m'.$$

The left side represents the agent's real money holdings on entering the night market plus income generated from production. The right side is consumption plus real balances carried

into the next day market. Using the budget constraint to eliminate  $H$ , the problem can be rewritten as

$$W_t(m) = \max_{X, m'} [U(X) + \phi_t m - X + \phi_t m' + \beta V_{t+1}(m')]. \quad (3.54)$$

The first-order conditions for an interior solution take the form<sup>31</sup>

$$U(X) = 1 \Rightarrow X = X^*, \quad (3.55)$$

$$\phi_t + \beta V'_{t+1}(m') = 0. \quad (3.56)$$

Equations (3.55) and (3.56) imply that  $X$  and  $m'$  are independent of  $m$ . This is a consequence of the assumption that utility is linear in  $H$ . Intuitively, the marginal value of accumulating an extra dollar in the centralized market is  $\beta V'_{t+1}(m')$ . The marginal cost of acquiring an extra dollar is  $\phi_t$  times the utility cost of the extra labor needed to produce and sell more output. But the marginal disutility of work is a constant (equal to 1). So the marginal cost of acquiring an extra dollar is just  $\phi_t$ , which is the same for all agents. But if all agents exit the night market holding the same level of money balances, that is, the same  $m'$ , the distribution of money holdings across agents at the start of each day will be degenerate. This is extremely useful in dealing with a model in which agents may have different market experiences, as they will in Lagos and Wright's day market, while still preserving the idea of a representative agent. Shi (1999) adopted the notion of a large family whose individual members may have different experiences during each period but who reunite into a representative family at the end of each period. This approach, originally introduced by Lucas (1990), is used in chapter 5 when discussing models that impose restrictions on access by some agents to credit markets.

A final useful result from (3.54) is that  $W$  can be written as

$$W_t(m) = \phi_t m + \max_{X, m'} [U(X) - X + \phi_t m' + \beta V_{t+1}(m')],$$

showing that  $W$  is linear in  $m$ .

The subperiods differ in the nature of the trading process that occurs in each. The day good  $x$  comes in different varieties, and agents each consume a different variety than the one they produce. Hence, there is a motive for trade. As in the night market, one unit of labor can be converted into one unit of the good. In the day market, agents search for trading partners. With probability  $\alpha$ , they meet another agent. One of three possible outcomes can occur as a result of this meeting. First, each consumes what the other produces. This corresponds to a double coincidence of wants; no money or credit is necessary for a trade to occur. Assume the probability of a double coincidence of wants is  $\delta$ . Second, there could be a single coincidence of wants; one agent consumes what the other produces, but not vice

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31. Because of the linearity of utility in  $H$ , Lagos and Wright (2005) needed to verify that  $H < \bar{H}$  in equilibrium, where  $\bar{H}$  is the maximum labor time an agent has available.

versa. Assume the probability of this occurring is  $2\sigma$ .<sup>32</sup> Finally, neither agent consumes what the other produces, an event that occurs with probability  $1 - \delta - 2\sigma$ .

Recall that  $V_t(m)$  is the value function for an agent with money holdings  $m$  who is entering the decentralized day market, and  $W_t(m)$  is the value function when entering the centralized night market. Let  $F_t(\tilde{m})$  be the fraction of agents at the beginning of day  $t$  with  $m \leq \tilde{m}$ . Then

$$\begin{aligned}
V_t(m) &= \alpha\delta \int B_t(m, \tilde{m}) dF_t(\tilde{m}) \\
&+ \alpha\sigma \int \{u[q_t(m, \tilde{m})] + W_t[m - d_t(m, \tilde{m})]\} dF_t(\tilde{m}) \\
&+ \alpha\sigma \int \{-c[q_t(\tilde{m}, m)] + W_t[m + d_t(\tilde{m}, m)]\} dF_t(\tilde{m}) \\
&+ (1 - \alpha\delta - 2\alpha\sigma) W_t(m), \tag{3.57}
\end{aligned}$$

where  $B_t(m, \tilde{m})$  is the payoff to an agent holding  $m$  who meets an agent holding  $\tilde{m}$  when there is a double coincidence of wants. The four terms in  $V_t(m)$  are (1) the probability of a double coincidence times the expected payoff; (2) the probability the agent meets another agent with  $\tilde{m}$ , there is a single coincidence of wants, and  $d_t(m, \tilde{m})$  is exchanged for  $q_t(m, \tilde{m})$  of the consumption good; (3) the probability of a single coincidence meeting in which the agent produces  $q_t(\tilde{m}, m)$  and receives  $d_t(\tilde{m}, m)$ ; and (4) the probability that no meeting (or trade) occurs and the agent enters the night market with  $m$ .

Because the day meetings each involve just two agents, the search literature has generally assumed the price and quantity exchanged,  $q_t$  and  $d_t$ , are determined by Nash bargaining between the agents. When a double coincidence of wants occurs, the joint surplus is maximized when  $q^*$  is exchanged, where

$$u'(q^*) = c'(q^*).$$

Hence,  $B_t(m, \tilde{m}) = u(q^*) - c(q^*) + W_t(m)$ .

When a single coincidence occurs, bargaining is more complicated. Let the buyer's share of the joint surplus from a bargain be  $\theta \in [0, 1]$ . The threat point of a buyer is  $W_t(m)$ ; that of the seller is  $W_t(\tilde{m})$ , where  $m$  and  $\tilde{m}$  are the buyer's and the seller's initial money holdings. The exchange of  $q$  for  $d$  units of money maximizes

$$[u(q) + W_t(m - d) - W_t(m)]^\theta [-c(q) + W_t(m + d) - W_t(m)]^{1-\theta}, \tag{3.58}$$

subject to  $d \geq 0, q \geq 0$ . Recall that  $W_t(m)$  is linear in  $m$ . Hence, (3.58) can be rewritten as

$$[u(q) - \phi_t d]^\theta [-c(q) + \phi_t d]^{1-\theta}. \tag{3.59}$$

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32. For agents  $i$  and  $j$ , the probability  $i$  consumes what  $j$  produces but not vice versa is  $\sigma$ ; the probability  $j$  consumes what  $i$  produces but not vice versa is also  $\sigma$ . Thus, the probability a meeting satisfies a single coincidence of wants is  $2\sigma$ .

If  $d \leq m$ , money holdings are not a binding constraint, and the first-order conditions with respect to  $d$  and  $q$  yield

$$-\theta\phi_t[u(q) - \phi_t d]^{-1} + (1 - \theta)\phi_t[-c(q) + \phi_t d]^{-1} = 0,$$

$$\theta u'(q)[u(q) - \phi_t d]^{-1} - (1 - \theta)c'(q)[-c(q) + \phi_t d]^{-1} = 0,$$

or

$$u'(q) = c'(q) \Rightarrow q_t = q^*,$$

$$\phi_t d^* = \theta c(q^*) + (1 - \theta)u(q^*).$$

The monetary cost of  $q, d^*$ , is a weighted average of the cost of producing it and the value of consuming it, with weights reflecting the bargaining power of the buyer and seller.

If  $d^* > m$ , then the buyer does not have the cash necessary to purchase  $q^*$ ; in effect, the cash-in-advance constraint is binding. In this case, Lagos and Wright (2005) showed that the seller receives all the buyer's money, so

$$\phi_t d_t = \phi_t m = z(q_t), \tag{3.60}$$

where  $q_t$  is the solution to a constrained Nash bargaining problem.<sup>33</sup> The quantity transacted and the price depend on the buyer's money holdings but do not depend on the seller's. This quantity can be expressed as a function of  $m$ :  $q_t = q_t(m)$ .

Lagos and Wright showed that  $m'$ , the amount of money agents carry out of the night market, is less than  $d^*$  whenever the inflation rate,  $(\phi_t/\phi_{t+1}) - 1$ , exceeds  $\beta - 1$ . Recall that an inflation rate of  $\beta - 1$  corresponds to the Friedman rule of a zero nominal interest rate. So, just as in the earlier CIA models, the cash-in-advance constraint is binding when the nominal rate of interest is positive. Of course, the constraint only binds for agents who find themselves as buyers in single coincidence of wants meetings. Sellers, or those in a double coincidence of wants meeting, or in no meeting, exit the period with unchanged money holdings.

Now consider the value to an agent of entering the day market with money holdings  $m$ . This value arises from the effects of  $m$  on price and quantity when the agent is the buyer in a single coincidence meeting. Since the probability this occurs is  $\alpha\sigma$ , it can be expressed, using (3.57), as

$$v_t(m) = \alpha\sigma \int \{u[q_t(m)] - \phi_t d_t(m)\} dF_t(\tilde{m}).$$

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33.  $\hat{q}(m)$  solves

$$\frac{\theta c(q)u'(q) + (1 - \theta)u(q)c'(q)}{\theta u'(q) + (1 - \theta)c'(q)} = \phi_t m.$$

See Lagos and Wright (2005) for details.

The value of money is then given by the pricing equation

$$\phi_t = \beta [v'_{t+1}(M) + \phi_{t+1}], \quad (3.61)$$

where  $M$  is the aggregate nominal quantity of money. Because  $d'_{t+1}(M) = 1$  (an increase in the quantity of money increases the number of dollars needed to purchase goods by the same amount),

$$v'_{t+1}(M) = \alpha\sigma [u' [q_{t+1}(M)] q'_{t+1}(M) - \phi_{t+1}].$$

Using this in (3.61),

$$\phi_t = \beta\alpha\sigma u' [q_{t+1}] q'_{t+1}(M) + \beta(1 - \alpha\sigma)\phi_{t+1}. \quad (3.62)$$

The value of money is determined by the marginal utility of the goods the agent is able to consume when faced with a single coincidence of wants trading opportunity. If such meetings are uncommon ( $\alpha\sigma$  is small), money will be less useful and therefore less valuable. This implication of search models of money emphasizes the importance of the trading environment for determining the value of money.

Equation (3.62) can be rewritten<sup>34</sup> using (3.60) as

$$\phi_t = \beta \left[ \alpha\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} + (1 - \alpha\sigma) \right] \phi_{t+1}.$$

Now consider a steady state in which the money stock grows at the rate  $\tau$ . The inflation rate will also equal  $\tau$ :  $(\phi_t/\phi_{t+1}) - 1 = \tau$ . Thus,

$$\phi_t = \beta \left[ \alpha\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} + (1 - \alpha\sigma) \right] \phi_{t+1} \Rightarrow 1 = \beta \left[ \alpha\sigma \frac{u'(q)}{z'(q)} + (1 - \alpha\sigma) \right] \left( \frac{1}{1 + \tau} \right)$$

using (3.60). Solving for  $u'/z'$ ,

$$\frac{u'(q)}{z'(q)} = \frac{1 + \tau - \beta(1 - \alpha\sigma)}{\beta\alpha\sigma} = 1 + \frac{1 + \tau - \beta}{\beta\alpha\sigma}.$$

The left side of this equation,  $u'/z'$ , is the marginal utility of consumption divided by the marginal cost of the good. The right side is 1 plus a term that can be written as  $\beta^{-1}(1 + \tau) - 1$  divided by  $\alpha\sigma$ . But since  $\beta^{-1}$  is the gross real interest rate and  $\tau$  is the inflation rate,  $\beta^{-1}(1 + \tau) - 1$  is the nominal rate of interest, so

$$\frac{u'(q)}{z'(q)} = 1 + \frac{i}{\alpha\sigma}. \quad (3.63)$$

This looks very similar to earlier results from a CIA model (see (3.33)). A positive nominal interest rate acts as a tax on consumption. But this tax now also depends on the nature of trading. An increase in the frequency of single coincidence meetings, by raising the usefulness of money, reduces the net cost of holding money.

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34. Equation (3.60) implies  $q' = \phi/z'$ .



### 3.4.2 The Welfare Costs of Inflation

While it is clear from (3.63) that the inflation tax is zero if the Friedman rule of a zero nominal interest rate is followed, Lagos and Wright showed that the equilibrium with  $i = 0$  is still not fully efficient because of the trading frictions associated with bargaining in the decentralized market. Efficiency requires that all the surplus go to the buyer ( $\theta = 1$ ).<sup>35</sup> In standard models such as the MIU model in chapter 2 or the CIA model in section 3.3, full efficiency is attained with  $i = 0$ . Then, since  $i = 0$  maximizes welfare, small deviations have small effects on welfare (basically an application of the envelope theorem). But if  $\theta < 1$ , the equilibrium with  $i = 0$  in the search model does not fully maximize utility. Hence, small deviations from the Friedman rule can have first-order effects on welfare. By calibrating their model, Lagos and Wright found much larger welfare costs of positive nominal interest rates than other authors had found.

The importance of the trading environment in determining the costs of inflation is further explored by Rocheteau and Wright (2005). They compared welfare costs in three settings: a search model similar to Lagos and Wright (2005), a competitive market model, and a search model with posted prices (rather than the bilateral bargaining of the basic search model). By allowing for endogenous determination of the number of market participants, Rocheteau and Wright introduced an extensive margin (the effects on the value of money as the number of traders varies) as well as an intensive margin (the effects for a given number of traders as individual agents' money holdings vary). The Friedman rule always ensures efficiency along the intensive margin, but the extensive margin may still generate a source of inefficiency. Interestingly, if the market makers in the competitive search version of the model internalize the effects of the prices they post on the number of traders they attract, the model endogenously ensures that the Hosios condition is satisfied, as shown by Moen (1997), and the equilibrium is fully efficient when the nominal rate of interest is zero. Lagos and Rocheteau (2005) explored the interactions of the pricing mechanism (bilateral bargaining versus posted pricing) and found that with directed search, inflation can increase search intensities when inflation is low but reduce them when inflation is high. Thus, at low inflation rates, an increase in inflation can raise output, but they showed that this actually reduces welfare, and the Friedman rule supports the efficient equilibrium. Craig and Rocheteau (2008) demonstrated the search approach can account for the estimates of the welfare costs of inflation obtained from examining the area under the money demand curve, as discussed in chapter 2.

The Lagos and Wright model has only one nominal asset money. If an interest-bearing nominal asset such as a bond were introduced into the analysis, it would dominate money whenever the nominal interest rate is positive. To explain the simultaneous existence of

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35. This is essentially the Hosios (1990) condition for this model; since the quantity transacted is independent of the seller's money holdings, all the surplus is due to the buyer, so efficiency would require  $\theta = 1$ .

interest-bearing nominal bonds and non-interest-bearing money, Shi (2005) employed a model with a decentralized goods market and a centralized bond market but in which there are assumed to be barriers to trading across markets. Households can use either bonds or money in the goods market, but only money can be used to purchase bonds. At the start of each period, households must allocate their money holdings between the two markets. Assume a fraction  $a$  is sent to the goods market and  $1 - a$  to the bond market. Let  $\omega_t^m$  denote the value of money at the end of period  $t$ . Then Shi showed that

$$\omega_t^m = \beta a \alpha \sigma \lambda_{t+1}^m + \beta \omega_{t+1}^m,$$

where  $\beta$  is the discount factor,  $\alpha$  and  $\sigma$  are the probability of meeting a potential trading partner and the probability there is single coincidence of wants, and  $\lambda^m$  is the Lagrangian multiplier on the constraint that the money payment from buyer to seller in the goods market must be less than the buyer's money holdings. Thus,  $a \alpha \sigma \lambda_{t+1}^m$  is the service value of money in facilitating a goods purchase. The current value of money is equal to this service value plus the discounted future value of money.

Money the household sends to the bond market cannot be used to purchase current goods, nor can the newly purchased bonds be used to exchange for goods. While bonds can, in future periods, be used to purchase goods, purchasing bonds initially entails a one-period loss of liquidity. Therefore, bonds must sell at a discount relative to money; if  $S$  is the money price of a bond,  $S < 1$ . Shi demonstrates that the nominal interest rate,  $(1 - S)/S$ , is given by

$$\frac{1 - S}{S} = \frac{a \alpha \sigma \lambda^m}{\omega^m} \quad (3.64)$$

which is positive if  $\lambda^m$  is positive. This expression for the nominal interest rate can be compared to (3.32), obtained in a basic cash-in-advance model. Similar to the result in other models in the search literature, (3.64) reveals how the nature of transactions in the decentralized market as reflected in the parameters  $\alpha$  and  $\sigma$  affects the value of money and the nominal interest rate.

In Shi's basic model, old bonds and money can both circulate in the goods market and be used in purchasing goods. Suppose, however, that the government also engages as a seller in the goods market, and assume the government only accepts money in payment for goods. Since there is a chance a household will encounter a government seller in the decentralized market, and frictions are assumed to prevent the household from locating another seller, there is a smaller probability of a successful trade if the household carries only bonds into the goods market than if it carries money. This difference drives bonds out of the goods market, and Shi showed that only money circulates as a means of payment.

The search-theoretic approach to monetary economics provides a natural framework for addressing a number of issues. Ritter (1995) used it to examine the conditions necessary

for fiat money to arise, linking it to the credibility of the issuer. Governments lacking credibility would be expected to overissue the currency to gain seigniorage. In this case, agents would be unwilling to hold the fiat money. Soller and Waller (2000) used a search-theoretic approach to study the coexistence of legal and illegal currencies. By stressing the role of money in facilitating exchange, the search-theoretic approach emphasizes the role of money as a medium of exchange. The approach also emphasizes the social aspect of valued money; agents are willing to accept fiat money only in environments in which they expect others to accept such money.<sup>36</sup>

### 3.5 Summary

The models studied in this chapter are among the basic frameworks monetary economists have found useful for studying the effects of inflation and the welfare implications of alternative rates of inflation. These models, and those examined in chapter 2, assume prices are perfectly flexible, adjusting to ensure that market equilibrium is continuously maintained. The MIU, CIA, shopping-time, and search models all represent means of introducing valued money into a general equilibrium framework. Each approach captures some aspects of the role that money plays in facilitating transactions.

Despite the different approaches, several conclusions are common to all. First, because the price level is completely flexible, the value of money, equal to 1 over the price of goods, behaves like an asset price.<sup>37</sup> The return money yields, however, differs in the various approaches. In the MIU model, the marginal utility of money is the direct return, while in the CIA model, this return is measured by the Lagrangian multiplier on the CIA constraint. In the shopping-time model, the return arises from the time savings provided by money in carrying out transactions, and the value of this time savings depends on the real wage. In search models, it depends on the probability of trading opportunities.

All these models have similar implications for the optimal rate of inflation. An efficient equilibrium is characterized by equality between social and private costs. Because the social cost of producing money is taken to be zero, the private opportunity cost of holding money must be zero in order to achieve optimality. The private opportunity cost is measured by the nominal interest rate, so the optimal rate of inflation in the steady state is the rate that achieves a zero nominal rate of interest. While this result is quite general, two important considerations have been ignored: the effects of inflation on government revenue and the interaction of inflation with other taxes in a nonindexed tax system. These are among the topics of chapter 4.

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36. Samuelson (1958) provided one of the earliest modern treatments of money as a social construct.

37. Of course, this is clearly not the case in the search models that assume fixed prices.

### 3.6 Appendix: The CIA Approximation

The method used to obtain a linear approximation around the steady state for the CIA model is discussed here. Since the approach is similar to the one followed for the MIU model, some details are skipped. The basic equations of the model are given by (3.44)–(3.53).

#### 3.6.1 The Steady State

With a binding CIA constraint,  $c^{ss} = \tau^{ss} + m^{ss}/(1 + \pi^{ss})$ , but in a steady state with  $m$  constant,  $\tau^{ss} + m^{ss}/(1 + \pi^{ss}) = m^{ss}$ . Thus,  $c^{ss} = m^{ss}$ , and  $m^{ss}/c^{ss} = 1$ .

From the first-order condition for the household's choice of  $n$ ,

$$\Psi(1 - n^{ss})^{-\eta} = (1 - \alpha) \left( \frac{y^{ss}}{n^{ss}} \right) \lambda^{ss}, \quad (3.65)$$

and since  $y^{ss}/k^{ss}$  takes on the same values as in the MIU model (because the production technology and the discount factor are identical), it only remains to determine the marginal utility of income  $\lambda^{ss}$ . From (3.38) and (3.41),  $(c^{ss})^{-\Phi} = \lambda^{ss} + \mu^{ss} = \lambda^{ss}(1 + i^{ss})$ . Using this relationship in (3.42) yields

$$\lambda^{ss} = \beta \left[ \frac{\lambda^{ss}(1 + i^{ss})}{1 + \theta^{ss}} \right] \Rightarrow 1 + i^{ss} = \frac{1 + \theta^{ss}}{\beta},$$

where  $\theta^{ss} = \pi^{ss}$ . This is the steady-state version of the Fisher equation, and it means one can write

$$\lambda^{ss} = \frac{(c^{ss})^{-\Phi}}{1 + i^{ss}} = \frac{\beta(c^{ss})^{-\Phi}}{1 + \theta^{ss}}.$$

Combining this with (3.65) and multiplying and dividing appropriately by  $k^{ss}$  and  $n^{ss}$ ,

$$\Psi(1 - n^{ss})^{-\eta} = (1 - \alpha) \left( \frac{\beta}{1 + \theta^{ss}} \right) \left( \frac{y^{ss}}{k^{ss}} \right) \left( \frac{c^{ss}}{k^{ss}} \right)^{-\Phi} \left( \frac{k^{ss}}{n^{ss}} \right)^{1-\Phi} (n^{ss})^{-\Phi}.$$

The production function implies that  $n^{ss}/k^{ss} = (y^{ss}/k^{ss})^{\frac{1}{1-\alpha}}$ , so one obtains

$$(1 - n^{ss})^{-\eta} (n^{ss})^{\Phi} = \left( \frac{1 - \alpha}{\Psi} \right) \left( \frac{\beta}{1 + \theta^{ss}} \right) \left( \frac{y^{ss}}{k^{ss}} \right)^{\frac{\Phi-\alpha}{1-\alpha}} \left( \frac{c^{ss}}{k^{ss}} \right)^{-\Phi}.$$

It is useful to note that the expressions for  $y^{ss}/k^{ss}$ ,  $c^{ss}/k^{ss}$ ,  $r^{ss}$ , and  $n^{ss}/k^{ss}$  are identical to those obtained in the MIU model. Only the equation determining  $n^{ss}$  differs from the one found in chapter 2.

#### 3.6.2 The Linear Approximation

Expressions linear in the percentage deviations around the steady state can be obtained for the economy's production function, and resource constraint, the definition of the marginal

product of capital, and the first-order conditions for consumption, money holdings, and labor supply, just as was done for the MIU model of chapter 2. The economy's production function, and resource constraint, the definition of the marginal product of capital, and the labor-leisure first-order condition are identical to those of the MIU model,<sup>38</sup> so they are simply stated here:

$$\hat{y}_t = \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t, \quad (3.66)$$

$$\hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \left( \frac{y^{ss}}{k^{ss}} \right) \hat{y}_t - \left( \frac{c^{ss}}{k^{ss}} \right) \hat{c}_t, \quad (3.67)$$

$$\hat{r}_t = \alpha \left( \frac{y^{ss}}{k^{ss}} \right) (E_t \hat{y}_{t+1} - \hat{k}_t), \quad (3.68)$$

$$\left( 1 + \eta \frac{n^{ss}}{l^{ss}} \right) \hat{n}_t = \hat{y}_t + \hat{\lambda}_t. \quad (3.69)$$

The Euler condition linking the marginal utility of income to its expected future value and the real return on capital, (3.40), becomes

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{r}_t. \quad (3.70)$$

Equations (3.38) and (3.41) imply

$$c_t^{-\Phi} = \lambda_t (1 + i_t).$$

When linearized, this yields

$$-\Phi \hat{c}_t = \hat{\lambda}_t + \hat{i}_t. \quad (3.71)$$

From (3.38) and (3.42),

$$\lambda_t = \beta E_t \left[ \frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right] = \beta E_t \left[ \frac{c_{t+1}^{-\Phi}}{1 + \pi_{t+1}} \right].$$

When this is linearized around the steady state, one obtains

$$\hat{\lambda}_t = -\Phi E_t \hat{c}_{t+1} - E_t \pi_{t+1}. \quad (3.72)$$

From the CIA constraint,

$$c_t = m_t \quad (3.73)$$

in an equilibrium with a positive nominal rate of interest.

Finally, define  $\hat{x}_t$  as the percentage deviation of investment around the steady state:

$$\hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \delta \hat{x}_t. \quad (3.74)$$

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38. See the chapter 2 appendix.

### Collecting All Equations

To summarize, the linearized model consists of equations (3.66), (3.67), (3.68), (3.69), (3.70), (3.71), (3.72), (3.73), and (3.74), together with the processes for the two exogenous shocks and the equation governing the evolution of real money balances. The resulting twelve equations solve for  $z_t$ ,  $u_t$ ,  $\hat{y}_t$ ,  $\hat{k}_t$ ,  $\hat{n}_t$ ,  $\hat{\lambda}_t$ ,  $\hat{c}_t$ ,  $\hat{x}_t$ ,  $\hat{m}_t$ ,  $\hat{r}_t$ ,  $\hat{l}_t$ , and  $\hat{\pi}_t$ . Collecting all the equilibrium conditions together, they are

$$z_t = \rho_z z_{t-1} + e_t,$$

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \varphi_t,$$

and (3.44)–(3.53).

Additional details on the derivation of the linearized CIA model and the MATLAB program used to simulate it are available at <http://people.ucsc.edu/~walshc/mtp4e/>.

### 3.7 Problems

1. Suppose the production function for shopping takes the form  $\psi = c = e^x (n^s)^a m^b$ , where  $a$  and  $b$  are both positive but less than 1, and  $x$  is a productivity factor. The agent's utility is given by  $v(c, l) = c^{1-\Phi}/(1-\Phi) + l^{1-\eta}/(1-\eta)$ , where  $l = 1 - n - n^s$ , and  $n$  is time spent in market employment.
  - a. Derive the transaction time function  $g(c, m) = n^s$ .
  - b. Derive the money-in-the-utility function specification implied by the shopping production function. How does the marginal utility of money depend on the parameters  $a$  and  $b$ ? How does it depend on  $x$ ?
  - c. Is the marginal utility of consumption increasing or decreasing in  $m$ ?
2. Using (3.5) and (3.8), show that

$$\frac{V_a(a_t, k_{t-1})}{P_t} = - \sum_{i=0}^{\infty} \beta^i \left( \frac{v_l(a_{t+i}, k_{t+i-1}) g_m(c_{t+i}, m_{t+i})}{P_{t+i}} \right).$$

Interpret this equation. How does it compare to (3.31)?

3. Show that, for the shopping-time model (section 3.2.1), the tax on consumption is given by

$$- \left( \frac{i_t}{1 + i_t} \right) \left( \frac{g_c}{g_m} \right).$$

(Recall that money reduced shopping time, so  $g_m \leq 0$ .) Provide an intuitive interpretation for this expression.

4. In the model of section 3.3.2, suppose the current CIA constraint is not binding. This implies  $\mu_t = 0$ . Use (3.41) and (3.42) to show that money still has value at time  $t$  (that is, the price level at time  $t$  is finite) as long as the CIA constraint is expected to bind in the future.
5. MIU and CIA models are alternative approaches to constructing models in which money has positive value in equilibrium.
  - a. What strengths and weaknesses do you see in each of these approaches?
  - b. Suppose you wanted to study the effects of the growth of credit cards on money demand. Which approach would you adopt? Why?
6. Modify the basic model of section 3.3.1 by assuming utility depends on the consumption of two goods,  $C_t^m$  and  $C_t^c$ . Purchases of  $C_t^m$  are subject to a cash-in-advance constraint; purchases of  $C_t^c$  are not. The two goods are produced by the same technology:  $C_t^m + C_t^c = Y_t = f(k_t)$ .
  - a. Write the household's decision problem.
  - b. Write the first-order conditions for the household's optimal choices for  $C_t^m$  and  $C_t^c$ . How are these affected by the cash-in-advance constraint?
  - c. Show that the nominal rate of interest acts as a tax on the consumption of  $C_t^m$ .
7. Assume the model of section 3.3.1 is modified so that only a fraction  $\psi$  of consumption must be purchased using cash. In this case, the cash-in-advance constraint takes the form
 
$$\psi c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t, 0 < \psi \leq 1.$$
  - a. Write the household's decision problem.
  - b. Write the household's first-order conditions. How are these affected by  $\psi$ ?
  - c. If  $\psi$  were a choice variable of the household, would it ever choose  $\psi > 0$ ?
8. Modify the model of section 3.3.2 so that only a fraction  $\psi$  of consumption is subject to the cash-in-advance constraint. How is the impact of a serially correlated shock to the money growth rate on real output affected by  $\psi$ ? (Use the programs available at <http://people.ucsc.edu/~walshc/mtp4e/> to answer this question, and compare the impulse response of output for  $\psi = 0.25, 0.5, 0.75$ , and 1.)
9. Consider the model of section 3.3.1. Suppose that money is required to purchase both consumption and investment goods. The CIA constraint then becomes  $c_t + x_t \leq m_{t-1}/(1 + \pi_t) + \tau_t$ , where  $x$  is investment. Assume that the aggregate production function takes the form  $y_t = e^{z_t} k_{t-1}^\alpha n_t^{1-\alpha}$ . Show that the steady-state capital-labor ratio is affected by the rate of inflation. Does a rise in inflation raise or lower the steady-state capital-labor ratio? Explain.

10. Consider the following model. Preferences are given by

$$E_t \sum_{i=0}^{\infty} \beta^i [\ln c_{t+i} + \theta \ln d_{t+i}],$$

and the budget and CIA constraints take the form

$$c_t + d_t + m_t + k_t = Ak_{t-1}^{\alpha} + (1 - \delta)k_{t-1} + \tau_t + \frac{m_{t-1}}{1 + \pi_t}, \quad (3.75)$$

$$c_t \leq \tau_t + \frac{m_{t-1}}{1 + \pi_t}, \quad (3.76)$$

where  $m$  denotes real money balances, and  $\pi_t$  is the inflation rate from period  $t - 1$  to period  $t$ . The two consumption goods,  $c$  and  $d$ , represent cash ( $c$ ) and credit ( $d$ ) goods. The net transfer  $\tau$  is viewed as a lump-sum payment (or tax) by the household.

- Does this model exhibit superneutrality? Explain.
  - What is the rate of inflation that maximizes steady-state utility?
11. Consider the following specification for the representative household. Preferences are given by

$$E_t \sum_{i=0}^{\infty} \beta^i [\ln c_{t+i} + \ln d_{t+i}],$$

and the budget constraint is

$$c_t + d_t + m_t + k_t = Ak_{t-1}^{\alpha} + \tau_t + \frac{m_{t-1}}{1 + \pi_t} + (1 - \delta)k_{t-1},$$

where  $m$  denotes real money balances, and  $\pi_t$  is the inflation rate from period  $t - 1$  to period  $t$ . Utility depends on the consumption of two types of good:  $c$  must be purchased with cash, while  $d$  can be purchased using either cash or credit. The net transfer  $\tau$  is viewed as a lump-sum payment (or tax) by the household. If a fraction  $\theta$  of  $d$  is purchased using cash, then the household also faces a CIA constraint that takes the form

$$c_t + \theta d_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t.$$

What is the relationship between the nominal rate of interest and whether the CIA constraint is binding? Explain. Will the household ever use cash to purchase  $d$  (i.e., will the optimal  $\theta$  ever be greater than zero)?

12. Suppose the representative household enters period  $t$  with nominal money balances  $M_{t-1}$  and receives a lump-sum transfer  $T_t$ . During period  $t$ , the bond market opens first, and the household receives interest payments and purchases nominal bonds in the



amount  $B_t$ . With its remaining money  $(M_{t-1} + T_t + (1 + i_{t-1})B_{t-1} - B_t)$ , the household enters the goods market and purchases consumption goods subject to

$$P_t c_t \leq M_{t-1} + T_t + (1 + i_{t-1})B_{t-1} - B_t.$$

The household receives income at the end of the period and ends period  $t$  with nominal money holdings  $M_t$ , given by

$$M_t = P_t e^{z_t} K_{t-1}^\alpha N_t^{1-\alpha} + (1 - \delta)P_t K_{t-1} - P_t K_t + M_{t-1} + T_t + (1 + i_{t-1})B_{t-1} - B_t - P_t c_t.$$

If the household's objective is to maximize

$$E_0 \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - N_{t+i}) = E_0 \sum_{i=0}^{\infty} \beta^i \left[ \frac{c_{t+i}^{1-\Phi}}{1 - \Phi} + \Psi \frac{(1 - N_{t+i})^{1-\eta}}{1 - \eta} \right],$$

do the equilibrium conditions differ from (3.38)–(3.42)?

13. Trejos and Wright (1993) found that if no search is allowed while bargaining takes place, output tends to be too low (the marginal utility of output exceeds the marginal production costs). Show that output is also too low in a basic CIA model. (For simplicity, assume that only labor is needed to produce output according to the production function  $y = n$ .) Does the same hold true for an MIU model?
14. For the bargaining problem of section 3.4.1, the buyer and seller exchange  $q$  for  $d$ , where these two values maximize (3.58). Verify that when money holdings are not a constraint,
 
$$\phi_t d^* = \theta c(q^*) + (1 - \theta)u(q^*).$$
15. Equation (3.63) shows how the nominal interest rate acts as a positive tax on consumption. Discuss how this condition compares to (3.33) from the basic CIA model. If the CIA model is interpreted as one in which trading takes place with certainty and always involves a single coincidence of wants, can the CIA model be viewed as a special case of the search model?
16. This question deals with the Lagos and Wright (2005) model.
  - a. Lagos and Wright divide each period into a decentralized market and a centralized market. What aspects of the model ensure that all agents leave the centralized market with the same money holdings, even though different agents enter the centralized market with different money holdings?
  - b. In the double coincidence of wants case analyzed by Lagos and Wright, show that the joint surplus is maximized without any money changing hands.
  - c. Show that  $m \leq [\theta c(q^*) + (1 - \theta)u(q^*)] / \phi$ , where  $q^*$  maximizes  $u(q) - c(q)$  (so that  $u'(q^*) = c'(q^*)$ ), when the gross inflation rate is greater than or equal to  $\beta$ .

17. In the bilateral bargaining problem of Lagos and Wright (2005), the quantity transacted  $q$  and the money exchanged  $d$  solve the following problem:

$$\max_{q,d} [u(q) + W_t(m-d) - W_t(m)]^\theta [-c(q) + W_t(\tilde{m}+d) - W_t(\tilde{m})]^{1-\theta} + \lambda(m-d),$$

where  $\lambda$  is the Lagrangian multiplier on the constraint  $d \leq m$ . Show that when the constraint binds,  $q$  solves

$$\phi_t m = \frac{\theta u'(q)c(q) + (1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)}.$$

18. Lagos and Wright (2005) showed that the solution to the bilateral bargaining problem when the cash constraint binds implies

$$\frac{z(q_t)}{M_t} = \beta \frac{z(q_{t+1})}{M_{t+1}} \left[ \frac{\alpha \sigma u'(q_{t+1})}{z'_{t+1}(q_{t+1})} + (1 - \alpha \sigma) \right],$$

where the quantity traded  $q$  satisfies  $z(q_t) = \phi_t M_t$ . Suppose the money stock grows at the rate  $\tau$  so that  $M_{t+1} = (1 + \tau)M_t$ . Show that in a steady-state with the real variables constant,

$$\frac{\phi_t}{\phi_{t+1}} = 1 + \tau \quad \text{and} \quad \frac{u'(q)}{z'(q)} = 1 + \frac{i}{\alpha \sigma},$$

where  $i$  is the nominal rate of interest.

19. Rocheteau and Wright (2005) consider three different market structures. For which ones does the Friedman rule deliver the first-best equilibrium? Explain. For which ones doesn't it? Explain.

# 4 Money and Public Finance

## 4.1 Introduction

Inflation is a tax. And as a tax, it both generates revenue for the government and distorts private sector behavior. Chapters 2 and 3 focused on these distortions. In the Sidrauski model, inflation distorts the demand for money, thereby generating welfare effects because real money holdings directly yield utility. In the cash-in-advance model, inflation serves as an implicit tax on consumption, so a higher inflation rate generates a substitution toward leisure, leading to lower labor supply, output, and consumption.

In the analysis of these distortions, the revenue side of the inflation tax was ignored except to note that the Friedman rule for the optimal rate of inflation may need to be modified if the government does not have lump-sum sources of revenue available. Any change in inflation that affects the revenue from the inflation tax will have budgetary implications for the government. If higher inflation allows other forms of distortionary taxation to be reduced, this fact must be incorporated into any assessment of the costs of the inflation tax. This chapter introduces the government sector's budget constraint and examines the revenue implications of inflation. This allows a more explicit focus on the role of inflation in a theory of public finance and draws on the literature on optimal taxation to analyze the effects of inflation.

A public finance approach yields several insights. Among the most important is the recognition that fiscal and monetary policies are linked through the government sector's budget constraint. Variations in the inflation rate can have implications for the fiscal authority's decisions about expenditures and taxes, and, conversely, decisions by the fiscal authority can have implications for money growth and inflation. When inflation is viewed as a distortionary revenue-generating tax, the degree to which it should be relied upon depends on the set of alternative taxes available to the government and on the reasons individuals hold money. Whether the most appropriate strategy is to think of money as entering the utility function as a final good or as an intermediate input into the production of transaction services can have implications for whether money should be taxed. The optimal tax perspective also has empirical implications for inflation.

In the next section, the consolidated government's budget identity is set out, and some of the revenue implications of inflation are examined. Section 4.3 introduces various assumptions that can be made about the relationship between monetary and fiscal policies. Section 4.4 discusses situations of fiscal dominance in which a fixed amount of revenue must be raised from the inflation tax. It then discusses the equilibrium relationship between money and the price level. Section 4.5 turns to recent theories that emphasize what has come to be called the *fiscal theory of the price level*. In section 4.6, inflation revenue (seigniorage) and other taxes are brought together to analyze the joint determination of the government's tax instruments. This theme is developed first in a partial equilibrium model, and then Friedman's rule for the optimal inflation rate is revisited. The implications of optimal Ramsey taxation for inflation are discussed. Finally, section 4.6.4 contains a brief discussion of some additional effects that arise when the tax system is not fully indexed.

## 4.2 Budget Accounting

To obtain goods and services, governments in market economies need to generate revenue. One way they can obtain goods and services is by printing money, which is then used to purchase resources from the private sector. However, to understand the revenue implications of inflation (and the inflation implications of the government's revenue needs), one must start with the government's budget constraint.<sup>1</sup>

Consider the following identity for the fiscal branch of a government:

$$G_t + i_{t-1}B_{t-1}^T = T_t + (B_t^T - B_{t-1}^T) + \text{RCB}_t, \quad (4.1)$$

where all variables are in nominal terms. The left side consists of government expenditures on goods, services, and transfers  $G_t$ , plus interest payments on the outstanding debt  $i_{t-1}B_{t-1}^T$  (the superscript  $T$  denoting total debt, assumed to be one period in maturity, where debt issued in period  $t - 1$  earns the nominal interest rate  $i_{t-1}$ ), and the right side consists of tax revenue  $T_t$ , plus new issues of interest-bearing debt  $B_t^T - B_{t-1}^T$ , plus any direct receipts from the central bank  $\text{RCB}_t$ . As an example of  $\text{RCB}_t$ , the U.S. Federal Reserve turns over to the Treasury almost all the interest earnings on its portfolio of government debt.<sup>2</sup> Equation (4.1) represents the *Treasury's budget constraint*.

The monetary authority, or central bank, also has a budget identity that links changes in its assets and liabilities. If the central bank's assets consist of government debt, its budget

1. Bohn (1992) provided a general discussion of government deficits and accounting.

2. In 2014 the Federal Reserve banks turned over \$96.9 billion to the Treasury (*101st Annual Report of the Federal Reserve System* 2014, 113). In contrast, the payment to the Treasury in 2007, before the huge expansion of the Federal Reserve's balance sheet during the financial crisis of 2008–2009 and the Great Recession, was only \$34.6 billion (*93rd Annual Report of the Federal Reserve System* 2007, 161). Klein and Neumann (1990) showed how the revenue generated by seigniorage and the revenue received by the fiscal branch may differ.

identity takes the form<sup>3</sup>

$$(B_t^M - B_{t-1}^M) + \text{RCB}_t = i_{t-1}B_{t-1}^M + (H_t - H_{t-1}), \quad (4.2)$$

where  $B_t^M - B_{t-1}^M$  is equal to the central bank's purchases of government debt,  $i_{t-1}B_{t-1}^M$  is the central bank's receipt of interest payments from the Treasury, and  $H_t - H_{t-1}$  is the change in the central bank's own liabilities. These liabilities are called *high-powered money*, or sometimes the *monetary base*, because they form the stock of currency held by the nonbank public plus bank reserves, and they represent the reserves private banks can use to back deposits. Changes in the stock of high-powered money lead to changes in broader measures of the money supply, measures that normally include various types of bank deposits as well as currency held by the public (see chapter 12).

By letting  $B = B^T - B^M$  be the stock of government interest-bearing debt held by the public, the budget identities of the Treasury and the central bank can be combined to produce the consolidated government sector budget identity:

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}). \quad (4.3)$$

From the perspective of the consolidated government sector, only debt held by the public (i.e., outside the government sector) represents an interest-bearing liability.

According to (4.3), the dollar value of government purchases  $G_t$ , plus its payment of interest on outstanding privately held liabilities  $i_{t-1}B_{t-1}$ , must be funded by revenue that can be obtained from one of three alternative sources. First,  $T_t$  represents revenue generated by taxes (other than inflation). Second, the government can obtain funds by borrowing from the private sector. This borrowing is equal to the change in the debt held by the private sector,  $B_t - B_{t-1}$ . Finally, the government can print currency to pay for its expenditures, and this is represented by the change in the outstanding stock of non-interest-bearing debt,  $H_t - H_{t-1}$ .<sup>4</sup>

Dividing (4.3) by the price level  $P_t$  yields

$$\frac{G_t}{P_t} + i_{t-1} \left( \frac{B_{t-1}}{P_t} \right) = \frac{T_t}{P_t} + \frac{B_t - B_{t-1}}{P_t} + \frac{H_t - H_{t-1}}{P_t}.$$

3. The Federal Reserve has paid interest on reserves since 2008, a factor ignored in (4.2). Accounting for it would add  $i'_{t-1}H_{t-1}$  to the left side of (4.2) if, for simplicity, one assumed the rate  $i'$  is paid on reserves plus currency. From 1985 to 2007 currency averaged just over 80 percent of high-powered money; since 2008, currency has fallen to less than half of high-powered money because of the tremendous increase in bank reserves. Also ignored here is any income to the central bank from interest charged on borrowed reserves. Chapter 12 discusses the implications of interest on reserves for monetary policy implementation. See Hall and Reis (2013).

4. If the central bank holds private sector assets on its balance sheet and pays interest on its liabilities, (4.3) becomes

$$G_t + i_{t-1}B_{t-1} + i'_{t-1}H_{t-1} + A_t - A_{t-1} = i'_{t-1}A_{t-1} + T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}),$$

where the left side now includes the cost of interest payments on the central bank's liabilities and net purchases of private sector assets  $A_t - A_{t-1}$ . On the right side the income from the private assets, denoted here by  $i'_{t-1}A_{t-1}$ , now appears as a revenue source.

Note that terms like  $B_{t-1}/P_t$  can be multiplied and divided by  $P_{t-1}$ , yielding

$$\frac{B_{t-1}}{P_t} = \left( \frac{B_{t-1}}{P_{t-1}} \right) \left( \frac{P_{t-1}}{P_t} \right) = b_{t-1} \left( \frac{1}{1 + \pi_t} \right),$$

where  $b_{t-1} = B_{t-1}/P_{t-1}$  represents real debt and  $\pi_t$  is the inflation rate.<sup>5</sup> Employing the convention that lowercase letters denote variables deflated by the price level, the government's budget identity is

$$g_t + \bar{r}_{t-1} b_{t-1} = t_t + (b_t - b_{t-1}) + h_t - \left( \frac{1}{1 + \pi_t} \right) h_{t-1}, \quad (4.4)$$

where  $\bar{r}_{t-1} = [(1 + i_{t-1}) / (1 + \pi_t)] - 1$  is the ex post real return from  $t - 1$  to  $t$ .

To highlight the respective roles of anticipated and unanticipated inflation, let  $r_t$  be the ex ante real rate of return, and let  $\pi_t^e$  be the expected rate of inflation; then  $1 + i_{t-1} = (1 + r_{t-1})(1 + \pi_t^e)$ . Adding  $(r_{t-1} - \bar{r}_{t-1})b_{t-1} = (\pi_t - \pi_t^e)(1 + r_{t-1})b_{t-1}/(1 + \pi_t)$  to both sides of (4.4) and rearranging, the budget constraint becomes

$$g_t + r_{t-1} b_{t-1} = t_t + (b_t - b_{t-1}) + \left( \frac{\pi_t - \pi_t^e}{1 + \pi_t} \right) (1 + r_{t-1}) b_{t-1} + \left[ h_t - \left( \frac{1}{1 + \pi_t} \right) h_{t-1} \right]. \quad (4.5)$$

The third term on the right side of this expression, involving  $(\pi_t - \pi_t^e)b_{t-1}$ , represents the revenue generated when unanticipated inflation reduces the real value of the government's outstanding interest-bearing nominal debt. To the extent that inflation is anticipated, this term will be zero;  $\pi_t^e$  will be reflected in the nominal interest rate that the government must pay. Inflation by itself does not reduce the burden of the government's interest-bearing debt; only unexpected inflation has such an effect.

The last bracketed term in (4.5) represents seigniorage, the revenue from money creation. Seigniorage can be written as

$$s_t \equiv \frac{H_t - H_{t-1}}{P_t} = (h_t - h_{t-1}) + \left( \frac{\pi_t}{1 + \pi_t} \right) h_{t-1}. \quad (4.6)$$

Seigniorage arises from two sources. First,  $h_t - h_{t-1}$  is equal to the change in real high-powered money holdings. Since the government is the monopoly issuer of high-powered money, an increase in the amount of high-powered money that the private sector is willing to hold allows the government to obtain real resources in return. In a steady-state equilibrium,  $h$  is constant, so this source of seigniorage then equals zero. The second term in (4.6) is normally the focus of analyses of seigniorage because it can be nonzero even in the steady state. To maintain a constant level of real money holdings, the private sector needs

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5. If one is dealing with a growing economy, it is appropriate to deflate nominal variables by the price level and the level of output, i.e., by  $P_t Y_t$ . If the growth rate of output is  $\mu_t$ , then  $B_{t-1}/P_t Y_t = b_{t-1} [1/(1 + \pi_t)(1 + \mu_t)]$ .

to increase its nominal holdings of money at the rate  $\pi$  (approximately) to offset the effects of inflation on real holdings. By supplying money to meet this demand, the government is able to obtain goods and services or reduce other taxes.

Denote the growth rate of the nominal monetary base  $H$  by  $\theta$ ; the growth rate of  $h$  is then  $(\theta - \pi)/(1 + \pi) \approx \theta - \pi$ .<sup>6</sup> In a steady state,  $h$  is constant, implying that  $\pi = \theta$ . In this case, (4.6) shows that seigniorage equals

$$\left(\frac{\pi}{1 + \pi}\right)h = \left(\frac{\theta}{1 + \theta}\right)h. \quad (4.7)$$

For small values of the rate of inflation,  $\pi/(1 + \pi)$  is approximately equal to  $\pi$ , so  $s$  can be thought of as the product of a tax rate of  $\pi$ , the rate of inflation, and a tax base of  $h$ , the real stock of base money. Since base money does not pay interest, its real value is depreciated by inflation whether or not inflation is anticipated.

The definition of  $s$  would appear to imply that the government receives no revenue if inflation is zero. But this inference neglects the real interest savings to the government of issuing  $h$ , which is non-interest-bearing debt, as opposed to  $b$ , which is interest-bearing debt. That is, for a given level of the government's total real liabilities  $d = b + h$ , interest costs are a decreasing function of the fraction of this total that consists of  $h$ . A shift from interest-bearing to non-interest-bearing debt would allow the government to reduce total tax revenue or increase transfers or purchases.

This observation suggests that one should consider the government's budget constraint expressed in terms of the total liabilities of the government. Using (4.5) and (4.6), the budget constraint can be rewritten<sup>7</sup> as

$$g_t + r_{t-1}d_{t-1} = t_t + (d_t - d_{t-1}) + \left(\frac{\pi_t - \pi_t^e}{1 + \pi_t}\right)(1 + r_{t-1})d_{t-1} + \left(\frac{i_{t-1}}{1 + \pi_t}\right)h_{t-1}. \quad (4.8)$$

Seigniorage, defined as the last term in (4.8), becomes

$$\bar{s} = \left(\frac{i_{t-1}}{1 + \pi_t}\right)h_{t-1}. \quad (4.9)$$

This shows that the relevant tax rate on high-powered money depends directly on the nominal rate of interest. Thus, under the Friedman rule for the optimal rate of inflation, which calls for setting the nominal rate of interest equal to zero (see chapters 2 and 3), the government collects no revenue from seigniorage. The budget constraint also illustrates that any change in seigniorage requires an offsetting adjustment in the other components of (4.8). Reducing the nominal interest rate to zero implies that the lost revenue must be replaced by

6. Problem 2 at this end of this chapter deals with the case in which there is population growth and real per capita income growth.

7. To obtain this, add  $r_{t-1}h_{t-1}$  to both sides of (4.5).

an increase in other taxes, real borrowing that increases the government's net indebtedness, or reductions in expenditures.

The various forms of the government's budget identity suggest at least three alternative measures of the revenue from money creation. First, the measure that might be viewed as appropriate from the perspective of the Treasury is simple RCB, total transfers from the central bank to the Treasury (see 4.1). Under this definition, shifts in the ownership of government debt between the private sector and the central bank affect the measure of seigniorage even if high-powered money remains constant. That is, from (4.2), if the central bank used interest receipts to purchase debt,  $B^M$  would rise, RCB would fall, and the Treasury would, from (4.1), need to raise other taxes, reduce expenditures, or issue more debt. But this last option means that the Treasury could simply issue debt equal to the increase in the central bank's debt holdings, leaving private debt holdings, government expenditures, and other taxes unaffected. Thus, changes in RCB do not represent real changes in the Treasury's finances and are therefore not the appropriate measure of seigniorage.

A second possible measure of seigniorage is given by (4.6), the real value of the change in high-powered money.<sup>8</sup> This measure of seigniorage equals the revenue from money creation for a given path of interest-bearing government debt. That is,  $s$  equals the total expenditures that could be funded, holding constant other tax revenue and the total private sector holdings of interest-bearing government debt. Finally, (4.9) provides a third definition of seigniorage as the nominal interest savings from issuing non-interest-bearing rather than interest-bearing debt.<sup>9</sup> This third definition equals the revenue from money creation for a given path of total (interest-bearing and non-interest-bearing) government debt; it equals the total expenditures that could be funded, holding constant other tax revenue and the total private sector holdings of real government liabilities.

The difference between  $s$  and  $\bar{s}$  arises from alternative definitions of fiscal policy. To understand the effects of monetary policy, one normally wants to consider changes in monetary policy while holding fiscal policy constant. Suppose tax revenue  $t$  is simply treated as lump-sum taxes. Then one definition of fiscal policy would be in terms of a time series for government purchases and interest-bearing debt:  $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$ . Changes in  $s$ , together with the changes in  $t$  necessary to maintain  $\{g_{t+i}, b_{t+i}\}_{i=0}^{\infty}$  unchanged, would constitute monetary policy. Under this definition, monetary policy would change the total liabilities of the government (i.e.,  $b + h$ ). An open market purchase by the central bank would, ceteris paribus, lower the stock of interest-bearing debt held by the public. The Treasury would then need to issue additional interest-bearing debt to keep the  $b_{t+i}$  sequence unchanged. Total government liabilities would rise. Alternatively, under the definition  $\bar{s}$ , fiscal policy

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8. Since the Fed began paying interest on reserves in 2008, the formulas for seigniorage need to be adjusted to reflect the payment of interest (which reduces seigniorage) and the income from Fed holdings of non-Treasury debt such as mortgage-backed securities.

9. These are not the only three possible definitions. See King and Plosser (1985) for an additional three.



sets the path  $\{g_{t+i}, d_{t+i}\}_{i=0}^{\infty}$  and monetary policy determines the division of  $d$  between interest-bearing and non-interest-bearing debt but not its total.

#### 4.2.1 Intertemporal Budget Balance

The budget relationships derived in the previous section link the government's choices concerning expenditures, taxes, debt, and seigniorage at each point in time. However, unless there are restrictions on the government's ability to borrow or to raise revenue from seigniorage, (4.8) places no direct constraint on expenditure or tax choices. If governments, like individuals, are constrained in their ability to borrow, then this constraint limits the government's choices. To see exactly how it does so requires focusing on the intertemporal budget constraint of the government.

Ignoring the effect of surprise inflation, the single-period budget identity of the government given by (4.5) can be written as

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + s_t.$$

Assuming the interest factor  $r$  is a constant (and is positive), this equation can be solved forward to obtain

$$(1+r)b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim_{i \rightarrow \infty} \frac{b_{t+i}}{(1+r)^i}. \quad (4.10)$$

The government's expenditure and tax plans are said to satisfy the requirement of intertemporal budget balance (the *no Ponzi condition*) if the last term in (4.10) equals zero:

$$\lim_{i \rightarrow \infty} \frac{b_{t+i}}{(1+r)^i} = 0. \quad (4.11)$$

In this case, the right side of (4.10) becomes the present discounted value of all current and future tax and seigniorage revenue, and this is equal to the left side, which is the present discounted value of all current and future expenditures plus current outstanding debt (principal plus interest). In other words, the government must plan to raise sufficient revenue, in present value terms, to repay its existing debt and finance its planned expenditures. Defining the primary deficit as  $\Delta = g - t - s$ , intertemporal budget balance implies, from (4.10), that

$$(1+r)b_{t-1} = - \sum_{i=0}^{\infty} \frac{\Delta_{t+i}}{(1+r)^i}. \quad (4.12)$$

Thus, if the government has outstanding debt ( $b_{t-1} > 0$ ), the present value of future primary deficits must be negative (i.e., the government must run a primary surplus in present value). This surplus can be generated through adjustments in expenditures, taxes, or seigniorage.

Is (4.12) a constraint on the government? Must the government (the combined monetary and fiscal authorities) pick expenditures, taxes, and seigniorage to ensure that (4.12) holds for all possible values of the initial price level and interest rates? Or is it an equilibrium condition that need only hold at the equilibrium price level and interest rate? Buiter (2002) argued strongly that the intertemporal budget balance condition represents a constraint on government behavior, and this is the perspective generally adopted here. However, Sims (1994), Woodford (1995; 2001a), and Cochrane (1999) argued that (4.12) is an equilibrium condition; this alternative perspective is taken up in section 4.5.

### 4.3 Money and Fiscal Policy Frameworks

Most analyses of monetary phenomena and monetary policy assume, usually without statement, that variations in the stock of money matter but that how that variation occurs does not. The nominal money supply could change because of a shift from tax-financed government expenditures to seigniorage-financed expenditures. Or it could change as the result of an open-market operation in which the central bank purchases interest-bearing debt, financing the purchase by an increase in non-interest-bearing debt, holding other taxes constant (see 4.2). Because these two means of increasing the money stock have differing implications for taxes and the stock of interest-bearing government debt, they may lead to different effects on prices and/or interest rates.

The government sector's budget constraint links monetary and fiscal policies in ways that can matter for determining how a change in the money stock affects the equilibrium price level.<sup>10</sup> The budget link also means that one needs to be precise about defining monetary policy as distinct from fiscal policy. An open-market purchase increases the stock of money, but by reducing the interest-bearing government debt held by the public, it has implications for the future stream of taxes needed to finance the interest cost of the government's debt. So an open-market operation potentially has a fiscal side to it, and this fact can lead to ambiguity in defining what one means by a change in monetary policy, *holding fiscal policy constant*.

The literature in monetary economics has analyzed several alternative assumptions about the relationship between monetary and fiscal policies. In most traditional analyses, fiscal policy is assumed to adjust to ensure that the government's intertemporal budget is always in balance, while monetary policy is free to set the nominal money stock or the nominal rate of interest. This situation is described as one of *monetary dominance* (Sargent 1982) or one in which fiscal policy is passive and monetary policy is active (Leeper 1991). The models of chapters 2 and 3 implicitly fall into this category in that fiscal policy was ignored and monetary policy determined the price level. Traditional quantity theory relationships were

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10. See, for example, Sargent and Wallace (1981) and Wallace (1981). The importance of the budget constraint for the analysis of monetary topics is clearly illustrated in Sargent (1987).

obtained, with one-time proportional changes in the nominal quantity of money leading to equal proportional changes in the price level.

If fiscal policy affects the real rate of interest, then the price level is not independent of fiscal policy, even under regimes of monetary dominance. A balanced budget increase in expenditures that raises the real interest rate raises the nominal interest rate and lowers the real demand for money. Given an exogenous path for the nominal money supply, the price level must jump to reduce the real supply of money.

A second policy regime is one in which the fiscal authority sets its expenditures and taxes without regard to any requirement of intertemporal budget balance. If the present discounted value of these taxes is not sufficient to finance expenditures (in present value terms), seigniorage must adjust to ensure that the government's intertemporal budget constraint is satisfied. This regime is one of fiscal dominance (or active fiscal policy) and passive monetary policy, as monetary policy must adjust to deliver the level of seigniorage required to balance the government's budget. Prices and inflation are affected by changes in fiscal policy because these fiscal changes, if they require a change in seigniorage, alter the current and/or future money supply. Any regime in which either taxes or seigniorage always adjust to ensure that the government's intertemporal budget constraint is satisfied is called a *Ricardian* regime (Sargent 1982). Regimes of fiscal dominance are analyzed in section 4.4.

A final regime leads to what has become known as the fiscal theory of the price level (Sims 1994; Woodford 1995; 2001a; Cochrane 1999). In this regime, the government's intertemporal budget constraint may not be satisfied for arbitrary price levels. Following Woodford (1995), these regimes are described as *non-Ricardian*. The discussion of non-Ricardian regimes is in section 4.5.

#### 4.4 Deficits and Inflation

The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses. One way to generate a surplus is to increase revenue from seigniorage, and for that reason, economists have been interested in the implications of budget deficits for future money growth. Two questions have formed the focus of studies of deficits and inflation. First, do fiscal deficits necessarily imply that inflation will eventually occur? Second, if inflation is not a necessary consequence of deficits, is it in fact a historical consequence?

The literature on the first question has focused on the implications for inflation if the monetary authority must act to ensure that the government's intertemporal budget is balanced. This interpretation views fiscal policy as set independently, so that the monetary authority is forced to generate enough seigniorage to satisfy the intertemporal budget balance condition. Leeper (1991) describes this as a situation with an active fiscal policy and a passive monetary policy. It is also described as a situation of *fiscal dominance*.

From (4.12), the government's intertemporal budget constraint takes the form

$$b_{t-1} = -R^{-1} \sum_{i=0}^{\infty} R^{-i} (g_{t+i} - t_{t+i} - s_{t+i}),$$

where  $R = 1 + r$  is the gross real interest rate,  $g_t - t_t - s_t$  is the primary deficit, and  $s_t$  is real seigniorage revenue. Let  $s_t^f \equiv t_t - g_t$  be the primary *fiscal* surplus (i.e., tax revenue minus expenditures but excluding interest payments and seigniorage revenue). Then the government's budget constraint can be written as

$$b_{t-1} = R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}. \quad (4.13)$$

The current real liabilities of the government must be financed, in present value terms, by either a fiscal primary surplus or seigniorage.

Given the real value of the government's liabilities  $b_{t-1}$ , (4.13) illustrates what Sargent and Wallace (1981) described as "unpleasant monetarist arithmetic" in a regime of fiscal dominance. If the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain (4.13). Or, for a given present value of  $s^f$ , an attempt by the monetary authority to reduce inflation and seigniorage today must lead to higher inflation and seigniorage in the future, because the present discounted value of seigniorage cannot be altered. The mechanism is straightforward; if current inflation tax revenues are lowered, the deficit grows and the stock of debt rises. This implies an increase in the present discounted value of future tax revenue, including revenue from seigniorage. If the fiscal authority does not adjust, the monetary authority will be forced eventually to produce higher inflation.<sup>11</sup>

The literature on the second question—has inflation been a consequence of deficits historically?—has focused on estimating empirically the effects of deficits on money growth. Joines (1985) found money growth in the United States to be positively related to major war spending but not to nonwar deficits. Grier and Neiman (1987) summarized a number of earlier studies of the relationship between deficits and money growth (and other measures of monetary policy) in the United States. That the results are generally inconclusive is perhaps not surprising, as the studies they reviewed were all based on postwar but pre-1980 data. Thus, the samples covered periods in which there was relatively little deficit variation and in which much of the existing variation arose from the endogenous response of deficits to the business cycle as tax revenue varied procyclically.<sup>12</sup> Grier and Neiman did find that the structural (high-employment) deficit is a determinant of money growth.

11. In a regime of monetary dominance, the monetary authority can determine inflation and seigniorage; the fiscal authority must then adjust either taxes or spending to ensure that (4.13) is satisfied.

12. For that reason, some of the studies cited by Grier and Neiman employed a measure of the high-employment surplus (i.e., the surplus estimated to occur if the economy had been at full employment). Grier and Neiman concluded, "The high employment deficit (surplus) seems to have a better 'batting average'" (204).

This finding is consistent with that of King and Plosser (1985), who reported that the fiscal deficit did help to predict future seigniorage for the United States. They interpreted this as mixed evidence for fiscal dominance.

Demopoulos, Katsimbris, and Miller (1987) provided evidence on debt accommodation for eight OECD countries. These authors estimated a variety of central bank reaction functions (regression equations with alternative policy instruments on the left side) in which the government deficit is included as an explanatory variable. For the post-Bretton Woods period, they found a range of outcomes, from no accommodation by the Federal Reserve and the Bundesbank to significant accommodation by the Bank of Italy and the Nederlandse Bank.

One objection to this empirical literature is that simple regressions of money growth on deficits, or unrestricted VAR used to assess Granger causality (i.e., whether deficits contain any predictive information about future money growth), ignore information about the long-run behavior of taxes, debt, and seigniorage that is implied by intertemporal budget balance. Intertemporal budget balance implies a cointegrating relationship between the primary deficit and the stock of debt. This link between the components of the deficit and the stock of debt restricts the time series behavior of expenditures, taxes, and seigniorage, and this fact in turn implies that empirical modeling of their behavior should be carried out within the framework of a vector error correction model (VECM).<sup>13</sup>

Suppose  $X_t = (g_t \quad T_t \quad b_{t-1})$ , where  $T = t + s$  is defined as total government receipts from taxes and seigniorage. If the elements of  $X$  are nonstationary, intertemporal budget balance implies that the deficit inclusive of interest, or  $(1 \quad -1 \quad r)X_t = \beta'X_t = g_t - T_t + rb_{t-1}$ , is stationary. Hence,  $\beta' = (1 \quad -1 \quad r)$  is a cointegrating vector for  $X$ . The appropriate specification of the time series process is then a VECM of the form

$$C(L)\Delta X_t = -\alpha\beta'X_t + e_t. \quad (4.14)$$

The presence of the deficit inclusive of interest,  $\beta'X_t$ , ensures that the elements of  $X$  cannot drift too far apart; doing so would violate intertemporal budget balance. A number of authors have tests for cointegration to examine the sustainability of budget policies (see Trehan and Walsh 1988; 1991 for one approach). However, Bohn (2007) argued that time series based on cointegration relationships are not capable of rejecting intertemporal budget balance.

Bohn (1991a) estimated a model of the form (4.14) using U.S. data from 1800 to 1988. Unfortunately for our purposes, Bohn did not treat seigniorage separately, and thus his results are not directly relevant for determining the effects of spending or tax shocks on the adjustment of seigniorage. He did find, however, that one-half to two-thirds of deficits initiated by a tax revenue shock were eventually eliminated by spending adjustments, while about one-third of spending shocks were essentially permanent and resulted in tax changes.

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13. See Engle and Granger (1987).

#### 4.4.1 Ricardian and (Traditional) Non-Ricardian Fiscal Policies

Changes in the nominal quantity of money engineered through lump-sum taxes and transfers (as in chapters 2 and 3) may have different effects than changes introduced through open-market operations in which non-interest-bearing government debt is exchanged for interest-bearing debt. In an early contribution, Metzler (1951) argued that an open-market purchase, that is, an increase in the nominal quantity of money held by the public and an offsetting reduction in the nominal stock of interest-bearing debt held by the public, would raise the price level less than proportionally to the increase in  $M$ . An open-market operation would therefore affect the real stock of money and lead to a change in the equilibrium rate of interest. Metzler assumed that households' desired portfolio holdings of bonds and money depended on the expected return on bonds. An open-market operation, by altering the ratio of bonds to money, requires a change in the rate of interest to induce private agents to hold the new portfolio composition of bonds and money. A price-level change proportional to the change in the nominal money supply would not restore equilibrium, because it would not restore the original ratio of nominal bonds to nominal money.

An important limitation of Metzler's analysis was its dependence on portfolio behavior that was not derived directly from the decision problem facing the agents of the model. The analysis was also limited in that it ignored the consequence for future taxes of shifts in the composition of the government's debt, a point made by Patinkin (1965). The government's intertemporal budget constraint requires the government to run surpluses in present value terms equal to its current outstanding interest-bearing debt. An open-market purchase by the monetary authority reduces the stock of interest-bearing debt held by the public, and this reduction has consequences for future expected taxes.

Sargent and Wallace (1981) showed that the backing for government debt, whether it is ultimately paid for by taxes or by printing money, is important in determining the effects of debt issuance and open-market operations. This finding can be illustrated following the analysis of Aiyagari and Gertler (1985). They used a two-period overlapping-generations model that allows debt policy to affect the real intergenerational distribution of wealth. This effect is absent from the representative agent model used here, but the representative agent framework can still be used to show how the specification of fiscal policy has important implications for conclusions about the link between the money supply and the price level.<sup>14</sup>

In order to focus on debt, taxes, and seigniorage, set government purchases equal to zero and ignore population and real income growth, in which case the government's budget constraint takes the simplified form

$$(1 + r_{t-1})b_{t-1} = t_t + b_t + s_t, \quad (4.15)$$

with  $s_t$  denoting seigniorage.

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14. See also Woodford (1995; 2001a) and section 4.5.2.

In addition to the government's budget constraint, one needs to specify the budget constraint of the representative agent. Assume that this agent receives an exogenous endowment  $y$  in each period and pays (lump-sum) taxes  $t_t$  in period  $t$ . The agent also receives interest payments on any government debt held at the start of the period; these payments, in real terms, equal  $(1 + i_{t-1})B_{t-1}/P_t$ , where  $i_{t-1}$  is the nominal interest rate in period  $t - 1$ ,  $B_{t-1}$  is the number of bonds held at the start of period  $t$ , and  $P_t$  is the period  $t$  price level. This can be written equivalently as  $(1 + r_{t-1})b_{t-1}$ , where  $r_{t-1} = (1 + i_{t-1})/(1 + \pi_t) - 1$  is the ex post real rate of interest. Finally, the agent has real money balances equal to  $M_{t-1}/P_t = (1 + \pi_t)^{-1}m_{t-1}$  that are carried into period  $t$  from period  $t - 1$ . The agent allocates these resources to consumption, real money holdings, and real bond purchases, subject to

$$c_t + m_t + b_t = y + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} - t_t. \quad (4.16)$$

Aiyagari and Gertler (1985) asked whether the price level will depend only on the stock of money or whether debt policy and the behavior of the stock of debt might also be relevant for price level determination. They assumed that the government sets taxes to back a fraction  $\psi$  of its interest-bearing debt liabilities, with  $0 \leq \psi \leq 1$ . If  $\psi = 1$ , government interest-bearing debt is completely backed by taxes in the sense that the government commits to maintaining the present discounted value of current and future tax receipts equal to its outstanding debt liabilities. Such a fiscal policy was called Ricardian by Sargent (1982).<sup>15</sup> If  $\psi < 1$ , Aiyagari and Gertler characterized fiscal policy as non-Ricardian. To avoid confusion with the more recent interpretations of non-Ricardian regimes (see section 4.5.2), regimes where  $\psi < 1$  are referred to here as *traditional non-Ricardian* regimes. In such regimes, seigniorage must adjust to maintain the present value of taxes plus seigniorage equal to the government's outstanding debt.

Let  $T_t$  now denote the present discounted value of taxes. Under the assumed debt policy, the government ensures that  $T_t = \psi(1 + r_{t-1})b_{t-1}$  because  $(1 + r_{t-1})b_{t-1}$  is the net liability of the government (including its current interest payment). Because  $T_t$  is a present value, one can also write

$$T_t = t_t + E_t \left( \frac{T_{t+1}}{1 + r_t} \right) = t_t + E_t \left[ \frac{\psi(1 + r_t)b_t}{(1 + r_t)} \right]$$

or  $T_t = t_t + \psi b_t$ . Now because  $T_t = \psi(1 + r_{t-1})b_{t-1}$ , it follows that

$$t_t = \psi (R_{t-1}b_{t-1} - b_t), \quad (4.17)$$

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15. It is more common for Ricardo's name to be linked with debt in the form of the Ricardian equivalence theorem, under which shifts between debt and tax financing of a given expenditure stream have no real effects. See Barro (1974) or Romer (2012). Ricardian equivalence holds in the representative agent framework used here; the issue is whether debt policy, as characterized by  $\psi$ , matters for price level determination.

where  $R = 1 + r$ . Similarly,  $s_t = (1 - \psi)(R_{t-1}b_{t-1} - b_t)$ . With taxes adjusting to ensure that the fraction  $\psi$  of the government's debt liabilities is backed by taxes, the remaining fraction,  $1 - \psi$ , represents the portion backed by seigniorage.

Using (4.17), the household's budget constraint (4.16) becomes

$$c_t + m_t + (1 - \psi)b_t = y + (1 - \psi)R_{t-1}b_{t-1} + \frac{m_{t-1}}{1 + \pi_t}.$$

In the Ricardian case ( $\psi = 1$ ), all terms involving the government's debt drop out; only the stock of money matters. If  $\psi < 1$ , however, debt does not drop out. One can then rewrite the budget constraint as  $y + R_{t-1}w_{t-1} = c_t + w_t + i_{t-1}m_{t-1}/(1 + \pi_t)$ , where  $w = m + (1 - \psi)b$ , showing that the relevant measure of household income is  $y + R_{t-1}w_{t-1}$  and this is then used to purchase consumption, financial assets, or money balances (where the opportunity cost of money is  $i/(1 + \pi)$ ). With asset demand depending on  $\psi$  through  $w_{t-1}$ , the equilibrium price level and nominal rate of interest generally depend on  $\psi$ .<sup>16</sup>

Having derived the representative agent's budget constraint and shown how it is affected by the means the government uses to back its debt, to actually determine the effects on the equilibrium price level and nominal interest rate, one must determine the agent's demand for money and bonds and then equate these demands to the (exogenous) supplies. To illustrate the role of debt policy, assume log separable utility,  $\ln c_t + \delta \ln m_t$ , and consider a perfect-foresight equilibrium. From chapter 2, the marginal rate of substitution between money and consumption is set equal to  $i_t/(1 + i_t)$ . With log utility, this implies  $m_t = \delta c_t(1 + i_t)/i_t$ . The Euler condition for the optimal consumption path yields  $c_{t+1} = \beta(1 + r_t)c_t$ . Using these in the agent's budget constraint,

$$\begin{aligned} y + R_{t-1}w_{t-1} &= c_t + w_t + \left(\frac{i_{t-1}}{1 + \pi_t}\right) \delta \left(\frac{1 + i_{t-1}}{i_{t-1}}\right) \frac{c_t}{\beta(1 + r_{t-1})} \\ &= \left(1 + \frac{\delta}{\beta}\right) c_t + w_t. \end{aligned}$$

In equilibrium,  $c_t = y$ , so this becomes  $R_{t-1}w_{t-1} = (\delta/\beta)y + w_t$ . In the steady state,  $w_t = w_{t-1} = w^{ss} = \delta y/\beta(R - 1)$ . But  $w = [M + (1 - \psi)B]/P$ , so the equilibrium steady-state price level is equal to

$$P^{ss} = \left(\frac{\beta r^{ss}}{\delta y}\right) [M + (1 - \psi)B]. \quad (4.18)$$

If government debt is entirely backed by taxes ( $\psi = 1$ ), one gets the standard result: the price level is proportional to the nominal stock of money. The stock of debt has no effect on the price level. With  $0 < \psi < 1$ , however, both the nominal money supply and the nominal stock of debt play a role in price level determination. Proportional changes in  $M$  and  $B$  produce proportional changes in the price level.

16. In this example,  $c = y$  in equilibrium, since there is no capital good that would allow the endowment to be transferred over time.



In a steady state, all nominal quantities and the price level must change at the same rate because real values are constant. Thus, if  $M$  grows, then  $B$  must also grow at the same rate. The real issue is whether the composition of the government's liabilities matters for the price level. To focus more clearly on that issue, let  $\lambda = M/(M + B)$  be the fraction of government liabilities that consists of non-interest-bearing debt. Since open-market operations affect the relative proportions of money and bonds in government liabilities, open-market operations determine  $\lambda$ . Equation (4.18) can then be written as

$$P^{ss} = \left( \frac{\beta r^{ss}}{\delta y} \right) [1 - \psi(1 - \lambda)](M + B).$$

Open market purchases (an increase in  $\lambda$ ) that substitute money for bonds but leave  $M + B$  unchanged raise  $P^{ss}$  when  $\psi > 0$ . The rise in  $P^{ss}$  is not proportional to the increase in  $M$ . Shifting the composition of its liabilities away from interest-bearing debt reduces the present discounted value of the private sector's tax liabilities by less than the fall in debt holdings; a rise in the price level proportional to the rise in  $M$  would leave households' real wealth lower (their bond holdings are reduced in real value, but the decline in the real value of their tax liabilities is only  $\psi < 1$  times as large).

Leeper (1991) argued that even if  $\psi = 1$  on average (that is, all debt is backed by taxes), the means used to finance shocks to the government's budget have important implications. He distinguished between active and passive policies; with an active monetary policy and a passive fiscal policy, monetary policy acts to target nominal interest rates and does not respond to the government's debt, while fiscal policy must then adjust taxes to ensure intertemporal budget balance. Conversely, with an active fiscal policy and a passive monetary policy, the monetary authority must adjust seigniorage revenue to ensure intertemporal budget balance, while fiscal policy does not respond to shocks to debt. Leeper showed that the inflation and debt processes are unstable if both policy authorities follow active policies, and there is price level indeterminacy if both follow passive policies.

#### 4.4.2 The Government Budget Constraint and the Nominal Rate of Interest

Earlier, Sargent and Wallace's "unpleasant monetarist" arithmetic was examined using (4.13). Given the government's real liabilities, the monetary authority would be forced to finance any difference between these real liabilities and the present discounted value of the government's fiscal surpluses. Fiscal considerations determine the money supply, but the traditional quantity theory holds and the price level is proportional to the nominal quantity of money. Suppose, however, that the initial nominal stock of money is set exogenously by the monetary authority. Does this mean that the price level is determined solely by monetary policy, with no effect of fiscal policy? The following example shows that the answer is no; fiscal policy can affect the initial equilibrium price level even when the initial nominal quantity of money is given and the government's intertemporal budget constraint must be satisfied at all price levels.

Consider a perfect-foresight equilibrium. In such an equilibrium, the government's budget constraint must be satisfied and the real demand for money must equal the real supply of money. The money-in-the-utility function (MIU) model of chapter 2 can be used, for example, to derive the real demand for money. That model implied that agents would equate the marginal rate of substitution between money and consumption to the cost of holding money, where this cost depended on the nominal rate of interest:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1 + i_t}.$$

With the utility function<sup>17</sup> employed in chapter 2, this condition implies that

$$m_t = \frac{M_t}{P_t} = \left[ \left( \frac{i_t}{1 + i_t} \right) \left( \frac{a}{1 - a} \right) \right]^{-\frac{1}{b}} c_t.$$

Evaluated at the economy's steady state, this can be written as

$$\frac{M_t}{P_t} = f(R_{m,t}), \quad (4.19)$$

where  $R_m = 1 + i$  is the gross nominal rate of interest and

$$f(R_m) = \left[ \left( \frac{R_m - 1}{R_m} \right) \left( \frac{a}{1 - a} \right) \right]^{-\frac{1}{b}} c_t.$$

Given the nominal interest rate, (4.19) implies a proportional relationship between the nominal quantity of money and the equilibrium price level. If the initial money stock is  $M_0$ , then the initial price level is  $P_0 = M_0/f(R_m)$ .

The government's budget constraint must also be satisfied. In a perfect-foresight equilibrium, there are no inflation surprises, so the government's budget constraint given by (4.5) can be written as

$$g_t + rb_{t-1} = t_t + (b_t - b_{t-1}) + m_t - \left( \frac{1}{1 + \pi_t} \right) m_{t-1}. \quad (4.20)$$

Now consider a stationary equilibrium in which government expenditures and taxes are constant, as are the real stocks of government interest-bearing debt and money. In such a stationary equilibrium, the budget constraint becomes

$$g + \left( \frac{1}{\beta} - 1 \right) b = t + \left( \frac{\pi_t}{1 + \pi_t} \right) m = t + \left( \frac{\beta R_m - 1}{\beta R_m} \right) f(R_m), \quad (4.21)$$

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17. In chapter 2 it was assumed that

$$u(c_t, m_t) = \frac{[ac_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1 - \Phi}.$$

which uses the steady-state results that the gross real interest rate is  $1/\beta$ ,  $R_m \equiv (1 + \pi_t)/\beta$ , and real money balances must be consistent with the demand given by (4.19).

Suppose the fiscal authority sets  $g$ ,  $t$ , and  $b$ . Then (4.21) determines the nominal interest rate  $R_m$ . With  $g$ ,  $t$ , and  $b$  given, the government needs to raise  $g + (1/\beta - 1)b - t$  in seigniorage. The nominal interest rate is determined by the requirement that this level of seigniorage be raised.<sup>18</sup> Because the nominal interest rate is equal to  $(1 + \pi)/\beta$ , one can alternatively say that fiscal policy determines the inflation rate. Once the nominal interest rate is determined, the initial price level is given by (4.19) as  $P_0 = M_0/f(R_m)$ , where  $M_0$  is the initial stock of money. In subsequent periods, the price level is equal to  $P_t = P_0 (\beta R_m)^t$ , where  $\beta R_m = (1 + \pi)$  is the gross inflation rate. The nominal stock of money in each future period is endogenously determined by  $M_t = P_t f(R_m)$ . In this case, even though the monetary authority has set  $M_0$  exogenously, the initial price level is determined by the need for fiscal solvency because the fiscal authority's budget requirement (4.21) determines  $R_m$  and therefore the real demand for money. The initial price level is proportional to the initial money stock, but the factor of proportionality,  $1/f(R_m)$ , is determined by fiscal policy, and both the rate of inflation and the path of the future nominal money supply are determined by the fiscal requirement that seigniorage equal  $g + (1/\beta - 1)b - t$ .

If the fiscal authority raises expenditures, holding  $b$  and  $t$  constant, then seigniorage must rise. The equilibrium nominal interest rate rises to generate this additional seigniorage.<sup>19</sup> With a higher  $R_m$ , the real demand for money falls, and this increases the equilibrium value of the initial price level  $P_0$ , even though the initial nominal quantity of money is unchanged.

#### 4.4.3 Equilibrium Seigniorage

Suppose that given its expenditures and other tax sources, the government has a fiscal deficit of  $\Delta^f$  that must be financed by money creation. When will it be feasible to raise  $\Delta^f$  in a steady-state equilibrium? And what will be the equilibrium rate of inflation?

The answers to these questions would be straightforward if there were a one-to-one relationship between the revenue generated by the inflation tax and the inflation rate. If this were the case, the inflation rate would be uniquely determined by the amount of revenue that must be raised. But the inflation rate affects the base against which the tax is levied. For a given base, a higher inflation rate raises seigniorage, but a higher inflation rate raises the opportunity cost of holding money and reduces the demand for money, thereby lowering the base against which the tax is levied. This raises the possibility that a given amount of

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18. The nominal interest rate that raises seigniorage equal to  $g + (1/\beta - 1)b - t$  may not be unique. A rise in  $R_m$  increases the tax rate on money, but it also erodes the tax base by reducing the real demand for money. A given amount of seigniorage may be raised with a low tax rate and a high base or a high tax rate and a low base.

19. This assumes that the economy is on the positively sloped portion of the Laffer curve so that raising the tax rate increases revenue; see section 4.4.3.

revenue can be raised by more than one rate of inflation. For example, the nominal rate of interest  $R_m$  that satisfies (4.21) may not be unique.

It will be helpful to impose additional structure so that one can say more about the demand for money. The standard approach used in most analyses of seigniorage is to specify directly a functional form for the demand for money as a function of the nominal rate of interest. An early example of this approach, and one of the most influential, is that of Cagan (1956). This approach is discussed in section 4.4.4, but here Calvo and Leiderman (1992) are followed in using a variant of the Sidrauski model of chapter 2 to motivate a demand for money. That is, suppose the economy consists of identical individuals, and the utility of the representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \quad (4.22)$$

where  $0 < \beta < 1$ ,  $c$  is per capita consumption,  $m$  is per capita real money holdings, and the function  $u(\cdot)$  is strictly concave and twice continuously differentiable. The representative agent chooses consumption, money balances, and holdings of interest-earning bonds to maximize the expected value of (4.22), subject to the following budget constraint:

$$c_t + b_t + m_t = y_t - \tau_t + (1 + r)b_{t-1} + \frac{m_{t-1}}{\Pi_t},$$

where  $b$  is the agent's holdings of bonds,  $y$  is real income,  $\tau$  is equal to the net taxes of the agent,  $r$  is the real rate of interest, assumed constant for simplicity, and  $\Pi_t \equiv P_t/P_{t-1} = 1 + \pi_t$ , where  $\pi_t$  is the inflation rate. Thus, the last term in the budget constraint,  $m_{t-1}/\Pi_t$ , is equal to the period  $t$  real value of money balances carried into period  $t$ , that is,  $M_{t-1}/P_t$ , where  $M$  represents nominal money holdings. In what follows, attention is restricted to perfect-foresight equilibria.

If  $w_t$  is the agent's real wealth in period  $t$ ,  $w_t = b_t + m_t$ , and let  $R_t = 1 + r_t$ , then the budget constraint can be rewritten as

$$\begin{aligned} c_t + w_t &= y_t - \tau_t + R_{t-1}w_{t-1} - \left( \frac{R_{t-1}\Pi_t - 1}{\Pi_t} \right) m_{t-1} \\ &= y_t - \tau_t + R_{t-1}w_{t-1} - \left( \frac{i_{t-1}}{\Pi_t} \right) m_{t-1} \end{aligned}$$

by using the fact that  $R\Pi = 1 + i$ , where  $i$  is the nominal rate of interest. Writing the budget constraint in this way, it is clear that the cost of holding wealth in the form of money rather than interest-earning bonds is  $i/\Pi$ .<sup>20</sup> The first-order condition for optimal money

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20. Recall from the derivation of (4.8) that the term for the government's revenue from seigniorage was  $(i_{t-1}/\Pi_t)h_{t-1}$ . Comparing this to the household's budget constraint (with  $h_{t-1} = m_{t-1}$ ) shows that the cost of holding money is exactly equal to the revenue obtained by the government.

holdings sets the marginal utility of money equal to the cost of holding money times the marginal utility of wealth. Since the interest forgone by holding money in period  $t$  is a cost that is incurred in period  $t + 1$ , this cost must be discounted back to period  $t$  using the discount factor  $\beta$  to compare with the marginal utility of money in period  $t$ . Thus,  $u_m(c_t, m_t) = \beta (i_t / \Pi_{t+1}) u_c(c_{t+1}, m_{t+1})$ . But the standard Euler condition for optimal consumption implies that  $u_c(c_t, m_t) = \beta R_t u_c(c_{t+1}, m_{t+1})$ . Combining these first-order conditions yields

$$u_m(c_t, m_t) = \left( \frac{i_t}{R_t \Pi_{t+1}} \right) u_c(c_t, m_t) = \left( \frac{i_t}{1 + i_t} \right) u_c(c_t, m_t). \quad (4.23)$$

Now suppose the utility function takes the form  $u(c_t, m_t) = \ln c_t + m_t(B - D \ln m_t)$ . Using the functional form in (4.23), one obtains

$$m_t = A e^{-\omega_t / D c_t}, \quad (4.24)$$

where  $A = e^{(\frac{B}{D}-1)}$  and  $\omega = i/(1+i)$ . Equation (4.24) provides a convenient functional representation for the demand for money.

Since the time of Cagan's seminal contribution to the study of seigniorage and hyperinflation (Cagan 1956, 158–161), many economists have followed him in specifying a money demand function of the form  $m = K e^{-\alpha \pi^e}$ ; (4.24) shows how something similar can be derived from an underlying utility function. As Calvo and Leiderman (1992) pointed out, the advantage is that one sees how the parameters  $K$  and  $\alpha$  depend on more primitive parameters of the representative agent's preferences and how they may actually be time-dependent. For example,  $\alpha$  depends on  $c_t$  and therefore will be time-dependent unless  $K$  varies appropriately or  $c$  itself is constant.

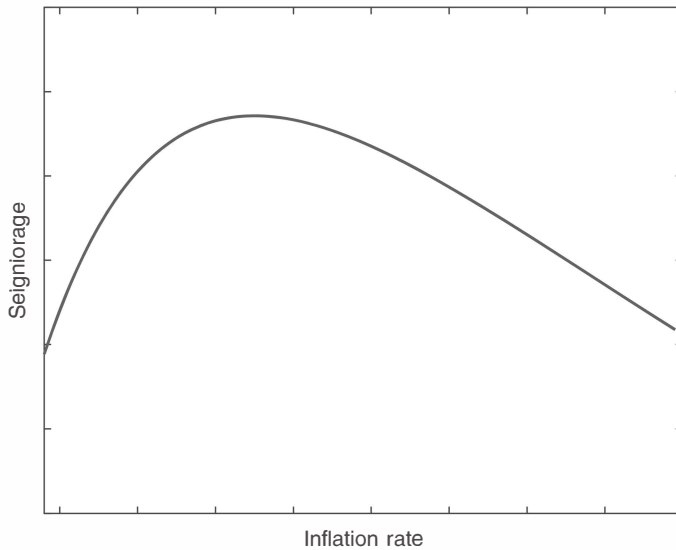
The reason for deriving the demand for money as a function of the rate of inflation is that, having done so, one can express seigniorage as a function of the rate of inflation. Recall from (4.9) that seigniorage was equal to  $i \times m / (1 + \pi) = (1 + r) i \times m / (1 + i)$ . Using the expression for the demand for money, steady-state seigniorage is equal to

$$\bar{s} = (1 + r) \left( \frac{i}{1 + i} \right) A \exp \left[ -\frac{i}{Dc(1 + i)} \right].$$

If superneutrality is assumed to characterize the model, then  $c$  is constant in the steady state and independent of the rate of inflation. The same is true of the real rate of interest.

To determine how seigniorage varies with the rate of inflation, think of choosing  $\omega = i/(1+i)$  through the choice of  $\pi$ . Then  $\bar{s} = (1 + r) \omega A e^{-\omega / Dc}$ , and  $\partial \bar{s} / \partial \pi = (\partial \bar{s} / \partial \omega) (\partial \omega / \partial i) (\partial i / \partial \pi) = (\partial \bar{s} / \partial \omega) (1 + r) / (1 + i)^2$ , so the sign of  $\partial \bar{s} / \partial \pi$  is determined by the sign of  $(\partial \bar{s} / \partial \omega)$ . Since

$$\frac{\partial \bar{s}}{\partial \omega} = (1 + r) A e^{-\omega / Dc} \left[ 1 - \frac{\omega}{Dc} \right] = \frac{\bar{s}}{\omega} \left[ 1 - \frac{\omega}{Dc} \right],$$



**Figure 4.1**  
Seigniorage as a function of inflation.

the sign of  $\partial \bar{s} / \partial \omega$  depends on the sign of  $1 - (\omega / Dc)$ . As illustrated in figure 4.1, seigniorage increases with inflation initially but eventually begins to decline with further increases in  $\pi$  as the demand for real balances shrinks.<sup>21</sup>

To determine the inflation rate that maximizes seigniorage, note that  $\partial \bar{s} / \partial \pi = 0$  if and only if

$$\omega = \frac{i}{1+i} = Dc, \quad \text{or} \quad \pi^{\max} = \left( \frac{1}{1+r} \right) \left( \frac{1}{1-Dc} \right) - 1.$$

For inflation rates less than  $\pi^{\max}$ , the government's revenue is increasing in the inflation rate. The effect of an increase in the tax rate dominates the effect of higher inflation in reducing the real demand for money. As inflation increases above  $\pi^{\max}$ , the tax base shrinks sufficiently that revenue from seigniorage declines. Consequently, governments face a seigniorage Laffer curve; raising inflation beyond a certain point results in lower real tax revenue.

21. Whether a Laffer curve exists for seigniorage depends on the specification of utility. For example, in chapter 2 it was noted that with a CES utility function the demand for money was given by  $m_t = A [i / (1+i)]^{-\frac{1}{b}} c_t$ , where  $A$  is a constant. Hence, seigniorage is  $A [i / (1+i)]^{1-\frac{1}{b}} c_t$ , which is monotonic in  $i$ .

#### 4.4.4 Cagan's Model

Since 1970 the consumer price index for the United States has risen just over sixfold; that's inflation.<sup>22</sup> In Hungary, the index of wholesale prices was 38,500 in January 1923 and 1,026,000 in January 1924, one year later, a 27-fold increase; that's hyperinflation (Sargent 1986).

Cagan (1956) provided one of the earliest studies of the dynamics of money and prices during hyperinflation. The discussion here follows Cagan in using continuous time. Suppose the real per capita fiscal deficit that needs to be financed is exogenously given and is equal to  $\Delta^f$ . This means that

$$\Delta^f = \frac{\dot{H}}{H} \frac{H}{PY} = \theta h,$$

where  $h$  has been expressed as real balances relative to income to allow for real economic growth. The demand for real balances depends on the nominal interest rate and therefore the expected rate of inflation. Treating real variables such as the real rate of interest and real output as constant (which is appropriate in a steady state characterized by superneutrality and is usually taken as reasonable during hyperinflations because all the action involves money and prices), write the demand for the real monetary base as  $h = \exp(-\alpha\pi^e)$ . Then the government's revenue requirement implies that

$$\Delta^f = \theta e^{-\alpha\pi^e}. \tag{4.25}$$

For  $h$  to be constant in equilibrium requires that  $\pi = \theta - \mu$ , where  $\mu$  is the growth rate of real income. And in a steady-state equilibrium,  $\pi^e = \pi$ , so (4.25) becomes

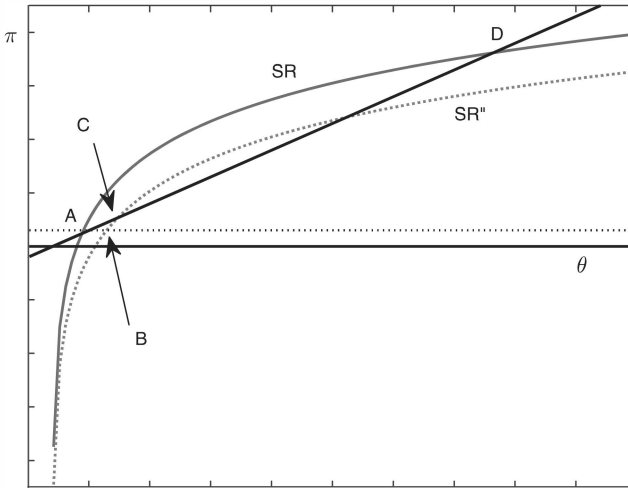
$$\Delta^f = \theta e^{-\alpha(\theta - \mu)}, \tag{4.26}$$

the solution(s) of which give the rates of money growth that are consistent with raising the amount  $\Delta^f$  through seigniorage. The right side of (4.26) equals zero when money growth is equal to zero, rises to a maximum at  $\theta = (1/\alpha)$ , and then declines.<sup>23</sup> That is, for rates of money growth above  $(1/\alpha)$  (and therefore inflation rates above  $(1/\alpha) - \mu$ ), higher inflation actually leads to lower revenues because the tax base falls sufficiently to offset the rise in inflation. Thus, any deficit less than  $\Delta^* = (1/\alpha) \exp(\alpha\mu - 1)$  can be financed by either a low rate of inflation or a high rate of inflation.

Figure 4.2, based on Bruno and Fischer (1990), illustrates the two inflation rates consistent with seigniorage revenue of  $\Delta^f$ . The curve  $SR$  is derived from (4.25) and shows, for

22. The CPI was equal to 37.9 in January 1970 and reached 238.0 in December 2015.

23. More generally, with  $h$  a function of the nominal interest rate and  $r$  a constant, seigniorage can be written as  $s = \theta h(\theta)$ . This is maximized at the point where the elasticity of real money demand with respect to  $\theta$  is equal to  $-1$ :  $\theta h'(\theta)/h = -1$ .



**Figure 4.2**

Money growth and seigniorage revenue.

each rate of money growth, the expected rate of inflation needed to generate the required seigniorage revenue.<sup>24</sup> The 45° line gives the steady-state inflation rate as a function of the money growth rate:  $\pi^e = \pi = \theta - \mu$ . The two points of intersection labeled *A* and *D* are the two solutions to (4.26).

What determines whether, for a given deficit, the economy ends up at the high inflation equilibrium or the low inflation equilibrium? Which equilibrium emerges depends on the stability properties of the economy. Determining this, in turn, requires a more complete specification of the dynamics of the model. Recall that the demand for money depends on expected inflation through the nominal rate of interest, while the inflation tax rate depends on actual inflation. In considering the effects of variations in the inflation rate, one needs to determine how expectations will adjust. Cagan (1956) addressed this by assuming that expectations adjust adaptively to actual inflation:

$$\frac{\partial \pi^e}{\partial t} = \dot{\pi}^e = \eta(\pi - \pi^e), \quad (4.27)$$

where  $\eta$  captures the “speed of adjustment” of expectations. A low  $\eta$  implies that expectations respond slowly to inflation forecast errors. Since  $h = \exp(-\alpha\pi^e)$ , differentiate this expression with respect to time, obtaining

$$\frac{\dot{h}}{h} = \theta - \mu - \pi = -\alpha\dot{\pi}^e.$$

24. That is,  $SR$  plots  $\pi^e = (\ln \theta - \ln \Delta^f)/\alpha$ . A reduction in  $\theta$  continues to yield  $\Delta^f$  only if money holdings rise, and this would require a fall in expected inflation.



Solving for  $\pi$  using (4.27) yields  $\pi = \theta - \mu + \alpha\dot{\pi}^e = \theta - \mu + \alpha\eta(\pi - \pi^e)$ , or  $\pi = (\theta - \mu - \alpha\eta\pi^e) / (1 - \alpha\eta)$ . Substituting this back into the expectations adjustment equation gives

$$\dot{\pi}^e = \frac{\eta(\theta - \mu - \pi^e)}{1 - \alpha\eta}, \quad (4.28)$$

which implies that the low inflation equilibrium will be stable as long as  $\alpha\eta < 1$ . This requires that expectations adjust sufficiently slowly ( $\eta < 1/\alpha$ ).

If expectations adjust adaptively and sufficiently slowly, what happens when the deficit is increased? Since the demand for real money balances depends on expected inflation, and because the adjustment process does not allow the expected inflation rate to jump immediately, the higher deficit can be financed by an increase in the rate of inflation (assuming the new deficit is still below the maximum that can be financed,  $\Delta^*$ ). Since actual inflation now exceeds expected inflation,  $\dot{\pi}^e > 0$ , and  $\pi^e$  begins to rise. The economy converges into a new equilibrium at a higher rate of inflation.

In terms of figure 4.2, an increase in the deficit shifts the  $SR$  curve to the right to  $SR''$  (for a given expected rate of inflation, money growth must rise in order to generate more revenue). Assume that initially the economy is at point  $A$ , the low inflation equilibrium. Budget balance requires that the economy be on the  $SR''$  line, so  $\theta$  jumps to the rate associated with point  $B$ . But now, at point  $B$ , inflation has risen and  $\pi^e < \pi = \theta - \mu$ . Expected inflation rises (as long as  $\alpha\eta < 1$ ; see (4.28)), and the economy converges to  $C$ . The high inflation equilibrium, in contrast, is unstable.

Adaptive expectations of the sort Cagan assumed disappeared from the literature under the onslaught of Lucas and Sargent's rational-expectations revolution of the early 1970s. If agents are systematically attempting to forecast inflation, then their forecasts will depend on the actual process governing the evolution of inflation; rarely will this imply an adjustment process such as (4.27). Stability in the Cagan model also requires that expectations not adjust too quickly ( $\eta < 1/\alpha$ ), and this requirement conflicts with the rational-expectations notion that expectations adjust quickly in response to new information. Bruno and Fischer (1990) showed that, to some degree, assuming agents adjust their holdings of real money balances slowly plays a role under rational expectations similar to the role played by the slow adjustment of expectations in Cagan's model in ensuring stability under adaptive expectations.

#### 4.4.5 Rational Hyperinflation

Why do countries find themselves in situations of hyperinflation? Most explanations of hyperinflation point to fiscal policy as the chief culprit. Governments that are forced to print money to finance real government expenditures often end up generating hyperinflations. In that sense, rapid money growth does lead to hyperinflation, consistent with the relationship between money growth and inflation implied by the models examined so far, but money

growth is no longer exogenous. Instead, it is endogenously determined by the need to finance a fiscal deficit.<sup>25</sup>

Two explanations for the development of hyperinflation suggest themselves. In the Cagan model with adaptive expectations, suppose that  $\alpha\eta < 1$ , so that the low inflation equilibrium is stable. Now suppose that a shock pushes the inflation rate above the high inflation equilibrium (above point  $D$  in figure 4.2). If that equilibrium is unstable, the economy continues to diverge, moving to higher and higher rates of inflation. So one explanation for hyperinflations is that they represent situations in which exogenous shocks push the economy into an unstable region.

Alternatively, suppose the deficit that needs to be financed with seigniorage grows. If it rises above  $\Delta^*$ , the maximum that can be financed by money creation, the government finds itself unable to obtain enough revenue, so it runs the printing presses faster, further reducing the real revenue it obtains and forcing it to print money even faster. Most hyperinflations have occurred after wars (and on the losing side). Such countries face an economy devastated by war and a tax system that no longer functions effectively. At the same time, there are enormous demands on the government for expenditures to provide the basics of food and shelter and to rebuild the economy. Revenue needs outpace the government's ability to raise tax revenue. The ends of such hyperinflations usually involve a fiscal reform that allows the government to reduce its reliance on seigniorage (see Sargent 1986).

When expected inflation falls in response to the reforms, the opportunity cost of holding money is reduced and the demand for real money balances rises. Thus, the growth rate of the nominal money supply normally continues temporarily at a very high rate after a hyperinflation has ended. A similar, if smaller-scale, phenomenon occurred in the United States in the mid-1980s. The money supply, as measured by  $M1$ , grew very rapidly. At the time, there were concerns that this growth would lead to a return of higher rates of inflation. Instead, it seemed to reflect the increased demand for money resulting from the decline in inflation from its peak levels in 1979–1980. The need for real money balances to grow as inflation is reduced often causes problems for establishing and maintaining the credibility of policies designed to reduce inflation. If a disinflation is credible, so that expected inflation falls, it may be necessary to increase the growth rate of the nominal money supply temporarily. But when inflation and rapid money growth are so closely related, letting money growth rise may be misinterpreted as a signal that the central bank has given up on its disinflation policy.

Fiscal theories of seigniorage, inflation, and hyperinflations are based on fundamentals—there really is a deficit that needs to be financed, and that is what leads to money creation. An alternative view of hyperinflations is that they are simply *bubbles*, similar to bubbles in financial markets. Such phenomena are based on the possibility of multiple equilibria in which expectations can be self-fulfilling.

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25. A recent modern example of such a fiscally driven hyperinflation was provided by Zimbabwe.

To illustrate this possibility, suppose the real demand for money is given by, in log terms,

$$m_t - p_t = -\alpha(E_t p_{t+1} - p_t),$$

where  $E_t p_{t+1}$  denotes the expectation formed at time  $t$  of time  $t + 1$  prices and  $\alpha > 0$ . This money demand function is the log version of Cagan's demand function. One can rearrange this equation to express the current price level as

$$p_t = \left( \frac{1}{1 + \alpha} \right) m_t + \left( \frac{\alpha}{1 + \alpha} \right) E_t p_{t+1}. \quad (4.29)$$

Suppose that the growth rate of the nominal money supply process is given by  $m_t = \theta_0 + (1 - \gamma)\theta_1 t + \gamma m_{t-1}$ . Since  $m$  is the log money supply, the growth rate of the money supply is  $m_t - m_{t-1} = (1 - \gamma)\theta_1 + \gamma(m_{t-1} - m_{t-2})$ , and the trend (average) growth rate is  $\theta_1$ . Given this process, and the assumption that agents make use of it and the equilibrium condition (4.29) in forming their expectations, one solution for the price level is given by

$$\begin{aligned} p_t &= \frac{\alpha [\theta_0 + (1 - \gamma)\theta_1(1 + \alpha)]}{1 + \alpha(1 - \gamma)} + \left[ \frac{\alpha(1 - \gamma)\theta_1}{1 + \alpha(1 - \gamma)} \right] t + \left[ \frac{1}{1 + \alpha(1 - \gamma)} \right] m_t \\ &= A_0 + A_1 t + A_2 m_t. \end{aligned}$$

That this is a solution can be verified by noting that it implies  $E_t p_{t+1} = A_0 + A_1(t + 1) + A_2 E_t m_{t+1} = A_0 + A_1(t + 1) + A_2 [\theta_0 + (1 - \gamma)\theta_1(t + 1) + \gamma m_t]$ ; substituting this into (4.29) yields the proposed solution. Under this solution, the inflation rate  $p_t - p_{t-1}$  converges to  $\theta_1$ , the average growth rate of the nominal supply of money.<sup>26</sup>

Consider, now, an alternative solution:

$$p_t = A_0 + A_1 t + A_2 m_t + B_t, \quad (4.30)$$

where  $B_t$  is time-varying. Does there exist a  $B_t$  process consistent with (4.29)? Substituting the new proposed solution into the equilibrium condition for the price level yields

$$A_0 + A_1 t + A_2 m_t + B_t = \frac{m_t}{1 + \alpha} + \frac{\alpha [A_0 + A_1(t + 1) + A_2 E_t m_{t+1} + E_t B_{t+1}]}{1 + \alpha},$$

which, to hold for all realizations of the nominal money supply, requires that, as before,  $A_0 = \alpha [\theta_0 + (1 - \gamma)\theta_1(1 + \alpha)] / [1 + \alpha(1 - \gamma)]$ ,  $A_1 = \alpha(1 - \gamma)\theta_1 / [1 + \alpha(1 - \gamma)]$ , and  $A_2 = 1 / [1 + \alpha(1 - \gamma)]$ . This then implies that the  $B_t$  process must satisfy

$$B_t = \left( \frac{\alpha}{1 + \alpha} \right) E_t B_{t+1},$$

which holds if  $B$  follows the explosive process

$$B_{t+1} = k B_t \quad (4.31)$$

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26. This follows, since  $p_t - p_{t-1} = A_1 + A_2(m_t - m_{t-1})$  converges to  $A_1 + A_2\theta_1 = \theta_1$ .

for  $k = (1 + \alpha)/\alpha > 1$ . In other words, (4.30) is an equilibrium solution for *any* process  $B_t$  satisfying (4.31). Since  $B$  grows at the rate  $k - 1 = 1/\alpha$ , and since  $\alpha$ , the elasticity of money demand with respect to expected inflation, is normally thought to be small, its inverse would be large. The actual inflation rate along a bubble solution path could greatly exceed the rate of money growth. As discussed in section 2.2.2, speculative hyperinflation in unbacked fiat money systems cannot generally be ruled out. Equilibrium paths may exist along which real money balances eventually converge to zero as the price level goes to  $+\infty$ . (See also section 4.5.1.)

The methods developed to test for bubbles are similar to those that have been employed to test for intertemporal budget balance. For example, if the nominal money stock is nonstationary, then the absence of bubbles implies that the price level will be nonstationary *but cointegrated with the money supply*. This is a testable implication of the no-bubble assumption. Equation (4.31) gives the simplest example of a bubble process. Evans (1991) showed how the cointegration tests can fail to detect bubbles that follow periodically collapsing processes. For more on asset prices and bubbles, see Shiller (1981), Matthey and Meese (1986), West (1987; 1988), Diba and Grossman (1988a; 1988b), and Barlevy (2007).

#### 4.5 The Fiscal Theory of the Price Level

A number of researchers have examined models in which fiscal factors replace the money supply as the key determinant of the price level. See, for example, Leeper (1991), Sims (1994), Woodford (1995; 1998; 2001a), Bohn (1999), Cochrane (1999; 2001), Kocherlakota and Phelen (1999), Daniel (2001), the excellent discussions by Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), Canzoneri, Cumby, and Diba (2011) and the references they list, and the criticisms of the approach by McCallum (2001), Buiter (2002), and McCallum and Nelson (2005). The *fiscal theory of the price level* raises some important issues for both monetary theory and monetary policy.<sup>27</sup>

There are two ways fiscal policy might matter for the price level. First, equilibrium requires that the real quantity of money equal the real demand for money. If fiscal variables affect the real demand for money, the equilibrium price level will also depend on fiscal factors (see section 4.4.2). This, however, is not the channel emphasized in fiscal theories of the price level. Instead, these theories focus on a second aspect of monetary models: there may be multiple price levels consistent with a given nominal quantity of money and equality between money supply and money demand. The possibility of multiple equilibria was discussed in section 2.2.2 in the context of the MIU model, but the same possibility arises in other models of the demand for money. Fundamentally, the real demand for money depends on the nominal interest rate, which in turn depends on the expected future price

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27. For an analysis of the 2008 financial crisis and its implications for monetary and fiscal policy from the perspective of the fiscal theory, see Cochrane (2011b).

level. There may be multiple paths for the price level at each point on which the real demand for money is equal to the real supply of money. When standard monetary models are consistent with multiple equilibrium values for the price level, fiscal policy may then determine which of these is the equilibrium price level. And in some cases, the equilibrium price level picked out by fiscal factors may be independent of the nominal supply of money.

In contrast to the standard monetary theories of the price level, the fiscal theory assumes that the government's intertemporal budget equation represents an equilibrium condition rather than a constraint that must hold for all price levels. At some price levels, the intertemporal budget constraint would be violated. Such price levels are not consistent with equilibrium. Given the stock of nominal debt, the equilibrium price level must ensure that the government's intertemporal budget is balanced. In that way, fiscal considerations may pin down the equilibrium price level.

#### 4.5.1 Multiple Equilibria

The traditional quantity theory of money highlights the role the nominal stock of money plays in determining the equilibrium price level. Using the demand for money given by (4.19), a proportional relationship is obtained between the nominal quantity of money and the equilibrium price level that depends on the nominal rate of interest. However, the nominal interest rate is also an endogenous variable, so (4.19) by itself may not be sufficient to determine the equilibrium price level. Because the nominal interest rate depends on the rate of inflation, (4.19) can be written as

$$\frac{M_t}{P_t} = f\left(R_t \frac{P_{t+1}}{P_t}\right),$$

where  $R$  is the gross real rate of interest. This forward difference equation in the price level may be insufficient to determine a unique equilibrium path for the price level.

Consider a perfect-foresight equilibrium with a constant nominal supply of money,  $M_0$ . Suppose the real rate of return is equal to its steady-state value of  $1/\beta$ , and the demand for real money balances is given by (4.19). One can then write the equilibrium between the real supply of money and the real demand for money as

$$\frac{M_0}{P_t} = g\left(\frac{P_{t+1}}{P_t}\right), \quad g' < 0.$$

Under suitable regularity conditions on  $g()$ , this condition can be rewritten as

$$P_{t+1} = P_t g^{-1}\left(\frac{M_0}{P_t}\right) \equiv \phi(P_t). \quad (4.32)$$

Equation (4.32) defines a difference equation in the price level. One solution is  $P_{t+i} = P^*$  for all  $i \geq 0$ , where  $P^* = M_0/g(1)$ . In this equilibrium, the quantity theory holds, and the price level is proportional to the money supply.

This constant price level equilibrium is not, however, the only possible equilibrium. As noted in sections 2.2.2 and 4.4.5, there may be equilibrium price paths starting from  $P_0 \neq P^*$  that are fully consistent with the equilibrium condition (4.32). This possibility was illustrated in figure 2.2. Thus, standard models in which equilibrium depends on forward-looking expectations of the price level, a property of the models discussed in chapters 2 and 3, generally have multiple equilibria. An additional equilibrium condition may be needed to uniquely determine the price level. The fiscal theory of the price level focuses on situations in which the government's intertemporal budget constraint may supply that additional condition.

#### 4.5.2 The Basic Idea of the Fiscal Theory

The fiscal theory can be illustrated in the context of a model with a representative household and a government, but with no capital. The implications of the fiscal theory are easiest to see if attention is restricted to perfect-foresight equilibria.

The representative household chooses its consumption and asset holdings optimally, subject to an intertemporal budget constraint. Suppose the period  $t$  budget constraint of the representative household takes the form

$$D_t + P_t y_t - T_t \geq P_t c_t + M_t^d + B_t^d = P_t c_t + \left( \frac{i_t}{1 + i_t} \right) M_t^d + \left( \frac{1}{1 + i_t} \right) D_{t+1}^d,$$

where  $D_t$  is the household's beginning-of-period financial wealth and  $D_{t+1}^d = (1 + i_t)B_t^d + M_t^d$ . The superscripts denote that  $M^d$  and  $B^d$  are the household's demand for money and interest-bearing debt. In real terms, this budget constraint becomes

$$d_t + y_t - \tau_t \geq c_t + m_t^d + b_t^d = c_t + \left( \frac{i_t}{1 + i_t} \right) m_t^d + \left( \frac{1}{1 + r_t} \right) d_{t+1}^d,$$

where  $\tau_t = T_t/P_t$ ,  $m_t^d = M_t^d/P_t$ ,  $1 + r_t = (1 + i_t)(1 + \pi_{t+1})$ , and  $d_t = D_t/P_t$ . Let

$$\lambda_{t,t+i} = \prod_{j=1}^i \left( \frac{1}{1 + r_{t+j}} \right)$$

be the discount factor, with  $\lambda_{t,t} = 1$ . Under standard assumptions, the household intertemporal budget constraint takes the form

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} (y_{t+i} - \tau_{t+i}) = \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ c_{t+i} + \left( \frac{i_{t+i}}{1 + i_{t+i}} \right) m_{t+i}^d \right]. \quad (4.33)$$

Household choices must satisfy this intertemporal budget constraint. The left side is the present discounted value of the household's initial real financial wealth and after-tax income. The right side is the present discounted value of consumption spending plus the real cost of holding money. This condition holds with equality because any path of

consumption and money holdings for which the left side exceeded the right side would not be optimal; the household could increase its consumption at time  $t$  without reducing consumption or money holdings at any other date. As long as the household is unable to accumulate debts that exceed the present value of its resources, the right side cannot exceed the left side.

The budget constraint for the government sector, in nominal terms, takes the form

$$P_t g_t + (1 + i_{t-1})B_{t-1} = T_t + M_t - M_{t-1} + B_t. \quad (4.34)$$

Dividing by  $P_t$ , this can be written as

$$g_t + d_t = \tau_t + \left( \frac{i_t}{1 + i_t} \right) m_t + \left( \frac{1}{1 + r_t} \right) d_{t+1}.$$

Recursively substituting for future values of  $d_{t+i}$ , this budget constraint implies that

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \rightarrow \infty} \lambda_{t,t+T} d_T, \quad (4.35)$$

where  $\bar{s}_t = i_t m_t / (1 + i_t)$  is the government's real seigniorage revenue. In previous sections, it was assumed that the expenditures, taxes, and seigniorage choices of the consolidated government (the combined monetary and fiscal authorities) were assumed to be constrained by the requirement that  $\lim_{T \rightarrow \infty} \lambda_{t,t+T} d_T = 0$  for all price levels  $P_t$ . Policy paths for  $(g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0}$  such that

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} [g_{t+i} - \tau_{t+i} - \bar{s}_{t+i}] = \lim_{T \rightarrow \infty} \lambda_{t,t+T} d_T = 0$$

for all price paths  $p_{t+i}$ ,  $i \geq 0$ , are called *Ricardian* policies. Policy paths for  $(g_{t+i}, \tau_{t+i}, \bar{s}_{t+i}, d_{t+i})_{i \geq 0}$  for which  $\lim_{T \rightarrow \infty} \lambda_{t,t+T} d_T$  may not equal zero for all price paths are called *non-Ricardian*.<sup>28</sup>

Now consider a perfect-foresight equilibrium. Regardless of whether the government follows a Ricardian or a non-Ricardian policy, equilibrium in the goods market in this simple economy with no capital requires that  $y_t = c_t + g_t$ . The demand for money must also equal the supply of money:  $m_t^d = m_t$ . Substituting  $y_t - g_t$  for  $c_t$  and  $m_t$  for  $m_t^d$  in (4.33) and rearranging yields

$$d_t + \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[ g_{t+i} - \tau_{t+i} - \left( \frac{i_{t+i}}{1 + i_{t+i}} \right) m_{t+i} \right] = 0. \quad (4.36)$$

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28. Notice that this usage differs somewhat from the way Sargent (1982) and Aiyagari and Gertler (1985) employed the terms. In these earlier papers, a Ricardian policy was one in which the fiscal authority fully adjusted taxes to ensure intertemporal budget balance for all price paths. A non-Ricardian policy was a policy in which the monetary authority was required to adjust seigniorage to ensure intertemporal budget balance for all price paths. Both these policies would be labeled Ricardian under the current section's use of the term.

Thus, an implication of the representative household's optimization problem and market equilibrium is that (4.36) must hold in equilibrium. Under Ricardian policies, (4.36) does not impose any additional restrictions on equilibrium because the policy variables are always adjusted to ensure that this condition holds. Under a non-Ricardian policy, however, it does impose an additional condition that must be satisfied in equilibrium. To see what this condition involves, one can use the definition of  $d_t$  and seigniorage to write (4.36) as

$$\frac{D_t}{P_t} = \sum_{i=0}^{\infty} \lambda_{t,t+i} [\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]. \quad (4.37)$$

At time  $t$ , the government's outstanding nominal liabilities  $D_t$  are predetermined by past policies. Given the present discounted value of the government's future surpluses (the right side of 4.37), the only endogenous variable is the current price level  $P_t$ . The price level must adjust to ensure that (4.37) is satisfied.

Equation (4.37) is an equilibrium condition under non-Ricardian policies, but it is not the only equilibrium condition. It is still the case that real money demand and real money supply must be equal. Suppose the real demand for money is given by (4.19), rewritten here as

$$\frac{M_t}{P_t} = f(1 + i_t). \quad (4.38)$$

Equations (4.37) and (4.38) must both be satisfied in equilibrium. However, which two variables are determined jointly by these two equations depends on the assumptions that are made about fiscal and monetary policies. For example, suppose the fiscal authority determines  $g_{t+i}$  and  $\tau_{t+i}$  for all  $i \geq 0$ , and the monetary authority pegs the nominal rate of interest  $i_{t+i} = \bar{i}$  for all  $i \geq 0$ . Seigniorage is equal to  $\bar{i}f(1 + \bar{i})/(1 + \bar{i})$  and so is fixed by monetary policy. With this specification of monetary and fiscal policies, the right side of (4.37) is given. Since  $D_t$  is predetermined at date  $t$ , (4.37) can be solved for the equilibrium price level  $P_t^*$  given by

$$P_t^* = \frac{D_t}{\sum_{i=0}^{\infty} \lambda_{t,t+i} [\tau_{t+i} + \bar{s}_{t+i} - g_{t+i}]}. \quad (4.39)$$

The current nominal money supply is then determined by (4.38):

$$M_t = P_t^* f(1 + \bar{i}).$$

One property of this equilibrium is that changes in fiscal policy ( $g$  or  $\tau$ ) directly alter the equilibrium price level, even though seigniorage as measured by  $\sum_{i=0}^{\infty} \lambda_{t,t+i} \bar{s}_{t+i}$  is unaffected.<sup>29</sup> The finding that the price level is uniquely determined by (4.39) contrasts with

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29. A change in  $g$  or  $\tau$  causes the price level to jump, and this transfers resources between the private sector and the government. This transfer can also be viewed as a form of seigniorage.



a standard conclusion that the price level is indeterminate under a nominal interest rate peg. This conclusion is obtained from (4.38): with  $i$  pegged, the right side of (4.38) is fixed, but this only determines the real supply of money. Any price level is consistent with equilibrium, as  $M$  then adjusts to ensure that (4.38) holds.

Critical to the fiscal theory is the assumption that (4.37), the government's intertemporal budget constraint, is an *equilibrium* condition that holds at the equilibrium price level and not a condition that must hold at all price levels. This means that at price levels not equal to  $P_t^*$ , the government is planning to run surpluses (including seigniorage) whose real value, in present discounted terms, is not equal to the government's outstanding real liabilities. Similarly, it means that the government could cut current taxes, leaving current and future government expenditures and seigniorage unchanged, and not simultaneously plan to raise future taxes.<sup>30</sup> When (4.37) is interpreted as a budget constraint that must be satisfied for all price levels, that is, under Ricardian policies, any decision to cut taxes today (and so lower the right side of (4.37)) must be accompanied by planned future tax increases to leave the right side unchanged.

In standard infinite-horizon representative agent models, a tax cut (current and future government expenditures unchanged) has no effect on equilibrium (i.e., Ricardian equivalence holds) because the tax reduction does not have a real wealth effect on private agents. Agents recognize that in a Ricardian regime, future taxes have risen in present value terms by an amount exactly equal to the reduction in current taxes. Alternatively expressed, the government cannot engineer a permanent tax cut unless government expenditures are also cut (in present value terms). Because the fiscal theory of the price level assumes that (4.37) holds only when evaluated at the equilibrium price level, the government can plan a permanent tax cut. If it does, the price level must rise to ensure that the new, lower value of discounted surpluses is again equal to the real value of government debt.

For (4.39) to define an equilibrium price level, it must hold that  $D_t \neq 0$ . Niepelt (2004) has argued that the fiscal theory cannot hold if there is no initial outstanding stock of nominal government debt. However, Daniel (2007) showed that one can define non-Ricardian policies in a consistent manner when the initial stock of debt is zero. Her argument is most clearly seen in a two-period example. If the monetary authority pegs the nominal rate of interest, then any initial value of the price level is consistent with equilibrium, a standard result under interest rate pegs (see chapter 10). The nominal interest rate peg does pin down the expected inflation rate, or equivalently, the expected price level in the second period. However, this policy does not pin down the actual price level in period 2. Under a Ricardian fiscal policy, *any* realization of the price level in period 2, consistent with the value expected, is an equilibrium. If the realized price level were to result in the government's budget constraint not balancing, then the Ricardian nature of policy means that

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30. However, as Bassetto (2002) emphasized, the ability of the government to run a deficit in any period under a non-Ricardian policy regime is constrained by the willingness of the public to lend to the government.

taxes and/or spending must adjust to ensure intertemporal budget balance at the realized price level. Under a non-Ricardian fiscal policy, only realizations of the price level that satisfy intertemporal budget balance can be consistent with an equilibrium. Thus, whatever quantity of nominal debt the government issued in the first period, the realized price level must ensure the real value of this debt in period 2 balances with the real value the government chooses for its primary surplus (including seigniorage). Under rational expectations, however, a non-Ricardian government cannot systematically employ price surprises in period 2 to finance spending because the monetary authority's interest peg has determined the expected value of the period 2 price level. Equilibrium must be consistent with those expectations.

An interest rate peg is just one possible specification for monetary policy. As an alternative, suppose as before that the fiscal authority sets the paths for  $g_{t+i}$  and  $\tau_{t+i}$ , but now suppose that the government adjusts tax revenue to offset any variations in seigniorage. In this case,  $\tau_{t+i} + \bar{s}_{t+i}$  becomes an exogenous process. Then (4.37) can be solved for the equilibrium price level, independent of the nominal money stock. Equation (4.38) must still hold in equilibrium. If the monetary authority sets  $M_t$ , this equation determines the nominal interest rate that ensures that the real demand for money is equal to the real supply. If the monetary authority sets the nominal rate of interest, (4.38) determines the nominal money supply. The extreme implication of the fiscal theory (relative to traditional quantity theory results) is perhaps most stark when the monetary authority fixes the nominal supply of money:  $M_{t+i} = \bar{M}$  for all  $i \geq 0$ . Then, under a fiscal policy that makes  $\tau_{t+i} + \bar{s}_{t+i}$  an exogenous process, the price level is proportional to  $D_t$  and, for a given level of  $D_t$ , is independent of the value chosen for  $\bar{M}$ .

### 4.5.3 Empirical Evidence on the Fiscal Theory

Under the fiscal theory of the price level, (4.37) holds at the equilibrium value of the price level. Under traditional theories of the price level, (4.37) holds for all values of the price level. If one only observes equilibrium outcomes, it is impossible empirically to distinguish between the two theories. As Sims (1994) put it, "Determinacy of the price level under any policy depends on the public's beliefs about what the policy authority would do under conditions that are never observed in equilibrium" (381). Canzoneri, Cumby, and Diba (2011) discussed the identification issues that arise in attempting to test whether fiscal policy is Ricardian or non-Ricardian.

Canzoneri, Cumby, and Diba (2001) examined VAR evidence on the response of U.S. liabilities to a positive innovation to the primary surplus. Under a non-Ricardian policy, a positive innovation to  $\tau_t + \bar{s}_t - g_t$  should increase  $D_t/P_t$  (see 4.37) unless it also signals future reductions in the surplus, that is, unless  $\tau_t + \bar{s}_t - g_t$  is negatively serially correlated. The authors argued that in a Ricardian regime, a positive innovation to the current primary

surplus will reduce real liabilities. This can be seen by writing the budget constraint (4.34) in real terms as

$$d_{t+1} = R[d_t - (\tau_t + s_t - g_t)]. \quad (4.40)$$

Examining U.S. data, the authors found the responses were inconsistent with a non-Ricardian regime. Increases in the surplus were associated with declines in current and future real liabilities, and the surplus did not display negative serial correlation.

Cochrane (1999) pointed out the fundamental problem with this test: both (4.40) and (4.37) must hold in equilibrium, so it can be difficult to develop testable restrictions that can distinguish between the two regimes. The two regimes have different implications only if one can observe nonequilibrium values of the price level. Cochrane (2011b) used the fiscal theory to analyze the role of monetary and fiscal policy during the Great Recession.

Bohn (1999) examined the U.S. deficit and debt processes and concluded that the primary surplus responds positively to the debt to GDP ratio. In other words, a rise in the debt to GDP ratio leads to an increase in the primary surplus. Thus, the surplus does adjust, and Bohn found that it responds enough to ensure that the intertemporal budget constraint is satisfied. This is evidence that the fiscal authority seems to act in a Ricardian fashion.

Finally, an older literature (see section 4.4) attempted to estimate whether fiscal deficits tended to lead to faster money growth. Such evidence might be interpreted to imply a Ricardian regime of fiscal dominance.

## 4.6 Optimal Taxation and Seigniorage

If the government can raise revenue by printing money, how much should it raise from this source? Suppose only distortionary revenue sources are available. To raise a given amount of revenue while causing the minimum deadweight loss from tax-induced distortions, the government should generally set its tax instruments so that the marginal distortionary cost per dollar of revenue raised is equalized across all taxes. As first noted by Phelps (1973), this suggests that an optimal tax package should include some seigniorage. This prescription links the optimal inflation tax to a more general problem of determining the optimal levels of all tax instruments. If governments are actually attempting to minimize the distortionary costs of raising revenue, then the optimal tax literature provides a positive theory of inflation.

This basic idea, which is developed in section 4.6.1 was originally used by Mankiw (1987) to explain nominal interest rate setting by the Federal Reserve. However, the implications of this approach are rejected for the industrialized economies (Poterba and Rotemberg 1990; Trehan and Walsh 1990), although this may not be too surprising because seigniorage plays a fairly small role as a revenue source for these countries. Calvo and

Leiderman (1992) used the optimal tax approach to examine the experiences of some Latin American economies, with more promising results. A survey of optimal seigniorage that links the topic with the issues of time inconsistency (see chapter 6) can be found in Herrendorf (1997). Section 4.6.2 considers the role inflation might play as an optimal response to the need to finance temporary expenditure shocks. Section 4.6.3 revisits Friedman's rule for the optimal rate of inflation in an explicit general equilibrium framework.

#### 4.6.1 A Partial Equilibrium Model

Assume a Ricardian regime in which the government has two revenue sources available to it. The government can also borrow. It needs to finance a constant, exogenous level of real expenditures  $g$ , plus interest on any borrowing. To simplify the analysis, the real rate of interest is assumed to be constant, and ad hoc descriptions of both money demand and the distortions associated with the two tax instruments are specified.

With these assumptions, the basic real budget identity of the government can be obtained by dividing (4.3) by the time  $t$  price level to obtain

$$b_t = Rb_{t-1} + g - \tau_t - s_t, \quad (4.41)$$

where  $R$  is the gross interest factor (i.e., 1 plus the rate of interest),  $\tau$  is nonseigniorage tax revenue, and  $s$  is seigniorage revenue. Seigniorage is given by

$$s_t = \frac{M_t - M_{t-1}}{P_t} = m_t - \frac{m_{t-1}}{1 + \pi_t}. \quad (4.42)$$

Taking expectations of (4.41) conditional on time  $t$  information and recursively solving forward yields the intertemporal budget constraint of the government:

$$E_t \sum_{i=0}^{\infty} R^{-i} (\tau_{t+i} + s_{t+i}) = Rb_{t-1} + \left( \frac{R}{R-1} \right) g. \quad (4.43)$$

Note that, given  $b_{t-1}$ , (4.43) imposes a constraint on the government, because  $E_t \lim_{i \rightarrow \infty} R^{-i} b_{t+i}$  has been set equal to zero. Absent this constraint, the problem of choosing the optimal time path for taxes and seigniorage becomes trivial. Just set both equal to zero and borrow continually to finance expenditures plus interest because debt never needs to be repaid.

The government is assumed to set  $\tau_t$  and the inflation rate  $\pi_t$  as well as planned paths for their future values to minimize the present discounted value of the distortions generated by these taxes, taking as given the inherited real debt  $b_{t-1}$ , the path of expenditures, and the financing constraint (4.43). The assumption that the government can commit to a planned path for future taxes and inflation is an important one. Much of chapter 6 deals with outcomes when governments cannot precommit to future policies.

In order to understand the key implications of the joint determination of inflation and taxes, assume that the distortions arising from income taxes are quadratic in the

tax rate:  $(\tau_t + \phi_t)^2/2$ , where  $\phi$  is a stochastic term that allows the marginal costs of taxes to vary randomly.<sup>31</sup> Similarly, costs associated with seigniorage are taken to equal  $(s_t + \varepsilon_t)^2/2$ , where  $\varepsilon$  is a stochastic shift in the cost function. Thus, the present discounted value of tax distortions is given by

$$\frac{1}{2}E_t \sum_{i=0}^{\infty} R^{-i} [(\tau_{t+i} + \phi_{t+i})^2 + (s_{t+i} + \varepsilon_{t+i})^2]. \quad (4.44)$$

The government's objective is to choose paths for the tax rate and inflation to minimize (4.44) subject to (4.43).

Letting  $\lambda$  represent the Lagrangian multiplier associated with the intertemporal budget constraint, the necessary first-order conditions for the government's setting of  $\tau$  and  $s$  take the form

$$E_t(\tau_{t+i} + \phi_{t+i}) = \lambda, \quad i \geq 0,$$

$$E_t(s_{t+i} + \varepsilon_{t+i}) = \lambda, \quad i \geq 0.$$

These conditions simply state that the government will arrange its tax collections to equalize the marginal distortionary costs across tax instruments, that is,  $E_t(\tau_{t+i} + \phi_{t+i}) = E_t(s_{t+i} + \varepsilon_{t+i})$  for each  $i \geq 0$ , and across time, that is,  $E_t(\tau_{t+i} + \phi_{t+i}) = E_t(\tau_{t+j} + \phi_{t+j})$  and  $E_t(s_{t+i} + \varepsilon_{t+i}) = E_t(s_{t+j} + \varepsilon_{t+j})$  for all  $i$  and  $j$ .

For  $i = 0$ , the first-order condition implies that  $\tau_t + \phi_t = s_t + \varepsilon_t = \lambda$ ; this represents an *intratemporal* optimality condition. Since the value of  $\lambda$  depends on the total revenue needs of the government, increases in  $Rg/(R-1) + Rb_{t-1}$  cause the government to increase the revenue raised from both tax sources. Thus, one would expect to observe  $\tau_t$  and  $s_t$  moving in similar directions (given  $\phi_t$  and  $\varepsilon_t$ ).

*Intertemporal* optimality requires that marginal costs be equated across time periods for each tax instrument:

$$E_t \tau_{t+1} = \tau_t - E_t \phi_{t+1} + \phi_t, \quad (4.45)$$

$$E_t s_{t+1} = s_t - E_t \varepsilon_{t+1} + \varepsilon_t. \quad (4.46)$$

These intertemporal conditions lead to standard *tax-smoothing* conclusions; for each tax instrument, the government will equate the expected marginal distortionary costs in different time periods. If the random shocks to tax distortions follow  $I(1)$  processes such that  $E_t \phi_{t+1} - \phi_t = E_t \varepsilon_{t+1} - \varepsilon_t = 0$ , these intertemporal optimality conditions imply that both  $\tau$  and  $s$  follow Martingale processes, an implication of the tax-smoothing model originally developed by Barro (1979a). If  $E_t \varepsilon_{t+1} - \varepsilon_t = 0$ , (4.46) implies that changes in seigniorage revenue should be unpredictable based on information available at time  $t$ .

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31. This approach follows that of Poterba and Rotemberg (1990), who specified tax costs directly, as done here, although they assumed a more general functional form for which the quadratic specification is a special case. See also Trehan and Walsh (1988).

Changes in revenue sources might be predictable and still be consistent with this model of optimal taxation if the expected  $t + 1$  values of  $\phi$  and/or  $\varepsilon$ , conditional on period  $t$  information, are nonzero. For example, if  $E_t \varepsilon_{t+1} - \varepsilon_t > 0$ , that is, if the distortionary cost of seigniorage revenue were expected to rise, it would be optimal to plan to reduce future seigniorage.

Using a form of (4.46), Mankiw (1987) argued that the near random walk behavior of inflation (actually nominal interest rates) is consistent with U.S. monetary policy having been conducted in a manner consistent with optimal finance considerations. Poterba and Rotemberg (1990) provided some cross-country evidence on the joint movements of inflation and other tax revenue. In general, this evidence was not favorable to the hypothesis that inflation (or seigniorage) has been set on the basis of optimal finance considerations. While Poterba and Rotemberg found the predicted positive relationship between tax rates and inflation for the United States and Japan, there was a negative relationship for France, Germany, and the United Kingdom.

The implications of the optimal finance view of seigniorage are, however, much stronger than simply that seigniorage and other tax revenue should be positively correlated. Since the unit root behavior of both  $s$  and  $\tau$  arises from the same source (their dependence on  $Rg/(R - 1) + Rb_{t-1}$  through  $\lambda$ ), the optimizing model of tax setting has the joint implication that both tax rates and inflation should contain unit roots (they respond to permanent shifts in government revenue needs) and that they should be cointegrated.<sup>32</sup> Trehan and Walsh (1990) showed that this implication is rejected for U.S. data.

The optimal finance view of seigniorage fails for the United States because seigniorage appears to behave more like the stock of debt than like general tax revenue. Under a tax-smoothing model, temporary variations in government expenditures should be met through debt financing. Variations in seigniorage should reflect changes in expected permanent government expenditures or, from (4.46), stochastic shifts in the distortions associated with raising seigniorage (because of the  $\varepsilon$  realizations). In contrast, debt should rise in response to a temporary revenue need (such as a war) and then gradually decline over time. However, the behavior of seigniorage in the United States, particularly during the World War II period, mimics that of the deficit much more than it does that of other tax revenue (Trehan and Walsh 1988).

One drawback of this analysis is that the specification of the government's objective function is ad hoc; the tax distortions were not related in any way to the underlying sources of the distortions in terms of the allocative effects of taxes or the welfare costs of inflation. These costs depend on the demand for money; therefore, the specification of the distortions should be consistent with the particular approach used to motivate the demand for money.

Calvo and Leiderman (1992) provided an analysis of optimal intertemporal inflation taxation using a money demand specification that is consistent with utility maximization.

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32. That is, if  $\phi$  and  $\varepsilon$  are  $I(0)$  processes, then  $\tau$  and  $s$  are  $I(1)$ , but  $\tau - s = \varepsilon - \phi$  is  $I(0)$ .

They showed that the government's optimality condition requires that the nominal rate of interest vary with the expected growth of the marginal utility of consumption. Optimal tax considerations call for high taxes when the marginal utility of consumption is low and low taxes when the marginal utility of consumption is high. Thus, models of inflation in an optimal finance setting generally imply restrictions on the joint behavior of inflation and the marginal utility of consumption, not just on inflation alone. Calvo and Leiderman estimated their model using data from three countries that have experienced periods of high inflation: Argentina, Brazil, and Israel. While the overidentifying restrictions implied by their model are not rejected for the first two countries, they are for Israel.

#### 4.6.2 Optimal Seigniorage and Temporary Shocks

The prescription to smooth marginal distortionary costs over time implies that tax levels are set on the basis of some estimate of permanent expenditure needs. Allowing tax rates to fluctuate in response to temporary and unanticipated fluctuations in expenditures would result in a higher total efficiency loss in present value terms because of the distortions induced by non-lump-sum taxes. As extended to seigniorage by Mankiw (1987), the same argument implies that seigniorage should be set on the basis of permanent expenditure needs, not adjusted in response to unanticipated temporary events.

The allocative distortions induced by the inflation tax, however, were shown in chapters 2 and 3 to be based on anticipated inflation. Consumption, labor supply, and money-holding decisions are made by households on the basis of expected inflation, and for this reason, variations in expected inflation generate distortions. In contrast, unanticipated inflation has wealth effects but no substitution effects. It therefore serves as a form of lump-sum tax. Given real money holdings, which are based on the public's expectations about inflation, a government interested in minimizing distortionary tax costs should engineer a surprise inflation. If sufficient revenue could be generated in this way, socially costly distortionary taxes could be avoided.<sup>33</sup>

Unfortunately, private agents are likely to anticipate that the government will have an incentive to attempt a surprise inflation; the outcome in such a situation is the major focus of chapter 6. But suppose the government can commit itself to, on average, only inflating at a rate consistent with its revenue needs based on average expenditures. That is, average inflation is set according to permanent expenditures, as implied by the tax-smoothing model. But if there are unanticipated fluctuations in expenditures, these should be met through socially costless unanticipated inflation.

Calvo and Guidotti (1993) made this argument rigorous. They showed that when the government can commit to a path for anticipated inflation, it is optimal for unanticipated

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33. Auernheimer (1974) provided a guide to seigniorage for an "honest" government, one that does not generate revenue by allowing the price level to jump unexpectedly, even though this would represent an efficient lump-sum tax.

inflation to respond flexibly to unexpected disturbances.<sup>34</sup> This implication is consistent with the behavior of seigniorage in the United States, which for most of the twentieth century followed a pattern that appeared to be more similar to that of the federal government deficit than to a measure of the average tax rate. During war periods, when most of the rise in expenditures could be viewed as temporary, taxes were not raised sufficiently to fund the war effort. Instead, the U.S. government borrowed heavily, just as the Barro tax-smoothing model implies. But the United States did raise the inflation tax; seigniorage revenue rose during the war, falling back to lower levels at the war's conclusion. This behavior is much closer to that implied by Calvo and Guidotti's theory than to the basic implications of Mankiw's.<sup>35</sup> Rockoff (2015) examined the U.S. evidence from the revolutionary war to the Iraq war, finding a common theme for when money creation was relied on to finance wars. He concluded that the United States used borrowing and taxes to finance wars against minor powers. In major wars, however, it resorted to inflationary finance in the face of perceived limits on further tax increases or when further borrowing would push interest rates to levels considered too high.

### 4.6.3 Friedman's Rule Revisited

The preceding analysis has gone partway toward integrating the choice of inflation with the general public finance choice of tax rates. The discussion was motivated by Phelps's conclusion that if only distortionary tax sources are available, some revenue should be raised from the inflation tax. However, this conclusion has been questioned by Kimbrough (1986a; 1986b), Faig (1988), Chari, Christiano, and Kehoe (1991; 1996), and Correia and Teles (1996; 1999).<sup>36</sup> They showed that there are conditions under which Friedman's rule for the optimal inflation rate—a zero nominal rate of interest—continues to be optimal even in the absence of lump-sum taxes. Mulligan and Sala-i-Martin (1997) provided a general discussion of the conditions necessary for taxing (or not taxing) money.

This literature integrates the question of the optimal inflation tax into the general problem of optimal taxation. By doing so, the analysis can build on findings in the optimal tax literature that identify situations in which the structure of optimal indirect taxes calls for different final goods to be taxed at the same rate or for the tax rate on goods that serve as intermediate inputs to be zero (see Diamond and Mirrlees 1971; Atkinson and Stiglitz 1972). Using an MIU approach, for example, treats money as a final good; in contrast, a shopping-time model, or a more general model in which money produces transaction services, treats money as an intermediate input. Thus, it is important to examine the implications of these alternative assumptions about the role of money have for the optimal tax

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34. See also Benigno and Woodford (2004) and Angeletos (2004).

35. Chapter 8 revisits the optimal choice of taxes and inflation in a new Keynesian model.

36. An early example of the use of optimal tax models to study the optimal inflation rate issue is Drazen (1979). See also Walsh (1984). Chari and Kehoe (1999) provided a survey.



approach to inflation determination, and how optimal inflation tax results might depend on particular restrictions on preferences or on the technology for producing transaction services.

### The Basic Ramsey Problem

The problem of determining the optimal structure of taxes to finance a given level of expenditures is called the *Ramsey problem*, after the classic treatment by Ramsey (1928). In the representative agent model studied here, the Ramsey problem involves setting taxes to maximize the utility of the representative agent, subject to the government's revenue requirement.

The following static Ramsey problem, based on Mulligan and Sala-i-Martin (1997), can be used to highlight the key issues.<sup>37</sup> The utility of the representative agent depends on consumption, real money balances, and leisure:

$$u = u(c, m, l).$$

Agents maximize utility subject to the following budget constraint:

$$f(n) \geq (1 + \tau)c + \tau_m m, \quad (4.47)$$

where  $f(n)$  is a standard production function,  $n = 1 - l$  is the supply of labor,  $c$  is consumption,  $\tau$  is the consumption tax,  $\tau_m = i/(1 + i)$  is the tax on money, and  $m$  is the household's holdings of real money balances. The representative agent picks consumption, money holdings, and leisure to maximize utility, taking the tax rates as given. Letting  $\lambda$  be the Lagrangian multiplier on the budget constraint, the first-order conditions from the agent's maximization problem are

$$u_c = \lambda(1 + \tau), \quad (4.48)$$

$$u_m = \lambda \tau_m, \quad (4.49)$$

$$u_l = \lambda f', \quad (4.50)$$

$$f(1 - l) - (1 + \tau)c - \tau_m m = 0. \quad (4.51)$$

From these first-order conditions and the budget constraint,  $c$ ,  $m$ , and  $l$  can be expressed as functions of the two tax rates:  $c(\tau, \tau_m)$ ,  $m(\tau, \tau_m)$ , and  $l(\tau, \tau_m)$ .

The government's problem is to set  $\tau$  and  $\tau_m$  to maximize the representative agent's utility, subject to three types of constraints. First, the government must satisfy its budget constraint; tax revenue must be sufficient to finance expenditures. This constraint takes the form

$$\tau c + \tau_m m \geq g, \quad (4.52)$$

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37. I thank Bo Sandemann for pointing out an error in my derivation of the model in earlier editions and for suggesting the approach taken in this edition. Fortunately, the key equation, (4.55) in the third edition, is not affected.

where  $g$  is real government expenditures. These expenditures are taken to be exogenous. Second, the government is constrained by the fact that consumption, labor supply, and real money must be consistent with the choices of private agents. That means that (4.48)–(4.51) represent constraints on the government's choices. Finally, the government is constrained by the economy's resource constraint:

$$f(1 - l) \geq c + g. \quad (4.53)$$

However, (4.51) and (4.52) imply (4.53) is redundant.

There are two approaches to solving this problem. The first approach, often called the *dual approach*, employs the indirect utility function to express utility as a function of taxes. These tax rates are treated as the government's control variables, and the optimal values of the tax rates are found by solving the first-order conditions from the government's optimization problem. The second approach, called the *primal approach*, treats quantities as the government's controls. The tax rates are found from the representative agent's first-order conditions to ensure that private agents choose the quantities that solve the government's maximization problem. The dual approach is presented first. The primal approach is employed later.

Substituting the solutions to the representative agent's decision problem into the utility function yields the indirect utility function:

$$v(\tau, \tau_m) \equiv u(c(\tau, \tau_m), m(\tau, \tau_m), l(\tau, \tau_m)). \quad (4.54)$$

From Roy's identity,<sup>38</sup>

$$c = -\frac{v_\tau}{\lambda},$$

$$m = -\frac{v_{\tau_m}}{\lambda}.$$

The government's problem is to pick  $\tau$  and  $\tau_m$  to maximize (4.54), subject to the government's budget constraint (4.52). Thus, the government's problem can be written as

$$\max_{\tau, \tau_m} \{v(\tau, \tau_m) + \mu [\tau_m m(\tau, \tau_m) + \tau c(\tau, \tau_m) - g]\},$$

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38. To see this, differentiate the indirect utility function with respect to  $\tau$  and use (4.48)–(4.50) to obtain

$$v_\tau = u_c c_\tau + u_m m_\tau + u_l l_\tau$$

$$= \lambda [(1 + \tau) c_l + \tau_m m_\tau + f' l_\tau].$$

From (4.51),

$$-f' l_\tau = c + (1 + \tau) c_l + \tau_m m_\tau.$$

Combining these two expressions implies

$$v_\tau = -\lambda c,$$

which when rearranged yields the desired result. A similar derivation yields  $m = -v_{\tau_m}/\lambda$ .

where  $\mu$  is the Lagrangian multiplier on the government's budget constraint. Notice that the constraints represented by (4.48)–(4.51) have been incorporated by writing consumption, money balances, and leisure as functions of the tax rates. The first-order conditions for the two taxes are

$$v_\tau + \mu (c + \tau c_\tau + \tau_m m_\tau) \leq 0, \quad (4.55)$$

$$v_{\tau_m} + \mu (\tau c_{\tau_m} + m + \tau_m m_{\tau_m}) \leq 0, \quad (4.56)$$

where  $v_\tau = u_c c_\tau + u_m m_\tau + u_l l_\tau$  and  $v_{\tau_m} = u_c c_{\tau_m} + u_m m_{\tau_m} + u_l l_{\tau_m}$ . These conditions will hold with equality if the solution is an interior one with positive taxes on both consumption and money. If the left side of the second first-order condition is negative when evaluated at a zero tax on money, then a zero tax on money ( $\tau_m = 0$ ) is optimal.

If the solution is an interior one with positive taxes on consumption and money holdings, (4.55) and (4.56) will both hold with equality and

$$\frac{v_{\tau_m}}{v_\tau} = \frac{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}{\tau_m m_\tau + c + \tau c_\tau}. \quad (4.57)$$

To interpret this condition, note that  $v_{\tau_m}$  is the effect of the tax on money on utility, while  $v_\tau$  is the effect of the consumption tax on utility. Thus, their ratio,  $v_{\tau_m}/v_\tau$ , is the marginal rate of substitution between the two tax rates, holding constant the utility of the representative agent.<sup>39</sup> The right side of (4.57) is the marginal rate of transformation, holding the government's revenue constant.<sup>40</sup> At an optimum, the government equates the marginal rates of substitution and transformation.

Given that  $g > 0$ , so that the government must raise some revenue, one can assume  $\tau > 0$ , which implies (4.55) holds with equality. When is the Friedman rule,  $\tau_m = 0$ , optimal? Assume, following Friedman, that at a zero nominal interest rate, the demand for money is finite. Evaluating (4.55) and (4.56) at  $\tau_m = 0$  and using Roy's identity yields

$$\lambda c = \mu (c + \tau c_\tau),$$

$$\lambda m \geq \mu (\tau c_{\tau_m} + m).$$

39. That is, if  $v(\tau, \tau_m)$  is the utility of the representative agent as a function of the two tax rates, then  $v_\tau d\tau + v_{\tau_m} d\tau_m = 0$  yields

$$\frac{d\tau}{d\tau_m} = -\frac{v_{\tau_m}}{v_\tau}.$$

40. That is, from the government's budget constraint,

$$(m + \tau_m m_{\tau_m} + \tau c_{\tau_m}) d\tau_m + (\tau_m m_\tau + c + \tau c_\tau) d\tau = 0,$$

yielding

$$\frac{d\tau_m}{d\tau} = -\frac{\tau_m m_\tau + c + \tau c_\tau}{m + \tau_m m_{\tau_m} + \tau c_{\tau_m}}.$$

Because  $\lambda > 0$ ,  $c > 0$ , and  $\mu > 0$ , these two expressions imply

$$\frac{m}{c} \geq \frac{\tau c_{\tau_m} + m}{c + \tau c_{\tau}}, \quad \text{or}$$

$$\frac{m}{\tau c_{\tau_m} + m} \geq \frac{c}{c + \tau c_{\tau}}. \quad (4.58)$$

The left side is proportional to the marginal impact of the inflation tax on utility per dollar of revenue raised, evaluated at  $\tau_m = 0$ . The right side is proportional to the marginal impact of the consumption tax on utility per dollar of revenue raised. If the inequality is strict at  $\tau_m = 0$ , then the distortion caused by using the inflation tax (per dollar of revenue raised) exceeds the cost of raising that same revenue using the consumption tax.

Noting that  $c_{\tau} < 0$ , (4.58) can be written as

$$\frac{m}{c} \leq \frac{c_{\tau_m}}{c_{\tau}}, \quad (4.59)$$

if  $\tau_m = 0$ .<sup>41</sup>

Mulligan and Sala-i-Martin (1997) considered (4.59) for a variety of special cases that have appeared in the literature. For example, if utility is separable in consumption and money holdings, then  $c_{\tau_m} = 0$ ; in this case, the right side of (4.59) is equal to zero, and the left side is positive. Hence, (4.59) cannot hold, and it is optimal to tax money.

A second case that leads to clear results occurs if  $c_{\tau_m} > 0$ . In this case, the right side of (4.59) is negative (because  $c_{\tau} < 0$ , an increase in the consumption tax reduces consumption). Since the left side is non-negative,  $m/c > c_{\tau_m}/c_{\tau}$  and money should always be taxed. This corresponds to a case in which money and consumption are substitutes so that an increase in the tax on money (which reduces money holdings) leads to an increase in consumption. Finally, if money and consumption are complements,  $c_{\tau_m} < 0$ . The ratio  $c_{\tau_m}/c_{\tau}$  is then positive, and whether money is taxed will depend on a comparison of  $m/c$  and  $c_{\tau_m}/c_{\tau}$ . Recall that the calibration exercises in chapter 2 used parameter values that implied that  $m$  and  $c$  were complements.

Chari, Christiano, and Kehoe (1996) examined the optimality of the Friedman rule in an MIU model with taxes on consumption, labor supply, and money. They showed that if preferences are homothetic in consumption and money balances and separable in leisure, the optimal tax on money is zero. When preferences satisfy these assumptions, one can write

$$u(c, m, l) = \bar{u}[s(c, m), l],$$

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41. This is proposition 2 in Mulligan and Sala-i-Martin (1997, 692).

where  $s(c, m)$  is homothetic.<sup>42</sup> Mulligan and Sala-i-Martin (1997) showed that in this case,

$$\frac{m}{c} = \frac{c\tau_m}{c_\tau},$$

so (4.59) implies that the optimal tax structure yields  $\tau_m = 0$ .

Chari, Christiano, and Kehoe related their results to the optimal taxation literature in public finance. Atkinson and Stiglitz (1972) showed that if two goods are produced under conditions of constant returns to scale, a sufficient condition for uniform tax rates is that the utility function is homothetic. With equal tax rates, the ratio of marginal utilities equals the ratio of producer prices. To see how this applies in the present case, suppose the budget constraint for the representative household takes the form

$$(1 + \tau_t^c)Q_t c_t + M_t + B_t = (1 - \tau_t^h)Q_t(1 - l_t) + (1 + i_{t-1})B_{t-1} + M_{t-1},$$

where  $M$  and  $B$  are the nominal money and bond holdings,  $i$  is the nominal rate of interest,  $Q$  is the producer price of output, and  $\tau^c$  and  $\tau^h$  are the tax rates on consumption ( $c$ ) and hours of work ( $1 - l$ ). In addition, it is assumed that the production function exhibits constant returns to scale and that labor hours,  $1 - l$ , are transformed into output according to  $y = 1 - l$ . Define  $P \equiv (1 + \tau^c)Q$ . Household real wealth is  $w_t = (M_t + B_t)/P_t = m_t + b_t$ , and the budget constraint can be written as

$$\begin{aligned} c_t + w_t &= \left( \frac{1 - \tau_t^h}{1 + \tau_t^c} \right) (1 - l_t) + (1 + r_{t-1})b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} \\ &= (1 - \tau_t)(1 - l_t) + (1 + r_{t-1})w_{t-1} - \left( \frac{i_{t-1}}{1 + \pi_t} \right) m_{t-1}, \end{aligned} \quad (4.60)$$

where  $1 - \tau_t \equiv (1 - \tau_t^h)/(1 + \tau_t^c)$  and  $(1 + r_{t-1}) = (1 + i_{t-1})/(1 + \pi_t)$ , and  $\pi_t = P_t/P_{t-1} - 1$ . Thus, the consumption and labor taxes only matter through the composite tax  $\tau$ , so without loss of generality, set the consumption tax equal to zero. If the representative household's utility during period  $t$  is given by  $\bar{u}[s(c, m), l]$ , and the household maximizes  $E_t \sum_{i=0}^{\infty} \beta^i \bar{u}[s(c_{t+i}, m_{t+i}), l_{t+i}]$  subject to the budget constraint given by (4.60), then the first-order conditions for the household's decision problem imply that consumption, money balances, and leisure will be chosen such that

$$\frac{\bar{u}_m(c_t, m_t, l_t)}{\bar{u}_c(c_t, m_t, l_t)} = \frac{s_m(c_t, m_t)}{s_c(c_t, m_t)} = \frac{i_t}{1 + i_t} \equiv \tau_{m,t}.$$

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42. Homothetic preferences imply that  $s(c, m)$  is homogeneous of degree 1 and that  $s_i$  is homogeneous of degree 0. With homothetic preferences, indifference curves are parallel to each other, with constant slope along any ray;  $s_2(c, m)/s_1(c, m) = f(m/c)$ .

With the production costs of money assumed to be zero, the ratio of marginal utilities differs from the ratio of production costs unless  $\tau_{m,t} = 0$ . Hence, with preferences that are homothetic in  $c$  and  $m$ , the Atkinson-Stiglitz result implies that it is optimal to set the nominal rate of interest equal to zero.

Correia and Teles (1999) considered other cases in which (4.59) holds so that the optimal tax on money equals zero. They follow M. Friedman (1969) in assuming a satiation level of money holdings  $m^*$  such that the marginal utility of money is positive for  $m < m^*$  and nonpositive for  $m \geq m^*$ . This satiation level can depend on  $c$  and  $l$ . Correia and Teles showed that the optimal tax on money is zero if  $m^* = \bar{k}c$  for a positive constant  $\bar{k}$ . They also showed that the optimal tax on money is zero if  $m^* = \infty$ . Intuitively, at an optimum, the marginal benefit of additional money holdings must balance the cost of the marginal effect on government revenue. This contrasts with the case of normal goods, where the marginal benefit must balance the costs of the marginal impact on the government's revenue *and* the marginal resource cost of producing the goods. Money, in contrast, is assumed to be costless to produce. At the satiation point, the marginal benefit of money is zero. The conditions studied by Correia and Teles ensure that the marginal revenue effect is also zero.

Friedman's rule for the optimal rate of inflation can be recovered even in the absence of lump-sum taxes. But it is important to recognize that the restrictions on preferences necessary to restore Friedman's rule are very strong, and as discussed by Braun (1991), different assumptions about preferences will lead to different conclusions. The assumption that the ratio of the marginal utilities of consumption and money is independent of leisure can certainly be questioned. However, it is very common in the literature to assume separability between leisure, consumption, and money holdings. The standard log utility specification, for example, displays this property and so would imply that a zero nominal interest rate is optimal.

### A CIA Model

The examples so far have involved MIU specifications. Suppose instead that the consumer faces a cash-in-advance (CIA) constraint on a subset of its purchases. Specifically, assume that  $c_1$  represents cash goods, and  $c_2$  represents credit goods. Let  $l$  denote leisure. The household's objective is to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i U(c_{1,t+i}, c_{2,t+i}, l_{t+i}),$$

subject to the budget constraint

$$(1 + \tau_t^c)Q_t(c_{1,t} + c_{2,t}) + M_t + B_t = (1 - \tau_t^h)Q_t(1 - l_t) + (1 + i_{t-1})B_{t-1} + M_{t-1},$$

where variables are as defined previously. In addition, the CIA constraint requires that

$$c_{1,t} \leq \frac{m_{t-1}}{1 + \pi_t}.$$

Before considering when the optimal inflation tax might be positive, suppose one ignores the credit good  $c_2$  for the moment so that the model is similar to the basic CIA model studied in chapter 3. Recall that inflation served as a tax on labor supply in that model. But according to the budget constraint, the government already has, in  $\tau^h$ , a tax on labor supply. Thus, the inflation tax is redundant.<sup>43</sup> Because it is redundant, the government can achieve an optimal allocation without using the inflation tax.

In a cash-and-credit-good economy, the inflation tax is no longer redundant if the government cannot set different commodity taxes on the two types of goods. So returning to the model with both cash and credit goods, the first-order conditions for the household's decision problem imply that consumption and leisure are chosen such that

$$\frac{U_1(c_{1,t}, c_{2,t}, l_t)}{U_2(c_{1,t}, c_{2,t}, l_t)} = 1 + i_t.$$

The analysis of Atkinson and Stiglitz (1972) implies that if preferences are homothetic in  $c_{1,t}$  and  $c_{2,t}$ , the ratio of the marginal utility of cash and credit goods should equal 1, the ratio of their production prices. This occurs only if  $i = 0$ ; hence, homothetic preferences imply that the nominal rate of interest should be set equal to zero. But this is just the Friedman rule for the optimal rate of inflation.

Thus, the optimal inflation tax should be zero if for all  $\lambda > 0$ ,

$$\frac{U_1(\lambda c_{1,t}, \lambda c_{2,t}, l_t)}{U_2(\lambda c_{1,t}, \lambda c_{2,t}, l_t)} = \frac{U_1(c_{1,t}, c_{2,t}, l_t)}{U_2(c_{1,t}, c_{2,t}, l_t)}, \quad (4.61)$$

in which case the utility function has the form

$$U(c_1, c_2, l) = V(\phi(c_1, c_2), l),$$

where  $\phi$  is homogeneous of degree 1. If this holds, the government should avoid using the inflation tax even though it must rely on other distortionary taxes. Positive nominal rates of interest impose an efficiency cost by distorting the consumer's choice between cash and credit goods.<sup>44</sup>

43. See Chari, Christiano, and Kehoe (1996).

44. As Chari, Christiano, and Kehoe (1996) noted, the preference restrictions are sufficient for the Friedman rule to be optimal but not necessary. For example, in the cash/credit model, suppose preferences are not homothetic and the optimal tax structure calls for taxing credit goods more heavily. A positive nominal interest rate taxes cash goods, and negative nominal rates are not feasible. Thus, a corner solution can arise in which the optimal nominal interest rate is zero. Note that this assumes that the government cannot impose separate goods taxes on cash and credit goods.

How reasonable is this condition? Recall that no explanation has been given for why one good is a cash good and the other is a credit good. This distinction has simply been assumed, and therefore it is difficult to argue intuitively why the preferences for cash and credit goods should (or should not) satisfy condition (4.61). In aggregate analysis, it is common to combine all goods into one composite good; this is standard in writing utility as  $u(c, l)$ , with  $c$  representing an aggregation over all consumption goods. Interpreting  $c$  as  $\phi(c_1, c_2)$ , where  $\phi$  is a homogeneous of degree 1 aggregator function, implies that preferences would satisfy the properties necessary for the optimal inflation tax to be zero. However, this is not an innocuous restriction. It requires, for example, that the ratio of the marginal utility of coffee at the local coffee cart (a cash good) to that of books at the bookstore (a credit good) remains constant if coffee and book consumption double.

### Money as an Intermediate Input

The approach in the previous sections motivated a demand for money by including real money balances as an element of the representative agent's utility function or by imposing a CIA constraint that applied to a subset of goods. If the role of money arises because of the services it provides in facilitating transactions, then it might be more naturally viewed as an intermediate good, a good used as an input in the production of the final goods that directly enter the utility function. The distinction between final goods and intermediate goods is important for determining the optimal structure of taxation; Diamond and Mirrlees (1971), for example, showed that under certain conditions it may be optimal to tax only final goods. In particular, when the government can levy taxes on each final good, intermediate goods should not be taxed.

The importance of money's role as an intermediate input was first stressed by Kimbrough (1986a; 1986b) and Faig (1988).<sup>45</sup> Their work suggested that the Friedman rule might apply even in the absence of lump-sum taxes, and conclusions to the contrary arose from the treatment of money as a final good that enters the utility function directly. Under conditions of constant returns to scale, the Diamond-Mirrlees result called for efficiency in production, implying that money and labor inputs into producing transactions should not be taxed. The MIU approach is usually used as a shortcut for modeling situations in which money serves as a medium of exchange by facilitating transactions, and the work of Kimbrough and Faig indicates that such shortcuts can have important implications. However, the requirement that taxes be available for every final good is not satisfied in practice, and the properties of the transaction technology of the economy are such that until these are better understood, there is no clear case for assuming constant returns to scale.

Correia and Teles (1996) provided further results on the applicability of the Friedman rule. They showed that Friedman's result holds for any shopping-time model in which shopping time is a homogeneous function of consumption and real money balances. To

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45. See also Guidotti and Vegh (1993).



investigate this result, and to illustrate the primal approach to the Ramsey problem, consider a generalized shopping-time model in which money and time are inputs into producing transaction services. Specifically, assume that the representative agent has a total time allocation, normalized to 1, that can be allocated to leisure ( $l$ ), market activity ( $n$ ), or shopping ( $n^s$ ):

$$l_t + n_t + n_t^s = 1. \quad (4.62)$$

Shopping time depends on the agent's choice of consumption and money holdings, with  $n_t^s$  increasing in  $c_t$  and decreasing in  $m_t$  according to the shopping production function

$$n_t^s = G(c_t, m_t).$$

Assume that  $G$  is homogeneous of degree  $\eta$  so that one can write  $G(\lambda_t c_t, \lambda_t m_t) = \lambda_t^\eta G(c_t, m_t)$ . Letting  $\lambda_t = 1/c_t$ ,

$$n_t^s = c_t^\eta G\left(1, \frac{m_t}{c_t}\right) \equiv c_t^\eta g\left(\frac{m_t}{c_t}\right).$$

In addition, assume that  $g$  is a convex function,  $g' \leq 0$ ,  $g'' \geq 0$ , which implies that shopping time is nonincreasing in  $m_t/c_t$  but real money balances exhibit diminishing marginal productivity. Constant returns to scale correspond to  $\eta = 1$ . Assume that there exists a level of real balances relative to consumption  $\bar{\mu}$  such that  $g'(x) = 0$  for  $x \geq \bar{\mu}$ , corresponding to a satiation level of real balances.

The representative agent chooses paths for consumption, labor supply, money holdings, and capital holdings to maximize

$$\sum_{i=0}^{\infty} \beta^i u\left[c_{t+i}, 1 - n_{t+i} - c_{t+i}^\eta g\left(\frac{m_{t+i}}{c_{t+i}}\right)\right], \quad (4.63)$$

subject to the following budget constraint:

$$w_t \equiv \left(\frac{1 + i_{t-1}}{1 + \pi_t}\right) d_{t-1} - \left(\frac{i_{t-1}}{1 + \pi_t}\right) m_{t-1} \geq c_t + d_t - (1 - \tau_t)f(n_t), \quad (4.64)$$

where  $f(n_t)$  is a standard neoclassical production function,  $\tau_t$  is the tax rate on income,  $d_t = m_t + b_t$  is total real asset holdings, equal to government interest-bearing debt holdings ( $b_t$ ) plus real money holdings,  $i_{t-1}$  is the nominal interest rate from  $t - 1$  to  $t$ , and  $\pi_t$  is the inflation rate from  $t - 1$  to  $t$ . Notice that capital accumulation has been ignored in this analysis. Further assume that initial conditions include  $M_{t-1} = B_{t-1} = 0$ , where these are the nominal levels of money and bond stocks. A final, important assumption in Correia and Teles's analysis is that production exhibits constant returns to scale, with  $f(n) = 1 - l - n^s$ .<sup>46</sup>

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46. Notice that the utility function in (4.63) can be written as  $v(c_{t+i}, m_{t+i}, n_{t+i})$  and so can be used to justify an MIU function (see also section 3.3.1). When the shopping-time function takes the form assumed here, Correia

The government's optimal tax problem is to pick time paths for  $\tau_{t+i}$  and  $i_{t+i}$  to maximize (4.63) subject to the economy resource constraint  $c_t + g_t \leq 1 - l_t - n_t^s$  and to the requirement that consumption and labor supply be consistent with the choices of private agents. Following Lucas and Stokey (1983), this problem can be recast by using the first-order conditions from the individual agent's decision problem to express, in terms of the government's tax instruments, the equilibrium prices that support the paths of consumption and labor supply that solve the government's problem. This leads to an additional constraint on the government's choices and can be summarized in terms of an *implementability condition*.

To derive this implementability condition, start with the first-order conditions for the representative agent's problem. Define the value function

$$v(w_t) = \max_{c_t, n_t, m_t, d_t} \left\{ u \left[ c_t, 1 - n_t - c_t^\eta g \left( \frac{m_t}{c_t} \right) \right] + \beta v(w_{t+1}) \right\},$$

where the maximization is subject to the budget constraint (4.64). Letting  $\lambda_t$  denote the Lagrangian multiplier associated with the time  $t$  budget constraint, the first-order conditions imply

$$u_c - u_l \left( \eta g - \frac{m}{c} g' \right) c_t^{\eta-1} = \lambda_t, \quad (4.65)$$

$$u_l = \lambda_t (1 - \tau_t), \quad (4.66)$$

$$-u_l g' c_t^{\eta-1} = \lambda_t I_t, \quad (4.67)$$

$$\lambda_t = \beta R_t \lambda_{t+1}, \quad (4.68)$$

where  $I_t = i_t / (1 + i_t)$  and the real interest rate is  $R_t = (1 + i_t) / (1 + \pi_{t+1})$ .

The next step is to recast the budget constraint (4.64). This constraint can be written as

$$R_{t-1} d_{t-1} = \sum_{i=0}^{\infty} D_i \left[ c_{t+i} - (1 - \tau_{t+i})(1 - l_{t+i} - n_{t+i}^s) + R_{t-1+i} I_{t-1+i} m_{t-1+i} \right], \quad (4.69)$$

where a no Ponzi condition has been imposed and the discount factor  $D_i$  is defined as  $D_i = 1$  for  $i = 0$  and  $D_i = \prod_{j=1}^i R_{t+j-1}^{-1}$  for  $i \geq 1$ . Since it is assumed that the initial stocks of money and bonds equal zero,  $d_{t-1} = 0$ , the right side of (4.69) must also equal zero.<sup>47</sup> The implementability condition is obtained by replacing the prices in this budget constraint using the first-order conditions of the agent's problem to express the prices in terms of quantities.<sup>48</sup>

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and Teles (1999) showed that  $m^* = \bar{k}c$  for a positive constant  $\bar{k}$ , where  $m^*$  is the satiation level of money balances such that  $g'(m^*/c) = 0$ . As noted earlier, the optimal tax on money is zero when  $m^* = \bar{k}c$ .

47. If the government's initial nominal liabilities were positive, it would be optimal to immediately inflate away their value, because this would represent a nondistortionary source of revenue. It is to avoid this outcome that the initial stocks are assumed to be zero.

48. The price of consumption is 1, the price of leisure is  $1 - \tau$ , and the price of real balances is  $I$ .

Recalling that  $c^\eta g = n^s$ , first multiply and divide the intertemporal budget constraint by  $\lambda_{t+i}$ ; then use the result from the first-order conditions (4.68) that  $D_i = \beta^i D_0 \lambda_{t+i} / \lambda_t$  to write (4.69)<sup>49</sup> as

$$\sum_{i=0}^{\infty} \beta^i [\lambda_{t+i} c_{t+i} - \lambda_{t+i} (1 - \tau_{t+i}) (1 - l_{t+i} - n_{t+i}^s) + \lambda_{t+i} I_{t+i} m_{t+i}] = 0.$$

Now use the first-order conditions (4.65)–(4.67) to obtain

$$\sum_{i=0}^{\infty} \beta^i \left\{ \left[ u_c - u_l \left( \eta g - u_l \frac{m}{c} g' \right) c_{t+i}^{\eta-1} \right] c_{t+i} - u_l (1 - l_{t+i} - n_{t+i}^s) - u_l \frac{m_{t+i}}{c_{t+i}} g' c_{t+i}^\eta \right\} = 0.$$

Since the term  $u_l \frac{m}{c} g' c_{t+i}^\eta$  appears twice, with opposite signs, these cancel, and this condition becomes

$$\sum_{i=0}^{\infty} \beta^i [u_c c_{t+i} - u_l (1 - l_{t+i}) + u_l (1 - \eta) n_{t+i}^s] = 0. \quad (4.70)$$

Equation (4.70) is the implementability condition. The government's problem now is to choose  $c_{t+i}$ ,  $m_{t+i}$ , and  $l_{t+i}$  to maximize the utility of the representative agent, subject to the economy's resource constraint, the production function for shopping time, and (4.70). That is,  $\max \sum \beta^i u(c_{t+i}, l_{t+i})$  subject to (4.70) and  $c_t + g_t \leq (1 - l_t - n_t^s)$ , where  $n_t^s = g(m_t/c_t)c_t^\eta$ . This formulation of the Ramsey problem illustrates the primal approach; the first-order conditions for the representative agent are used to eliminate prices from the agent's intertemporal budget constraint.

Since  $m$  appears in this problem only in the production function for shopping time, the first-order condition for the optimal choice of  $m$  is

$$[\beta^i \psi u_l (1 - \eta) - \mu] g' = 0, \quad (4.71)$$

where  $\psi \geq 0$  is the Lagrangian multiplier on the implementability constraint (4.70), and  $\mu \geq 0$  is the Lagrangian multiplier on the resource constraint. Correia and Teles showed that  $\beta^i \psi u_l (1 - \eta) - \mu = 0$  cannot characterize the optimum, so for (4.71) to be satisfied requires that  $g' = 0$ . From the first-order conditions in the representative agent's problem,

49. This uses the fact that  $m_{t-1} = 0$ , so we can write

$$\begin{aligned} \sum_{i=0}^{\infty} D_i R_{t-1+i} I_{t-1+i} m_{t-1+i} &= \sum_{i=1}^{\infty} D_i R_{t-1+i} I_{t-1+i} m_{t-1+i} = \sum_{i=1}^{\infty} D_{i-1} I_{t-1+i} m_{t-1+i} \\ &= \sum_{i=1}^{\infty} \frac{D_{i-1}}{\lambda_{t-1+i}} \lambda_{t-1+i} I_{t-1+i} m_{t-1+i} = \sum_{i=0}^{\infty} \frac{D_i}{\lambda_{t+i}} \lambda_{t+i} I_{t+i} m_{t+i} \\ &= \frac{1}{\lambda_t} \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} I_{t+i} m_{t+i}. \end{aligned}$$

$-u_l g' c^{\eta-1} = \lambda I$ ; this implies that  $g' = 0$  requires  $I = 0$ . That is, the nominal rate of interest should equal zero, and the optimal tax on money should be zero.

The critical property of money, according to Correia and Teles, is its status as a free primary good. *Free* in this context means that it can be produced at zero variable cost. The costless production assumption is standard in monetary economics, and it provided the intuition for Friedman's original result. With a zero social cost of production, optimality requires that the private cost also be zero. This occurs only if the nominal rate of interest is zero.

It is evident that there are general cases in which Phelps's conclusion does not hold. Even in the absence of lump-sum taxation, optimal tax policy should not distort the relative price of cash and credit goods or distort money holdings. But as discussed by Braun (1991) and Mulligan and Sala-i-Martin (1997), different assumptions about preferences or technology can lead to different conclusions. Correia and Teles (1999) attempted to quantify the deviations from the Friedman rule when the preference and technology restrictions required for a zero nominal interest rate to be optimal do not hold. They found that the optimal nominal rate of interest is still close to zero.

#### 4.6.4 Nonindexed Tax Systems

Up to this point, the discussion has assumed that the tax system is indexed so that taxes are levied on real income; a one-time change in all nominal quantities and the price level would leave the real equilibrium unchanged. This assumption requires that a pure price change have no effect on the government's real tax revenue or the tax rates faced by individuals and firms in the private sector. Most actual tax systems, however, are not completely indexed to ensure that pure price level changes leave real tax rates and real tax revenue unchanged. Inflation-induced distortions generated by the interaction of inflation and the tax system have the potential to be much larger than the revenue-related effects on which most of the seigniorage and optimal inflation literature has focused. Feldstein (1998) analyzed the net benefits of reducing inflation from 2 percent to zero,<sup>50</sup> concluding that for his preferred parameter values, the effects due to reducing distortions related to the tax system are roughly twice those associated with the change in government revenue.

One important distortion arises when nominal interest income, not real interest income, is taxed. After-tax real rates of return will be relevant for individual agents in making savings and portfolio decisions, and if nominal income is subject to a tax rate of  $\tau$ , the real after-tax return will be

$$\begin{aligned} r_a &= (1 - \tau)i - \pi \\ &= (1 - \tau)r - \tau\pi, \end{aligned}$$

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50. Feldstein allowed for an upward bias in the inflation rate, as measured by the consumer price index, so his estimates apply to reducing consumer price inflation from 4 percent to 2 percent.

where  $i = r + \pi$  is the nominal return and  $r$  is the before-tax real return. Thus, for a given pretax real return  $r$ , the after-tax real return is decreasing in the rate of inflation.

To see how this distortion affects the steady-state capital-labor ratio, consider the basic MIU model of chapter 2 with an income tax of  $\tau$  on total nominal income. Nominal income is assumed to include any nominal capital gain on capital holdings:

$$Y_t \equiv P_t f'(k_{t-1}) + i_{t-1} B_{t-1} + P_t T_t + (P_t - P_{t-1})(1 - \delta)k_{t-1}.$$

The representative agent's budget constraint becomes

$$(1 - \tau)Y_t = P_t c_t + P_t k_t - P_t(1 - \delta)k_{t-1} + (B_t - B_{t-1}) + (M_t - M_{t-1}),$$

where  $M$  is the agent's nominal money holdings,  $B$  is the agent's bond holdings, and  $P_t T_t$  is a nominal transfer payment.<sup>51</sup> In real terms, the budget constraint becomes<sup>52</sup>

$$\begin{aligned} (1 - \tau) \left[ f(k_{t-1}) + \frac{i_{t-1} b_{t-1}}{1 + \pi_t} + T_t \right] - \tau \left( \frac{\pi_t}{1 + \pi_t} \right) (1 - \delta)k_{t-1} \\ = c_t + k_t - (1 - \delta)k_{t-1} + \left( b_t - \frac{b_{t-1}}{1 + \pi_t} \right) + \left( m_t - \frac{m_{t-1}}{1 + \pi_t} \right). \end{aligned}$$

Assuming the agent's objective is to maximize the present discounted value of expected utility, which depends on consumption and money holdings, the first-order conditions for capital and bonds imply, in the steady state,

$$(1 - \tau)f'_k(k) + \left[ \frac{1 + (1 - \tau)\pi}{1 + \pi} \right] (1 - \delta) = \frac{1}{\beta}, \quad (4.72)$$

$$(1 - \tau) \left( \frac{1 + i}{1 + \pi} \right) + \frac{\tau}{1 + \pi} = \frac{1}{\beta}. \quad (4.73)$$

The steady-state capital-labor ratio is determined by

$$f'_k(k^{ss}) = \left( \frac{1}{1 - \tau} \right) \left\{ \frac{1}{\beta} - \left[ \frac{1 + (1 - \tau)\pi}{1 + \pi} \right] (1 - \delta) \right\}.$$

Since  $[1 + (1 - \tau)\pi]/(1 + \pi)$  is decreasing in  $\pi$ ,  $k^{ss}$  is decreasing in the inflation rate. Higher inflation leads to larger nominal capital gains on existing holdings of capital, and because these are taxed, inflation increases the effective tax rate on capital.

51. For simplicity, assume that  $T$  is adjusted in a lump-sum fashion to ensure that variations in inflation and the tax rate on income leave the government's budget balanced. Obviously, if lump-sum taxes actually were available, the optimal policy would involve setting  $\tau = 0$  and following Friedman's rule for the optimal rate of inflation. The purpose here is to examine the effects of a nonindexed tax system on the steady-state capital stock in the easiest possible manner.

52. This formulation assumes that real economic depreciation is tax deductible. If depreciation allowances are based on historical nominal cost, a further inflation-induced distortion is introduced.

Equation (4.73) can be solved for the steady-state nominal rate of interest to yield

$$1 + i^{ss} = \frac{1}{\beta} \left( \frac{1 + \pi}{1 - \tau} \right) - \frac{\tau}{1 - \tau}.$$

Thus, the pretax real return on bonds,  $(1 + i)/(1 + \pi)$ , increases with the rate of inflation, implying that nominal rates rise more than proportionately with an increase in inflation.

It is important to recognize that only one aspect of the effects of inflation and the tax system has been examined.<sup>53</sup> Because of the taxation of nominal returns, higher inflation distorts the individual's decisions, but it also generates revenue for the government that, with a constant level of expenditures (in present value terms), would allow other taxes to be reduced. Thus, the distortions associated with the higher inflation are potentially offset by the reduction in the distortions caused by other tax sources. As noted earlier, however, Feldstein (1998) argued that the offset is only partial, leaving a large net annual cost of positive rates of inflation. Feldstein identified the increased effective tax rate on capital that occurs because of the treatment of depreciation and the increased subsidy on housing associated with the deductibility of nominal mortgage interest in the United States as important distortions generated by higher inflation interacting with a nonindexed tax system. Including these effects with an analysis of the implications for government revenue and consequently possible adjustments in other distortionary taxes, Feldstein estimated that a 2 percent reduction in inflation (from 2 percent to zero) increases net welfare by 0.63 percent to 1.01 percent of GDP annually.<sup>54</sup> Since these are annual gains, the present discounted value of permanently reducing inflation to zero would be quite large.

#### 4.7 Summary

Monetary and fiscal actions are linked through the government's budget constraint. Under Ricardian regimes, changes in the money stock or its growth rate require some other variable in the budget constraint—taxes, expenditures, or borrowing—to adjust. With fiscal dominance, changes in government taxes or expenditures can require changes in inflation. Under non-Ricardian regimes, changes in government debt can affect prices even if monetary policy is exogenous. A complete analysis of price level determinacy requires a specification of the relationship between fiscal and monetary policies.

Despite this, and despite the emphasis budget relationships have received in the work of Sargent and Wallace and the work on the fiscal theory of the price level initiated by

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53. Feldstein, Green, and Sheshinski (1978) employed a version of Tobin's money and growth model (Tobin 1965) to explore the implications of a nonindexed tax system when firms use both debt and equity to finance capital.

54. These figures assume an elasticity of savings with respect to the after-tax real return of 0.4 and a deadweight loss of taxes of between 40 cents for every dollar of revenue (leading to the 0.63 percent figure) and \$1.50 per dollar of revenue (leading to the 1.01 percent figure).

Sims and Woodford, much of monetary economics ignores the implications of the budget constraint. This is valid in the presence of lump-sum taxes; any effects on the government's budget can simply be offset by an appropriate variation in lump-sum taxes. Traditional analyses that focus only on the stock of high-powered money are also valid when governments follow a Ricardian policy of fully backing interest-bearing debt with tax revenue, either now or in the future. In general, though, one should be concerned with the fiscal implications of any analysis of monetary policy, because changes in the quantity of money that alter the interest payments of the government have implications for future tax liabilities.

#### 4.8 Problems

1. Suppose the central bank pays interest  $i_t^r$  on high-powered money. How would (4.5) be affected?
2. Suppose the rate of population growth is  $n$  and the rate of growth of real per capita income is  $\lambda$ . Show that (4.6) becomes

$$s_t = (h_t - h_{t-1}) + \left[ \frac{(1 + \pi_t)(1 + \mu) - 1}{(1 + \pi_t)(1 + \mu)} \right] h_{t-1},$$

where  $1 + \mu = (1 + n)(1 + \lambda)$ . Now consider the steady state in which  $h_t = h_{t-1}$  and inflation is constant. Does seigniorage depend on  $\mu$ ? Explain.

3. Suppose utility is given by  $u(c_t, m_t) = w(c_t) + v(m_t)$ , with  $w(c_t) = \ln c_t$  and  $v(m_t) = m_t(B - D \ln m_t)$ , where  $B$  and  $D$  are positive parameters. Approximate steady-state revenue from seigniorage is given by  $\theta m$ , where  $\theta$  is the growth rate of the money supply.
  - a. Is there a Laffer curve for seigniorage (i.e., is revenue increasing in  $\theta$  for all  $\theta \leq \theta^*$  and decreasing in  $\theta$  for all  $\theta > \theta^*$  for some  $\theta^*$ )?
  - b. What rate of money growth maximizes steady-state revenue from seigniorage?
  - c. Assume that the economy's rate of population growth is  $\lambda$ , and reinterpret  $m$  as real money balances per capita. What *rate of inflation* maximizes seigniorage? How does it depend on  $\lambda$ ?
4. Suppose the demand for real money balances is  $m = f(R_m)$ , where  $R_m$  is the gross nominal rate of interest. Assume the gross real interest rate is fixed at its steady-state value of  $1/\beta$ , so that  $R_m = (1 + \pi)/\beta$ , where  $\pi$  is the rate of inflation. Using the definition of seigniorage revenue given in (4.21), what rate of inflation maximizes steady-state seigniorage?
5. Suppose the government faces the following budget identity:

$$b_t = Rb_{t-1} + g_t - \tau_t y_t - s_t,$$

where the terms are one-period debt, gross interest payments, government purchases, income tax receipts, and seigniorage. Assume seigniorage is given by  $f(\pi_t)$ , where  $\pi$  is the rate of inflation. The interest factor  $R$  is constant, and the expenditure process  $\{g_{t+i}\}_{i=0}^{\infty}$  is exogenous. The government sets time paths for the income tax rate and for inflation to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i [h(\tau_{t+i}) + k(\pi_{t+i})],$$

where the functions  $h$  and  $k$  represent the distortionary costs of the two tax sources. Assume that the functions  $h$  and  $k$  imply positive and increasing marginal costs of both revenue sources.

- What is the *intra-temporal* optimality condition linking the choices of  $\tau$  and  $\pi$  at each point in time?
  - What is the *inter-temporal* optimality condition linking the choice of  $\pi$  at different points in time?
  - Suppose  $y = 1$ ,  $f(\pi) = a\pi$ ,  $h(\tau) = b\tau^2$ , and  $k(\pi) = c\pi^2$ . Evaluate the inter- and intra-temporal conditions. Find the optimal settings for  $\tau_t$  and  $\pi_t$  in terms of  $b_{t-1}$  and  $\sum R^{-i}g_{t+i}$ .
  - Using your results from part (c), when will optimal financing imply constant planned tax rates and inflation over time?
- Suppose utility is given by  $U = c^{1-\sigma}/(1-\sigma) + m^{1-\theta}/(1-\theta)$ . Find the function  $\phi(P)$  defined in (4.32) and verify that it has the shape shown in figure 2.2. Solve for the stationary equilibrium price level  $P^*$  such that  $P^* = \phi(P^*)$ .
  - The model of section 4.6.1 assumed the distortions of taxes and seigniorage were quadratic functions of the level of taxes and the government desired to minimize

$$\frac{1}{2} E_t \sum_{i=0}^{\infty} R^{-i} [(\tau_{t+i} + \phi_{t+i})^2 + (s_{t+i} + \varepsilon_{t+i})^2],$$

subject to

$$E_t \sum_{i=0}^{\infty} R^{-i} (\tau_{t+i} + s_{t+i}) = Rb_{t-1} + \left( \frac{R}{R-1} \right) g,$$

where  $s$  is seigniorage revenue,  $\tau$  represents other tax revenue,  $b_{t-1}$  is the initial stock of outstanding government debt,  $g$  is the fixed level of expenditures the government needs to finance each period, and  $\phi$  and  $\varepsilon$  are stochastic shocks to the distortionary costs of each tax source. Suppose  $\phi_t = \rho_\phi \phi_{t-1} + z_t$  and  $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + e_t$ , where  $z$  and  $e$  are mutually and serially uncorrelated white noise innovations to  $\phi$  and  $\varepsilon$ . Derive the intra-temporal and inter-temporal optimality conditions for the two taxes. How does the behavior of each tax depend on  $\rho_\phi$  and  $\rho_\varepsilon$ ?



8. Mankiw (1987) suggested that the nominal interest rate should evolve as a random walk under an optimal tax policy. Suppose that the real rate of interest is constant and that the equilibrium price level is given by (4.29). Suppose that the nominal money supply is given by  $m_t = m_t^p + v_t$ , where  $m_t^p$  is the central bank's planned money supply and  $v_t$  is a white noise control error. Let  $\theta$  be the optimal rate of inflation. There are different processes for  $m^p$  that lead to the same average inflation rate but different time series behavior of the nominal interest rate. For each of the processes for  $m_t^p$  given here, demonstrate that average inflation is  $\theta$ . In each case, is the nominal interest rate a random walk?
- $m_t^p = \theta(1 - \gamma)t + \gamma m_{t-1}$ ,
  - $m_t^p = m_{t-1} + \theta$ .
9. Consider (4.37) implied by the fiscal theory of the price level. Seigniorage  $\bar{s}_t$  was defined as  $i_t m_t / (1 + i_t)$ . Assume that the utility function of the representative agent takes the form  $u(c, m) = \ln c + b \ln m$ . Show that  $\bar{s}_t = bc_t$  and that the price level is independent of the nominal supply of money as long as  $\tau_t - g_t + bc_t$  is independent of  $M_t$ .
10. Consider the optimal tax problem of section 4.6.3. The government wishes to maximize  $u(c, m, l) = v(c, m) + \phi(l)$ , subject to the economy's resource constraint,  $f(1 - l) \geq c + g$ .
- Derive the implementability constraint by using the first-order conditions (4.48)–(4.50) to eliminate the tax rates from the representative agent's budget constraint (4.47).
  - Set up the government's optimization problem and derive the first-order conditions.
  - Show that the first-order condition for  $m$  is satisfied if  $v_m = v_{mc} = v_{mm} = 0$ . Argue that these conditions are met if the satiation level  $m^*$  is equal to  $\infty$ .
11. Suppose the Correia-Teles model of section 4.6.3 is modified so that output is equal to  $f(n)$ , where  $f$  is a standard neoclassical production function exhibiting positive but diminishing marginal productivity of  $n$ . Show that if  $f(n) = n^a$  for  $a > 0$ , the optimality condition given by (4.71) continues to hold.



# 5 Informational and Portfolio Rigidities

## 5.1 Introduction

The empirical evidence from the United States is consistent with the notion that positive monetary shocks lead to a hump-shaped positive response of output that persists for appreciable periods of time, and Sims (1992) found similar patterns for other OECD economies. The models of chapters 2–4 did not seem capable of producing such an effect. So why does money matter?<sup>1</sup> Is it only through the tax effects that arise from inflation? Or are there other channels through which monetary actions have real effects? This question is critical for any normative analysis of monetary policy because designing good policy requires understanding how monetary policy affects the real economy and how changes in the way policy is conducted might affect economic behavior.

In the models examined in earlier chapters, monetary disturbances did cause output movements, but these movements arose from substitution effects induced by expected inflation. Most analyses suggest that these effects are too small to account for the empirical evidence on the output responses to monetary shocks. In addition, the evidence in many countries is that inflation responds only slowly to monetary shocks.<sup>2</sup> If actual inflation responds gradually, so should expectations. Thus, the evidence does not appear to support theories that require monetary shocks to affect labor supply decisions and output only by causing shifts in expected inflation.

In this chapter, the focus shifts away from the role of inflation as a tax and toward the effects of policy-induced changes in real interest rates that affect aggregate spending decisions. Monetary models designed to capture the real effects of money in the short run incorporate frictions that fall into one of three classes: informational frictions, portfolio frictions, and nominal price or wage rigidities. This chapter discusses the first two of these classes; nominal wage and price rigidities are the focus of chapters 7 and 8.

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1. For a survey on this topic, see Blanchard (1990). See also Romer (2012, Ch.6).

2. For example, see Nelson (1998) or Christiano, Eichenbaum, and Evans (2005) for evidence on the United States. Sims (1992) and Taylor (1993b) provide evidence for other countries.

## 5.2 Informational Frictions

This section reviews two attempts to resolve the tension between the long-run neutrality of money and the empirical evidence on the short-run real effects of monetary shocks while maintaining the assumption that wages and prices are flexible. The first approach focuses on misperceptions about aggregate economic conditions; the second focuses on delays in information acquisition.

### 5.2.1 Imperfect Information

During the 1960s the need to reconcile the long-run neutrality of money with the apparent short-run non-neutrality of money was not considered a major research issue in macroeconomics. Models used for policy analysis incorporated a Phillips curve relationship between wage (or price) inflation and unemployment that allowed for a long-run trade-off between the two. In 1968, Milton Friedman and Edmund Phelps independently argued on theoretical grounds that the inflation-unemployment trade-off was only a short-run trade-off at best; attempts to exploit the trade-off by engineering higher inflation to generate lower unemployment would ultimately result only in higher inflation.

Friedman (1968) reconciled the apparent short-run trade-off with the long-run neutrality of money by distinguishing between actual real wages and perceived real wages.<sup>3</sup> The former were relevant for firms making hiring decisions; the latter were relevant for workers making labor supply choices. In a long-run equilibrium, the two would coincide; the real wage would adjust to clear the labor market. Since economic decisions depend on real wages, the same labor market equilibrium would be consistent with any level of nominal wages and prices or any rate of change of wages and prices that left the real wage equal to its equilibrium level.

An unexpected increase in inflation would disturb this real equilibrium. As nominal wages and prices rose more rapidly than previously expected, workers would see their nominal wages rising but would initially not realize that the prices of all the goods and services they consumed were also rising more rapidly. They would misinterpret the nominal wage increase as a rise in their real wage. Labor supply would increase, shifting the labor market equilibrium to a point of higher employment and lower actual real wages. As workers then engaged in shopping activities, they would discover that not only the nominal price of their labor services had risen unexpectedly but all prices had risen. Real wages had actually fallen, not risen. The labor supply curve would shift back, and equilibrium would eventually be restored at the initial levels of employment and real wages.

The critical insight is that changes in wages and prices that are unanticipated generate misperceptions about relative prices (the real wage, in Friedman's version). Economic agents, faced with what they perceive to be changes in relative prices, alter their real

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3. A nice exposition of Friedman's model is provided by Rasche (1973). See also M. Friedman (1977).

economic decisions, and the economy's real equilibrium is affected. Once expectations adjust, however, the economy's natural equilibrium is reestablished. Expectations, and the information on which they are based, become central to understanding the short-run effects of money.

### 5.2.2 The Lucas Model

Friedman's insight was given an explicit theoretical foundation by Lucas (1972). Lucas showed how unanticipated changes in the money supply could generate short-run transitory movements in real economic activity. He did so by analyzing the impact of monetary fluctuations in an overlapping-generations environment with physically separated markets. The demand for money in each location was made random by assuming that the allocation of the population to each location was stochastic.<sup>4</sup> The key features of this environment can be illustrated by employing the analogy of an economy consisting of a large number of individual islands. Agents are randomly reallocated among islands after each period, so individuals care about prices on the island they currently are on and prices on other islands to which they may be reassigned. Individuals on each island are assumed to have imperfect information about aggregate economic variables such as the nominal money supply and price level. Thus, when individuals observe changes in the prices on their island, they must decide whether these reflect purely nominal changes in aggregate variables or island-specific relative price changes.

To illustrate how variations in the nominal quantity of money can have real effects when information is imperfect, assume a basic money-in-the-utility function (MIU) model such as the one developed in chapter 2, but simplify it in three ways. First, ignore capital. This choice implies that only labor is used to produce output and, with no investment, equilibrium requires that output equal consumption. Second, assume that money is the only available asset. Third, assume monetary transfers associated with changes in the nominal quantity of money are viewed by agents as being proportional to their own holdings of cash. This change has substantive implications and is not done just to simplify the model. It implies that the transfers will appear to money holders as interest payments on their cash holdings. This approach eliminates inflation tax effects so that one can concentrate on the role of imperfect information.<sup>5</sup>

Suppose the aggregate economy consists of several islands, indexed by  $i$ ; thus,  $x_t^i$  denotes the value of variable  $x$  on island  $i$ , and  $x_t$  denotes its economywide average value. Since information differs across islands, let  $E^i x_t$  denote the expectations of a variable  $x_t$  based on

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4. In Lucas's formulation, agents had two-period lives; young agents were distributed randomly to each location.

5. Recall that in chapter 2 transfers were viewed as lump-sum. With higher inflation, the transfers rose (as the seigniorage revenues were returned to private agents), but each individual viewed these transfers as unrelated to his or her own money holdings. If the transfers are viewed as interest payments, higher inflation does not raise the opportunity cost of holding money because the interest payment on cash also rises. In this case, money is superneutral.

the information available on island  $i$ . Using the model from the chapter 2 appendix, express equilibrium deviations from the steady state on each island by the following conditions:<sup>6</sup>

$$y_t^i = (1 - \alpha)n_t^i, \quad (5.1)$$

$$\eta \left( \frac{n^{ss}}{1 - n^{ss}} \right) n_t^i = y_t^i - n_t^i + \lambda_t^i, \quad (5.2)$$

$$m_t^i - p_t^i = y_t^i + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) E^i [\tau_{t+1} - (p_{t+1} - p_t^i) + (\lambda_{t+1} - \lambda_t^i)], \quad (5.3)$$

$$\lambda_t^i = \Omega_1 y_t^i + \Omega_2 (m_t^i - p_t^i), \quad (5.4)$$

where  $\lambda_t^i$  is the marginal utility of consumption on island  $i$ , and  $\Omega_1$  and  $\Omega_2$  depend on parameters from the utility function.<sup>7</sup> Note that the goods market equilibrium condition,  $y_t = c_t$ , has been used, and in contrast to chapters 2–4,  $m^i$  now denotes the *nominal* supply of money on island  $i$ . Equation (5.1) is the production function linking labor input ( $n_t^i$ ) to output.<sup>8</sup> Equation (5.2) comes from the first-order condition linking the marginal utility of leisure, the marginal utility of consumption, and the real wage.<sup>9</sup> Equation (5.3) is derived from the first-order condition for the individual agent's holdings of real money balances. This first-order condition requires that reducing consumption at time  $t$  slightly, thereby carrying higher money balances into period  $t + 1$  and then consuming them, must at the margin have no effect on total utility over the two periods. In the present context, the cost of reducing consumption in period  $t$  is the marginal utility of consumption; the additional money balances yield the marginal utility of money in period  $t$  and a gross return of  $T_{t+1}/\Pi_{t+1}$  in period  $t + 1$ , where  $T_{t+1}$  is the gross nominal transfer per dollar on money holdings and  $\Pi_{t+1}$  is 1 plus the inflation rate from  $t$  to  $t + 1$ . This return can be consumed at  $t + 1$ , yielding, in terms of period  $t$  utility,  $\beta(T_{t+1}/\Pi_{t+1})$  times the marginal utility of consumption, where  $\beta$  is the representative household's discount factor. Linearizing the result around the steady state leads to (5.3). Finally, (5.4) defines the marginal utility of consumption as a function of output (consumption) and real money balances; see the chapter appendix.

If agents are reallocated randomly across islands in each period, then the relevant period  $t + 1$  variables in (5.3) are the aggregate price level  $p_{t+1}$ , marginal utility of consumption  $\lambda_{t+1}$ , and nominal transfer  $\tau_{t+1}$ .

6. All variables are expressed as natural log deviations around steady-state values. Since all values will be in terms of deviations, the “hat” notation of chapters 2–4 is dropped for convenience. For an early exposition of a linearized version of Lucas's model, see McCallum (1984a).

7. The underlying utility function that leads to (5.1)–(5.4) is the same as employed in chapter 2. Details can be found in that chapter's appendix.

8. Note that any productivity disturbance has been eliminated; the focus is on monetary disturbances.

9. Equation (5.2) arises from the requirement that the marginal utility of leisure ( $[\eta n^{ss}/(1 + n^{ss})] n_t$  in percentage deviation around the steady state) equal the real wage times the marginal utility of consumption. The marginal product of labor (the real wage) is equal to  $(1 - \alpha)Y/N$ , or  $y - n$  in terms of percentage deviations.

The final component of the model is the specification of the nominal money supply process. Assume the aggregate average nominal money supply evolves<sup>10</sup> as

$$m_t = \rho_m m_{t-1} + v_t + u_t, \quad 0 \leq \rho_m < 1. \quad (5.5)$$

The aggregate supply is assumed to depend on two serially uncorrelated shocks,  $v_t$  and  $u_t$ , assumed to have zero means and variances  $\sigma_v^2$  and  $\sigma_u^2$ . The difference between the two is that  $v_t$  is public information, whereas  $u_t$  is not. Including both helps to illustrate how imperfect information (in this case about  $u$ ) influences the real effects of money shocks. The nominal money stock on island  $i$  is given by

$$m_t^i = m_t + u_t^i = \rho_m m_{t-1} + v_t + u_t + u_t^i,$$

where  $u_t^i$  is a serially uncorrelated island-specific money shock that averages to zero across all islands and has variance  $\sigma_i^2$ . If the aggregate money stock at time  $t - 1$ , as well as  $v_t$ , is public information, then observing the island-specific nominal money stock  $m_t^i$  allows individuals on island  $i$  to infer  $u_t + u_t^i$  but not  $u_t$  and  $u_t^i$  separately. This is important because from (5.5) only  $u_t$  affects the aggregate money stock, and, as long as  $\rho_m \neq 0$ , knowledge about  $u_t$  would be useful in forecasting  $m_{t+1}$ .

Since  $m_{t+1} = \rho_m m_t + v_{t+1} + u_{t+1}$ , the expectation of the time  $t + 1$  money supply, conditional on the information available on island  $i$ , is  $E^i m_{t+1} = \rho_m E^i m_t = \rho_m^2 m_{t-1} + \rho_m v_t + \rho_m E^i u_t$ . If expectations are equated with linear least squares projections,

$$E^i u_t = \kappa (u_t + u_t^i),$$

where  $\kappa = \sigma_u^2 / (\sigma_u^2 + \sigma_i^2)$ ,  $0 \leq \kappa \leq 1$ . If aggregate money shocks are large relative to island-specific shocks (i.e.,  $\sigma_u$  is large relative to  $\sigma_i$ ),  $\kappa$  will be close to 1; movements in  $u_t + u_t^i$  are interpreted as predominantly reflecting movements in the aggregate shock  $u_t$ . In contrast, if the variance of the island-specific shocks is large relative to that of aggregate shocks,  $\kappa$  will be close to zero; movements in  $u_t + u_t^i$  are interpreted as predominantly reflecting island-specific shocks.

Using (5.1)–(5.4), the chapter appendix shows that the equilibrium solutions for the price level and employment are given by

$$p_t = \rho_m m_{t-1} + v_t + \left( \frac{\kappa + K}{1 + K} \right) u_t, \quad (5.6)$$

$$n_t = A(m_t - p_t) = A \left( \frac{1 - \kappa}{1 + K} \right) u_t, \quad (5.7)$$

where  $A$  and  $K$  depend on the underlying parameters of the model.

10. With money supply changes engineered via transfers,

$$\tau_t = m_t - m_{t-1} = (\rho_m - 1)m_{t-1} + v_t + u_t.$$

Equation (5.7) reveals Lucas's basic result; aggregate monetary shocks, represented by  $u_t$ , have real effects on employment (and therefore output) if and only if there is imperfect information ( $\kappa < 1$ ), and their effect depends on the aggregate errors agents make in inferring  $u_t$ :  $u_t - \int E^i u_t di = (1 - \kappa)u_t$ , where  $\int E^i u_t di$  is the aggregate average (over all islands) of the expected value of  $u_t$ . Publicly announced changes in the money supply, represented by the  $v_t$  shocks, have no real effects on output ( $v_t$  does not appear in (5.7)) but simply move the price level one-to-one ( $v_t$  has a coefficient equal to 1 in (5.6)). But the  $u_t$  shocks will affect employment and output if private agents are unable to determine whether the money stock movements they observe on island  $i$  reflect aggregate or island-specific movements. Predictable movements in money (captured here by  $\rho_m m_{t-1}$ ) or announced changes (captured by  $v_t$ ) have no real effects. Only unanticipated changes in the money supply have real effects.

Equation (5.7) can be rewritten in a form that emphasizes the role of money surprises in producing employment and output effects. From (5.5),  $u_t = m_t - E(m_t | \Gamma_{t-1}, v_t)$ , where  $E(m_t | \Gamma_{t-1}, v_t)$  denotes the expectation of  $m_t$  conditional on aggregate information on variables dated  $t - 1$  or earlier, summarized by the information set  $\Gamma_{t-1}$  and the announced money injection  $v_t$ . Thus,

$$n_t = A \left( \frac{1 - \kappa}{1 + K} \right) [m_t - E(m_t | \Gamma_{t-1}, v_t)].$$

Equations of this form provided the basis for the empirical work of Barro (1977; 1978) and others in testing whether unanticipated or anticipated changes in money matter for real output.

In writing employment as a function of money surprises, it is critically important to specify correctly the information set on which agents base their expectations. In empirical work, this information set was often assumed to consist simply of lagged values of the relevant variables. But in the example here,  $E(m_t | \Gamma_{t-1}) = \rho_m m_{t-1}$  and  $m_t - E(m_t | \Gamma_{t-1}) = u_t + v_t \neq u_t$ . Misspecifying the information set can create difficulties in testing models that imply only surprises matter.

Because (5.7) was derived directly from a model consistent with optimizing behavior, the effects of an unanticipated money supply shock on employment can be related to the basic parameters of the production and utility functions.<sup>11</sup> Using the basic parameter values given in section 2.5.4,  $A/[1 + K] = 0.007$ . This implies that even if  $\kappa$  is close to zero, the elasticity of employment with respect to a money surprise is tiny; a 10 percent surprise

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11. McCallum (1984a) presented a linearized approximation to Lucas's model within an overlapping-generations framework. See also Romer (2012). However, both simply postulated some of the basic behavioral relationships of the model.



increase in the money supply would raise employment by 0.07 percent and output by less than  $(1 - \alpha) \times 0.07 = 0.64 \times 0.07 \approx 0.05$  percent.<sup>12</sup>

The impact of money surprises in this example works through labor supply decisions. An increase in real money balances raises the marginal utility of consumption and induces agents to increase consumption and labor supply (since  $\Omega_2 > 0$ ). This effect is larger the more willing agents are to substitute consumption over time. Thus, the impact of a money surprise is larger when the degree of intertemporal substitution is larger.<sup>13</sup> The effect of a money surprise on output is increasing in the wage elasticity of labor supply.

The basic idea behind Lucas's island model is that unpredicted variations in money generate price movements that agents may misinterpret as relative price movements. If a general price rise is falsely interpreted to be a rise in the relative price of what the individual or firm sells, the price rise will induce an increase in employment and output. Once individuals and firms correctly perceive that the price rise was part of an increase in all prices, output returns to its former equilibrium level.

Lucas's model makes clear the important distinction between expected and unexpected variations in money. Economic agents face a signal extraction problem because they have imperfect information about the current money supply. If changes in the nominal supply of money were perfectly predictable, money would have no real effects. Short-run fluctuations in the money supply are likely to be at least partially unpredictable, so they will cause output and employment movements. In this way, Lucas was able to reconcile the neutrality of money in the long run with its important real effects in the short run. Sargent and Wallace (1975) and Barro (1976) provided important early contributions that employed the general approach pioneered by Lucas to examine its implications for monetary policy issues.

Lucas's model has several important testable implications, and these were the focus of a great deal of empirical work in the late 1970s and early 1980s. A first implication is that the distinction between anticipated and unanticipated money matters. Barro (1977; 1978; 1979b) was the first to directly examine whether output was related to anticipated or unanticipated money. He concluded that the evidence supported Lucas's model, but subsequent empirical work by Mishkin (1982) and others showed that both anticipated and unanticipated money appear to influence real economic activity. A survey of the general approach motivated by Lucas's work and of the empirical literature can be found in Barro (1981 ch. 2).

A second implication is that the short-run relationship between output and inflation depends on the relative variance of real and nominal disturbances. The parameter  $\kappa$  in (5.7) depends on the predictability of aggregate changes in the money supply, and this

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12. Because the calibration employed in chapter 2 differs from that used in earlier editions, the value reported for  $A/(1 + K)$  also differs.

13. See Barro and King (1984).

can vary across time and across countries. Lucas (1973) examined the slopes of short-run Phillips curves in a cross-country study and showed that, as predicted by his model, there was a positive correlation between the slope of the Phillips curve and the relative variance of nominal aggregate volatility. A rise in aggregate volatility (an increase in  $\sigma_u^2$  in the version of Lucas's model developed here) implies that an observed increase in prices is more likely to be interpreted as resulting from an aggregate price increase. A smaller real response occurs as a result, and aggregate money surprises have smaller real effects.

A third influential implication of Lucas's model was demonstrated by Sargent and Wallace (1975) and became known as the *policy irrelevance hypothesis*. If changes in money have real effects only when they are unanticipated, then any policy that generates systematic, predictable variations in the money supply will have no real effect. For example, (5.7) shows that employment and therefore output are independent of the degree of serial correlation in  $m$  as measured by  $\rho_m$ . Because the effects of lagged money on the current aggregate money stock are completely predictable, no informational confusion is created and the aggregate price level simply adjusts, leaving real money balances unaffected (see 5.6). A similar conclusion would hold if policy responded to lagged values of  $u$  (or to lagged values of anything else) as long as private agents knew the rule being followed by the policymaker.

The empirical evidence that both anticipated and unanticipated money affect output implies, however, that the policy irrelevance hypothesis does not hold. Systematic responses to lagged variables seem to matter, and therefore the choice of policy rule is not irrelevant for the behavior of real economic activity.

Lucas's misperceptions model was popularized by Sargent and Wallace (1975) and Barro (1976), who employed tractable log-linear versions of the basic model. Although these models are no longer viewed as providing an adequate explanation for the short-run real effects of monetary policy, they have had enormous influence on modern monetary economics. For example, these models play an important role in the analysis of the time inconsistency of optimal policy (see chapter 6). And the finding that announced changes in money (the  $v$  term in the example here) have no real effects implies that inflation could be reduced at no output cost simply by announcing a reduction in money growth. But such announcements must be credible so that expectations are actually reduced as money growth falls; disinflations will be costly if announcements are not credible. This point has produced a large literature on the role of credibility (see chapter 6).

### 5.2.3 Sticky Information

As an alternative to the misperceptions view of imperfect information (and in contrast to the models of sticky prices discussed in chapter 7), Mankiw and Reis (2002) argued that *sticky information*—the slow dispersal of information about macroeconomic conditions—can help account for the sluggish adjustment of prices and for the real effects that occur in response to monetary shocks. The implications of sticky information have been developed

in a number of papers, including Ball, Mankiw, and Reis (2005), Mankiw and Reis (2006a; 2006b), and Reis (2006a; 2006b).<sup>14</sup>

Mankiw and Reis developed a simple model in which each firm adjusts its price every period but its decision may be based on outdated information. In every period, a fraction of firms update their information, so that, over time, new information reaches all firms in a delayed manner. To illustrate the implications of sticky information, assume that in period  $t$ , firm  $j$ 's optimal price is, in log terms,

$$p_t^*(j) = p_t + \alpha x_t, \quad (5.8)$$

where  $p_t$  is the log aggregate price level and  $x_t$  is an output gap measure of output relative to the natural rate of output. Equation (5.8) reflects the fact that individual firms care about their price relative to other firms,  $p_t^*(j) - p_t$ , and variation in the output gap leads to variation in the firm's marginal costs, which affects its optimal price. Notice that if all firms are identical, as is assumed here,  $p_t^*(j) = p_t^*$  for all  $j$  and (5.8) can be written as  $p_t^* = p_t + \alpha x_t$ . Further, if all firms set their price equal to  $p_t^*$ , then the aggregate average price level is  $p_t = p_t^*$ , from which it follows that  $x_t = 0$ ; output is equal to its natural rate. The effect of sticky information is to cause firms to set different prices, even if, under full information, they all have the same desired price.

Specifically, suppose a firm that updated its information  $i$  periods in the past sets the price

$$p_t^i = E_{t-i} p_t^*.$$

All firms with information sets that are  $i$  period old will set the same price, so it is not necessary to index  $p_t^i$  by  $j$ . Now suppose that in each period a fraction  $\lambda$  of all firms are randomly selected to update their information.<sup>15</sup> This assumption implies that, at time  $t$ ,  $\lambda$  of all firms will set their price equal to  $p_t^*$  because they have fully updated information. Of the remaining  $1 - \lambda$  fraction of all firms that do not update their information at time  $t$ ,  $\lambda$  of them will have updated their information in  $t - 1$ . These firms, of whom there are  $(1 - \lambda)\lambda$ , set their price at time  $t$  equal to  $E_{t-1} p_t^*$ . Following a similar logic, there remain  $1 - \lambda - (1 - \lambda)\lambda = (1 - \lambda)^2$  of firms that do not update at either  $t$  or  $t - 1$ . However,  $\lambda$  of these firms update at  $t - 2$  and, at time  $t$ , set price equal to  $E_{t-2} p_t^*$ ; there are  $(1 - \lambda)^2 \lambda$  such firms. For any period  $i$  in the past, there will be  $(1 - \lambda)^i \lambda$  firms that have not updated their information since period  $t - i$ . It follows that the average aggregate price log price level will be

$$p_t = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-i} p_t^* = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-i} (p_t + \alpha x_t). \quad (5.9)$$

14. See also Sims (2003).

15. This structure borrows from a common modeling strategy employed to deal with sticky prices, originally due to Calvo (1983) (see chapters 7 and 8).

The parameter  $\lambda$  provides a measure of the degree of information stickiness. If  $\lambda$  is large, most firms update frequently; if  $\lambda$  is small, many firms will be basing time  $t$  decisions on old information.

To derive an expression for the inflation rate from (5.9), let  $z_t = p_t + \alpha x_t$ . Then write (5.9) as

$$\begin{aligned} p_t &= \lambda z_t + \lambda \sum_{i=1}^{\infty} (1-\lambda)^i \mathbf{E}_{t-i} z_t \\ &= \lambda z_t + \lambda(1-\lambda) \mathbf{E}_{t-1} z_t + \lambda(1-\lambda)^2 \mathbf{E}_{t-2} z_t + \dots, \\ p_{t-1} &= \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \mathbf{E}_{t-1-i} z_{t-1} \\ &= \lambda \mathbf{E}_{t-1} z_{t-1} + \lambda(1-\lambda) \mathbf{E}_{t-2} z_{t-1} + \lambda(1-\lambda)^2 \mathbf{E}_{t-3} z_{t-1} + \dots \end{aligned}$$

Subtracting the second equation from the first yields

$$\pi_t = p_t - p_{t-1} = \lambda z_t + \lambda \sum_{i=1}^{\infty} (1-\lambda)^{i-1} \mathbf{E}_{t-i} \Delta z_t - \lambda^2 \sum_{i=0}^{\infty} (1-\lambda)^i \mathbf{E}_{t-1-i} z_t, \quad (5.10)$$

where  $\Delta z_t \equiv z_t - z_{t-1}$ . Recalling that  $z_t = p_t + \alpha x_t$ , (5.9) also implies

$$\begin{aligned} p_t &= \left( \frac{\lambda}{1-\lambda} \right) \alpha x_t + \left( \frac{\lambda}{1-\lambda} \right) \sum_{i=1}^{\infty} (1-\lambda)^i \mathbf{E}_{t-i} z_t \\ &= \left( \frac{\lambda}{1-\lambda} \right) \alpha x_t + \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \mathbf{E}_{t-1-i} z_t. \end{aligned}$$

This means that the last term in (5.10) is equal to  $\lambda p_t - \left( \frac{\lambda^2}{1-\lambda} \right) \alpha x_t$ . Making this substitution, (5.10) becomes

$$\begin{aligned} \pi_t &= \lambda z_t + \lambda \sum_{i=1}^{\infty} (1-\lambda)^{i-1} \mathbf{E}_{t-i} \Delta z_t - \lambda p_t + \left( \frac{\lambda^2}{1-\lambda} \right) \alpha x_t \\ &= \left( \frac{\lambda}{1-\lambda} \right) \alpha x_t + \lambda \sum_{i=1}^{\infty} (1-\lambda)^{i-1} \mathbf{E}_{t-i} (\pi_t + \alpha \Delta x_t). \end{aligned} \quad (5.11)$$

Equation (5.11) is the sticky-information Phillips curve (SIPC). The coefficient on the current output gap is increasing in  $\lambda$ ; the more frequently firms update their information, the more sensitive current pricing decisions are to current economic conditions. The key aspect of the SIPC is the presence of expectations of current variables based on lagged information sets. The presence of these terms means that a shock that occurs at time  $t$  will only gradually affect inflation as the information on which expectations are based is only

gradually updated. The faster information is updated (the larger is  $\lambda$ ), the more rapidly will inflation respond to current movements in real output.

To complete the determination of inflation and the output gap, Mankiw and Reis (2002) assumed a simple quantity theory equation and an exogenous AR(1) process for the growth rate of the money supply. In log terms, the quantity theory relation is  $m_t + v_t = p_t + x_t$ . Taking first differences, this becomes

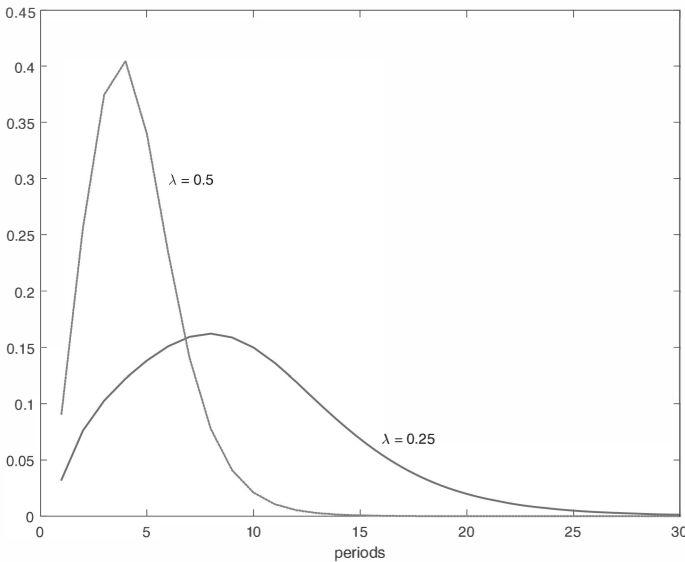
$$m_t - m_{t-1} + \Delta v_t = \pi_t + \Delta x_t.$$

Assume that the money process is given by

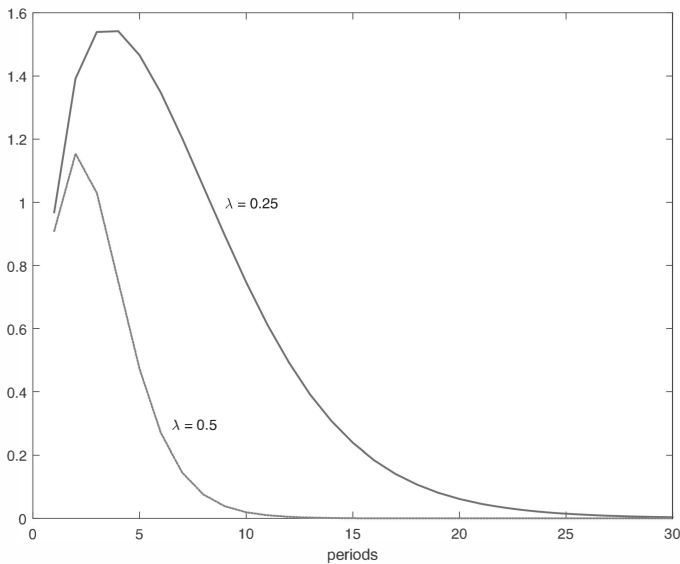
$$m_t - m_{t-1} = \rho (m_{t-1} - m_{t-2}) + u_t,$$

and velocity  $v_t$  follows a random walk, with  $\Delta v_t$  a mean zero, serially uncorrelated process. Figure 5.1 illustrates the response of inflation to a unit realization of  $u_t$  for two different values of  $\lambda$  when  $\rho = 0.5$ . Mankiw and Reis set  $\lambda = 0.25$  in their baseline calibration;  $\lambda = 0.5$  corresponds to faster information updating. Figure 5.2 shows the corresponding behavior of output.

These impulse responses display the hump-shaped pattern observed in estimated VARs, and Mankiw and Reis argued that sticky information can provide an explanation for the real effects of monetary policy shocks and for the persistence seen in the inflation process.



**Figure 5.1** Response of inflation to a unit innovation in money growth in the sticky-information model.



**Figure 5.2**

Response of output to a unit innovation in money growth in the sticky-information model.

In contrast to the Lucas model of imperfect information, a key aspect of the sticky-information model is the presence of heterogeneity in information sets across firms, heterogeneity that persists over time because of the staggered updating of information.

In their original development of sticky information, Mankiw and Reis (2002) used a calibrated version of their model to argue that sticky information was better able to capture inflation and output dynamics than were the models based on sticky prices (see chapter 7). Khan and Zu (2006) estimated an SIPC using quarterly U.S. data from the period 1980–2000. To generate expectations of current inflation and output based on old information, they employed VAR forecasts. They found the average duration of information stickiness ranges from three to seven quarters, consistent with the findings of Mankiw and Reis (2002).

A number of authors have estimated sticky-information Phillips curves and compared them to the inflation equations based on sticky prices, for example, Coibion (2010), Klenow and Willis (2007), and Coibion and Gorodnichenko (2015). This empirical work is discussed in chapter 7.

#### 5.2.4 Learning

Standard rational-expectations models assume agents know the true model of the economy. Typically, only information on the current innovations to exogenous shocks may be

incomplete. Once these innovations become known to all agents, after one period in the Lucas islands model or only as part of a staggered multiperiod process in the Mankiw-Reis model of sticky information, the model is characterized by complete information. A growing literature has investigated situations in which the underlying state of the economy may never be known or in which the structure of the economy is unknown and agents must engage in a process of learning.

Brunner, Meltzer, and Cukierman (1980) provided an early example of a model in which observed disturbances are composed of permanent and transitory components. These individual components are not directly observed, so agents must estimate them based on the past history of the observed disturbance. This signal extraction problem leads to richer dynamics than would be generated in the basic islands model. For instance, Brunner, Meltzer, and Cukierman showed that the rational expectation of the permanent component is a weighted average of all current and past realizations of the observed disturbance, with weights depending on the relative variance of the permanent and transitory innovations. When a realization of the permanent shock occurs, agents initially interpret part of the change in the observed disturbance as due to the transitory component. They underestimate the change in the permanent component and, as a consequence, underestimate future realizations of the disturbance. Since the true values of the two components are never observed, forecast errors can be serially correlated. In the case of a money supply disturbance, money surprises are serially correlated, leading to real effects that persist for several periods.

In the Brunner, Meltzer, and Cukierman model, agents know the model structure but each period update their beliefs about the value of the persistent disturbance. In contrast, the adaptive learning literature pioneered by Evans and Honkapohja (2001) assumed agents do not know the true model structure. However, agents have beliefs about the true model, and they update their beliefs using recursive least squares as new data become available. A key question is whether the adaptive learning process converges to the rational-expectations equilibrium. If it does, the model is said to be e-stable under learning.

To illustrate the adaptive learning approach, consider a general model of the form

$$y_t = \alpha + ME_t y_{t+1} + \delta y_{t-1} + \phi e_t, \quad (5.12)$$

and  $e_t = \rho e_{t-1} + \varepsilon_t$ . The minimum state-variable, rational-expectations solution (McCallum 1983a) takes the form

$$y_t = a + b y_{t-1} + c e_t. \quad (5.13)$$

Assume agents know the solution takes this general form, and given values for the parameters  $a$ ,  $b$ , and  $c$ , they treat (5.13) as their perceived law of motion (PLM) for  $y_t$ . Agents use the PLM to form expectations:

$$E_t y_{t+1} = a + b y_t + c \rho e_t. \quad (5.14)$$

Given these expectations, the actual law of motion (ALM) for  $y_t$  is obtained by substituting (5.14) into (5.12). Solving for  $y_t$  yields

$$y_t = \alpha + M[a + by_t + c\rho e_t] + \delta y_{t-1} + \phi e_t$$

$$= \frac{\alpha + Ma}{1 - Mb} + \left(\frac{\delta}{1 - Mb}\right)y_{t-1} + \left(\frac{\phi + Mc\rho}{1 - Mb}\right)e_t.$$

The mapping from the PLM to the ALM is defined by

$$T(a, b, c) = \left[ \frac{\alpha + Ma}{1 - Mb}, \left(\frac{\delta}{1 - Mb}\right), \left(\frac{\phi + Mc\rho}{1 - Mb}\right) \right].$$

Evans and Honkapohja (2001) showed that if the mapping  $T(a, b, c) \rightarrow (a, b, c)$  is locally asymptotically stable at the fixed point  $T(a, b, c) = (a, b, c)$  that corresponds to the minimum state-variable solution, the system is e-stable. Evans and Honkapohja then showed that this ensures stability under real-time learning in which the PLM is

$$y_t = a_{t-1} + b_{t-1}y_{t-1} + c_{t-1}e_t$$

and the coefficients are updated by running recursive least squares.

Because agents are using macroeconomic outcomes to update their beliefs about the structure of the economy, and these beliefs then influence both expectations and macroeconomic outcomes, learning can have important implications for economic dynamics. For example, Erceg and Levin (2003) showed that accounting for learning about the central bank's inflation goals can be important for understanding the real effects on the economy during periods of disinflation such as the early 1980s in the United States.

Much of the recent literature on learning in the context of monetary policy has employed the new Keynesian model (see chapter 8). Evans and Honkapohja (2009) provided a survey of some of the important implications of learning for monetary policy as well as references to the relevant literature.

### 5.3 Limited Participation and Liquidity Effects

The impact of a monetary disturbance on market interest rates can be decomposed into its effect on the expected real rate of return and its effect on the expected inflation rate. If money growth is positively serially correlated, an increase in money growth will be associated with higher future inflation and therefore higher *expected* inflation. As noted in chapters 2 and 3, the flexible-price MIU and CIA models implied that faster money growth would immediately increase nominal interest rates.

Most economists, and certainly monetary policymakers, believe that central banks can reduce short-term nominal interest rates by employing policies that lead to faster growth in the money supply. This belief is often interpreted to mean that faster money growth will initially cause nominal interest rates to fall, an impact called the *liquidity effect*. This effect



is usually viewed as an important channel through which a monetary expansion affects real consumption, investment, and output.<sup>16</sup>

A number of authors have explored flexible-price models in which monetary injections reduce nominal interest rates. See, for example, Lucas (1990), Christiano (1991), Christiano and Eichenbaum (1992b; 1995), Fuerst (1992), Dotsey and Ireland (1995), King and Watson (1996), Cooley and Quadrini (1999), Alvarez, Lucas, and Weber (2001), Alvarez, Atkeson, and Kehoe (2002), and Williamson (2004; 2005). These models generate effects of monetary shocks on real interest rates by imposing restrictions on the ability of agents to engage in certain types of financial transactions.<sup>17</sup> For example, Lucas modified a basic CIA framework to study effects that arise when monetary injections are not distributed equally across a population of otherwise representative agents. If a monetary injection affects agents differentially, a price level increase proportional to the aggregate change in the money stock will not restore the initial real equilibrium. Some agents will be left with higher real money holdings, others with lower real balances.<sup>18</sup>

Fuerst (1992) and Christiano and Eichenbaum (1995) introduce a liquidity effect by modifying a basic CIA model to distinguish between households, firms, and financial intermediaries. Households can allocate resources between bank deposits and money balances that are then used to finance consumption. Intermediaries lend out their deposits to firms that borrow to finance purchases of labor services from households. After households have made their choice between money and bank deposits, financial intermediaries receive lump-sum monetary injections. Only firms and intermediaries interact in financial markets after the monetary injection.<sup>19</sup>

In a standard representative agent CIA model, monetary injections are distributed proportionately to all agents. Thus, a proportional rise in the price level leaves all agents with the same level of real money balances as previously. In contrast, if the injections initially affect only the balance sheets of the financial intermediaries, a new channel is

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16. A thorough discussion of possible explanations of liquidity effects is provided by Ohanian and Stockman (1995) and Hoover (1995).

17. The first limited-participation models were due to Grossman and Weiss (1983) and Rotemberg (1984). Models that restrict financial transactions can be viewed as variants of the original Baumol-Tobin models with infinite costs for certain types of transactions rather than the finite costs of exchanging money and interest-earning assets assumed by Baumol (1952) and Tobin (1956).

18. Cooley and Quadrini (1999) combined a limited-participation model with a search and matching model of the labor market of the type discussed in section 8.5.

19. Allowing for heterogeneity greatly complicates the analysis, but these limited-participation models overcome this problem by following the modeling strategy introduced by Lucas (1980a), in which each representative family consists of a household supplying labor and purchasing goods; a firm hiring labor, producing goods, and borrowing from the intermediary; and an intermediary. At the end of each period, the various units of the family are reunited and pool resources. As a result, there can be heterogeneity within periods as the new injections of money affect only firms and intermediaries, but between periods all families are identical, so the advantages of the representative agent formulation are preserved.

introduced by which employment and output are affected. As long as the nominal interest rate is positive, intermediaries wish to increase their lending in response to a positive monetary injection. To induce firms to borrow the additional funds, the interest rate on loans must fall. Hence, a liquidity effect is generated; interest rates decline in response to a positive monetary injection.<sup>20</sup> The restrictions on trading mean that cash injections create a wedge between the value of cash in the hands of household members shopping in the goods market and the value of cash in the financial market.<sup>21</sup> Because Fuerst and Christiano and Eichenbaum assumed that firms must borrow to fund their wage bill, the appropriate marginal cost of labor to firms is the real wage times the gross rate of interest on loans. The interest rate decline generated by the liquidity effect lowers the marginal cost of labor; at each real wage, labor demand increases. As a result, equilibrium employment and output rise.

### 5.3.1 A Basic Limited-Participation Model

The real effects of money in a limited-participation model can be illustrated in a version of the model of Fuerst (1992). The basic model follows Lucas (1990) in assuming that each representative household consists of several members. The household members play different roles within each period, thus allowing for heterogeneity, but because all members reunite at the end of each period, all households remain identical in equilibrium. Specifically, the household consists of a shopper, a firm manager, a worker, and a financial intermediary (a bank). The household enters the period with money holdings  $M_t$ . An amount equal to  $D_t$  in nominal terms is deposited in the bank, while the shopper takes  $M_t - D_t$  to be used in the goods market to purchase consumption goods. The purchase of such goods is subject to a cash-in-advance constraint:

$$P_t C_t \leq M_t - D_t.$$

The worker sells labor services  $N_t^s$  to firms, but firms must pay wages prior to receiving the receipts from production. To accomplish this, firms must take out bank loans to pay workers. If  $N_t^d$  is the firm's demand for labor hours and  $L_t$  equals nominal bank loans, then the wages-in-advance constraint in nominal terms is

$$P_t \omega_t N_t^d \leq L_t,$$

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20. Expected inflation effects will also be at work, so the net impact on nominal interest rates will depend on, among other things, the degree of positive serial correlation in the growth rate of the money supply.

21. In Fuerst (1992), this wedge was measured by the difference between the Lagrangian multiplier on the household's CIA constraint and that on the firm's CIA constraint. A cash injection lowers the value of cash in the financial market and lowers the nominal rate of interest. Similarly, positive nominal interest rates arise in the search model of Shi (2005) because money balances taken to the bond market cannot be used in the goods market within the same period (see section 3.4).

where  $\omega_t$  is the real wage. Firm profits, expressed in nominal terms, are

$$\Pi_t^f = P_t Y(N_t^d) - P_t \omega_t N_t^d - R_t^L L_t,$$

where  $Y(N^d)$  is the firm's production technology and  $R^L$  is the interest rate charged on bank loans.

Banks accept deposits from households and pay interest  $R^D$  on them. Banks make loans to firms, charging  $R^L$ . Finally, the central bank makes transfers to banks. The balance sheet of the representative bank is

$$L_t = D_t + H_t,$$

where  $H$  represent transfers from the central bank. Profits for the representative bank are

$$\Pi_t^b = R_t^L L_t + H_t - R_t^D D_t = (R_t^L - R_t^D) D_t + (1 + R_t^L) H_t.$$

Competition and profit maximization in the banking sector ensure

$$R_t^L = R_t^D \equiv R,$$

so bank profits are  $(1 + R_t) H_t$ .

Key to the structure of this model is the assumption that households must make their financial portfolio decision in choosing  $D_t$  prior to learning the current realization of the central bank transfer  $H_t$ . Hence, households are unable to adjust their portfolio in response to the monetary injection. Banks and firms are able to respond after  $H_t$  is realized. Thus, the effects of  $H_t$  on the supply of bank loans affects the equilibrium interest rate on loans needed to balance loan supply and loan demand.

Before writing the decision problem of the representative household and deriving the equilibrium conditions, it is useful to divide all nominal variables by the aggregate price level and let lowercase letters denote the resulting real quantities. Hence,  $m_t$  equals real money holdings of the representative household. Thus, the cash-in-advance constraint becomes  $C_t \leq m_t - d_t$  and the wage-in-advance constraint becomes  $\omega_t N_t^d \leq l_t$ . The household's budget constraint is, in nominal terms,

$$P_t \omega_t N_t^s + M_t - D_t + (1 + R_t) D_t + \Pi_t^b + \Pi_t^f - P_t C_t = M_{t+1},$$

so that after using the expressions for  $\Pi_t^b$  and  $\Pi_t^f$  and dividing by  $P_t$ , this becomes

$$\omega_t N_t^s + m_t + R_t d_t + (1 + R_t) h_t + \left[ Y(N_t^d) - \omega_t N_t^d - R_t l_t \right] - C_t = \left( \frac{P_{t+1}}{P_t} \right) m_{t+1}.$$

In equilibrium,  $m_t = M_t^s / P_t$ , where  $M_t^s$  is the nominal supply of money,  $N_t^s = N_t^d = N_t$ , and  $l_t = d_t + h_t$ .

Let the household's preferences over consumption and hours of work be given by

$$u(C_t) - v(N_t^s),$$

where  $u_c, v_N \geq 0, u_{cc} \leq 0, v_{NN} \geq 0$ . The value function for the household can be written as

$$V(m_t) = \max_d \mathbb{E} \left\{ \max_{C_t, N_t^s, N_t^d, l_t, m_{t+1}} [u(C_t) - v(N_t^s) + \beta V(m_{t+1})] \right\}$$

where the maximization is subject to

$$m_t - d_t \geq C_t,$$

$$\omega_t N_t^s + m_t + R_t^D d_t + \pi_t^b + [Y(N_t^d) - \omega_t N_t^d - R_t^L l_t] - C_t - \left( \frac{P_{t+1}}{P_t} \right) m_{t+1} = 0,$$

$$l_t \geq \omega_t N_t^d.$$

Let  $\lambda_1, \lambda_2$ , and  $\lambda_3$  be the Lagrangian multipliers associated with these three constraints. Note that  $d_t$  is chosen before the household knows the current level of transfers and so must be picked based on expectations but knowing that the other variables will subsequently be chosen optimally. The first-order necessary conditions for the optimal choice of  $d_t, C_t, N_t^s, N_t^d, l_t$ , and  $m_{t+1}$  include

$$d: \mathbb{E}_h [-\lambda_{1t} + R_t^D \lambda_{2t}] = 0, \quad (5.15)$$

$$C: u'(C_t) = \lambda_{1t} + \lambda_{2t}, \quad (5.16)$$

$$N^s: -v'(N_t^s) + \omega_t \lambda_{2t} = 0, \quad (5.17)$$

$$N^d: \lambda_{2t} Y'(N_t^d) - \omega_t (\lambda_{2t} + \lambda_{3t}) = 0, \quad (5.18)$$

$$l: \lambda_{3t} - R_t^L \lambda_{2t} = 0, \quad (5.19)$$

$$m: -\lambda_{2t} \left( \frac{P_{t+1}}{P_t} \right) + \beta V_m(m_{t+1}) = 0, \quad (5.20)$$

$$V_m(m_t) = \mathbb{E}_h (\lambda_{1t} + \lambda_{2t}). \quad (5.21)$$

The operator  $\mathbb{E}_h$  denotes expectations with respect to the distribution of  $h_t$  and is applied to (5.15) and the envelope condition (5.21) because  $d_t$  is chosen before observing  $h_t$ . Conditions such as (5.16) are familiar from cash-in-advance models (see chapter 3). The marginal utility of consumption can differ from the marginal utility of income ( $\lambda_{2t}$ ) if the cash-in-advance constraint binds (i.e., when  $\lambda_{1t} > 0$ ).

The multipliers  $\lambda_{1t}$  and  $\lambda_{3t}$  measure the value of liquidity in the goods market and the loan market, respectively. Adding (5.16) to (5.19) yields

$$u'(C_t) = (1 + R_t) \lambda_{2t} + \lambda_{1t} - \lambda_{3t},$$

and (5.16), (5.20), and (5.21) imply

$$\lambda_{2t} = \beta \left( \frac{P_t}{P_{t+1}} \right) V_m(m_{t+1}) = \beta \left( \frac{P_t}{P_{t+1}} \right) E_h u'(C_{t+1}). \quad (5.22)$$

These last two equations imply

$$u'(C_t) = \beta \left( \frac{1 + R_t}{1 + \pi_{t+1}} \right) E_h u'(C_{t+1}) - (\lambda_{3t} - \lambda_{1t}), \quad (5.23)$$

where  $1 + \pi_{t+1} = P_{t+1}/P_t$ . This expression would, in the absence of the last term, simply be a standard Euler condition linking the marginal utility of consumption at  $t$  and  $t + 1$  with the real return on the bond. When the value of cash in the goods market differs from its value in the loan market,  $\lambda_{3t} - \lambda_{1t} \neq 0$ , and a wedge is created between the current marginal utility of consumption and its future value adjusted for the expected real return.

From (5.19) and the earlier result that  $R_t^D = R_t^L$ , (5.15) can be written as

$$E_h \lambda_{1t} = E_h \lambda_{3t}.$$

When the household makes its portfolio choice, the value of sending money to the goods market (as measured by  $\lambda_{1t}$ ) and of sending it to the loan market by depositing it in a bank (as measured by  $\lambda_{3t}$ ) must be equal. Ex post, the two multipliers can differ because households cannot reallocate funds between the two markets during the period.

Turning to the labor market, (5.18) and (5.19) imply

$$Y'(N_t^d) = (1 + R_t^L)\omega_t; \quad (5.24)$$

the firm equates the marginal product of labor to the marginal cost of labor, but this is greater than the real wage because of the cost of borrowing funds to finance the firm's wage bill. Thus, the nominal interest rate drives a wedge between the real wage and the marginal product of labor.<sup>22</sup>

From the perspective of labor suppliers, wages earned in period  $t$  cannot be used to purchase consumption goods until period  $t + 1$ . Thus, from (5.17), the marginal rate of substitution between leisure and income is set equal to the real wage:

$$\frac{v'(N_t^s)}{\lambda_{2t}} = \omega_t.$$

Combining this expression with the labor demand condition and noting that  $N_t^d = N_t^s = N_t$  in equilibrium,

$$\frac{v'(N_t)}{\lambda_{2t}} = \frac{Y'(N_t)}{1 + R_t}, \quad (5.25)$$

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22. A rise in the interest rate increases labor costs for each value of the wage and has a negative effect on aggregate employment and output. This effect of interest rates is usually called the cost channel of monetary policy (see Ravenna and Walsh 2006).

revealing how the nominal interest rate drives a wedge between the marginal rate of substitution and the marginal product of labor.

Now consider what happens when there is an unexpected monetary injection  $H_t$ . Since the injection is received initially by banks, it increases the supply of loans  $D_t + H_t$  because  $D_t$  is predetermined by the household's portfolio choice. Equilibrium requires a rise in loan demand, and this is induced by a fall in the interest rate on loans. From (5.24), the fall in  $R_t^L$  increases the demand for labor at each real wage. This increase in labor demand leads to a rise in the real wage, which in turn induces households to supply more labor. In equilibrium, both employment and the real wage rise until the demand for loans,  $\omega_t N_t$ , has increased to absorb the rise in the supply of loans. Hence, both employment and the real wage rise in response to the monetary injection.

Monetary injections have real effects in this model because households must make their portfolio choices before observing the current monetary shock. Any change in the money supply that is anticipated would not have real effects, since it would be factored into the household's portfolio choice. Once households are able to reallocate their money and bond holdings, changes in the level of the money supply are neutral, affecting only the level of prices.<sup>23</sup>

### 5.3.2 Endogenous Market Segmentation

The standard limited-participation model assumes all agents participate in all markets, just not all the time. Some agents make portfolio choices before all information is revealed and are then restricted until the next period from reallocating their portfolio once the information is available. An alternative approach is to assume some agents access some markets infrequently. For example, Alvarez, Atkeson, and Kehoe (2002) developed a model of endogenous market segmentation. Fixed costs of exchanging bonds for money leads agents to trade infrequently.<sup>24</sup> Monetary injections into the bond market cause distributional effects because the new money must be held by the subset of agents active in the bond market. These effects depend on the level of inflation, however. When inflation is low, the opportunity cost of holding money is low, and few agents find it worthwhile to pay the fixed cost of exchanging money for bonds; market segmentation is high, and monetary shocks have large distributional effects. When inflation is high, the opportunity cost of holding money is also high, and most agents find it worthwhile to pay the fixed cost of exchanging money for bonds; market segmentation is low, and monetary shocks will have small distributional effects.

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23. To produce more persistent real effects of monetary shocks, Chari, Christiano, and Eichenbaum (1995) introduced quadratic costs of portfolio adjustment. A similar mechanism is employed by Cooley and Quadrini (1999).

24. Alvarez, Lucas, and Weber (2001) provided a simplified model with segmented markets in which an exogenous fraction of agents do not participate in the bond market.

The basic structure of the Alvarez, Atkeson, and Kehoe (2002) model consists of a goods market and an asset market. Changes in the supply of money are engineered via open-market operations in the asset market. Purchases in the goods market are subject to a cash-in-advance constraint. Suppose a household's desired consumption is greater than the real value of its initial cash holdings. The household can sell bonds to obtain additional cash, but each transfer of cash between the asset and the goods market incurs a fixed cost of  $\gamma$ . Let  $m$  be the household's real money balances, and let  $c$  denote the consumption level it chooses if it incurs the fixed cost of obtaining additional money. Then it will pay the fixed cost and make an asset exchange if

$$h(c, m) \equiv U(c) - U(m) - U_c(c)(c + \gamma - m) > 0, \quad (5.26)$$

where  $U(c)$  is utility of consumption and  $U_c$  is marginal utility. To understand this condition, note that if the household does not make an asset transfer, the cash-in-advance constraint implies it can consume  $m$ , yielding utility  $U(m)$ . If it does make a transfer, it can consume  $c$ , yielding utility  $U(c)$ , but it must also pay the fee  $\gamma$ ; the last term in (5.26) is the cost of the fee in terms of utility.<sup>25</sup>

Importantly, the function  $h$  defined in (5.26) is minimized when  $m = c$ , in which case  $h(c, c) = -U_c(c)\gamma < 0$ . Because  $h$  is continuous, for  $m$  near  $c$ ,  $h$  is negative, the condition in (5.26) is not satisfied, and the household is in what Alvarez, Atkeson, and Kehoe described as a zone of inactivity. The gain from an asset exchange is not sufficient to justify the fixed cost, so the household does not participate in the asset market. If  $m$  is further away from  $c$ , the gains are larger. Alvarez, Atkeson, and Kehoe showed that  $m_L$  and  $m_H$  can be defined such that if  $m < m_L$ , or  $m > m_H$ , the household will find it worthwhile to be active in the asset market. In the former case (i.e., when  $m < m_L$ ), the household will sell bonds for money; in the latter case (i.e., when  $m > m_H$ ), the household will use money to buy bonds. Thus, markets are segmented in that some households are actively participating in the asset markets while others are inactive, but this segmentation is endogenously determined.

Given market segmentation, a monetary injection has real effects, just as in the earlier limited-participation models that treated segmentation or lack of market access as an exogenous characteristic of the environment. An increase in the growth rate of money increases expected inflation, and this raises the nominal interest rate. However, the increase in real balances raises the consumption of the households that are active in the asset market. As in all the models that have been examined, the real return links the marginal utility of consumption today with the expected future marginal utility of consumption, but it is

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25. The cost of consuming  $c$  is  $c + \gamma$ , since the fee must be paid. The cost of consuming  $m$  is just  $m$ , so the extra cost of consuming  $c$  is  $c + \gamma - m$ , which carries a utility cost of  $U_c(c)(c + \gamma - m)$ . The gain is  $U(c) - U(m)$ .

only the marginal utility of the active households that is relevant in the asset market.<sup>26</sup> Since their current consumption has risen relative to their expected future consumption, the real interest rate falls. Alvarez, Atkeson, and Kehoe called this the segmentation effect. Thus, nominal interest rates may fall with an increase in the growth rate of money if the segmentation effect is larger than the expected inflation effect.

To illustrate the competing effects on the nominal interest rate, Alvarez, Atkeson, and Kehoe assumed money growth rates, expressed as deviations around the steady state, following an AR(1) process:

$$\hat{\mu}_t = \rho \hat{\mu}_{t-1} + e_t,$$

where  $\hat{\mu}_t$  denotes the log deviation of  $\mu_t$  from its steady-state value. They then showed that expected inflation is equal to  $E_t \hat{\mu}_{t+1}$ , while the marginal utility of active households is  $-\phi \hat{\mu}_t$ , with  $\phi > 0$ . Let  $\hat{u}_c(t)$  denote the log deviation of the marginal utility of active households at date  $t$ . Then, the log-linear approximation to the Euler condition implies the real interest rate is

$$\hat{r}_t = - [E_t \hat{u}_c(t+1) - \hat{u}_c(t)] = \phi (E_t \hat{\mu}_{t+1} - \hat{\mu}_t) = \phi (\rho - 1) \hat{\mu}_t.$$

The effect of money growth on the nominal interest rate is then equal to

$$\hat{i}_t = \hat{r}_t + E_t \hat{\pi}_{t+1} = \phi (\rho - 1) \hat{\mu}_t + \rho \hat{\mu}_t = [\phi (\rho - 1) + \rho] \hat{\mu}_t,$$

which is negative (faster money growth lowers the nominal interest rate) if  $\phi > \rho / (1 - \rho)$ . The authors discussed possible calibrations of  $\phi$  consistent with their model. For example, if half of all households are inactive and the coefficient of relative risk aversion is 2, then  $\phi = 1$ . In this case, the nominal interest rate falls in response to a rise in money growth as long as  $\rho < 1/2$ .

### 5.3.3 Assessment

Models that generate real effects of money by restricting financial transactions can account for nominal (and real) interest rate declines in response to monetary policy shocks. But as Dotsey and Ireland (1995) showed, this class of models does not account for interest rate effects of the magnitude actually observed in the data. Similarly, King and Watson (1996) found that monetary shocks do not produce significant business cycle fluctuations in their version of a limited-participation model (which they call a *liquidity-effect model*). Christiano, Eichenbaum, and Evans (1997) showed that their limited-participation model is able to match evidence on the effects of monetary shocks on prices, output, real wages, and profits only if the labor supply wage elasticity is assumed to be very high. They argued that this outcome is due, in part, to the absence of labor market frictions in their model.

26. The standard Euler condition,  $U_c(t) = \beta(1 + r_t)E_t U_c(t+1)$ , still holds for active households, but it no longer holds for the representative household.



Because limited-participation models were developed to account for the observation that monetary injections lower market interest rates, the real test of whether they have isolated an important channel through which monetary policy operates must come from evaluating their other implications. Christiano, Eichenbaum, and Evans (1997) examined real wage and profit movements to test their models. They argued that limited-participation models are able to account for the increase in profits that follows a monetary expansion. A further implication of such models relates to the manner in which the impact of monetary injections will change over time as financial sectors evolve and the cost of transactions falls. Financial markets today are very different than they were 25 years ago, and these differences should show up in the way money affects interest rates now as compared to 25 years ago.<sup>27</sup>

Financial market frictions are important for understanding the impact effects of monetary policy actions on market interest rates. The general role of such frictions is discussed in chapter 10. Various models that incorporate limited participation by some agents in financial markets or other forms of financial market segmentation have played an important role in recent analyses of central bank balance sheet policies. These models are reviewed in chapter 11.

#### 5.4 Summary

Monetary economists generally agree that the models discussed in chapters 2–4, while useful for examining issues such as the welfare cost of inflation and the optimal inflation tax, need to be modified to account for the short-run effects of monetary factors on the economy. In this chapter, two such modifications were explored: informational frictions, and frictions that limit the ability of some agents to adjust their portfolios. Aggregate informational or portfolio frictions can allow money to have real effects in the short run even when prices are completely flexible, but most monetary models designed to address short-run monetary issues assume that wages and/or prices do not adjust instantaneously in response to changes in economic conditions (sticky prices and wages). Models of nominal rigidities are discussed in detail in chapter 7.

#### 5.5 Appendix: An Imperfect-Information Model

This appendix provides details on the derivation of equilibrium in the Lucas imperfect-information model (section 5.2.2). Additional details on the derivations employed in this appendix can be found at <http://people.ucsc.edu/~walshc/mtp4e/>.

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27. Cole and Ohanian (2002) argued that the impact of money shocks in the United States has declined with the ratio of  $M1$  to nominal GDP, a finding consistent with the implications of limited-participation models.

Using (5.4) to eliminate the marginal utility of consumption from (5.1)–(5.3), the equilibrium in local market  $i$ , or island  $i$ , can be represented by the following three equations, where the goods equilibrium condition  $y^i = c^i$  has been used:

$$y_t^i = (1 - \alpha)n_t^i, \quad (5.27)$$

$$\eta \left( \frac{n^{ss}}{1 - n^{ss}} \right) n_t^i = y_t^i - n_t^i + \lambda_t^i, \quad (5.28)$$

$$\lambda_t^i = \Omega_1 y_t^i + \Omega_2 (m_t^i - p_t^i), \quad (5.29)$$

where<sup>28</sup>

$$\Omega_1 = [\gamma(b - \Phi) - b],$$

$$\Omega_2 = (b - \Phi)(1 - \gamma),$$

$$m_t^i - p_t^i = y_t^i + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) E^i [\tau_{t+1} - (p_{t+1} - p_t^i) + (\lambda_{t+1} - \lambda_t^i)], \quad (5.30)$$

$$\gamma \equiv \frac{a(C^{ss})^{1-b}}{a(C^{ss})^{1-b} + (1 - a)[(M/P)^{ss}]^{1-b}}.$$

The chapter 2 appendix contains a more complete derivation of the basic MIU model. Equation (5.28) is derived from the condition that the marginal utility of leisure divided by the marginal utility of consumption must equal the marginal product of labor. Equation (5.29) defines the marginal utility of consumption. Equation (5.30) is derived from the first-order condition that, for an agent on island  $i$ ,

$$u_c^i(t) = u_m^i(t) + \beta E^i \left( \frac{T_{t+1}}{\Pi_{t+1}} \right) u_c(t + 1), \quad (5.31)$$

where the left side is the utility cost of reducing consumption marginally in order to hold more money, and the right side is the return from higher money holdings. This return consists of the direct utility yield  $u_m^i(t)$  plus the utility from using the real balances to increase consumption in period  $t + 1$ . With transfers viewed as proportional to money holdings,

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28. The parameters  $\beta$ ,  $\Phi$ ,  $b$ , and  $\eta$  are from the utility function of the representative agent:

$$u \left( C_t, \frac{M_t}{P_t}, 1 - N_t \right) = \frac{\left[ aC_t^{1-b} + (1 - a) \left( \frac{M_t}{P_t} \right)^{1-b} \right]^{\frac{1-\Phi}{1-b}}}{1 - \Phi} + \Psi \frac{[(1 - N_t)^{1-\eta}]}{1 - \eta}.$$

the individual treats money as if it yielded a real return of  $T_{t+1}/\Pi_{t+1}$ . Given the assumed utility function, both sides of (5.31) can be divided by  $u_c^i(t) = u_c(1 + \lambda_t^i)$  and written as

$$1 = \left(\frac{1-a}{a}\right) \left(\frac{M_t^i/P_t^i}{C_t^i}\right)^{-b} + \beta E^i \left(\frac{T_{t+1}}{\Pi_{t+1}}\right) \left(\frac{1 + \lambda_{t+1}^i}{1 + \lambda_t^i}\right).$$

Expressed in terms of percentage deviations around the steady state (denoted by lowercase letters), the two terms on the right side become

$$\left(\frac{1-a}{a}\right) \left(\frac{M_t^i/P_t^i}{C_t^i}\right)^{-b} \approx \left(\frac{1-a}{a}\right) \left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b} [1 + bc_t^i - b(m_t^i - p_t^i)]$$

and

$$\beta E^i \left(\frac{T_{t+1}}{\Pi_{t+1}}\right) \left(\frac{1 + \lambda_{t+1}^i}{1 + \lambda_t^i}\right) \approx \beta E^i [1 + \tau_{t+1} - p_{t+1} + p_t^i + (\lambda_{t+1} - \lambda_t^i)],$$

where the fact that in the steady state  $T^{ss} = \Pi^{ss}$  has been used. This condition also implies

$$1 = \left(\frac{1-a}{a}\right) \left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b} + \beta \left(\frac{T^{ss}}{\Pi^{ss}}\right) = \left(\frac{1-a}{a}\right) \left[\left(\frac{M/P}{C}\right)^{ss}\right]^{-b} + \beta, \quad (5.32)$$

so the first-order condition becomes

$$0 = (1 - \beta) [bc_t^i - b(m_t^i - p_t^i)] + \beta E^i [\tau_{t+1} - p_{t+1} + p_t^i + (\lambda_{t+1} - \lambda_t^i)],$$

which can be rearranged to yield (5.30), since  $c^i = y^i$ .

The nominal money supply on island  $i$  is assumed to evolve according to

$$m_t^i = \rho_m m_{t-1} + v_t + u_t + u_t^i.$$

The value of  $v$  is announced (or observed) at the start of period  $t$ . Individuals on island  $i$  observe the island-specific nominal money stock  $m_t^i$ . This allows them to infer  $u_t + u_t^i$  but not  $u$  and  $u^i$  separately. The expectation of the time  $t+1$  money supply, conditional on the information available on island  $i$ , will be  $E^i m_{t+1} = \rho_m^2 m_{t-1} + \rho_m v_t + \rho_m E^i u_t$ . Equating expectations with linear least squares projections,  $E^i u_t = \kappa(u_t + u_t^i)$ , where  $\kappa = \sigma_u^2 / (\sigma_u^2 + \sigma_i^2)$ .

The time  $t$  transfer  $\tau_t$  is  $\tau_t = m_t - m_{t-1} = (\rho_m - 1)m_{t-1} + v_t + u_t$ , so

$$\begin{aligned} E_t \tau_{t+1} &= (\rho_m - 1) E^i m_t + E^i (v_{t+1} + u_{t+1}) \\ &= (\rho_m - 1) [\rho_m m_{t-1} + v_t + \kappa(u_t + u_t^i)]. \end{aligned}$$

Eliminating output, the marginal utility of consumption, and the expected transfer from equations (5.27)–(5.30), these equations yield the following two equations for employment and prices:

$$n_t^i = \left[ \frac{\Omega_2}{1 + \eta \left( \frac{n_t^{ss}}{l_t^{ss}} \right) - (1 + \Omega_1)(1 - \alpha)} \right] (m_t^i - p_t^i) = A (m_t^i - p_t^i), \quad (5.33)$$

$$\begin{aligned} m_t^i - p_t^i &= (1 - \alpha)n_t^i + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) \Omega_2 E^i [m_{t+1} - p_{t+1} - m_t^i + p_t^i] \\ &\quad + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) (1 - \alpha) \Omega_1 E^i \Delta n_{t+1} \\ &\quad + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) [(\rho_m - 1)E^i m_t - E^i p_{t+1} + p_t^i]. \end{aligned} \quad (5.34)$$

$\Delta$  is the first difference operator ( $\Delta n_{t+1} = n_{t+1} - n_t^i$ ).

By substituting (5.33) into (5.34), one obtains a single equation that involves the price process and the exogenous nominal money supply process:

$$\begin{aligned} (m_t^i - p_t^i) &= (1 - \alpha)A (m_t^i - p_t^i) \\ &\quad + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) [\Omega_2 + (1 - \alpha)\Omega_1 A] E^i [m_{t+1} - p_{t+1} - m_t^i + p_t^i] \\ &\quad + \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) [(\rho_m - 1)E^i m_t - E^i p_{t+1} + p_t^i]. \end{aligned} \quad (5.35)$$

Equation (5.35) can be solved using the method of undetermined coefficients (see McCallum 1989; Attfield, Demery, and Duck 1991). This method involves guessing a solution for  $p_t^i$  and then verifying that the solution is consistent with (5.35). Since  $m_t$  depends on  $m_{t-1}$ ,  $v_t$ ,  $u_t$ , and  $u_t^i$ , a guess for the minimum state-variable solution (McCallum 1983b) for the equilibrium price level takes the following form:

$$p_t^i = a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i, \quad (5.36)$$

where the  $a_j$  coefficients are yet to be determined parameters. Equation (5.36) implies the aggregate price level is  $p_t = a_1 m_{t-1} + a_2 v_t + a_3 u_t$ , so

$$\begin{aligned} E^i p_{t+1} &= a_1 E^i m_t = a_1 (\rho_m m_{t-1} + v_t + E^i u_t) \\ &= a_1 [\rho_m m_{t-1} + v_t + \kappa(u_t + u_t^i)]. \end{aligned}$$

$$E^i m_{t+1} = \rho_m^2 m_{t-1} + \rho_m v_t + \rho_m E^i u_t.$$

Now all the terms in (5.35) can be evaluated. The left side of (5.35) is equal to

$$(m_t^i - p_t^i) = (\rho_m - a_1)m_{t-1} + (1 - a_2)v_t + (1 - a_3)u_t + (1 - a_4)u_t^i$$

while the terms on the right side equal

$$(1 - \alpha)A [(\rho_m - a_1)m_{t-1} + (1 - a_2)v_t + (1 - a_3)u_t + (1 - a_4)u_t^i],$$

$$B \left[ \rho_m^2 m_{t-1} + \rho_m v_t + \rho_m \kappa (u_t + u_t^i) \right] - Ba_1 [\rho_m m_{t-1} + v_t + \kappa (u_t + u_t^i)] \\ - B [\rho_m m_{t-1} + v_t + u_t + u_t^i] + B (a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i),$$

where  $B = (\beta/b(1 - \beta)) [\Omega_2 + (1 - \alpha)\Omega_1 A]$ , and

$$\left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) (\rho_m - 1) [\rho_m m_{t-1} + v_t + \kappa (u_t + u_t^i)] \\ - \left( \frac{1}{b} \right) \left( \frac{\beta}{1 - \beta} \right) [a_1 (\rho_m m_{t-1} + v_t + \kappa (u_t + u_t^i)) - (a_1 m_{t-1} + a_2 v_t + a_3 u_t + a_4 u_t^i)].$$

For the two sides of (5.35) to be equal for all possible realizations of  $m_{t-1}$ ,  $v_t$ ,  $u_t$ , and  $u_t^i$  requires that the following hold. First, the coefficient on  $m_{t-1}$  on the right side must equal the coefficient of  $m_{t-1}$  on the left side, which holds if  $a_1 = \rho_m$ . Second, the coefficient of  $v_t$  on the right side must equal the coefficient of  $v_t$  on the left side, or  $a_2 = 1$  (since  $a_1 = \rho_m$ ). Third, the coefficient of  $u_t$  on the right side must equal the coefficient of  $u_t$  on the left side, or

$$a_3 = \frac{\kappa + K}{1 + K} < 1,$$

where

$$K = b \left( \frac{1 - \beta}{\beta} \right) [1 - (1 - \alpha)A + B].$$

Finally, the coefficient on  $u_t^i$  on the right side must be equal to its coefficient on the left side, or  $a_4 = a_3$ .

Combining these results, one obtains the expressions for the equilibrium economywide price level and employment given by (5.6) and (5.7).

## 5.6 Problems

1. In chapter 2, monetary transfers were treated as lump-sum. With higher inflation, transfers rose (as the seigniorage revenue was returned to private agents), but each individual viewed these transfers as unrelated to his or her own money holdings. If individual agents view the transfers as interest payments that are proportional to their own money holdings, show that higher inflation does not raise the opportunity cost of holding money.

2. Assume  $\Omega_2 = 0$  in the model consisting of (5.1)–(5.4) and (5.5).
- Show that  $y_t^i = n_t^i = y_t = n_t = 0$ . Explain why  $\Omega_2 = 0$  implies the real variables (output and employment) on every island equal their steady-state values.
  - Now also assume  $\rho_m = 1$  in (5.5), so that the nominal money supply is a random walk. Show that the aggregate price level satisfies

$$p_t = \left( \frac{1}{1+a} \right) m_t + \left( \frac{a}{1+a} \right) \int E_t^i p_{t+1} di - \left( \frac{a}{1+a} \right) \int E_t^i m_t di,$$

where  $a \equiv [\beta/(1-\beta)](1/b)$  and the integrals are the average expectations over all islands.

- Guess that  $p_t = a_1 m_{t-1} + a_2 v_t + a_3 u_t$ . Find the equilibrium values of  $a_1$ ,  $a_2$ , and  $a_3$  assuming rational expectations.
- Suppose the central bank becomes more transparent in that  $\sigma_u^2$  falls so that the aggregate money stock becomes more predictable. Using the Lucas model (section 5.2.2), explain how the variance of the price level and the variance of employment would be affected by this change.
  - According to the sticky-information model of section 5.2.3, the impact of the output gap on inflation (holding constant expectations), is equal to  $\alpha\lambda/(1-\lambda)$ , where  $\lambda$  is the fraction of firms that update their information. Explain why the impact of the output gap on inflation is increasing in  $\lambda$ .
  - Using the model of section 5.2.3 and the calibrated values used to obtain figures 5.1 and 5.2, construct impulse responses for inflation and output to innovations to the money growth rate for  $\rho = 0, 0.25, 0.5$ , and  $0.75$ . How are the responses affected by the degree of serial correlation in the growth rate of money?
  - Using the limited-participation model of section 5.3.1 and ignoring uncertainty, show that when the household makes its portfolio decision, it anticipates that the marginal rate of substitution between leisure and consumption equals the marginal product of labor divided by  $(1 + R_t)^2$ . Explain the intuition for this result.

# 6 Discretionary Policy and Time Inconsistency

## 6.1 Introduction

Macroeconomic equilibrium depends on both the current and expected future behavior of monetary policy.<sup>1</sup> If policymakers behave according to a systematic rule, the rule can be used to determine rational expectations of future policy actions under the assumption that the central bank continues to behave according to the rule. In principle, one could derive an optimal policy rule by specifying an objective function for the central bank and then determining the values of the parameters in the policy rule that maximize the expected value of the objective function.

But what ensures that the central bank will find it desirable to follow such a rule? Absent enforcement, it may be optimal to deviate from the rule once private agents have made commitments based on the expectation that the rule will be followed. Firms and workers may agree to set nominal wages or prices based on the expectation that monetary policy will be conducted in a particular manner, but once these wage and price decisions have been made, the central bank may have an incentive to deviate from actions called for under the rule. If deviations from a strict rule are possible, that is, if the policymakers can exercise discretion, agents will need to consider the policymakers' incentive to deviate; they can no longer simply base their expectations on the rule the policymakers say they will follow.

A large literature has focused on the incentives central banks face when actually setting their policy instrument. Following the seminal contribution of Kydland and Prescott (1977), attention has been directed to issues of central bank credibility and the ability to precommit to policies. Absent some means of committing in advance to take specific policy actions, central banks may find that they face incentives to act in ways that are inconsistent with their earlier plans and announcements.

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1. This dependence is illustrated in the equilibrium expressions for the price level in the money-in-the-utility function (MIU) and cash-in-advance (CIA) models (chapters 2 and 3) and in the discussion of the new Keynesian model (chapter 8).

A policy is *time-consistent* if an action planned at time  $t$  for time  $t + i$  remains optimal to implement when time  $t + i$  actually arrives. The policy can be state-contingent, that is, the action promised for time  $t + i$  can depend on the realization of events that were unknown at time  $t$  when the policy was originally planned. But a time-consistent policy is one in which the planned response to new information remains the optimal response once the new information arrives. A policy is *time-inconsistent* if at time  $t + i$  it will not be optimal to respond as originally planned. This chapter explores the average level of inflation when the monetary policy optimal for the central bank to promise is time-inconsistent, and how discretionary action, that is, doing what is optimal at that time regardless of past promises, may lead to excessive inflation.<sup>2</sup>

Analyzing time inconsistency in monetary policy is important for two reasons. First, it forces one to examine the incentives faced by central banks. Just as with a study of private sector behavior, an understanding of systematic behavior by the central bank requires an examination of the incentives the policymaker faces. And by focusing on these incentives, models of time inconsistency have had an important influence as *positive* theories of observed rates of inflation.

Second, if time inconsistency is important, then models that help us to understand the incentives faced by policymakers and the nature of the decision problems they face are important for the normative task of designing policymaking institutions. For this purpose, monetary economists need models that show how institutional structures affect policy outcomes.

The next section develops a framework, originally due to Barro and Gordon (1983a), that despite its simplicity proved extremely useful for studying problems of time inconsistency in monetary policy. The discretionary conduct of policy, meaning that the central bank is free at any time to alter its instrument setting, is shown to produce an average inflation bias; equilibrium inflation exceeds the socially desired rate. This bias arises from a desire for economic expansions above the economy's equilibrium output level (or for unemployment rates below the economy's natural rate) and the inability of the central bank to commit credibly to a low rate of inflation. Section 6.3 examines solutions that have been proposed for overcoming this inflation bias. Central banks very often seem to be concerned with their reputations, and section 6.3.1 examines how such a concern might reduce or even eliminate the inflation bias. Section 6.3.2 considers the possibility that society or government might wish to delegate responsibility for monetary policy to a central banker with preferences between employment and inflation fluctuations that differ from those of society as a whole. Since the inflation bias can be viewed as arising because the central bank faces the wrong incentives, a third approach to solving the inflation bias problem is to design mechanisms for creating the right incentives. This approach is discussed in section 6.3.3. Section 6.3.4

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2. A stabilization bias can arise under discretionary policy regimes when inflation depends on forward-looking expectations (see chapter 8).



considers the role of institutional structures in solving the inflation bias problem. Finally, the role of explicit targeting rules is studied in section 6.3.5.

The models of sections 6.2 and 6.3, with their focus on the inflation bias that can arise under discretion, have played a major role in the academic literature on inflation. The success of these models as positive theories of inflation—that is, as explanations for the actual historical variations of inflation over time and across countries—is open to debate. Section 6.4 discusses the empirical importance of the inflation bias in accounting for episodes of inflation.

## 6.2 Inflation under Discretionary Policy

If inflation is costly (even a little), if there is no real benefit to having, say, 5 percent rather than 0 or 1 percent inflation on average, why have average rates of inflation in most countries consistently been positive? Many explanations of positive average rates of inflation have built on the time-inconsistency analysis of Kydland and Prescott (1977) and Calvo (1978).<sup>3</sup> The basic insight is that while it may be optimal to achieve a low average inflation rate, such a policy is not time consistent. If the public were to expect low inflation, the central bank would face an incentive to inflate at a higher rate. Understanding this incentive, and believing the policymaker will succumb to it, the public correctly anticipates a higher inflation rate. The policymaker then finds it optimal to deliver the inflation rate the public anticipated.

### 6.2.1 Policy Objectives

To determine the central bank's actions, one needs to specify the preferences of the central bank. It is standard to assume that the central bank's objective function involves output (or employment) and inflation. The exact manner in which output enters the objective function has been posited in two different forms. Barro and Gordon (1983b) proposed that the central bank's objective is to maximize the expected value of

$$U = \lambda(y - y_n) - \frac{1}{2}\pi^2, \quad (6.1)$$

where  $y$  is output,  $y_n$  is the economy's natural rate of output, and  $\pi$  is the inflation rate. More output is preferred to less output with constant marginal utility, so output enters linearly, and inflation is assumed to generate increasing marginal disutility and so enters quadratically. The parameter  $\lambda$  governs the relative weight the central bank places on

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3. For a survey dealing with time-inconsistency problems in the design of both monetary and fiscal policies, see Persson and Tabellini (1990) or Stokey (2002). Cukierman (1992) also provided an extensive discussion of the theoretical issues related to the analysis of inflation in models in which time inconsistency plays a critical role. The Persson and Tabellini (1999) survey of political economy covered many of the issues discussed in this chapter. See also Driffill (1988) and Stokey (2002).

output expansions relative to inflation stabilization. Often the desire for greater output is motivated by political pressure because of the effects of economic expansions on the reelection prospects of incumbent politicians.<sup>4</sup> Alternatively, distortions due to taxes, monopoly unions, or monopolistic competition may lead  $y_n$  to be inefficiently low. For discussions of alternative motivations for this type of loss function, see Cukierman (1992). Thus, the central bank would like to expand output, but it will be able to do so only by creating surprise inflation (see section 7.2).

The other standard specification for preferences assumes that the central bank desires to minimize the expected value of a loss function that depends on output and inflation fluctuations. Thus, the *loss* function is quadratic in both output and inflation and takes the form

$$V = \frac{1}{2}\lambda(y - y_n - k)^2 + \frac{1}{2}\pi^2. \quad (6.2)$$

The key aspect of this loss function is the parameter  $k$ . The assumption is that the central bank desires to stabilize both output and inflation, inflation around zero but output around  $y_n + k$ , a level that exceeds the economy's equilibrium output  $y_n$  by the constant  $k$ .<sup>5</sup> Because the expected value of  $V$  involves the variance of output, the loss function (6.2) will generate a role for stabilization policy that is absent when the central bank cares only about the level of output, as in (6.1).

There are several common explanations for a positive  $k$ , and these parallel the arguments for the output term in the linear preference function (6.1). Most often, some appeal is made to the presence of imperfect competition, as in the new Keynesian model (see chapter 8), or labor market distortions (e.g., a wage tax) that lead the economy's natural rate of output to be inefficiently low. Attempting to use monetary policy to stabilize output around  $y_n + k$  then represents a second-best solution (the first-best would involve eliminating the original distortion). An alternative interpretation is that  $k$  arises from political pressure on the central bank: officials have a bias for economic expansions because these tend to increase the probability of reelection. The political interpretation motivates institutional reforms designed to minimize political pressures on the central bank.

The two alternative objective functions (6.1) and (6.2) are clearly closely related. Expanding the term involving output in the quadratic loss function, (6.2) can be written as

$$V = -\lambda k(y - y_n) + \frac{1}{2}\pi^2 + \frac{1}{2}\lambda(y - y_n)^2 + \frac{1}{2}\lambda k^2.$$

The first two terms are the same as the linear utility function (with signs reversed because  $V$  is a loss function), showing that the assumption of a positive  $k$  is equivalent to the presence

4. The influence of reelections on the central bank's policy choices was studied by Fratianni, von Hagen, and Waller (1997) and Herrendorf and Neumann (2003).

5. See (6.3). Note that the inflation term in (6.1) and (6.2) can be replaced by  $\frac{1}{2}(\pi - \pi^*)^2$  if the monetary authority has a target inflation rate  $\pi^*$  that differs from zero.

of a utility gain from output expansions above  $y_n$ . In addition,  $V$  includes a loss arising from deviations of output around  $y_n$  (the  $\lambda(y - y_n)^2$  term). This introduces a role for stabilization policies that is absent when the policymaker's preferences are assumed to be strictly linear in output. The final term, involving  $k^2$ , is simply a constant and so has no effect on the central bank's decisions.<sup>6</sup>

The alternative formulations reflected in (6.1) and (6.2) produce many of the same insights. The discussion follows Barro and Gordon (1983b) in beginning with the loss function (6.1) that is linear in output. The equilibrium concept in the basic Barro-Gordon model is noncooperative Nash. Given the public's expectations, the central bank's policy choice maximizes its objective function (or equivalently, minimizes its loss function). The assumption of rational expectations implicitly defines the loss function for private agents as  $L^P = E(\pi - \pi^e)^2$ ; given the public's understanding of the central bank's decision problem, their choice of  $\pi^e$  is optimal.

### 6.2.2 The Economy

The specification of the economy is quite simple and follows the analysis of Barro and Gordon (1983a; 1983b). Aggregate output is given by a Lucas-type aggregate supply function (see chapter 5) of the form

$$y = y_n + a(\pi - \pi^e) + e, \quad (6.3)$$

where  $e$  is a stochastic shock. This supply function can be motivated as arising from imperfect information about aggregate monetary disturbances, as in the Lucas islands model of chapter 5, or by the presence of one-period nominal wage contracts set at the beginning of each period based on the public's expectation of the rate of inflation. If actual inflation exceeds the expected rate, real wages will be eroded and firms will expand employment.<sup>7</sup> If actual inflation is less than the expected rate, realized real wages will exceed the level expected and employment will be reduced. A critical discussion of this basic aggregate supply relationship can be found in Cukierman (1992, ch.3).<sup>8</sup>

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6. See Cukierman (1992) for more detailed discussions of alternative motivations that might lead to objective functions of the form given by either (6.1) or (6.2). For an open-economy framework, Bohn (1991c) showed how the incentives for inflation depend on foreign-held debt denominated in the domestic currency. In chapter 8 the objective function for the central bank is derived as an approximation to the utility of the represented agent. Under certain conditions, such an approximation yields an objective function similar to (6.2).

7. The model with one-period sticky nominal wages is developed in section 7.2.1.

8. If the aggregate supply equation is substituted into the central bank's preference function, both (6.1) and (6.2) can be written in the form  $\bar{U}(\pi - \pi^e, \pi, e)$ . Thus, the general framework is one in which the central bank's objective function depends on both surprise inflation and actual inflation. In addition to the employment motives mentioned, one could emphasize the desire for seigniorage as leading to a similar objective function, since surprise inflation, by depreciating the real value of both interest-bearing and non-interest-bearing liabilities of the government, produces larger revenue gains for the government than does anticipated inflation (which only erodes non-interest-bearing liabilities).

Recall that the tax distortions of inflation analyzed in chapter 4 were a function of anticipated inflation. Fluctuations in unanticipated inflation caused neutral price level movements, while expected inflation altered nominal interest rates and the opportunity cost of money, leading to tax effects on money holdings, the consumption of cash goods, and the supply of labor. If the costs of inflation arise purely from expected inflation, while surprise inflation generates economic expansions, then a central bank would perceive only benefits from attempting to produce unexpected inflation. Altering the specification of the central bank's objective function in (6.1) or (6.2) to depend only on output and expected inflation would, given (6.3), then imply that the equilibrium inflation rate could be infinite (see Auernheimer 1974; Calvo 1978; and problem 7 at the end of this chapter).

The rest of the model is a simple link between inflation and the policy authority's actual policy instrument:

$$\pi = \Delta m + v, \tag{6.4}$$

where  $\Delta m$  is the growth rate of the money supply (the first difference of the log nominal money supply), assumed to be the central bank's policy instrument, and  $v$  is a velocity disturbance. The private sector's expectations are assumed to be determined prior to the central bank's choice of a growth rate for the nominal money supply. Thus, in setting  $\Delta m$ , the central bank will take  $\pi^e$  as given. Also assume that the central bank can observe  $e$  (but not  $v$ ) prior to setting  $\Delta m$ ; this assumption generates a role for stabilization policy. Finally, assume  $e$  and  $v$  are uncorrelated.

The sequence of events is important. First, the private sector sets nominal wages based on its expectations of inflation. Thus, in the first stage,  $\pi^e$  is set. Then the supply shock  $e$  is realized. Because expectations have already been determined, they do not respond to the realization of  $e$ . Policy can respond, however, and the policy instrument  $\Delta m$  is set after the central bank has observed  $e$ . The velocity shock  $v$  is then realized, and actual inflation and output are determined.

Several important assumptions have been made here. First, as with most models involving expectations, the exact specification of the information structure is important. Most critically, it is assumed that private agents must commit to nominal wage contracts before the central bank sets the rate of growth of the nominal money supply. This means that the central bank has the opportunity to surprise the private sector by acting in a manner that differs from what private agents had expected when they locked themselves into nominal contracts. Second, in keeping with the literature based on Barro and Gordon (1983a), it is assumed the central bank sets money growth as its policy instrument. If the main objective is to explain the determinants of average inflation rates, the distinction between money and interest rates as the policy instrument is not critical. Third, the basic model assumes the central bank can react to realization of the supply shock  $e$  while the public commits to wage contracts prior to observing this shock. This informational advantage on the part of the central bank introduces a role for stabilization policy and is meant to capture the fact

that policy decisions can be made more frequently than are most wage and price decisions. It means the central bank can respond to economic disturbances before private agents have had the chance to revise nominal contracts.

The assumption that  $v$  is observed after  $\Delta m$  is set is not critical. It is easy to show that the central bank will always adjust  $\Delta m$  to offset any observed or forecastable component of the velocity shock, and this is why the rate of inflation itself is often treated as the policy instrument. Output and inflation will only be affected by the component of the velocity disturbance that was unpredictable at the time policy was set.

### 6.2.3 Equilibrium Inflation

Since the central bank is assumed to act before observing the disturbance  $v$ , its objective will be to maximize the expected value of  $U$ , where the central bank's expectation is defined over the distribution of  $v$ . Substituting (6.3) and (6.4) into the central bank's objective function yields

$$U = \lambda[a(\Delta m + v - \pi^e) + e] - \frac{1}{2}(\Delta m + v)^2.$$

The first-order condition for the optimal choice of  $\Delta m$ , conditional on  $e$  and taking  $\pi^e$  as given, is

$$a\lambda - \Delta m = 0, \quad \text{or}$$

$$\Delta m = a\lambda > 0. \tag{6.5}$$

Given this policy, actual inflation equals  $a\lambda + v$ . Because private agents are assumed to understand the incentives facing the central bank—that is, they are rational—they use (6.5) in forming their expectations about inflation. With private agents forming expectations prior to observing the velocity shock  $v$ , (6.4) and (6.5) imply

$$\pi^e = E[\Delta m] = a\lambda.$$

Thus, average inflation is fully anticipated. From (6.3), output is  $y_n + av + e$  and is independent of the central bank's policy.

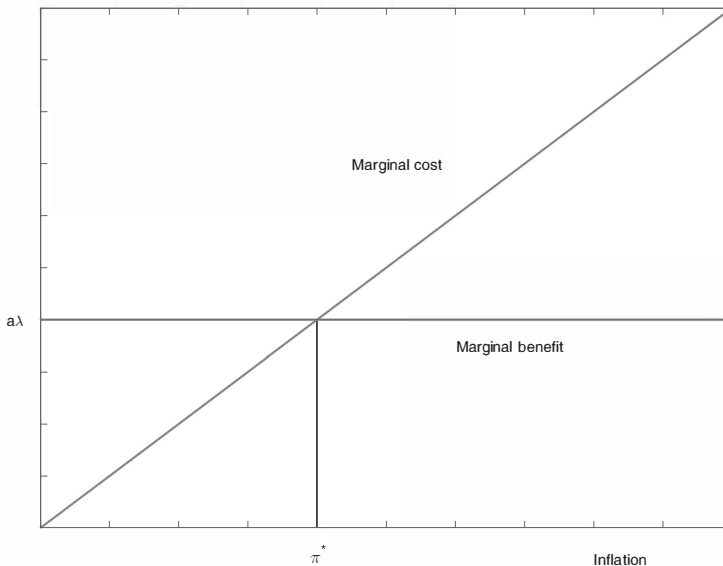
When the central bank acts with discretion in setting  $\Delta m$ , equilibrium involves a positive average rate of inflation equal to  $a\lambda$ . This has no effect on output, since the private sector completely anticipates inflation at this rate ( $\pi^e = a\lambda$ ). The economy suffers from a positive average inflation bias that yields no benefit in terms of greater output. The size of the bias is increasing in the effect of a money surprise on output,  $a$ , since this parameter governs the marginal benefit in the form of extra output that can be obtained from an inflation surprise. The larger is  $a$ , the greater is the central bank's incentive to inflate. Recognizing this fact, private agents anticipate a higher rate of inflation. The inflation bias is also increasing in the weight the central bank places on its output objective,  $\lambda$ . A small  $\lambda$  implies that the

gains from economic expansion are low relative to achieving inflation objectives, so the central bank has less incentive to generate inflation.

Why does the economy end up with positive average inflation even though it confers no benefits and the central bank dislikes inflation? The central bank is acting systematically to maximize the expected value of its objective function, so it weighs the costs and benefits of inflation in setting its policy. At a zero rate of inflation, the marginal benefit of generating a little inflation is positive because, with wages set, the effect of an incremental rise in inflation on output is equal to  $a > 0$ . The value of this output gain is  $a\lambda$ . This is illustrated in figure 6.1 by the horizontal line at a height equal to  $a\lambda$ . The marginal cost of inflation is equal to  $\pi$ . At a planned inflation rate of zero, this marginal cost is zero, so the marginal benefit of inflation exceeds the marginal cost. But the marginal cost rises (linearly) with inflation, as illustrated in the figure. At an expected inflation rate of  $a\lambda$ , the marginal cost equals the marginal benefit.

Under this discretionary policy outcome, expected utility of the central bank is equal to

$$\begin{aligned} E[U^d] &= E\left[\lambda(av + e) - \frac{1}{2}(a\lambda + v)^2\right] \\ &= -\frac{1}{2}(a^2\lambda^2 + \sigma_v^2), \end{aligned}$$



**Figure 6.1**

Equilibrium inflation under discretion (linear objective function).

where  $E[v] = E[e] = 0$  and  $\sigma_v^2$  is the variance of the random inflation control error  $v$ . Expected utility is decreasing in the variance of the random control error  $v$  and decreasing in the weight placed on output relative to inflation objectives ( $\lambda$ ) because a larger  $\lambda$  increases the average rate of inflation. Although the control error is unavoidable, the loss due to the positive average inflation rate arises from the monetary authority's fruitless attempt to stimulate output.

The outcome under discretion can be contrasted with the situation in which the monetary authority is able to commit to setting money growth always equal to zero:  $\Delta m = 0$ . In this case,  $\pi = v$  and expected utility would equal

$$E[U^c] = E\left[\lambda(av + e) - \frac{1}{2}v^2\right] = -\frac{1}{2}\sigma_v^2 > E[U^d].$$

The central bank (and society, if the central bank's utility is interpreted as a social welfare function) would be better off if it were possible to commit to a policy of zero money growth. Discretion, in this case, generates a cost.

As noted earlier, an alternative specification of the central bank's objectives focuses on the loss associated with output and inflation fluctuations around desired levels. This alternative formulation, given by the loss function (6.2), leads to the same basic conclusions. Discretion produces an average bias toward positive inflation and lower expected utility. In addition, specifying the loss function so that the central bank cares about output fluctuations means that there will be a potential role for policy to reduce output volatility caused by the supply shock  $e$ .

Substituting (6.3) and (6.4) into the quadratic loss function (6.2) yields

$$V = \frac{1}{2}\lambda[a(\Delta m + v - \pi^e) + e - k]^2 + \frac{1}{2}(\Delta m + v)^2.$$

If  $\Delta m$  is chosen after observing the supply shock  $e$ , but before observing the velocity shock  $v$ , to minimize the expected value of the loss function, the first-order condition for the optimal choice of  $\Delta m$ , conditional on  $e$  and taking  $\pi^e$  as given, is

$$a\lambda[a(\Delta m - \pi^e) + e - k] + \Delta m = 0, \quad \text{or}$$

$$\Delta m = \frac{a^2\lambda\pi^e + a\lambda(k - e)}{1 + a^2\lambda}. \quad (6.6)$$

There are two important differences to note in comparing (6.5), the optimal setting for money growth from the model with a linear objective function, to (6.6). First, the aggregate supply shock appears in (6.6); because the central bank wants to minimize the variance of output around its target level, it will make policy conditional on the realization of the supply shock. Thus, an explicit role for stabilization policies arises that will involve trading off some inflation volatility for reduced output volatility. Second, the optimal policy depends on private sector expectations about inflation.

Private agents are assumed to understand the incentives facing the central bank, so they use (6.6) in forming their expectations. However, private agents are atomistic; they do not take into account the effect their choice of expected inflation might have on the central bank's decision.<sup>9</sup> With expectations formed prior to observing the aggregate supply shock, (6.4) and (6.6) imply

$$\pi^e = E[\Delta m] = \frac{a^2\lambda\pi^e + a\lambda k}{1 + a^2\lambda}.$$

Solving for  $\pi^e$  yields  $\pi^e = a\lambda k > 0$ . Substituting this back into (6.6) and using (6.4) gives an expression for the equilibrium rate of inflation:

$$\pi^d = \Delta m + v = a\lambda k - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e + v, \quad (6.7)$$

where the superscript  $d$  stands for discretion. Note that the equilibrium when the central bank acts with discretion implies a positive average rate of inflation equal to  $a\lambda k$ . This has no effect on output because the private sector completely anticipates this rate ( $\pi^e = a\lambda k$ ). The size of the inflation bias is increasing in the distortion ( $k$ ), the effect of a money surprise on output ( $a$ ), and the weight the central bank places on its output objective ( $\lambda$ ).<sup>10</sup>

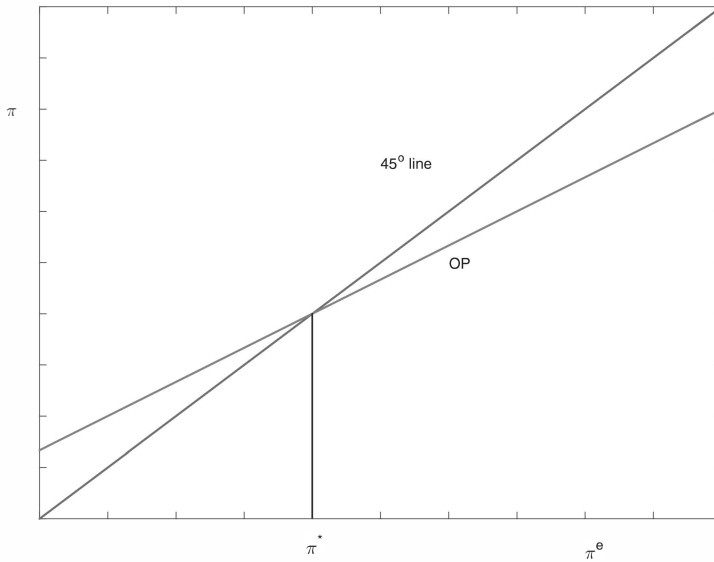
If, for the moment, one ignores the random disturbances  $e$  and  $v$ , the equilibrium with the quadratic loss function can be illustrated using figure 6.2. Equation (6.6) is shown, for  $e = 0$ , as the straight line OP (for optimal policy), giving the central bank's reaction function for its optimal inflation rate as a function of the public's expected rate of inflation. The slope of this line is  $a^2\lambda/(1 + a^2\lambda) < 1$ , with intercept  $a\lambda k/(1 + a^2\lambda) > 0$ . An increase in the expected rate of inflation requires that the central bank increase actual inflation by the same amount in order to achieve the same output effect, but because this action raises the cost associated with inflation, the central bank finds it optimal to raise  $\pi$  by less than the increase in  $\pi^e$ . Hence the slope is less than 1. The positive intercept reflects the fact that, if  $\pi^e = 0$ , the central bank's optimal policy is to set a positive rate of inflation. In equilibrium, expectations of private agents must be consistent with the behavior of the central bank. In the absence of any random disturbances, this requires that  $\pi^e = \pi$ . Thus, equilibrium must lie along the 45° line in figure 6.2.

An increase in  $k$ , the measure of the output distortion, shifts the OP line upward and leads to a higher rate of inflation in equilibrium. An increase in  $a$ , the impact of an inflation surprise on real output, has two effects. First, it increases the slope of the OP line; by

9. This assumption is natural in the context of individual firms and workers determining wages and prices. If nominal wages are set in a national bargaining framework, for example, by a monopoly union and employer representatives, then it may be more appropriate to assume wages are set strategically, taking into account the impact of the wage decision on the incentives faced by the central bank. The case of a monopoly union has been analyzed by Tabellini (1988) and Cubitt (1992). See also Cukierman and Lippi (2001).

10. In a model with monetary and fiscal policy authorities, Dixit and Lambertini (2003) showed that if fiscal policy is optimally designed to eliminate the distortions behind  $k$ , the central bank's objective function can be reduced to  $\frac{1}{2}\lambda(y - y_n)^2 + \frac{1}{2}\pi^2$ . This would eliminate the average inflation bias.





**Figure 6.2**  
Equilibrium inflation under discretion (quadratic loss function).

increasing the output effects of an inflation surprise, it raises the marginal benefit to the central bank of more inflation. By increasing the impact of an inflation surprise on output, however, a rise in  $a$  reduces the inflation surprise needed to move output to  $y_n + k$ , and if  $\lambda$  is large, the intercept of OP could actually fall. The net effect of a rise in  $a$ , however, is to raise the equilibrium inflation rate (see (6.7), which shows that the equilibrium inflation rate when  $e = 0$  is  $a\lambda k$ , which is increasing in  $a$ ).

The coefficient on  $e$  in (6.7) is negative; a positive supply shock leads to a reduction in money growth and inflation. This response reduces the impact of  $e$  on output (the coefficient on  $e$  in the output equation becomes  $1/(1 + a^2\lambda)$ , which is less than 1). The larger the weight on output objectives ( $\lambda$ ), the smaller the impact of  $e$  on output. In contrast, a central bank that places a larger relative weight on inflation objectives (a small  $\lambda$ ) stabilizes output less.

Using (6.7), the loss function under discretion is

$$V^d = \frac{1}{2}\lambda \left[ \left( \frac{1}{1 + a^2\lambda} \right) e + av - k \right]^2 + \frac{1}{2} \left[ a\lambda k - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e + v \right]^2. \tag{6.8}$$

The unconditional expectation of this loss is

$$E[V^d] = \frac{1}{2}\lambda (1 + a^2\lambda) k^2 + \frac{1}{2} \left[ \left( \frac{\lambda}{1 + a^2\lambda} \right) \sigma_e^2 + (1 + a^2\lambda) \sigma_v^2 \right], \tag{6.9}$$

where  $\sigma_x^2$  denotes the variance of  $x$ .

Now suppose that the central bank is able to precommit to a policy rule prior to the formation of private expectations. Because there is a role for stabilization policy in the present case (i.e., the monetary authority would like to respond to the supply shock  $e$ ), the policy rule is not simply a fixed growth rate for  $\Delta m$ , as it was in the case when the central bank's objective function was a linear function of output. Instead, suppose the central bank is able to commit to a policy rule of the form

$$\Delta m^e = b_0 + b_1 e.$$

In the present linear-quadratic framework, a linear rule such as this is optimal. Given this rule,  $\pi^e = b_0$ . Now substituting this into the loss function gives

$$V^c = \frac{1}{2}\lambda [a(b_1 e + v) + e - k]^2 + \frac{1}{2}[b_0 + b_1 e + v]^2. \quad (6.10)$$

Under a commitment policy, the central bank commits itself to particular values of the parameters  $b_0$  and  $b_1$  *prior* to the formation of expectations by the public and *prior* to observing the particular realization of the shock  $e$ . Thus,  $b_0$  and  $b_1$  are chosen to minimize the unconditional expectation of the loss function. Solving the minimization problem, the optimal policy under precommitment is

$$\Delta m^c = - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e. \quad (6.11)$$

Note that average inflation under precommitment is zero ( $b_0 = 0$ ), but the response to the aggregate supply shock is the same as under discretion (see 6.7). The unconditional expectation of the loss function under precommitment is

$$E[V^c] = \frac{1}{2}\lambda k^2 + \frac{1}{2} \left[ \left( \frac{\lambda}{1 + a^2\lambda} \right) \sigma_e^2 + (1 + a^2\lambda)\sigma_v^2 \right], \quad (6.12)$$

which is strictly less than the loss under discretion. Comparing (6.9) and (6.12), one sees that the cost of discretion is equal to  $(a\lambda k)^2/2$ , which is simply the loss attributable to the nonzero average rate of inflation.

The inflation bias that arises under discretion occurs for two reasons. First, the central bank has an incentive to inflate once private sector expectations are set. Second, the central bank is unable to precommit to a zero average inflation rate. To see why it cannot commit, suppose the central bank announces that it will deliver zero inflation. If the public believes the announced policy, and therefore  $\pi^e = 0$ , it is clear from (6.5) or (6.6) that the optimal policy for the central bank to follow would involve setting a positive average money growth rate, and the average inflation rate would be positive. So the central bank's announcement would not be believed in the first place. The central bank cannot believably commit to a zero inflation policy because under such a policy (i.e., if  $\pi = \pi^e = 0$ ) the marginal cost of a little inflation is  $\partial \frac{1}{2}\pi^2 / \partial \pi = \pi = 0$ , while the marginal benefit is  $a\lambda > 0$  under the linear objective function formulation, or  $-a^2\lambda(\pi - \pi^e) + a\lambda k = a\lambda k > 0$  under the quadratic

formulation. Because the marginal benefit exceeds the marginal cost, the central bank has an incentive to break its commitment.

Society is clearly worse off under the discretionary policy outcome because it experiences positive average inflation with no systematic improvement in output performance. This result fundamentally alters the long-running debate in economics over rules versus discretion in the conduct of policy. Prior to Kydland and Prescott's analysis of time inconsistency, economists had debated whether monetary policy should be conducted according to a simple rule, such as Milton Friedman's  $k$  percent growth rate rule for the nominal supply of money, or whether central banks should have the flexibility to respond with discretion. With the question posed in this form, the answer is clearly that discretion is better. After all, if following a simple rule is optimal, under discretion one could always choose to follow such a rule. Thus, one could do no worse under discretion, and one might do better. But as the Barro-Gordon model illustrates, one might actually do worse under discretion. Restricting the flexibility of monetary policy may result in a superior outcome. To see this, suppose the central bank is forced (somehow) to set  $\Delta m = 0$ . This avoids any average inflation bias, but it also prevents the central bank from engaging in any stabilization policy. With the loss function given by (6.2), the unconditional expected loss under such a policy rule is  $\frac{1}{2}\lambda(\sigma_e^2 + k^2) + \frac{1}{2}(1 + a^2\lambda)\sigma_v^2$ . If this is compared to the unconditional expected loss under discretion,  $E[V^d]$ , given in (6.8), the zero money growth rule will be preferred if

$$\left( \frac{a^2\lambda^2}{1 + a^2\lambda} \right) \sigma_e^2 < (a\lambda k)^2.$$

The left side measures the gains from stabilization policy under discretion; the right side measures the cost of the inflation bias that arises under discretion. If the latter is greater, expected loss is lower if the central bank is forced to follow a fixed money growth rule.

By focusing on the strategic interaction of the central bank's actions and the public's formation of expectations, the Barro-Gordon model provides a simple but rich game-theoretic framework for studying monetary policy outcomes. The approach emphasizes the importance of understanding the incentives faced by the central bank in order to understand policy outcomes. It also helps to highlight the role of credibility, illustrating why central bank promises to reduce inflation may not be believed. The viewpoint provided by models of time inconsistency contrasts sharply with the traditional analysis of policy outcomes as either exogenous or as determined by a rule that implicitly assumes an ability to precommit.

A more formal treatment of the economic structure that could motivate the adhoc specifications provided by (6.1) or (6.2) and the aggregate supply function (6.3) is contained in Albanesi, Chari, and Christiano (2003). They assumed imperfect competition in the goods market and that a fraction of firms set prices before current-period information is revealed. The presence of sticky prices provides the central bank with a means of affecting aggregate output; imperfect competition implies average output is inefficiently low, and this

provides the central bank with an incentive to boost output. In addition, Albanesi, Chari, and Christiano introduced the distinction between cash and credit goods (see chapter 3). Cash goods can only be purchased with money. As a consequence, the relative price of cash and credit goods depends on the nominal rate of interest, and inflation alters households' choice between these two types of goods. The central bank faces a trade-off: higher inflation that was not anticipated increases welfare by raising output, whereas higher expected inflation lowers welfare by distorting the choice between cash and credit goods. Multiple equilibria can arise in this framework, leading to what the authors described as expectational traps. If the public expects high inflation, the best policy for the central bank is to validate those expectations.

### 6.3 Solutions to the Inflation Bias

Following Barro and Gordon (1983a), a large literature developed to examine alternative solutions to the inflationary bias under discretion.<sup>11</sup> Because the central bank is assumed to set the inflation rate so that the marginal cost of inflation (given expectations) is equal to the marginal benefit, most solutions alter the basic model to raise the marginal cost of inflation as perceived by the central bank. For example, the first class of solutions incorporates notions of reputation into a repeated-game version of the basic framework. Succumbing to the temptation to inflate today worsens the central bank's reputation for delivering low inflation; as a consequence, the public expects more inflation in the future, and this lowers the expected value of the central bank's objective function. By punishing the central bank, the loss of reputation raises the marginal cost of inflation.

The second class of solutions can also be interpreted in terms of the marginal cost of inflation. Rather than viewing inflation as imposing a reputational cost on the central bank, one could allow the central bank to have preferences that differ from those of society at large so that the marginal cost of inflation as perceived by the central bank is higher. One way to do this is simply to select as the policymaker an individual who places a larger-than-normal weight on achieving low inflation and then give that individual the independence to conduct policy. Another way involves thinking of the policymaker as an executive whose compensation package is structured so as to raise the marginal cost of inflation. Or, if the inflation bias arises from political pressures on the central bank, institutions might be designed to reduce the effect of such pressures on the conduct of monetary policy.

Finally, a third class of solutions involves imposing limitations on the central bank's flexibility. The most common such restriction is a targeting rule that requires the central bank to achieve a preset rate of inflation or imposes a cost related to deviations from this

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11. See Persson and Tabellini (1990) for an in-depth discussion of much of this literature. Many of the most important papers are collected in Persson and Tabellini (1994).

target. An analysis of inflation targeting is important because many central banks have adopted inflation targeting as a framework for the conduct of policy.<sup>12</sup>

Before considering these solutions, however, it is important to note that the tradition in the monetary policy literature has been to assume that the underlying cause of the bias, the desire for economic expansion captured either by the presence of output in the case of the linear objective function (6.1) or by the parameter  $k$  in the quadratic loss function (6.2), is given. Clearly, policies that might eliminate the factors that create a wedge between the economy's equilibrium output and the central bank's desired level would lead to the first-best outcome in the Barro-Gordon model.

### 6.3.1 Reputation

One potential solution to an inflationary bias is to force the central bank to bear some cost if it deviates from its announced policy of low inflation, thereby raising the marginal cost of inflation as perceived by the central bank. One form such a cost might take is a lost reputation. The central bank might, perhaps through its past behavior, demonstrate that it will deliver zero inflation despite the apparent incentive to inflate. If the central bank then deviates from the low-inflation solution, its credibility is lost and the public expects high inflation in the future. That is, the public employs a trigger strategy. The folk theorem for infinite-horizon repeated games (Fudenberg and Maskin 1986) suggests that equilibria exist in which inflation remains below the discretionary equilibrium level as long as the central bank's discount rate is not too high. Hence, as long as the central bank cares enough about the future, a low-inflation equilibrium can be supported.

An alternative approach is to consider situations in which the public may be uncertain about the true preferences of the central bank. In the resulting imperfect-information game, the public's expectations concerning inflation must be based on its beliefs about the central bank's preferences or *type*. Based on observed outcomes, these beliefs evolve over time, and central banks may have incentives to affect these beliefs through their actions. A central bank willing to accept some inflation in return for an economic expansion may still find it optimal initially to build a reputation as an anti-inflation central bank.

### A Repeated Game

The basic Barro-Gordon model is a one-shot game; even if the central bank's objective is to maximize  $E_t \sum_{i=0} \beta^i U_{t+i}$ , where  $U_t$  is defined by (6.1) and  $\beta$  is a discount factor ( $0 < \beta < 1$ ), nothing links time  $t$  decisions with future periods.<sup>13</sup> Thus, the inflation rate in each period  $t + s$  is chosen to maximize the expected value of  $U_{t+s}$ , and the discretionary

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12. More than 30 countries have adopted inflation targeting (Roger 2010; Rose 2014). For evaluations of inflation targeting, see Mishkin and Schmidt-Hebbel (2002; 2007); Carare and Stone (2006); Batini and Laxton (2007); and Walsh (2009; 2011).

13. The same clearly applies to the case of a quadratic objective function of the form (6.2).

equilibrium of the one-shot game is a noncooperative Nash equilibrium of the repeated game. Barro and Gordon (1983b) evaluated the role of reputation by considering a repeated game in which the choice of inflation at time  $t$  can affect expectations about future inflation. They examined whether inflation rates below the one-shot discretionary equilibrium rate can be sustained in a trigger strategy equilibrium.

To illustrate their approach, suppose that the central bank's objective is to maximize the expected present discounted value of (6.1) and that the public behaves in the following manner. If in period  $t - 1$  the central bank delivered an inflation rate equal to what the public had expected (i.e., the central bank did not fool them in the previous period), the public expects an inflation rate in period  $t$  of  $\bar{\pi} < a\lambda$ . But if the central bank did fool them, the public expects the inflation rate that would arise under pure discretion,  $a\lambda$ . The hypothesized behavior of the public is summarized by

$$\pi_t^e = \bar{\pi} < a\lambda \quad \text{if } \pi_{t-1} = \pi_{t-1}^e,$$

$$\pi_t^e = a\lambda \quad \text{otherwise.}$$

It is important to note that this trigger strategy involves a one-period punishment. If, after deviating and inflating at a rate that differs from  $\bar{\pi}$ , the central bank delivers an inflation rate of  $a\lambda$  for one period, the public again expects the lower rate  $\bar{\pi}$  in the following period.<sup>14</sup>

The central bank's objective is to maximize

$$\sum_{i=0}^{\infty} \beta^i \mathbf{E}_t (U_{t+i}),$$

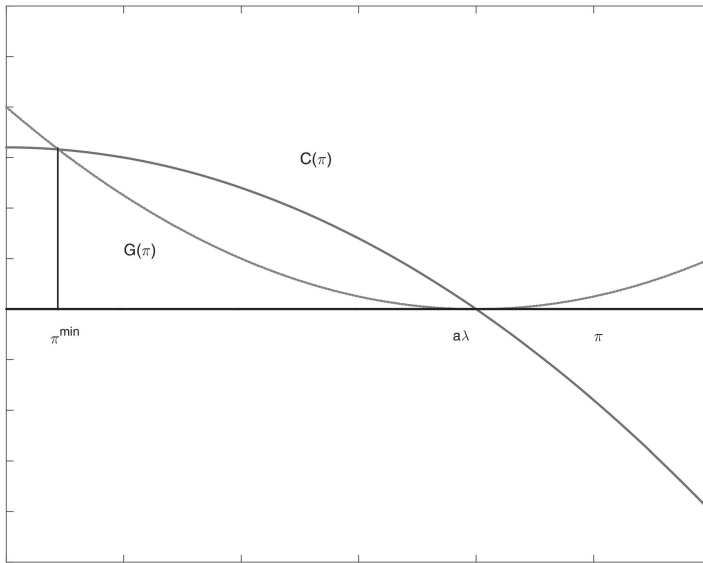
where  $U_t$  is given by (6.1). Previously, the central bank's actions at time  $t$  had no effects in any other period. Consequently, the problem simplified to a sequence of one-period problems, a situation that is no longer true in this repeated game with reputation. Inflation at time  $t$  affects expectations at time  $t + 1$  and therefore the expected value of  $U_{t+1}$ . The question is whether equilibria exist for inflation rates  $\bar{\pi}$  that are less than the outcome under pure discretion.

Suppose that the central bank has set  $\pi_s = \bar{\pi}$  for all  $s < t$ . Under the hypothesis about the public's expectations,  $\pi_t^e = \bar{\pi}$ . What can the central bank gain by deviating from the  $\bar{\pi}$  equilibrium? Ignoring any aggregate supply shocks (i.e.,  $e \equiv 0$ ), assume the central bank controls inflation directly. Then setting inflation a little above  $\bar{\pi}$ , say, at  $\pi_t = \varepsilon > \bar{\pi}$ , increases the time  $t$  value of the central bank's objective function by

$$\left[ a\lambda(\varepsilon - \bar{\pi}) - \frac{1}{2}\varepsilon^2 \right] - \left[ -\frac{1}{2}\bar{\pi}^2 \right] = a\lambda(\varepsilon - \bar{\pi}) - \frac{1}{2}(\varepsilon^2 - \bar{\pi}^2).$$

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14. This type of one-period punishment strategy has little to commend it in terms of plausibility. It does, however, provide a useful starting point for analyzing a situation in which the central bank might refrain from inflating at the discretionary rate because it recognizes that the public will subsequently expect higher inflation.



**Figure 6.3**  
Temptation and enforcement.

This is maximized for  $\varepsilon = a\lambda$ , the inflation rate under discretion. So if the central bank deviates, it will set inflation equal to  $a\lambda$  and gain

$$G(\bar{\pi}) \equiv a\lambda(a\lambda - \bar{\pi}) - \frac{1}{2} \left[ (a\lambda)^2 - \bar{\pi}^2 \right] = \frac{1}{2} (a\lambda - \bar{\pi})^2 \geq 0.$$

Barro and Gordon referred to this as the *temptation* to cheat. The function  $G(\bar{\pi})$ , shown as a dotted line in figure 6.3, is non-negative for all  $\bar{\pi}$  and reaches a minimum at  $\bar{\pi} = a\lambda$ .

Cheating carries a cost because, in the period following a deviation, the public will punish the central bank by expecting an inflation rate of  $a\lambda$ . Since  $a\lambda$  maximizes the central bank's one-period objective function for any expected rate of inflation, the central bank sets  $\pi_{t+1} = a\lambda$ . The subsequent loss, relative to the  $\bar{\pi}$  inflation path, is given by

$$C(\bar{\pi}) \equiv \beta \left( -\frac{1}{2} \bar{\pi}^2 \right) - \beta \left( -\frac{1}{2} a^2 \lambda^2 \right) = \frac{\beta}{2} \left[ (a\lambda)^2 - \bar{\pi}^2 \right]. \tag{6.13}$$

Since the loss occurs in period  $t + 1$ , multiply it by the central bank's discount factor  $\beta$ . Barro and Gordon referred to this as the *enforcement*. The function  $C(\bar{\pi})$ , shown as a solid line in figure 6.3, is decreasing for  $\bar{\pi} > 0$ .

The central bank will deviate from the proposed equilibrium if the gain (the temptation) exceeds the loss (the enforcement). Any  $\bar{\pi}$  such that  $C(\bar{\pi}) \geq G(\bar{\pi})$  can be supported as an equilibrium; with the loss exceeding the gain, the central bank has no incentive to deviate.

As shown in figure 6.3,  $C(\pi) < G(\pi)$  for inflation rates less than  $\pi^{\min} \equiv (1 - \beta)a\lambda / (1 + \beta) < a\lambda$ . Because  $\pi^{\min} > 0$ , the trigger strategy cannot support the socially optimal, zero inflation outcome. However, any inflation rate in the interval  $[\pi^{\min}, a\lambda]$  is sustainable. The minimum sustainable inflation rate  $\pi^{\min}$  is decreasing in  $\beta$ ; the greater the weight the central bank places on the future, the greater the enforcement mechanism provided by the public's expectations and the lower the inflation rate that can be sustained.<sup>15</sup>

This example is a simple illustration of a trigger strategy. The public expects one rate of inflation ( $\bar{\pi}$  in this example) as long as the central bank "behaves," and it expects a different, higher rate of inflation if the central bank misbehaves. But how does the public coordinate on this trigger strategy? If the public is atomistic, each member would take the expectations of others as given in forming its own expectations, and the notion of public coordination makes little sense. This problem is even more severe when multiperiod punishment periods are considered, in which the public expects high inflation for some fixed number of periods greater than one. Again, how is this expectation determined?

One way to solve the coordination problem is to assume that the central bank plays a game against a monopoly union.<sup>16</sup> With only one agent in the private sector (the union), the issue of atomistic agents coordinating on a trigger strategy no longer arises. Of course, the coordination problem has, in some sense, been solved by simply assuming it away, but it is also the case that many countries do have labor markets that are dominated by national unions and business organizations that negotiate over wages.<sup>17</sup>

The general point, though, is that the reputational solution works because the loss of reputation represents a cost to the central bank. Raising the marginal cost of inflation lowers the equilibrium rate of inflation. If  $C(\bar{\pi}) > G(\bar{\pi})$ , the central bank will not have an incentive to cheat, and inflation at the rate  $\bar{\pi}$  can be supported. But suppose the central bank does cheat. Will it be in the interests of a private sector that has somehow coordinated on a trigger strategy to actually punish the central bank? If by punishing the central bank the private sector also punishes itself, the threat to punish may not be credible. If punishment is not credible, the central bank is not deterred from cheating in the first place.

The credibility of trigger strategies in the context of the Barro-Gordon model (with the utility function (6.1)) has been examined by al-Nowaihi and Levine (1994). They considered the case of a single monopoly union and showed that if one requires that the punishment hurt the central bank but not the private sector (i.e., consider only equilibria that are

15. With the central bank's objective given by (6.2), a zero inflation rate can be supported with a one-period punishment trigger strategy of the type considered as long as the central bank places sufficient weight on the future. In particular, zero inflation is an equilibrium if  $\beta/(\alpha^2 + \beta) > 1$ . See problem 10 at the end of this chapter.

16. Tabellini (1988) studied the case of a monopoly union in the Barro-Gordon framework, although he focuses on imperfect information about the central bank's type, a topic discussed later. See also Cubitt (1992).

17. al-Nowaihi and Levine (1994) provided an interpretation in terms of a game involving successive governments rather than a monopoly union. See also Herrendorf and Lockwood (1997).



renegotiation-proof), then the only equilibrium is the high-inflation discretionary equilibrium. Thus, it would appear that trigger strategies will not support a low-inflation equilibrium.

Requiring that the punishment hurt only the central bank imposes strong restrictions on the possible equilibria. Adopting a weaker notion of renegotiation, al-Nowaihi and Levine introduced the concept of *chisel-proof credibility* by asking, if the central bank cheats just a little, will the public be better off simply acquiescing, or will it be better off punishing? They showed that the lowest inflation rate that can be supported in a chisel-proof equilibrium is positive but less than the discretionary rate.

This discussion of trigger strategy equilibria assumed that the trigger was pulled whenever inflation deviated from its optimal value. If inflation differed from  $\bar{\pi}$ , this outcome revealed to the public that the central bank had cheated. But for such strategies to work, the public must be able to determine whether the central bank cheated. If inflation depends not just on the central bank's policy but also on the outcome of a random disturbance, as in (6.4), then the trigger strategy must be based directly on the central bank's policy instrument rather than on the realized rate of inflation. Simply observing the actual rate of inflation may only reveal the net effects of both the central bank's policy actions and the realizations of a variety of random effects that influence the inflation rate.

This consequence raises a difficulty, one first analyzed by Canzoneri (1985). Suppose that inflation is given by  $\pi = \Delta m + v$ . In addition, suppose that the central bank has a private, unverifiable forecast of  $v$  (call it  $v^f$ ) and that  $\Delta m$  can be set conditional on  $v^f$ . Reputational equilibria will now be harder to sustain. Recall that the trigger strategy equilibrium required that the public punish the central bank whenever the central bank deviated from the low-inflation policy. In the absence of private information, the public can always determine whether the central bank deviated by simply looking at the value of  $\Delta m$ . When the central bank has private information on the velocity shock, it should adjust  $\Delta m$  to offset  $v^f$ . So if the central bank forecasts a negative  $v$ , it should raise  $\Delta m$ . Simply observing ex post a high value of  $\Delta m$ , therefore, will not allow the public to determine if the central bank cheated; the central bank can always claim that  $v^f$  was negative and that it had not cheated.<sup>18</sup>

Canzoneri showed that a trigger strategy equilibrium can be constructed in which the public assumes that the central bank cheated whenever the implicit forecast error of the central bank is too large. That is, a policy designed to achieve a zero rate of inflation would call for setting  $\Delta m = -v^f$ , and this might involve a positive rate of money growth. Whenever money growth is too high, that is, whenever  $\Delta m > -\bar{v}$  for some  $\bar{v}$ , the public assumes that the central bank has cheated. The public then expects high inflation in the

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18. Herrendorf (1999) considered situations in which  $v$  has a bounded support  $[\underline{v}, \bar{v}]$ . If the optimal commitment policy is  $\Delta m = 0$ , then as long as  $\underline{v} \leq \pi \leq \bar{v}$ , the public cannot tell whether the central bank cheated. However, if  $\pi > \bar{v}$ , the public knows the central bank cheated. Thus, the probability of detection is  $\text{Prob}(v > \bar{v} - m)$ .

subsequent period; high expected inflation punishes the central bank. The constant  $\bar{v}$  is chosen to ensure that the central bank has no incentive to deviate from the zero inflation policy. This equilibrium leads to a situation in which there are occasional periods of inflation; whenever the central bank's forecast for the random variable  $v$  takes on a value such that  $\Delta m = -v^f > -\bar{v}$ , expected inflation (and actual inflation) rises. One solution to this problem may involve making policy more transparent by establishing targets that allow deviations to be clearly observed by the public. Herrendorf (1999), for example, argued that a fixed exchange rate policy may contribute to credibility, since any deviation is immediately apparent. This solves the Canzoneri problem; the public does not need to verify the central bank's private information about velocity. If the central bank has private information on the economy that under the optimal commitment policy would call for a change in the exchange rate, a fixed exchange rate regime will limit the flexibility of the central bank to act on this information. Changing the exchange rate would signal to the public that the central bank was attempting to cheat. As a result, a trade-off between credibility and flexibility in conducting stabilization policy can arise.

The basic model of time inconsistency under discretion characterized the equilibrium in terms of a sequence of single-period equilibria that depend only on the current state. In particular, the actions by the private sector in forming expectations do not depend on the past history of policy actions. Chari and Kehoe (1990) introduced the notion of sustainable plans under discretion, where a sustainable plan is a policy that is optimal from the perspective of the policymaker when the impact of the policy on future histories, and the impact of these histories on future private sector decisions, is taken into account. To characterize the set of possible equilibria that are sustainable, Chari and Kehoe followed Abreu (1988) in finding the worst sustainable outcome. In the simple Barro-Gordon model, this worst outcome is the one with the average inflation bias that led, in the case of a quadratic loss function, to the expected loss given in (6.9). Ireland (1997b) applied the concept of sustainable equilibrium to study the Barro-Gordon average inflation bias in a well-specified model that allows policies to be ranked according to their implications for the welfare of the representative agent.<sup>19</sup> He showed that if the policymaker places a sufficiently large weight on future outcomes, any inflation rate between a deflation associated with the Friedman rule (a zero nominal rate of interest) and the rate that arises under discretion can be sustained as an equilibrium.

### Central Bank Types

According to Canzoneri (1985), the central bank has private information about the economy in the form of an unverifiable forecast of an economic disturbance. The public doesn't know what the central bank knows about the economy, and more important, the public

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19. Kurozumi (2008) examined optimal sustainable monetary policies within the context of a new Keynesian model in which discretionary policy generates a stabilization bias.

cannot ex post verify the central bank's information. An alternative aspect of asymmetric information involves situations in which the public is uncertain about the central bank's true preferences. Backus and Driffill (1985), Barro (1986), Cukierman and Meltzer (1986), Vickers (1986), Tabellini (1988), Andersen (1989), Mino and Tsutsui (1990), Cukierman and Liviatan (1991), Cukierman (1992), Garcia de Paso (1993), Drazen and Masson (1994), Ball (1995), al-Nowaihi and Levine (1996), Briault, Haldane, and King (1996), Nolan and Schaling (1996), Herrendorf (1998), and Walsh (2000), among others, studied models in which the public is uncertain about the central bank's type, which is usually identified as its preference between output and inflation stabilization or as its ability to commit. In these models, the public must attempt to infer the central bank's type from its policy actions, and equilibria in which central banks may deviate from one-shot optimal policies in order to develop reputations have been studied (for a survey, see Rogoff 1989). In choosing its actions, a central bank must take into account the uncertainty faced by the public, and it may be advantageous for one type of bank to mimic the other type to conceal (possibly only temporarily) its true type from the public.

In one of the earliest reputational models of monetary policy, Backus and Driffill (1985) assumed that governments (or central banks) come in two types: optimizers who always act to maximize the expected present discounted value of a utility function of the form (6.1) and single-minded inflation fighters who always pursue a policy of zero inflation. Alternatively, the inflation fighter types can be described as having access to a precommitment technology. The government in office knows which type it is, but this information is unverifiable by the public. Simply announcing it is a zero inflation government would not be credible because the public realizes that an optimizing central bank would also announce that it is a strict inflation fighter to induce the public to expect low inflation.<sup>20</sup>

Initially, the public is assumed to have prior beliefs about the current government's type (where these beliefs come from is unspecified, and therefore there will be multiple equilibria, one for each set of initial beliefs). If the government is actually an optimizer and ever chooses to inflate, its identity is revealed, and from then on the public expects the equilibrium inflation rate under discretion. To avoid this outcome, the optimizing government may have an incentive to conceal its true identity by mimicking the zero inflation type, at least for a while. Equilibrium may involve pooling, in which both types behave the same way. In a finite-period game, the optimizer always inflates in the last period because there is no future gain from further attempts at concealment.

Backus and Driffill solved for the equilibrium in their model by employing the concept of a *sequential equilibrium* (Kreps and Wilson 1982) for a finitely repeated game. Let  $\pi_t^d$  equal the inflation rate for period  $t$  set by a zero inflation ("dry") government, and let

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20. Vickers (1986) assumed the types differed with respect to the weight placed on inflation in the loss function. In the work of Tabellini (1988) the "tough" type has  $\lambda = 0$  (i.e., no weight on output), while the "weak" type is characterized by a  $\lambda > 0$ . Cukierman and Liviatan (1991) assumed the types differ in their ability to commit.

$\pi_t^w$  be the rate set by an optimizing (“wet”) government. Start in the final period  $T$ . The zero inflation type always sets  $\pi_T^d = 0$ , and the optimizing type always inflates in the last period at the discretionary rate  $\pi_T^w = a\lambda$ . With no further value in investing in a reputation, a wet government just chooses the optimal inflation rate derived from the one-period Barro-Gordon model analyzed earlier.

In periods prior to  $T$ , however, the government’s policy choice affects its future reputation, and it may therefore benefit a wet government to choose a zero rate of inflation in order to build a reputation as a dry government. Thus, equilibrium may consist of an initial series of periods in which the wet government mimics the dry government, and inflation is zero. For suitable values of the parameters, the sequential equilibrium concept that Backus and Driffill employed also leads to mixed strategies in which the wet government inflates with some probability. So the wet government randomizes; if the outcome calls for it to inflate, the government is revealed as wet, and from then on, inflation is equal to  $a\lambda$ . If it doesn’t inflate, the public updates its beliefs about the government’s type using Bayes’s rule.

Ball (1995) developed a model of inflation persistence based on the same notion of central bank types used by Backus and Driffill (1985) and Barro (1986). That is, one type, type  $D$ , always sets inflation equal to zero, while type  $W$  acts opportunistically to minimize the expected discounted value of a quadratic loss function of the form

$$L^W = \sum_{i=0}^{\infty} \beta^i \left[ \lambda (y_{t+i} - y_n - k)^2 + \pi_{t+i}^2 \right], \quad (6.14)$$

where  $0 < \beta < 1$ . To account for shifts in policy, Ball assumes that the central bank type follows a Markov process. If the central bank is of type  $D$  in period  $t$ , then the probability that the central bank is still type  $D$  in period  $t + 1$  is  $d$ ; the probability that the bank switches to type  $W$  in  $t + 1$  is  $1 - d$ . Similarly, if the period  $t$  central bank is type  $W$ , then the  $t + 1$  central bank is type  $W$  with probability  $w$  and type  $D$  with probability  $1 - w$ .

The specification of the economy is standard, with output a function of inflation surprises and an aggregate supply shock:

$$y_t = y_n + a(\pi_t - \pi_t^e) + e_t. \quad (6.15)$$

To capture the idea that economies are subject to occasional discrete supply shocks, Ball assumed that  $e$  takes on only two possible values: 0 with probability  $1 - q$  and  $\bar{e} < 0$  with probability  $q$ . If shifts in policy and supply shocks are infrequent, then  $1 - d$ ,  $1 - w$ , and  $q$  are all small.

The timing in this game has the public forming expectations of inflation; then the supply shock and the central bank type are determined. It is assumed that the realization of  $e$  but not of the central bank type is observable. Finally, the central bank sets  $\pi$ . In this game, there are many possible equilibria, depending on how the public is assumed to form its expectations about the central bank type. Ball considered a perfect Nash equilibrium

concept in which actions depend only on variables that directly affect current payoffs. Such equilibria are Markov perfect equilibria (Maskin and Tirole 1988) and rule out the types of trigger strategy equilibria considered, for example, by Barro (1986).<sup>21</sup> Ball then showed that such an equilibrium exists and involves the  $W$  type setting  $\pi = 0$  as long as  $e = 0$ ; if  $e = \bar{e}$ , the  $W$  type inflates at the discretionary rate. Since this reveals the identity of the central bank (i.e., as a type  $W$ ), inflation remains at the discretionary rate  $\alpha k$  until such time as a type  $D$  central bank takes over. At this point, inflation drops to zero, remaining there until a bad supply shock is again realized.<sup>22</sup> Ball's model predicts periodic and persistent bouts of inflation in response to adverse economic disturbances.

One undesirable aspect of the Backus-Driffill framework is its assumption that one government, the dry government, is simply an automaton, always playing zero inflation. While serving a useful purpose in allowing one to characterize how beliefs about type might affect the reputation and the behavior of a government that would otherwise like to inflate, the myopic behavior of the dry government is unsatisfactory; such a government might also wish to signal its type to the public or otherwise attempt to differentiate itself from a wet type.

One way a dry government might distinguish itself would be to announce a planned or target rate of inflation and then build credibility by actually delivering on its promises. In the Backus-Driffill model, the dry government could be thought of as always announcing a zero target for inflation, but as Cukierman and Liviatan (1991) noted, even central banks that seem committed to low inflation often set positive inflation targets, in part because low inflation is not perfectly credible. That is, if the public expects a positive rate of inflation because the central bank's true intentions are unknown, then even a dry central bank may feel the need to partially accommodate these expectations. Doing otherwise would produce a recession.

To model this type of situation, Cukierman and Liviatan assumed that there are two potential government or central bank types,  $D$  and  $W$ , that differ in their ability to commit. Type  $D$  commits to its announced policy; type  $W$  cannot precommit. In contrast to Backus and Driffill, Cukierman and Liviatan allow their central banks to make announcements, and the  $D$  type is not simply constrained always to maintain a zero rate of inflation. If the public assigns some prior probability to the central bank being type  $W$ , type  $D$ 's announcement will not be fully credible. As a result, a type  $D$  central bank may find it optimal to announce a positive rate of inflation.

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21. In the trigger strategy equilibria, current actions depend on  $\pi_{t-1}$  even though payoffs do not depend directly on lagged inflation.

22. For this to be an equilibrium, the discount factor must be large but not too large. As in standard reputational models, the type  $W$  central bank must place enough weight on the future to be willing to mimic the type  $D$  in order to develop a reputation for low inflation. However, if the future receives too much weight, the type  $W$  will be unwilling to separate, that is, inflate, when the bad shock occurs. See Ball (1995).

To show the effect on inflation of the public's uncertainty about the type of central bank in office, the basic points can be illustrated within the context of a two-period model. To determine the equilibrium behavior of inflation, one needs to solve the model backward by first considering the equilibrium during the last period.

Assume that both central bank types share a utility function that is linear in output and quadratic in inflation, as given by (6.1). With utility linear in output, stabilization will not play a role, so let output be given by (6.3) with  $e \equiv 0$ . In the second period, reputation has no further value, so the type  $W$  central bank will simply set inflation at the optimal discretionary rate  $a\lambda$ . To determine  $D$ 's strategy, however, one needs to consider whether the equilibrium will be a separating, pooling, or mixed-strategy equilibrium. In a separating equilibrium, the behavior of the central bank during the first period reveals its identity; in a pooling equilibrium, both types behave the same way during the first period, so the public will remain uncertain as to the true identity of the bank. A mixed-strategy equilibrium would involve type  $W$  mimicking type  $D$ , with a positive probability less than 1.

Since a separating equilibrium is a bit simpler to construct, that case is considered first. With first-period behavior revealing the bank's type, the public in period 2 knows the identity of the central bank. Since type  $D$  is able to commit, its optimal policy is to announce a zero rate of inflation for period 2. The public, knowing that a type  $D$  is truthful, expects a zero inflation rate, and in equilibrium  $\pi_2^D = 0$ .

In the first period of a separating equilibrium, the public is uncertain about the type of central bank actually in power. Suppose the public assigns an initial probability  $q$  that the central bank is type  $D$ . In a separating equilibrium in which the  $W$  type reveals itself by inflating at a rate that differs from the announced rate, the type  $W$  will choose to inflate at the rate  $a\lambda$  because this value maximizes its utility function.<sup>23</sup> So if the type  $D$  announces  $\pi^a$ , then the public will expect an inflation rate of  $\pi_1^e = q\pi^a + (1 - q)a\lambda$ .<sup>24</sup> The last step to fully characterize the separating equilibrium is to determine the optimal announcement (since the  $D$  type actually inflates at the announced rate and the  $W$  type inflates at the rate  $a\lambda$ ).

If future utility is discounted at the rate  $\beta$ , the utility of the type  $D$  central bank is given by

$$\begin{aligned} U_{\text{sep}}^D &= \lambda(y_1 - y_n) - \frac{1}{2}\pi_1^2 + \beta \left[ \lambda(y_2 - y_n) - \frac{1}{2}\pi_2^2 \right] \\ &= a\lambda(\pi_1 - \pi_1^e) - \frac{1}{2}\pi_1^2, \end{aligned}$$

23. Recall that with the utility function (6.1), the central bank's optimal period 1 inflation rate is independent of the expected rate of inflation.

24. The  $W$  type will also announce the same inflation rate as the type  $D$ , since doing otherwise would immediately raise the public's expectations about first-period inflation and lower type  $W$ 's utility.

since, in period 2,  $y_2 = y_n$  and  $\pi_2^D = 0$ . The type  $D$  picks first-period inflation subject to  $\pi_1 = \pi^a$  and  $\pi_1^e = q\pi^a + (1 - q)a\lambda$ . This yields

$$\pi_1^D = (1 - q)a\lambda \leq a\lambda.$$

The role of credibility is clearly illustrated in this result. If the central bank were known to be of type  $D$ , that is, if  $q = 1$ , it could announce and deliver a zero rate of inflation. The possibility that the central bank might be of type  $W$ , however, forces the  $D$  type to actually announce and deliver a positive rate of inflation. The public's uncertainty leads it to expect a positive rate of inflation; the type  $D$  central bank could announce and deliver a zero rate of inflation, but doing so would create a recession whose cost outweighs the gain from a lower inflation rate.

To summarize, in a separating equilibrium, the type  $W$  inflates at the rate  $a\lambda$  in each period, and the type  $D$  inflates at the rate  $(1 - q)a\lambda$  during the first period and zero during the second period. Since expected inflation in the first period is  $q(1 - q)a\lambda + (1 - q)a\lambda = (1 - q^2)a\lambda$ , which is less than  $a\lambda$  but greater than  $(1 - q)a\lambda$ , output is above  $y_n$  if the central banker is actually type  $W$  and below  $y_n$  if the bank is type  $D$ .

What happens in a pooling equilibrium? A pooling equilibrium requires that the  $W$  type not only make the same first-period announcement as the  $D$  type but also that it pick the same actual inflation rate in period 1 (otherwise, it would reveal itself). In this case, the  $D$  type faces period 2 expectations  $\pi_2^e = q\pi_2^a + (1 - q)a\lambda$ .<sup>25</sup> Since this is just like the problem analyzed for the first period of the separating equilibrium,  $\pi_2^D = \pi_2^a = (1 - q)a\lambda > 0$ . The type  $D$  inflates at a positive rate in period 2, since its announcement lacks complete credibility. In the first period of a pooling equilibrium, however, things are different. In a pooling equilibrium, the  $D$  type knows that the  $W$  type will mimic whatever the  $D$  type does. And the public knows this also, so both types will inflate at the announced rate of inflation and  $\pi_1^e = \pi_1^a$ . In this case, with the announcement fully credible, the  $D$  type will announce and deliver  $\pi_1 = 0$ .

To summarize, in the pooling equilibrium, inflation will equal zero in period 1 and either  $(1 - q)a\lambda$  or  $a\lambda$  in period 2, depending on which type is actually in office. In the separating equilibrium, inflation will equal  $(1 - q)a\lambda$  in period 1 and zero in period 2 if the central bank is of type  $D$ , and  $a\lambda$  in both periods if the central bank is of type  $W$ .

Which equilibrium will occur? If the type  $W$  separates by inflating at the rate  $a\lambda$  during period 1, its utility will be  $a\lambda[a\lambda - (1 - q^2)a\lambda] - \frac{1}{2}(a\lambda)^2 - \beta\frac{1}{2}(a\lambda)^2$ , or

$$U_{\text{sep}}^W = (a\lambda)^2 \left[ q^2 - \frac{1}{2}(1 + \beta) \right].$$

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25. In the pooling equilibrium, first-period outcomes do not reveal any information about the identity of the central bank type, so the public continues to assess the probability of a type  $D$  as equal to  $q$ . This would not be the case if the equilibrium involved the  $W$  type following a mixed strategy in which it inflates in period 1 with probability  $p < 1$ . In a sequential Bayesian equilibrium, the public updates the probability of a  $D$  type on the basis of the period 1 outcomes using Bayes's rule.

If type  $W$  deviates from the separating equilibrium and mimics type  $D$  instead by only inflating at the rate  $(1 - q)a\lambda$  during period 1, it will achieve a utility payoff of  $a\lambda[(1 - q)a\lambda - (1 - q^2)a\lambda] - \frac{1}{2}[(1 - q)a\lambda]^2 + \beta a\lambda(a\lambda - 0) - \beta \frac{1}{2}(a\lambda)^2$ , or

$$U_m^W = \frac{1}{2}(a\lambda)^2(q^2 - 1 + \beta),$$

because mimicking fools the public into expecting zero inflation in period 2. Type  $W$  will separate if and only if  $U_{\text{sep}}^W > U_m^W$ , which occurs when

$$\beta < q^2/2 \equiv \underline{\beta}. \quad (6.16)$$

Thus, the separating equilibrium occurs if the public places a high initial probability on the central bank's being type  $D$  ( $q$  is large). In this case, type  $D$  is able to set a low first-period rate of inflation and the  $W$  type does not find it worthwhile to mimic. Only if the type  $W$  places a large weight on being able to engineer a surprise inflation in period 2 (i.e.,  $\beta$  is large) would deviating from the separating equilibrium be profitable.<sup>26</sup>

Suppose  $\beta \geq \underline{\beta}$ ; will pooling emerge? Not necessarily. If the type  $W$  pools, its utility payoff will be

$$a\lambda[0] - \frac{1}{2}[0]^2 + \beta a\lambda[a\lambda - \pi_2^e] - \beta \frac{1}{2}(a\lambda)^2$$

or, since  $\pi_2^e = q\pi_2^a + (1 - q)a\lambda = (1 - q^2)a\lambda$ ,

$$U_p^W = \beta(a\lambda)^2 \left( q^2 - \frac{1}{2} \right).$$

If the type  $W$  deviates from the pooling equilibrium, it will generate an output expansion in period 1, but by revealing its identity, period 2 inflation is fully anticipated and output equals  $y_n$ . Thus, deviating gives the type  $W$  a payoff of  $a\lambda[a\lambda] - \frac{1}{2}[a\lambda]^2 + \beta a\lambda[0] - \beta \frac{1}{2}[a\lambda]^2$ , or

$$U_{\text{dev}}^W = \frac{1}{2}(a\lambda)^2(1 - \beta).$$

By comparing the incentive for  $W$  to deviate from a pooling equilibrium, the pooling outcome is an equilibrium whenever

$$\beta > \frac{1}{2q^2} \equiv \bar{\beta} \quad (6.17)$$

because in this case  $U_p^W > U_{\text{dev}}^W$ . If  $\beta$  is large enough, meaning  $\beta > 1/(2q^2)$ , type  $W$  places enough weight on the future that it is willing to forgo the temptation to inflate immediately,

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26. Walsh (2000) showed that a separating equilibrium is less likely if inflation is determined by the type of forward-looking new Keynesian Phillips curves discussed in chapters 7 and 8. When current inflation depends on expected future inflation, a type  $W$  whose identity is revealed in the first period suffers an immediate rise in inflation as expected future inflation rises.



and zero inflation is the equilibrium in period 1. Of course, in period 2, there is no further value in maintaining a reputation, so type  $W$  inflates at the rate  $a\lambda$ . Equation (6.17) shows that the critical cutoff value for  $\beta$  depends on  $q$ , the prior probability the public assigns to a type  $D$  setting policy. A larger  $q$  makes pooling an equilibrium for more values of  $\beta$ , so that even a less patient type  $W$  will find it advantageous not to deviate from the pooling equilibrium. If  $q$  is large, then the public thinks it likely that the central bank is a type  $D$ . This leads them to expect low inflation in period 2, so the output gains of inflating at the rate  $a\lambda$  will be large. By pooling during period 1, a type  $W$  can then benefit from causing a large expansion in period 2. If the type  $W$  deviates and reveals its type during period 1, the first-period output gain is independent of  $q$ .<sup>27</sup> So a rise in  $q$  leaves the period 1 advantage of deviating unchanged while increasing the gain from waiting until period 2 to inflate.

Comparing (6.16) and (6.17) shows that  $\underline{\beta} < \bar{\beta}$ , so there will be a range of values for the discount factor for which neither a separating nor a pooling outcome is an equilibrium. For  $\beta$  in this range, there are mixed-strategy equilibria (see Cukierman and Liviatan 1991).

This model reveals how public uncertainty about the intentions of the central bank affects the equilibrium inflation rate. In both the separating equilibrium and the mixed-strategy equilibrium, the type  $D$  central bank inflates in the first period even though it is (by assumption) capable of commitment and always delivers on its announcements.

The formulation of Cukierman and Liviatan provides a nice illustration of the role that announcements can play in influencing the conduct of policy. It also illustrates why central banks might be required to make announcements about their inflation plans. The type  $D$  central bank is clearly better off making announcements; as long as  $q > 0$ , making an announcement allows the type  $D$  to influence expectations and reduce the first-period inflation rate (this occurs in separating and pooling equilibria and also in mixed-strategy equilibria). Even when there may be incentives to manipulate announcements, they can constrain the subsequent conduct of policy. They may also convey information about the economy if the central bank has private and unverifiable information such as its own internal forecast of economic conditions.<sup>28</sup>

### 6.3.2 Preferences

An alternative approach to solving the inflationary bias of discretion focuses directly on the preferences of the central bank. This branch of the literature has closer connections with the extensive empirical work that has found, at least for the industrialized economies, that average inflation rates across countries are negatively correlated with measures of the

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27. This is because expected inflation equals zero during the first period of a pooling equilibrium. Consequently, the output expansion of inflating at the rate  $a\lambda$  is  $a(a\lambda - 0) = a^2\lambda$ , which is independent of  $q$ .

28. See Persson and Tabellini (1993); Muscatelli (1999); and Walsh (1999).

degree to which a central bank is independent of the political authorities.<sup>29</sup> If the central bank is independent, then one can begin to think of the preferences of the central bank as differing from those of the elected government. And if they can differ, then one can ask how they might differ and how the government, through its appointment process, might influence the preferences of the central bank(er).

Rogoff (1985) was the first to analyze explicitly the issue of the optimal preferences of the central banker.<sup>30</sup> He did so in terms of the relative weight the central banker places on the inflation objective. In the objective function (6.2),  $\lambda$  measures the weight on output relative to a weight normalized to 1 on inflation objectives. Rogoff concluded that the government should appoint as central banker someone who places greater relative weight on the inflation objective than does society (the government) as a whole. That is, the central banker should have preferences that are of the form given by (6.2) but with a weight on inflation of  $1 + \delta > 1$ . Rogoff characterized such a central banker as more *conservative* than society as a whole. This is usefully described as *weight conservatism* (Svensson 1997b) because there are other interpretations of conservatism; for example, the central bank might have a target inflation rate that is lower than that of the government. In most of the literature, however, *conservative* is interpreted in terms of the weight placed on inflation objectives relative to output objectives.

The intuition behind Rogoff's result is easily understood by referring back to (6.7), which showed the inflation rate under discretion for the quadratic loss function (6.2). If the central banker conducting monetary policy has a loss function that differs from (6.2) only by placing weight  $1 + \delta$  on inflation rather than 1, then inflation under discretion will equal

$$\pi^d(\delta) = \Delta m + v = \frac{a\lambda k}{1 + \delta} - \left( \frac{a\lambda}{1 + \delta + a^2\lambda} \right) e + v. \quad (6.18)$$

The equilibrium inflation rate is a function  $\delta$ . Two effects are at work. First, the average inflation bias is reduced, because  $1 + \delta > 1$ . This tends to reduce the social loss function (the loss function with weight 1 on inflation and  $\lambda$  on output). But the coefficient on the aggregate supply shock is also reduced; stabilization policy is distorted, and the central bank responds too little to  $e$ . As a consequence, output fluctuates more than is socially optimal in response to supply shocks. The first effect (lower average inflation) makes it

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29. The empirical literature on central bank independence and inflation and other macroeconomic outcomes is large. See Cukierman (1992) for an excellent treatment. Carlstrom and Fuerst (2009) argued that central bank independence accounts for two-thirds of the better inflation performance among industrialized economies over the past 20 years. I surveyed this literature in previous editions (see section 8.5 of the second edition). That material is available at <http://people.ucsc.edu/~walshc/mtp4e/>.

30. Interestingly, Barro and Gordon (1983a) recognized that outcomes could be improved under discretion by distorting the central banker's preferences so that "there is a divergence in preferences between the principal (society) and its agent (the policymaker)" (607, n. 19). This insight is also relevant for the contracting approach (see section 6.3.3).

optimal to appoint a central banker who places more weight on inflation than does society; this is usually interpreted to mean that society should appoint a conservative to head the central bank. But the second effect (less output stabilization) limits how conservative the central banker should be.

Using (6.18), one can evaluate the government's loss function  $V$  as a function of  $\delta$ . By then minimizing the government's expected loss function with respect to  $\delta$ , one can find the *optimal preferences* for a central banker. The expected value of the government's objective function is

$$\begin{aligned} E[V] &= \frac{1}{2} E \left( \lambda \left\{ a \left[ \pi^d(\delta) - \pi^e \right] + e - k \right\}^2 + \left[ \pi^d(\delta) \right]^2 \right) \\ &= \frac{1}{2} \left[ \lambda k^2 + \lambda \left( \frac{1 + \delta}{1 + \delta + a^2 \lambda} \right)^2 \sigma_e^2 + a^2 \lambda \sigma_v^2 \right] \\ &\quad + \frac{1}{2} \left[ \left( \frac{a \lambda k}{1 + \delta} \right)^2 + \left( \frac{a \lambda}{1 + \delta + a^2 \lambda} \right)^2 \sigma_e^2 + \sigma_v^2 \right], \end{aligned}$$

where (6.18) is used to replace  $\pi^e$  with  $a \lambda k / (1 + \delta)$  under the assumption that the public knows  $\delta$  when forming its expectations. Minimizing this expression with respect to  $\delta$  yields, after some manipulation, the following condition that must be satisfied by the optimal value of  $\delta$ :

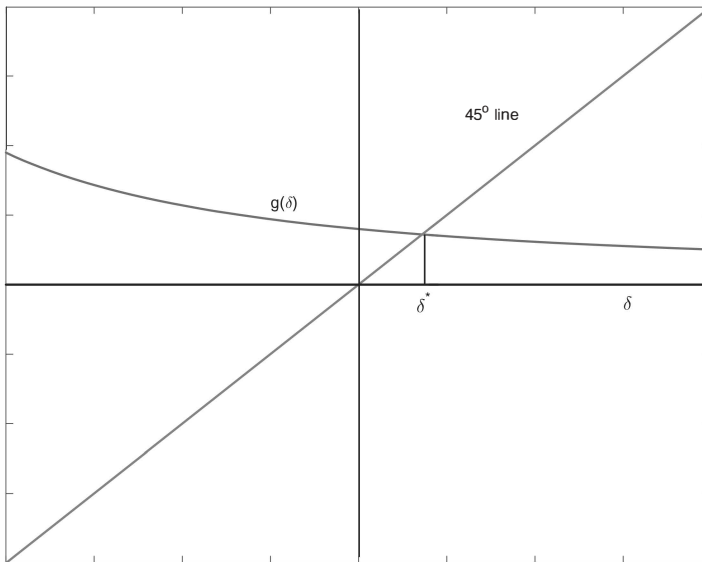
$$\delta = \left( \frac{k^2}{\sigma_e^2} \right) \left( \frac{1 + \delta + a^2 \lambda}{1 + \delta} \right)^3 \equiv g(\delta). \quad (6.19)$$

The function  $g(\delta)$  is shown in figure 6.4.<sup>31</sup> Equation (6.19) is satisfied where  $g(\delta)$  crosses the 45° line. Since  $g(0) > 0$  and  $\lim_{\delta \rightarrow \infty} g(\delta) = k^2 / \sigma_e^2 > 0$ , the intersection always occurs in the range  $\delta \in (0, \infty]$ ; given the trade-off between distorting the response of policy to aggregate supply shocks and reducing the average inflation bias, it is always optimal to appoint a central banker who places more weight ( $\delta > 0$ ) on inflation objectives than the government itself does.

Rogoff's solution is often characterized as involving the appointment of a conservative to head an independent central bank. The concept of *independence* means that, once appointed, the central banker is able to set policy without interference or restriction and will do so to minimize his or her own assessment of social costs. Thus, the inflation bias problem is solved partly through delegation; the government delegates responsibility for monetary policy to an independent central bank. The benefit of this independence is lower average inflation; the cost depends on the realization of the aggregate supply shock. If

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31. See Eijffinger, Hoerichts, and Schaling (1995) for a discussion of this graphical representation of the determinants of the optimal degree of conservatism. Eijffinger and Schaling (1995) extended the framework to an open-economy context.



**Figure 6.4**

The optimal degree of conservatism.

shocks are small, the gain in terms of low inflation clearly dominates the distortion in stabilization policy; if shocks are large, the costs associated with the stabilization distortion can dominate the gain from low inflation.<sup>32</sup>

Lohmann (1992) showed that the government can do even better if it appoints a weight-conservative central banker but limits the central bank's independence. If the aggregate supply shock turns out to be too large, the government overrides the central banker, where the critical size determining what is too large is determined endogenously as a function of the costs of overriding. The knowledge that the government can override also affects the way the central banker responds to shocks that are less than the threshold level that triggers an override. By responding more actively to large shocks, the central banker is able to extend the range of shocks over which independence is maintained.

Rogoff's solution highlights a trade-off: one can reduce the bias but only at the cost of distorting stabilization policy. One implication is that countries with central banks that place a high weight on inflation objectives should have, on average, lower inflation, but they should also experience greater output variance. The variance of output is equal to

$$\left( \frac{1 + \delta}{1 + \delta + a^2 \lambda} \right)^2 \sigma_e^2 + a^2 \sigma_v^2,$$

32. Since society is better off appointing a conservative, the expected gain from low inflation exceeds the expected stabilization cost, however.

and this is increasing in  $\delta$ . Highly independent central banks are presumed to place more weight on achieving low inflation, and a large literature has investigated the finding that measures of central bank independence are negatively correlated with average inflation, at least for the industrialized economies (see Cukierman 1992; Eijffinger and de Haan 1996; Carlstrom and Fuerst 2009). Alesina and Summers (1993) showed, however, that such measures do not appear to be correlated with the variance of real output. This runs counter to the implications of the Rogoff model.

Solving the inflationary bias of discretionary policy through the appointment of a conservative central banker raises several issues. First, how does the government identify the preference parameter  $\delta$ ? Second, how does it commit to a  $\delta$ ? Once expectations are set, the government has an incentive to fire the conservative central banker and appoint a replacement who shares the government's preferences. Finally, the focus on preferences rather than incentives clouds the model's implications for institutional structure and design. Should institutions be designed to generate appropriate incentives for policymakers? Or does good policy simply require putting the right people in charge?

### 6.3.3 Contracts

The problems that occur under discretion arise because central banks respond optimally to the incentive structure they face, but the incentives are wrong. This perspective suggests that rather than relying on the central banks having the right preferences, one might try to affect the incentives the central banks face. But this requires first determining what incentives central banks *should* face.

The appropriate perspective for addressing such issues is provided by the principal agent literature.<sup>33</sup> A key insight that motivated the large literature expanding on the analysis of the time inconsistency of optimal plans was the recognition that central banks respond to the incentives they face. These incentives may be shaped by the institutional structure within which policy is conducted. For example, as has been noted, Lohmann showed how policy is affected when the central banker knows the government will override the central bank if the economy is subject to a disturbance that is too large. Rogoff (1985) argued that targeting rules might be enforced by making the monetary authority's budget depend on adherence to the rule. In a similar vein, Garfinkel and Oh (1993) suggested that a targeting rule might be enforced by legislation punishing the monetary authority if it fails to achieve the target. Such institutional aspects of the central bank's structure and its relationship with the government can be thought of as representing a *contract* between the government and the monetary authority. The conduct of monetary policy is then affected by the contract the government offers to the central bank.

The government's (or perhaps society's) problem can be viewed as that of designing an optimal incentive structure for the central bank. Following Walsh (1995b), the

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33. This section draws heavily on Walsh (1995b). See also Persson and Tabellini (1993) and Waller (1995).

most convenient way to determine an optimal incentive structure is to assume that the government can offer the head of the central bank a state-contingent wage contract. Such a contract allows one to derive explicitly the manner in which the bank's incentives should depend on the state of the economy. While there are numerous reasons to question the effectiveness and implementability of such employment contracts in the context of monetary policy, a (possibly) state-contingent wage contract for the central banker represents a useful fiction for deriving the optimal incentive structure the central banker should face and provides a convenient starting point for the analysis of optimal central bank incentives.<sup>34</sup>

The basic structure of the model is identical to that used earlier, consisting of an aggregate supply relationship given by (6.3), a link between money growth and inflation given by (6.4), and an objective function that depends on output fluctuations and inflation variability, as in (6.2). The private sector's expectations are assumed to be determined prior to the central bank's choice of a growth rate for the nominal money supply. Thus, in setting  $\Delta m$ , the central bank will take  $\pi^e$  as given. Assume the central bank can observe the supply shock  $e$  prior to setting  $\Delta m$ ; this generates a role for stabilization policy. The disturbance  $v$  in the link between money growth and inflation is realized after the central bank sets  $\Delta m$ . Finally, assume that  $e$  and  $v$  are uncorrelated.

Monetary policy is conducted by an independent central bank, one that shares the government's preferences,  $V$ , but that also receives a monetary transfer payment from the government. This payment can be thought of either as the direct income of the central banker or as the budget of the central bank. Or the transfer payment can be viewed more broadly as reflecting legislated performance objectives for the central bank. Let  $t$  represent the transfer to the central bank, and assume that the central bank's utility is given by

$$U = t - V.$$

That is, the central bank cares about both the transfer it receives and the social loss generated by inflation and output fluctuations. The central bank sets  $\Delta m$  to maximize the expected value of  $U$ , conditional on the realization of  $e$ . The problem faced by the government (the principal) is to design a transfer function  $t$  that induces the central bank to choose  $\Delta m = \Delta m^c(e)$ , where  $\Delta m^c$  is the socially optimal commitment policy. As already noted, the optimal commitment policy in this framework is  $\Delta m^c(e) = -a\lambda e / (1 + a^2\lambda)$  (see 6.11).

If the government can verify  $e$  ex post, there are clearly many contracts that would achieve the desired result. For example, any contract that imposes a large penalty on the central bank if  $\Delta m$  deviates from  $\Delta m^c$  will ensure that  $\Delta m^c$  is chosen. However, the difficulty of determining both the possible states of nature ex ante and the actual realization of shocks ex post makes such contracts infeasible. This task is particularly difficult

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34. Walsh (2002) demonstrated that a dismissal rule can, in some circumstances, substitute for a state-contingent wage contract in affecting the central bank's incentives.

if the central bank must respond to a forecast of  $e$ , as its internal forecast might be difficult to verify ex post, leading to the problems of private information highlighted by the analysis of Canzoneri (1985). Therefore, consider a transfer function  $t(\pi)$  that makes the government's payment to the central bank contingent on the observed rate of inflation. The transfer function implements the optimal policy  $\pi^c(e) = \Delta m^c(e) + v$  if  $\pi^c$  maximizes  $E^{cb} [t(\pi(e)) - V]$  for all  $e$ , where  $E^{cb} [ \ ]$  denotes the central bank's expectation conditional on  $e$ .

The first-order condition for the central banker's problem can be solved for  $\Delta m^{cb}(e)$ , the optimal discretionary policy:

$$\Delta m^{cb}(e) = \frac{a\lambda k}{1 + a^2\lambda} + \left( \frac{a^2\lambda}{1 + a^2\lambda} \right) \pi^e + \frac{E^{cb}(t')}{1 + a^2\lambda} - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e, \quad (6.20)$$

where  $t' = \partial t(\pi)/\partial \pi$ . The last term in (6.20) shows that the optimal discretionary policy response to the supply shock is equal to the response under the optimal commitment policy  $\Delta m^c$ . This is important because it implies that the government's objective will be to design a contract that eliminates the inflationary bias while leaving the central bank free to respond with discretion to  $e$ . Taking expectations of (6.20) and letting  $E[ \ ]$  denote the public's expectation, one obtains

$$E \left[ \Delta m^{cb}(e) \right] = \pi^e = a\lambda k + E \left[ t'(\pi) \right].$$

When this is substituted back into (6.20), one obtains

$$\Delta m^{cb}(e) = a\lambda k + E \left[ t'(\pi(e)) \right] - \frac{E \left[ t' \right] - E^{cb} \left[ t' \right]}{1 + a^2\lambda} - \frac{a\lambda}{1 + a^2\lambda} e.$$

Setting  $\Delta m^{cb}(e)$  equal to the optimal commitment policy  $\Delta m^c(e)$  for all  $e$  requires that the first three terms vanish. They will vanish if  $t(\pi)$  satisfies

$$t' = \frac{\partial t}{\partial \pi} = -a\lambda k.$$

The optimal commitment policy can therefore be implemented by the linear transfer function

$$t = t_0 - a\lambda k\pi.$$

The constant  $t_0$  is set to ensure that the expected return to the central banker is sufficient to ensure participation.<sup>35</sup> Presenting the central banker with this incentive contract achieves the dual objectives of eliminating the inflationary bias while still ensuring optimal

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35. This is known as the *individual rationality constraint*. Since  $\partial \pi / \partial m = 1$ , a contract of the form  $t_0 - akm$  based on the observed rate of money growth would also work. Chortareas and Miller (2007) analyzed the case in which the government also cares about the cost of the contract.

stabilization policy in response to the central bank's private information about the aggregate supply shock.

Why does the transfer function take such a simple linear form? Recall that the time-consistent policy under discretion resulted in an inflationary bias of  $a\lambda k$ . The key insight is that this is constant; it does not vary with the realization of the aggregate supply shock. Therefore, the incentive structure for the central bank just needs to raise the marginal cost of inflation (from the perspective of the central bank) by a constant amount; that is what the linear transfer function does. Because the bias is independent of the realization of the underlying state of nature, it is not necessary for the government to actually verify the state, and so the presence of private information about the state of the economy on the part of the central bank does not affect the ability of the linear contract to support the optimal policy. This case contrasts sharply with the one in which reputation is relied upon to achieve low average inflation (Canzoneri 1985).

One interpretation of the linear inflation contract result is that it simply points out that the Barro-Gordon framework is too simple to adequately capture important aspects of monetary policy design. In this view, there really is a trade-off between credibility and flexibility, and the fact that this trade-off can be made to disappear so easily represents a methodological criticism of the Barro-Gordon model.<sup>36</sup> Several authors have explored modifications to the Barro-Gordon model that allow this trade-off to be reintroduced. They do so by making the inflation bias state-dependent. In this way, the linear contract, which raises the marginal cost of inflation by a constant amount for all state realizations, cannot achieve the socially optimal commitment policy. If the penalty cannot be made state-contingent, then average inflation can be eliminated, but inflation will remain too volatile. For example, Walsh (1995a), Canzoneri, Nolan, and Yates (1997), and Herrendorf and Lockwood (1997) introduced a state-contingent bias by modifying the basic model structure. Walsh assumed there exists a flexible wage sector in addition to a nominal wage contract sector. Herrendorf and Lockwood assumed labor market participants can observe a signal that reveals information about aggregate supply shocks prior to forming nominal wage contracts. Canzoneri, Nolan, and Yates assumed the central bank has an interest rate-smoothing objective. Herrendorf and Lockwood (1997) and Muscatelli (1998) showed that when the inflation bias is state-contingent rather than constant, there can be a role for a linear inflation contract, as in Walsh (1995b), and a conservative central banker, as in Rogoff (1985). Schellekens (2002) considered delegation to a central bank with preferences that are generalized from the standard quadratic form. He examined the connection between optimal conservatism and cautionary policy arising from model uncertainty.

Chortareas and Miller (2003) showed that the linear inflation contract would not fully offset the inflation bias when the government cares about the cost of the contract, a cost

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36. This argument is made by Canzoneri, Nolan, and Yates (1997).



that was ignored by Walsh (1995b).<sup>37</sup> However, as Chortareas and Miller (2007) demonstrate, the original linear inflation contract remains optimal if the government must ensure the central banker's participation constraint is satisfied, even when the government also cares about the costs of the contract. Intuitively, with a linear inflation contract of the form  $a + b\pi$ , the government can always set  $b$  to generate the correct incentives for the central bank (because the value of the constant term  $a$  does not alter the first-order conditions of the central banker's optimality conditions). Then  $a$  can be set to minimize the cost of the contract to the government, where this minimum cost is determined by the central banker's outside opportunity.

The contracting approach was developed further in Persson and Tabellini (1993). Walsh (1995a; 2002) showed how the properties of a linear inflation contract can be mimicked by a dismissal rule under which the central banker is fired if inflation ever rises above a critical level. Lockwood (1997), Jonsson (1995; 1997), and Svensson (1997b) showed how linear inflation contracts are affected when the inflation bias is time-dependent because of persistence in the unemployment process. *Persistence* means that a surprise expansion in period  $t$  reduces unemployment (increases output) in period  $t$  but also leads to lower expected unemployment in periods  $t + 1$ ,  $t + 2$ , and so on. Thus, the benefits of a surprise inflation are larger, leading to a higher average inflation rate under discretionary policy. The bias at time  $t$ , though, will depend on the unemployment rate at  $t - 1$ , because, with persistence, unemployment at  $t - 1$  affects the average unemployment rate expected for period  $t$ . Therefore, the inflation bias will be time-varying. The simple linear contract with a fixed weight on inflation will no longer be optimal if the inflation bias is state-dependent. However, a state-contingent contract can support the optimal commitment policy.

Like Rogoff's conservative central banker solution, the contracting solution *relocates* the commitment problem that gives rise to the inflation bias in the first place.<sup>38</sup> Jensen (1997) showed how the ability of an incentive contract for the central banker to solve an inflation bias is weakened when the government can undo the contract *ex post*. In the case of the conservative central banker, the proposed solution assumes that the government cannot commit to a specific inflation policy but can commit to the appointment of an agent with specific preferences. In the contracting case, the government is assumed to be able to commit to a specific contract. Both of these assumptions are plausible; relocating the commitment problem is often a means of solving the problem. Confirmation processes, together with long terms of office, can reveal the appointee's preferences and ensure that policy is actually conducted by the appointed agent. Incentives called for in the contracting approach can similarly be thought of as aspects of the institutional structure and may therefore be more difficult to change than actual policy instrument settings.

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37. See also Candel-Sanchez and Campoy-Minarroy (2004).

38. McCallum (1995; 1997a) emphasized the relocation issue with respect to the contracting approach. A similar criticism applies to the conservative central banker solution.

As al-Nowaihi and Levine (1996) argued, relocation can allow the government to commit credibly to a contract or to a particular appointee if the process is public. If contract renegotiations or the firing of the central banker are publicly observable, then it may be in the interest of the government to forgo any short-term incentive to renegotiate in order to develop a reputation as a government that can commit. Thus, the transparency of any renegotiation serves to support a low-inflation equilibrium; relocating the time-inconsistency problem can solve it.<sup>39</sup>

The type of policy transparency emphasized by al-Nowaihi and Levine characterizes the policy process established under the 1989 central banking reform in New Zealand. There the government and the Reserve Bank establish short-run inflation targets under a Policy Targets Agreement (PTA). The PTA can be renegotiated, and once current economic disturbances have been observed, both the government and the Reserve Bank have incentives to renegotiate the target (Walsh 1995a). Because this renegotiation must be public, however, reputational considerations may sustain an equilibrium in which the targets are not renegotiated. Svensson (1997b) showed that publicly assigning an inflation target to the central bank may also replicate the optimal incentives called for under the linear inflation contract.

Dixit and Lambertini (2001) extended the contracting approach to the case of a monetary union in which member governments offer the common central bank incentive contracts designed to influence monetary policy. They showed that if the central bank cares about the incentives it receives and about the unionwide inflation rate, the central bank implements a policy that leads to average inflation that is too low and stable. The central bank implements a weighted average of each member country's desired policy only if the central bank cares only about the contract incentives. Hence, mandating that the central bank achieve price stability would result in a deflationary bias under discretion.

Athey, Atkeson, and Kehoe (2005) reexamined the optimal delegation of monetary policy by employing mechanism design theory in an environment similar to the one studied originally by Canzoneri (1985), in which the central bank has private information. They showed that under certain conditions, the optimal scheme involves an inflation cap—a maximum inflation rate the central bank is allowed to choose. The greater the time-inconsistency problem, the lower is the cap. If the central bank has little private information, then the optimal design calls for giving no discretion to the central bank.

### 6.3.4 Institutions

One interpretation of the contracting approach is that the incentive structures might be embedded in the institutional structure of the central bank. If institutions are costly to

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39. See also Herrendorf (1998; 1999), who developed a similar point using inflation targeting, and Walsh (2002), who showed that the government will find it advantageous to carry out a dismissal rule policy under which the central banker is fired if inflation exceeds a critical level.

change, then institutional reforms designed to raise the costs of inflation can serve as commitment devices. Incorporating a price stability objective directly into the central bank's charter legislation, for example, might raise the implicit penalty (in terms of institutional embarrassment) the central bank would suffer if it failed to control inflation. Most discussions of the role of institutional structure and inflation have, however, focused on the effects of alternative structures on the extent to which political pressures affect the conduct of monetary policy.

A starting point for such a focus is Alesina's model of policy in a two-party system.<sup>40</sup> Suppose there is uncertainty about the outcome of an approaching election, and suppose the parties differ in their economic policies, so that inflation in the postelection period will depend on which party wins the election. Let the parties be denoted *A* and *B*. The inflation rate expected if party *A* wins the election is  $\pi^A$ ; inflation under party *B* will be  $\pi^B$ . Assume  $\pi^A > \pi^B$ . If the probability that party *A* wins the election is  $q$ , then expected inflation prior to the election will be  $\pi^e = q\pi^A + (1 - q)\pi^B$ . Since  $q$  is between 0 and 1, expected inflation falls in the interval  $[\pi^B, \pi^A]$ . If postelection output is equal to  $y = a(\pi - \pi^e)$ , where  $\pi$  is actual inflation, then the election of party *A* will generate an economic expansion (because  $\pi^A - \pi^e = (1 - q)(\pi^A - \pi^B) > 0$ ), whereas the election of party *B* will lead to an economic contraction ( $\pi^B - \pi^e = q(\pi^B - \pi^A) < 0$ ).

This very simple framework provides an explanation for a political business cycle that arises because of policy differences between parties and electoral uncertainty. Because parties are assumed to exploit monetary policy to get their desired inflation rate, and because election outcomes cannot be predicted with certainty, inflation surprises will occur after an election. Alesina and Sachs (1988) provided evidence for this theory based on U.S. data, and Alesina and Roubini (1992) examined OECD countries. Faust and Irons (1999), however, concluded there was little evidence from the United States to support the hypothesis that political effects generate monetary policy surprises.

Waller (1989; 1992) showed how the process used to appoint members of the central bank's policy board can influence the degree to which partisan political factors are translated into monetary policy outcomes. If policy is set by a board whose members serve overlapping but noncoincident terms, the effect of policy shifts resulting from changes in government is reduced. In a two-party system in which nominees forwarded by the party in power are subject to confirmation by the out-of-power party, the party in power will nominate increasingly moderate candidates as elections near. Increasing the length of terms of office for central bank board members also reduces the role of partisanship in monetary policymaking.<sup>41</sup> Waller and Walsh (1996) considered a partisan model of monetary

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40. For a discussion of this model, see Alesina (1987; 1989); Alesina and Sachs (1988); Alesina and Roubini (1992); Alesina, Roubini, with Cohen (1997); and Drazen (2000).

41. See also Havrilesky and Gildea (1992) and Garcia de Paso (1994). For some empirical evidence in support of these models, see Mixon Jr. and Gibson (2002).

policy. They focused on the implications for output of the degree of partisanship in the appointment process and the term length of the central banker. Similarly, Alesina and Gatti (1995) showed that electorally induced business cycles can be reduced if political parties jointly appoint the central banker.

While most work has focused on the appointment of political nominees to the policy board, the Federal Reserve's policy board (the FOMC) includes both political appointees (the governors) and nonappointed members (the regional bank presidents).<sup>42</sup> Faust (1996) provided an explanation for this structure by developing an overlapping-generations model in which inflation has distributional effects. If monetary policy is set by majority vote, excessive inflation results as the (larger) young generation attempts to transfer wealth from the old generation. If policy is delegated to a board consisting of one representative from the young generation and one from the old, the inflationary bias is eliminated. Faust argued that the structure of the FOMC takes its shape because of the advantages of delegating to a board in which the relative balance of different political constituencies differs from that of the voting public as a whole.

Who makes policy and who appoints the policymakers can affect policy outcomes, but institutional design also includes mechanisms for accountability, and these can affect policy as well. Minford (1995), in fact, argued that democratic elections can enforce low-inflation outcomes if voters punish governments that succumb to the temptation to inflate, and Lippi (1997) developed a model in which rational voters choose a weight-conservative central banker. O'Flaherty (1990) showed how finite term lengths can ensure accountability, and Walsh (1995a) showed that the type of dismissal rule incorporated into New Zealand's Reserve Bank Act of 1989 can partially mimic an optimal contract. Walsh (2015) evaluated the use of a policy goal such as inflation as a means of measuring the central bank's performance and ensuring policy accountability versus the use of deviations of the central bank's instrument from a simple rule as a method for assessing performance.

The launch of the European Central Bank in 2000 helped to focus attention on the role institutions and their formal structure play in affecting policy outcomes. Because the individual member countries in a monetary union may face different economic conditions, disagreements about the common central bank's policies may arise. Dixit (2000) used a principal agent approach to study policy determination in a monetary union. With a single central bank determining monetary policy for a union of countries, the central bank is the agent of many principals. Each principal may try to influence policy outcomes, and the central bank may need to appease its principals to avoid noncooperative outcomes.

Dixit showed that the central bank's decision problem must take into account the individual incentive compatibility constraints that require all principals to accept a continuation

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42. Havrilesky and Gildea (1991; 1995) argued that the voting behavior of regional bank presidents and board governors differs, with regional bank presidents tending to be tougher on inflation; this conclusion is disputed by Tootell (1991).

of the policy the central bank chooses. For example, if one country has a large adverse shock, the central bank may have to raise inflation above the optimal commitment level to ensure the continued participation in the union of the affected country. When the incentive constraint binds, policy will diverge from the full commitment case in order to secure the continued participation of the union members. Dixit showed that when countries are hit by different shocks, it is the incentive constraint of the worst-hit country that is binding; policy must shade toward what that country wants. If the costs of overturning the central bank's policy (and thereby reverting to the discretionary equilibrium) are high enough, there will be some range of asymmetric shocks within which it is possible to sustain the full commitment policy.

### 6.3.5 Targeting Rules

The contracting approach focuses on the incentive structure faced by the central bank; once the incentives are correct, complete flexibility in the actual conduct of policy is allowed. This allows the central bank to respond to new and possibly unverifiable information. An alternative approach acts to reduce the problems arising from discretion by *restricting* policy flexibility. The gold standard or a fixed exchange rate regime provides examples of situations in which policy flexibility is deliberately limited; Milton Friedman's proposal that the Fed be required to maintain a constant growth rate of the money supply is another famous example. A wide variety of rules designed to restrict the flexibility of the central bank have been proposed and analyzed. The cost of reduced flexibility depends on the nature of the economic disturbances affecting the economy and the original scope for stabilization policies in the first place, and the gain from reducing flexibility takes the form of a lower average inflation rate.

Targeting rules are rules under which the central bank is judged in part on its ability to achieve a prespecified value for some macroeconomic variable. Inflation targeting is currently the most commonly discussed form of targeting, and some form of inflation targeting has been adopted in over 30 developed and developing economies.<sup>43</sup> Fixed or target zone exchange rate systems also can be interpreted as targeting regimes. The central bank's ability to respond to economic disturbances, or to succumb to the temptation to inflate, is limited by the need to maintain an exchange rate target. When the lack of credibility is a problem for the central bank, committing to maintaining a fixed nominal exchange rate against a low-inflation country can serve to import credibility. Giavazzi and Pagano (1988) provided an analysis of the advantages of "tying one's hands" by committing to a fixed exchange rate.

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43. See Ammer and Freeman (1995); Haldane (1995); McCallum (1997b); Mishkin and Posen (1997); Bernanke et al. (1998); and the papers in Leiderman and Svensson (1995) and Lowe (1997) for earlier discussions of inflation targeting. Walsh (2009) provided an extensive list of references on the topic. Most analyses of inflation targeting have been done using the new Keynesian model (see section 8.4.6). Chapter 11 discusses alternative targeting regimes when the central bank's nominal interest rate instrument is at zero.

### Flexible Targeting Rules

Suppose the central bank cares about output and inflation stabilization but is, in addition, penalized for deviations of actual inflation from a target level.<sup>44</sup> In other words, the central bank's objective is to minimize

$$V^{cb} = \frac{1}{2}\lambda E_t(y_t - y_n - k)^2 + \frac{1}{2}E_t(\pi_t - \pi^*)^2 + \frac{1}{2}hE_t(\pi_t - \pi^T)^2, \quad (6.21)$$

where this differs from (6.2) in that  $\pi^*$  now denotes the socially optimal inflation rate (which may differ from zero), and the last term represents the penalty related to deviations from the target inflation rate  $\pi^T$ . The parameter  $h$  measures the weight placed on deviations from the target inflation rate. Targeting rules of this form are known as *flexible targeting rules*. They do not require that the central bank hit its target exactly; instead, one can view the last term as representing a penalty suffered by the central bank based on how large the deviation from the target turns out to be. This type of targeting rule allows the central bank to trade off achieving its inflation target for achieving more desired values of its other goals.

The rest of the model consists of an aggregate supply function and a link between the policy instrument, the growth rate of money, and inflation:

$$y_t = y_n + a(\pi_t - \pi^e) + e_t,$$

$$\pi_t = \Delta m_t + v_t,$$

where  $v$  is a velocity disturbance. It is assumed that the public's expectations are formed prior to observing either  $e$  or  $v$ , but the central bank can observe  $e$  (but not  $v$ ) before setting  $\Delta m$ .

Before deriving the policy followed by the central bank, note that the socially optimal commitment policy is given by<sup>45</sup>

$$\Delta m_t^S = \pi^* - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e_t. \quad (6.22)$$

Now consider policy under discretion. Using the aggregate supply function and the link between inflation and money growth, the loss function (6.21) can be written as

$$V^{cb} = \frac{1}{2}\lambda E \left[ a(\Delta m + v - \pi^e) + e - k \right]^2 + \frac{1}{2}E(\Delta m + v - \pi^*)^2 + \frac{1}{2}hE(\Delta m + v - \pi^T)^2.$$

The first-order condition for the optimal choice of  $\Delta m$ , taking expectations as given, is

$$a^2\lambda(\Delta m - \pi^e) + a\lambda(e - k) + (\Delta m - \pi^*) + h(\Delta m - \pi^T) = 0.$$

44. The central bank might be required to report on its success or failure in achieving the target, with target misses punished by public censoring and embarrassment or by some more formal dismissal procedure.

45. This is obtained by substituting the commitment policy  $\Delta m = b_0 + b_1 e$  into the social objective function

$$\frac{1}{2} \left[ \lambda E(y - y_n - k)^2 + E(\pi - \pi^*)^2 \right]$$

and minimizing the unconditional expectation with respect to  $b_0$  and  $b_1$ .

Solving yields

$$\Delta m = \frac{a^2\lambda\pi^e - a\lambda e + a\lambda k + \pi^* + h\pi^T}{1 + h + a^2\lambda}. \quad (6.23)$$

Assuming rational expectations,  $\pi^e = \Delta m^e = (a\lambda k + \pi^* + h\pi^T)/(1 + h)$  because the public forms expectations prior to knowing  $e$ . Substituting this result into (6.23) yields the time-consistent money growth rate:

$$\begin{aligned} \Delta m^T &= \frac{a\lambda k + \pi^* + h\pi^T}{1 + h} - \left( \frac{a\lambda}{1 + h + a^2\lambda} \right) e \\ &= \pi^* + \frac{a\lambda k}{1 + h} + \frac{h(\pi^T - \pi^*)}{1 + h} - \left( \frac{a\lambda}{1 + h + a^2\lambda} \right) e. \end{aligned} \quad (6.24)$$

If the target inflation rate is equal to the socially optimal inflation rate ( $\pi^T = \pi^*$ ), (6.24) reduces to

$$\Delta m^T = \pi^* + \frac{a\lambda k}{1 + h} - \left( \frac{a\lambda}{1 + h + a^2\lambda} \right) e. \quad (6.25)$$

Setting  $h = 0$  yields the time-consistent discretionary solution *without* targeting:

$$\Delta m^{\text{NT}} = \pi^* + a\lambda k - \left( \frac{a\lambda}{1 + a^2\lambda} \right) e, \quad (6.26)$$

with the inflation bias equal to  $a\lambda k$ .

Comparing (6.22), (6.25), and (6.26) reveals that the targeting penalty reduces the inflation bias from  $a\lambda k$  to  $a\lambda k/(1 + h)$ . The targeting requirement imposes an additional cost on the central bank if it allows inflation to deviate too much from  $\pi^T$ ; this raises the marginal cost of inflation and reduces the time-consistent inflation rate. The cost of this reduction in the average inflation bias is the distortion that targeting introduces into the central bank's response to the aggregate supply shock  $e$ . Under pure discretion, the central bank responds optimally to  $e$  (note that the coefficient on the supply shock is the same in (6.26) as in (6.22)), but the presence of a targeting rule distorts the response to  $e$ . Comparing (6.25) with (6.22) shows that the central bank will respond too little to the supply shock (the coefficient falls from  $a\lambda/(1 + a^2\lambda)$  to  $a\lambda/(1 + h + a^2\lambda)$ ).

This trade-off between bias reduction and stabilization response was seen earlier in discussing Rogoff's model.<sup>46</sup> Note that if  $\pi^T = \pi^*$ , the central bank's objective function can be written as

$$V^{\text{cb}} = \frac{1}{2}\lambda E(y_t - y_n - k)^2 + \frac{1}{2}(1 + h)E(\pi - \pi^*)^2. \quad (6.27)$$

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46. Canzoneri (1985); Garfinkel and Oh (1993); and Garcia de Paso (1993; 1994); considered multiperiod targeting rules as solutions to this trade-off between stabilization and inflation bias. Defining money growth or inflation targets as averages over several periods restricts average inflation while allowing the central bank more flexibility in each period to respond to shocks.

It is apparent from (6.27) that the parameter  $h$  plays exactly the same role that Rogoff's degree of conservatism played. From the analysis of Rogoff's model, the optimal value of  $h$  will be positive, so that the total weight placed on the inflation objective exceeds society's weight, which is equal to 1. A flexible inflation target, interpreted here as a value for  $h$  that is positive, leads to an outcome that dominates pure discretion.<sup>47</sup>

The connection between an inflation-targeting rule and Rogoff's conservative central banker approach has just been highlighted. Svensson (1997b) showed that a similar connection exists between inflation targeting and the optimal linear inflation contract. Svensson demonstrated that the optimal linear inflation contract can be implemented if the central bank is required to target an inflation rate  $\pi^T$  that is actually less than the socially optimal rate of inflation. To see how this result is obtained, let  $H = 1 + h$ , replace  $\pi^*$  with  $\pi^T$  in (6.27), and expand the resulting second term so that the expression becomes

$$\begin{aligned} V^{\text{cb}} &= \frac{1}{2}\lambda E(y_t - y_n - k)^2 + \frac{1}{2}HE(\pi - \pi^* + \pi^* - \pi^T)^2 \\ &= \frac{1}{2}\lambda E(y_t - y_n - k)^2 + \frac{1}{2}HE(\pi - \pi^*)^2 + DE(\pi - \pi^*) + C, \end{aligned}$$

where  $D = H(\pi^* - \pi^T)$  and  $C = \frac{1}{2}H(\pi^* - \pi^T)^2$ . Since  $C$  is a constant, it does not affect the central bank's behavior. Notice that  $V^{\text{cb}}$  is equal to  $V + \frac{1}{2}hE(\pi - \pi^*)^2 + DE(\pi - \pi^*) + C$ . This is exactly equivalent to the incentive structure established under the optimal linear inflation contract if and only if  $h = 0$  and  $D = a\lambda k$ . The condition  $h = 0$  is achieved if the central banker is not a weight-conservative but instead shares society's preferences (so  $H = 1$ ); the condition  $D = a\lambda k$  is then achieved if

$$\pi^T = \pi^* - a\lambda k < \pi^*.$$

Thus, the optimal linear contract can be implemented by assigning to the central bank an inflation target that is actually below the rate that is socially preferred. But at the same time, policy should be assigned to an agent who has the same preferences between inflation and output stabilization as society in general.

### Strict Targeting Rules

The preceding analysis considered a flexible targeting rule. The central bank was penalized for deviations of  $\pi$  around a targeted level but was not required to achieve the target precisely. This flexibility allowed the central bank to trade off the objective of meeting the target against achieving its other objectives. Often, however, targeting is analyzed in terms of strict targets; the central bank is required to achieve a specific target outcome regardless of the implications for its other objectives. For an early analysis of strict targeting regimes, see Aizenman and Frankel (1986).

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47. That is, of course, unless  $h$  is too large.



As an example, consider a strict money growth rate target under which the central bank is required to set the growth rate of the money supply equal to some constant:<sup>48</sup>

$$\Delta m = \Delta m^T.$$

Since the desired rate of inflation is  $\pi^*$ , it makes sense to set  $\Delta m^T = \pi^*$ , and the public will set  $\pi^e = \pi^*$ . With this rule in place, the social loss function can be evaluated. If social loss is given by

$$V = \frac{1}{2}\lambda E_t(y_t - y_n - k)^2 + \frac{1}{2}E_t(\pi_t - \pi^*)^2,$$

then under a strict money growth rate target it takes the value

$$V(\Delta m^T) = \frac{1}{2} \left[ \lambda k^2 + \lambda \sigma_e^2 + (1 + a^2 \lambda) \sigma_v^2 \right].$$

Recall that under pure discretion the expected value of the loss function was, from (6.9),

$$V^d = \frac{1}{2}\lambda (1 + a^2 \lambda) k^2 + \frac{1}{2} \left[ \left( \frac{\lambda}{1 + a^2 \lambda} \right) \sigma_e^2 + (1 + a^2 \lambda) \sigma_v^2 \right].$$

Comparing these two, one obtains

$$V(\Delta m^T) - V^d = -\frac{1}{2} (a\lambda k)^2 + \frac{1}{2} \left( \frac{a^2 \lambda^2}{1 + a^2 \lambda} \right) \sigma_e^2.$$

Notice that this can be either positive or negative. It is more likely to be negative (implying that the strict money growth rate target is superior to discretion) if the underlying inflationary bias under discretion,  $a\lambda k$ , is large. Since the strict targeting rule ensures that average inflation is  $\pi^*$ , it eliminates any inflationary bias, so the gain is larger, the larger the bias that arises under discretion. However, discretion is more likely to be preferred to the strict rule when  $\sigma_e^2$  is large. The strict targeting rule eliminates any stabilization role for monetary policy. The cost of doing so will depend on the variance of supply shocks. Eliminating the central bank's flexibility to respond to economic disturbances increases welfare if

$$k > \sigma_e \sqrt{\frac{1}{1 + a^2 \lambda}}.$$

If  $\sigma_e^2$  is large, pure discretion, even with its inflationary bias, may still be the preferred policy (Flood and Isard 1988).

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48. Alternatively, the targeting rule could require the central bank to minimize  $E(\Delta m - \Delta m^T)^2$ . However, this occurs if the central bank sets policy such that  $E(\Delta m) = \Delta m^T$ . If  $\Delta m$  is controlled exactly, this is equivalent to  $\Delta m = \Delta m^T$ .

Another alternative targeting rule that has often been proposed focuses on nominal income (e.g., Hall and Mankiw 1994). If  $y - y_n$  is interpreted as the percentage output deviation from trend, one can approximate a nominal income rule as requiring that

$$(y - y_n) + \pi = g^*,$$

where  $g^*$  is the target growth rate for nominal income. Since the equilibrium growth rate of  $y - y_n$  is zero (because it is a deviation from trend) and the desired rate of inflation is  $\pi^*$ , one should set  $g^* = 0 + \pi^* = \pi^*$ . Under this rule, expected inflation is  $\pi^e = g^* - E(y - y_n) = g^* - 0 = g^* = \pi^*$ . Aggregate output is given by

$$y = y_n + a(\pi - \pi^e) + e = y_n + a(y_n - y) + e \Rightarrow y - y_n = \left(\frac{1}{1+a}\right)e,$$

as  $\pi = g^* - (y - y_n) = \pi^e - (y - y_n)$  under the proposed rule. A positive supply shock that causes output to rise will induce a contraction designed to reduce the inflation rate to maintain a constant rate of nominal income growth. The decline in inflation (which is unanticipated because it was induced by the shock  $e$ ) acts to reduce output and partially offset the initial rise. With the specification used here, exactly  $a/(1+a)$  of the effect of  $e$  is offset. Substituting this result back into the policy rule implies that  $\pi = \pi^* - e/(1+a)$ .

Using these results, the expected value of the social loss function is

$$V(g^*) = \frac{1}{2}\lambda k^2 + \frac{1}{2}\left[\frac{1+\lambda}{(1+a)^2}\right]\sigma_e^2.$$

In this model, nominal income targeting stabilizes real output more than pure discretion (and the optimal commitment policy) if  $a\lambda < 1$ . In this example, it is assumed that the central bank controls nominal income growth exactly. If, as is more realistic, this is not the case, a term due to control errors will also appear in the expected value of the loss function.

Nominal income targeting imposes a particular trade-off between real income growth and inflation in response to aggregate supply disturbances. The social loss function does not weigh output fluctuations and inflation fluctuations equally unless  $\lambda = 1$ , but nominal income targeting does. Nevertheless, nominal income targeting is often proposed as a “reasonably good rule for the conduct of monetary policy” (Hall and Mankiw 1994). For analyses of nominal income targeting, see Bean (1983), Frankel and Chinn (1995), McCallum (1988), Taylor (1985), and West (1986).<sup>49</sup>

#### 6.4 Is the Inflation Bias Important?

Despite the large academic literature that has focused on the inflationary bias of discretionary monetary policy, some have questioned whether this whole approach has anything to do with explaining actual episodes of inflation. Do these models provide useful

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49. See Billi (2015) for an analysis of nominal income targeting in the new Keynesian model (chapter 8).

frameworks for positive theories of inflation? Since monetary models generally imply that the behavior of real output should be the same whether average inflation is zero or 10 percent, the very fact that most economies have consistently experienced average inflation rates well above zero for extended periods of time might be taken as evidence for the existence of an inflation bias.<sup>50</sup> However, earlier chapters examined theories of inflation based on optimal tax considerations that might imply nonzero average rates of inflation, although few argue that tax considerations alone could account for the level of inflation observed during the 1970s in most industrialized economies (or for the observed variations in inflation). There are several reasons for questioning the empirical relevance of time inconsistency as a factor in monetary policy. Some economists have argued that time inconsistency just isn't a problem. For example, Taylor (1983) pointed out that society finds solutions to these sorts of problems in many other areas (patent law, for example) and that there is no reason to suppose that the problem is particularly severe in the monetary policy arena.<sup>51</sup> Others, such as Blinder and Rudd (2013), attributed the rise in inflation during the 1970s to supply shocks rather than to any inherent bias of discretionary policies. Institutional solutions, such as separating responsibility for monetary policy from the direct control of elected political officials, may reduce or even eliminate the underlying bias toward expansions that leads to excessively high average inflation under discretion.<sup>52</sup>

McCallum (1995; 1997a) argued that central banks can be trusted not to succumb to the incentive to inflate because they know that succumbing leads to a bad equilibrium. But such a view ignores the basic problem; even central banks that want to do the right thing may face the choice of either inflating or causing a recession. In such circumstances, the best policy may not be to cause a recession. For example, consider Cukierman and Liviatan's type *D* policymaker. Such a policymaker is capable of committing to and delivering a zero inflation policy, but if the public assigns some probability to the possibility that a type *W* might be in office, even the type *D* ends up inflating. If central banks were to define their objectives in terms of stabilizing output around the economy's natural rate (i.e.,  $k \equiv 0$ ), there would be no inflationary bias; central banks would deliver the socially optimal policy. However, this corresponds to a situation in which there is no bias, not to one in which an incentive to inflate exists but the central bank resists it.

An alternative criticism of the time-inconsistency literature questions the underlying assumption that the central bank cannot commit. Blinder (1995), for example, argued that

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50. While most monetary models do not display superneutrality (so that inflation does affect real variables even in the steady state), most policy-oriented models satisfy a natural rate property in that average values of real variables such as output are assumed to be independent of monetary policy.

51. As Taylor (1983) puts it, "In the Barro-Gordon inflation-unemployment model, the superiority of the zero inflation policy is just as obvious to people as the well-recognized patent problem is in the real world. It is therefore difficult to see why the zero inflation policy would not be adopted in such a world" (125).

52. For material from the second edition surveying this empirical literature on central bank institutional structure and macro outcomes, see <http://people.ucsc.edu/~walshc/mtp4e/>.

the inherent lags between a policy action and its effect on inflation and output serve as a commitment technology. Inflation in period  $t$  is determined by policy actions taken in earlier periods, so if the public knows past policy actions, the central bank can never produce a surprise inflation. The presence of lags does serve as a commitment device. If outcomes today are entirely determined by actions taken earlier, the central bank is clearly committed; nothing it can do will affect today's outcome. And few would disagree that monetary policy acts with a (long) lag. But appealing to lags solves the time-inconsistency problem by eliminating *any* real effects of monetary policy in a Barro-Gordon type model. That is, there is no incentive to inflate because anticipated expansionary monetary policy does not affect real output or unemployment. If this were the case, central banks could costlessly disinflate; seeing a shift in policy, private agents could all revise nominal wages and prices before any real effects occurred.

If monetary policy does have real effects, even if these occur with a lag, the inflationary bias under discretion will reappear. In the models that have been used in the time-inconsistency literature, monetary policy affects real output through its effect on inflation, more specifically, by creating inflation surprises. The empirical evidence from most countries, however, indicates that policy actions affect output before inflation is affected.<sup>53</sup> Policy actions can be observed long before the effects on inflation occur. But for this to represent a commitment technology that can overcome the time-inconsistency problem requires that the observability of policy eliminate its ability to affect real output. It is the ability of monetary policy to generate real output effects that leads to the inflationary bias under discretion, and the incentive toward expansionary policies exists as long as monetary policy can influence real output. The fact that the costs of an expansion in terms of higher inflation only occur later actually increases the incentive for expansion if the central bank discounts the future.

There has been relatively little empirical work testing directly for the inflation bias. One relevant piece of evidence is provided by Romer (1993). He argued that the average inflation bias should depend on the degree to which an economy is open. A monetary expansion produces a real depreciation, raising the price of foreign imports. This increases inflation as measured by the consumer price index, raising the inflationary cost of the monetary expansion.<sup>54</sup> As a result, a given output expansion caused by an unanticipated rise in the domestic price level brings with it a larger inflation cost in terms of an increase in consumer price

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53. Kiley (2002b) presented evidence for the United States, Canada, Great Britain, France, and Germany. Kilponen and Leitemo (2011) examined the role of transmission lags on the discretionary policy bias in a new Keynesian model of the type developed in chapter 8, but their focus was on a bias in the way the central bank responds to shocks, not on the implications for average inflation.

54. That is, output depends on domestic price inflation  $\pi_d$  and is given by

$$y = y_n + \alpha(\pi_d - \pi_d^e),$$

while consumer price inflation is equal to

$$\pi_{cpi} = \theta\pi_d + (1 - \theta)s,$$

inflation. In addition, the output gain from such an expansion will be reduced if domestic firms use imported intermediate goods or if nominal wages are indexed. In terms of the basic model, this could be interpreted either as a lowering of the benefits of expansion relative to the costs of inflation or as a reduction in the output effects of unanticipated inflation. Consequently, the weight on output,  $\lambda$ , should be smaller (or the weight on inflation larger) in more open economies, and the coefficient of the supply curve,  $a$ , should be smaller. Since the inflation bias is increasing in  $a\lambda$  (see, e.g., equation 6.7), the average inflation rate should be lower in more open economies.

Romer tested these implications using data on 114 countries for the post-1973 period. Using the import share as a measure of openness, he found the predicted negative association between openness and average inflation. The empirical results, however, did not hold for the OECD economies. For the highly industrialized, high-income countries, openness was unrelated to average inflation.<sup>55</sup>

Temple (2002) examined the link between inflation and the slope of the Phillips curve linking inflation and output (represented by the value of the parameter  $a$  in (6.3)) and found little evidence that  $a$  is smaller in more open economies. To account for Romer's finding that openness is associated with lower inflation, he suggested inflation may be more costly in open economies because it is associated with greater real exchange rate variability. In this case, the parameter  $\lambda$  would be smaller in a more open economy, as the central bank places relatively more weight on inflation objectives. As a result, average inflation would be lower in open economies, as Romer found.

Romer's test focuses on one of the factors (openness) that might affect the incentive to inflate. If central banks respond systematically to the costs and benefits of inflation, variations in the incentive to inflate across countries should be reflected, *ceteris paribus*, in variations in actual inflation rates.

Ireland (1999) argued that the behavior of inflation in the United States is consistent with the Barro-Gordon model if one allows for time variation in the natural rate of unemployment. Ireland assumed the central bank's objective is to minimize

$$V = \frac{1}{2}\lambda(u - ku_n)^2 + \frac{1}{2}\pi^2,$$

where  $u$  is the unemployment rate and  $u_n$  is the natural rate of unemployment. It is assumed that  $k < 1$  so that the central bank attempts to target an unemployment rate below the economy's natural rate. Ireland assumed that  $u_n$  is unobservable but varies over time and

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where  $s$  is the rate of change of the nominal exchange rate and  $\theta$  is the share of domestic output in the consumer price index.

55. Terra (1998) argued that Romer's results were driven by the countries in his sample that were severely indebted. However, Romer (1998) noted in reply that the relationship between indebtedness and the openness-inflation correlation disappears when one controls for central bank independence. This suggests that both indebtedness and inflation are more severe in countries that have not solved the policy commitment problem.

is subject to permanent stochastic shifts. As a result, the average inflation rate varies with these shifts in  $u_n$ ; when  $u_n$  rises, average inflation also rises (see problem 9 at the end of this chapter). Ireland found support for a long-run (cointegrating) relationship between unemployment and inflation in the United States. However, this is driven by the rise in inflation in the 1970s that coincided with the rise in the natural rate of unemployment as the baby boom generation entered the labor force. Whether the latter was the cause of the former is more difficult to determine, and Europe in the 1990s experienced a rise in average unemployment rates accompanied by a fall rather than an rise in average inflation.

A serious criticism of explanations of actual inflation episodes based on the Barro-Gordon approach relates to the assumption that the central bank and the public understand that there is no long-run trade-off between inflation and unemployment. The standard aggregate supply curve, relating output movements to inflation surprises, implies that the behavior of real output (and unemployment) will be independent of the average rate of inflation. However, many central banks in the 1960s and into the 1970s did not accept this as an accurate description of the economy. Phillips curves were viewed as offering a menu of inflation-unemployment combinations from which policymakers could choose. Actual inflation may have reflected policymakers' misconceptions about the economy rather than their attempts to engineer surprise inflations that would not be anticipated by the public. For example, Romer and Romer (2002) attributed policy mistakes and high inflation in the United States during the late 1960s and the 1970s to the use of a wrong model. Specifically, they argued that policymakers during the 1960s believed the Phillips curve offered a permanent trade-off between average unemployment and average inflation. They then argued that once inflation had reached high levels, policymakers came to believe that inflation was insensitive to recessions, implying the cost of reducing inflation would be extremely high. Thus, inflation was allowed to rise and policymakers delayed reducing it because they based decisions on models that are now viewed as incorrect. Levin and Taylor (2013) blamed stop-start monetary policies, in part attributable to political pressures on the Federal Reserve, for the Great Inflation between 1965 and 1980, while Blinder and Rudd (2013) focused on the role of supply shocks such as the oil embargoes and price increases of the 1970s rather than issues of policy time inconsistency in accounting for the behavior of inflation.

These criticisms, while suggesting that the simple models of time inconsistency may not account for all observed inflation, do not mean that time-inconsistency issues are unimportant. Explaining actual inflationary experiences will certainly involve consideration of the incentives faced by policymakers and the interaction of the factors such as uncertainty over policy preferences, responses to shocks, and a bias toward expansions that play a key role in models of discretionary policy.<sup>56</sup> So the issues that are central to the time-inconsistency literature do seem relevant for understanding the conduct of monetary policy. At the same

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56. And in reputational solutions, observed inflation may remain low for extended periods of time even though the factors highlighted in the time-inconsistency literature play an important role in determining the equilibrium.

time, important considerations faced by central banks are absent from the basic models generally used in the literature. For example, the models have implications for average inflation rates but usually do not explain variations in average inflation over longer time periods.<sup>57</sup> Yet one of the most important characteristics of inflation during the past 60 years in the developed economies is that it has varied; it was low in the 1950s and early 1960s, much higher in the 1970s, and lower again in the mid-1980s and 1990s. Thus, average inflation changes, but it also displays a high degree of persistence.

This persistence does not arise in the models examined so far. Reputational models can display a type of inflation persistence; inflation may remain low in a pooling equilibrium; then, once the high-inflation central bank reveals itself, the inflation rate jumps and remains at a higher level. But this description does not seem to capture the manner in which a high degree of persistence is displayed in the response of actual inflation to economic shocks that, in principle, should cause only one-time price level effects. For example, consider a negative supply shock. When the central bank is concerned with stabilizing real output, such a shock leads to a rise in the inflation rate. This reaction seems consistent with the early 1970s, when the worldwide oil price shock is generally viewed as being responsible for the rise in inflation. In the models of the previous sections, the rise in inflation lasts only one period. The shock may have a permanent effect on the price level, but it cannot account for persistence in the inflation rate. Ball (1991; 1995) argued, however, that inflation results from an adverse shock and that once inflation increases, it remains high for some time. Eventually, policy shifts do bring inflation back down. Models of unemployment persistence based on labor market hysteresis, such as those developed by Lockwood and Philippopoulos (1994), Jonsson (1995), Lockwood (1997), and Svensson (1997b), also imply some inflation persistence. A shock that raises unemployment now also raises expected unemployment in the future. This increases the incentive to generate an expansion and so leads to a rise in inflation both now and in the future. But these models imply that inflation gradually returns to its long-run average and so cannot account for the shifts in policy that often seem to characterize disinflations.

One model that does display such shifts was discussed earlier. Ball (1995) accounted for shifts in policy by assuming that the central bank type can change between a zero inflation type and an optimizing type according to a Markov process. With imperfect information, the public must attempt to infer the current central bank's type from inflation outcomes. The wet type mimics the zero inflation type until an adverse disturbance occurs. If such a shock occurs and the central bank is a wet type, inflation rises. This increase reveals the central bank's type, so the public expects positive inflation, and in equilibrium inflation remains high until a dry type takes over. As a result, the model predicts the type of

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57. Potential sources of shifts in the discretionary average rate of inflation would be changes in labor market structure that affect the output effects of inflation (the  $a$  parameter in the basic model), shifts in the relative importance of output expansions or output stabilization in the policymaker's objective functions (the  $\lambda$  parameter), or changes in the percentage gap between the economy's natural rate of output (unemployment) and the socially desired level (the parameter  $k$ ).

periodic and persistent bouts of inflation that seem to have characterized inflation in many developed economies. The model of Albanesi, Chari, and Christiano (2003) displayed multiple equilibria and so could account for shifts between low and high inflation equilibria.

A number of authors have suggested that central banks now understand the dangers of having an overly ambitious output target and, as a consequence, these central banks now target the output gap  $y_t - y_n$ ; in other words,  $k = 0$ . With the standard quadratic loss function, the inflation bias under discretion is zero when  $k = 0$ . Cukierman (2002), however, showed that an inflation bias reemerges if central bank preferences are asymmetric. He argued that central banks are not indifferent between  $y_t - y_n > 0$  and  $y_t - y_n < 0$  even if the deviations are of equal magnitude. A central bank that views a 1 percent fall in output below  $y_n$  as more costly than a 1 percent rise above  $y_n$  will tend to err in the direction of an overly expansionary policy. As a result, an average inflation bias reemerges even though  $k = 0$ . Ruge-Murcia (2003), in contrast, considered the case of a central bank with an inflation target and asymmetric preferences over target misses. He showed that if the central bank prefers undershooting its target rather than overshooting it, average inflation will tend to be less than the target inflation rate.<sup>58</sup>

Ruge-Murcia nested both Cukierman's asymmetric preferences and the standard quadratic preferences of the Barro-Gordon into a model he is able to take to the data. He does so by specifying preferences over inflation and unemployment by the following mixture of a quadratic function in inflation and a linex function in unemployment:

$$L_t = \frac{1}{2} (\pi_t - \pi^*)^2 + \left( \frac{\phi}{\gamma^2} \right) \left\{ \exp [\gamma (u_t - u_t^*)] - \gamma (u_t - u_t^*) - 1 \right\},$$

where  $\pi^*$  is a constant inflation target and  $u_t^*$  is the policymaker's desired rate of unemployment. For  $\gamma > 0$ , positive deviations of unemployment above target are viewed as more costly than negative deviations. If  $u_t^* = kE_{t-1}u_t^n$ , where  $u_t^n$  is the natural rate of unemployment and  $0 < k < 1$ , Ruge-Murcia showed that the Barro-Gordon model is obtained by letting  $\gamma \rightarrow 0$  and a version similar to Cukierman's asymmetric preferences is obtained when  $\gamma > 0$  and  $k = 1$ , that is, when the target unemployment rate equals the natural rate. In the standard Barro-Gordon model, the inflation bias would disappear when  $k = 1$ . Using U.S. data, Ruge-Murcia found that the Barro-Gordon model is rejected, but Cukierman's model is not, suggesting that a stronger aversion to unemployment rate increases relative to unemployment rate decreases may have been important in generating the observed pattern of inflation in the United States.

Cukierman and Gerlach (2003) also found support for the importance of asymmetric preferences. They showed that if central banks are uncertain about economic developments and have asymmetric preferences over output, average inflation should be positively correlated with the volatility of output. They found evidence supporting this implication from a cross-section consisting of 22 OECD countries.

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58. This may describe the case of the European Central Bank, whose objective is described in an asymmetric manner as an inflation rate of 2 percent or lower.



Finally, in an important contribution, Sargent (1999) studied the case of a central banker in a Barro-Gordon world that must learn about the structure of the economy. Initially, if the central bank believes it faces a Phillips curve trade-off between output and inflation, it will attempt to expand the economy. The equilibrium involves the standard inflation bias. As new data reveal to the central bank that the Phillips curve is vertical and that it has not gained an output expansion despite the inflation, the equilibrium can switch to a zero inflation path. However, the apparent conquest of inflation is temporary, and the equilibrium can alternate between periods of high inflation and periods of low inflation.

## 6.5 Summary

Many countries experience, for long periods of time, average inflation rates that clearly exceed what would seem to be reasonable estimates of the socially desired inflation rate. The time-inconsistency literature originated as a positive attempt to explain this observation. In the process, the approach has made important methodological contributions to monetary policy analysis by emphasizing the need to treat central banks as responding to the incentives they face. The factors emphasized in this chapter—central bank preferences, the short-run real effects of surprise inflation, the rate at which the central bank discounts the future, the effects of political influences on the central bank—are quite different from the factors that receive prominence in the optimal taxation models of inflation of chapter 4.

Perhaps the most important contribution of the literature on time inconsistency, however, has been to provide theoretical frameworks for thinking formally about credibility issues, on the one hand, and the role of institutions and political factors, on the other, in influencing policy choices. By emphasizing the interactions of the incentives faced by policymakers, the structure of information about the economy and about the central bank's preferences, and the public's beliefs, the models examined in this chapter provide a critical set of insights that have influenced debates over rules, discretion, and the design of monetary policy institutions.

## 6.6 Problems

1. Consider the following simple economy. Output is given by

$$y_t = \bar{y} + a(\pi_t - \pi_t^e) + e_t,$$

where  $y$  is output,  $\pi$  is inflation,  $\pi^e$  is expected inflation, and  $e$  is a productivity shock. Private sector expectations are formed before observing  $e$ , while the central bank can act after observing  $e$ . Suppose the central bank controls inflation and does so to minimize

$$L_t = \left(\frac{1}{2}\right) \left[ (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right],$$

where  $y^* > \bar{y}$ .

- a. Solve for the rational-expectations equilibrium for inflation and output if the central bank acts with discretion.
  - b. Solve for the rational-expectations equilibrium for inflation and output under the optimal commitment policy.
  - c. Explain (in words) how the inflation bias under discretion depends on  $a$ ,  $\lambda$ , and  $y^* - \bar{y}$ .
2. Suppose output is given by

$$y_t = \bar{y} + a(\pi_t - \pi_t^e) + e_t,$$

where  $y$  is output,  $\pi$  is inflation,  $\pi^e$  is expected inflation, and  $e$  is a productivity shock. Private sector expectations are formed after receiving a signal  $v$  on the productivity shock, where

$$v_t = e_t + n_t,$$

and  $n_t$  is white noise. Let  $\sigma_x^2$  denote the variance of  $x$ , and the public's forecast of  $e_t$  conditional on  $v_t$  is  $sv_t$ , where  $s = \sigma_e^2 / (\sigma_e^2 + \sigma_n^2)$ . The central bank can act after observing  $e$ . Suppose the central bank controls inflation and does so to minimize

$$L_t = \left(\frac{1}{2}\right) \left[ (\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 \right],$$

where  $y^* > \bar{y}$ .

- a. Solve for the rational-expectations equilibrium for inflation and output if the central bank acts with discretion. How does the central bank's reaction to  $e_t$  depend on how noisy the public's signal is, as measured by  $\sigma_n^2$ ?
  - b. Solve for the rational-expectations equilibrium for inflation and output under the optimal commitment policy. How does the central bank's reaction to  $e_t$  depend on how noisy the public's signal is, as measured by  $\sigma_n^2$ ?
  - c. Calculate expected loss under discretion and under commitment. Does  $\sigma_n^2$  influence the expected gains from commitment? Explain.
3. Suppose the central bank dislikes inflation variability around a target level  $\pi^*$ . It also prefers to keep unemployment stable around an unemployment target  $u^*$ . These objectives can be represented in terms of minimizing

$$V = \lambda(u - u^*)^2 + \frac{1}{2}(\pi - \pi^*)^2,$$

where  $\pi$  is the inflation rate and  $u$  is the unemployment rate. The economy is described by

$$u = u_n - a(\pi - \pi^e) + v,$$

where  $u_n$  is the natural rate of unemployment and  $\pi^e$  is expected inflation. Expectations are formed by the public before observing the disturbance  $v$ . The central bank can set inflation after observing  $v$ . Assume  $u^* < u_n$ .

- a. What is the equilibrium rate of inflation under discretion? What is the equilibrium unemployment rate?
  - b. Is equilibrium unemployment under discretion affected by  $u^*$ ? Explain.
  - c. Is equilibrium inflation under discretion affected by  $u^*$ ? Explain.
  - d. How is equilibrium inflation under discretion affected by  $v$ ? Explain.
  - e. What is the equilibrium rate of inflation under commitment? What is the equilibrium unemployment rate under commitment? How are they affected by  $u^*$ ? Explain.
4. Suppose an economy is characterized by the following three equations:

$$\pi = \pi^e + ay + e,$$

$$y = -br + u,$$

$$\Delta m - \pi = -di + y + v,$$

where the first equation is an aggregate supply function written in the form of an expectations-augmented Phillips curve, the second is an investment-saving (IS) or aggregate demand relationship, and the third is a money demand equation, where  $\Delta m$  denotes the growth rate of the nominal money supply. The real interest rate is denoted by  $r$  and the nominal rate by  $i$ , with  $i = r + \pi^e$ . Let the central bank implement policy by setting  $i$  to minimize the expected value of  $\frac{1}{2}[\lambda(y - k)^2 + \pi^2]$ , where  $k > 0$ . Assume that the policy authority has forecasts  $e^f$ ,  $u^f$ , and  $v^f$  of the shocks but that the public forms its expectations prior to the setting of  $i$  and without any information on the shocks.

- a. Assume that the central bank can commit to a policy of the form  $i = c_0 + c_1 e^f + c_2 u^f + c_3 v^f$  prior to knowing any of the realizations of the shocks. Derive the optimal commitment policy (i.e., the optimal values of  $c_0$ ,  $c_1$ ,  $c_2$ , and  $c_3$ ).
  - b. Derive the time-consistent equilibrium under discretion. How does the nominal interest rate compare to the case under commitment? What is the average inflation rate?
5. Verify that the optimal commitment rule that minimizes the unconditional expected value of the loss function given by (6.10) is  $\Delta m^c = -a\lambda e / (1 + a^2\lambda)$ .
6. Suppose the central bank acts under discretion to minimize the expected value of (6.2). The central bank can observe  $e$  prior to setting  $\Delta m$ , but  $v$  is observed only after policy is set. Assume, however, that  $e$  and  $v$  are correlated and that the expected value of  $v$ ,

conditional on  $e$ , is  $E[v|e] = qe$ , where  $q = \sigma_{v,e}/\sigma_e^2$  and  $\sigma_{v,e}$  is the covariance between  $e$  and  $v$ .

- a. Find the optimal policy under discretion. Explain how policy depends on  $q$ .
  - b. What is the equilibrium rate of inflation? Does it depend on  $q$ ?
7. Since the tax distortions of inflation are related to expected inflation, suppose the loss function (6.2) is replaced by

$$L = \lambda(y - y_n - k)^2 + (\pi^e)^2,$$

where  $y = y_n + a(\pi - \pi^e)$ . How is figure 6.2 modified by this change in the central bank's loss function? Is there an equilibrium inflation rate? Explain.

8. Based on Jonsson (1995) and Svensson (1997b). Suppose (6.3) is modified to incorporate persistence in the output process:

$$y_t = (1 - \theta)y_n + \theta y_{t-1} + a(\pi_t - \pi_t^e) + e, \quad 0 < \theta < 1.$$

Suppose the policymaker has a two-period horizon with objective function given by

$$L = \min E [L_t + \beta L_{t+1}],$$

$$\text{where } L_i = \frac{1}{2} [\lambda(y_i - y_n - k)^2 + \pi_i^2].$$

- a. Derive the optimal commitment policy.
  - b. Derive the optimal policy under discretion without commitment.
  - c. How does the presence of persistence ( $\theta > 0$ ) affect the inflation bias?
9. Suppose the central bank's objective is to minimize

$$V = \frac{1}{2} \lambda (u - k u_n)^2 + \frac{1}{2} \pi^2,$$

where  $u$  is the unemployment rate and  $u_n$  is the natural rate of unemployment, with  $k < 1$ . If the economy is described by

$$u = u_n - a(\pi - \pi^e),$$

what is the equilibrium rate of inflation under discretion? How does a fall in  $u_n$  affect the equilibrium rate of inflation?

10. Suppose that the private sector forms expectations according to

$$\begin{aligned} \pi_t^e &= \pi^* & \text{if } \pi_{t-1} &= \pi_{t-1}^e, \\ \pi_t^e &= a\lambda k & \text{otherwise.} \end{aligned}$$

If the central bank's objective function is the discounted present value of the single-period loss function given by (6.2), and its discount rate is  $\beta$ , what is the minimum value of  $\pi^*$  that can be sustained in equilibrium?

11. Based on Cukierman and Liviatan (1991). Assume that there are two central bank types with common preferences given by (6.1), but type  $D$  always delivers what it announces, while type  $W$  acts opportunistically. Assume that output is given by (6.3), with  $e \equiv 0$ . Using a two-period framework, show how output behaves under each type in (a) a pooling equilibrium, and (b) a separating equilibrium. Are there any values of  $\beta$  such that welfare is higher if a type  $W$  central bank is setting policy?
12. Suppose there are two possible policymaker types: a commitment type ( $C$ ), whose announced target represents a commitment, and an opportunistic type ( $O$ ), who is not necessarily bound by the target. The objective of both types is to maximize

$$A(\pi_1 - \pi_1^e) - \frac{\pi_1^2}{2} + \beta \left[ A(\pi_2 - \pi_2^e) - \frac{\pi_2^2}{2} \right], \quad (6.28)$$

where  $\pi_j$  and  $\pi_j^e$ ,  $j = 1, 2$ , are actual and expected inflation in period  $j$ , respectively,  $0 \leq \beta \leq 1$  is a discount factor, and  $A$  is a positive parameter. Assume that the type in office in the first period remains in office also in the second period. The public does not know which type is in office but believes, at the beginning of period 1, that the probability that a preannounced inflation target has been issued by type  $C$  is  $0 \leq p_1 \leq 1$  (this is the first-period reputation of policymakers). The timing of moves within each period is as follows. First the policymaker in office announces the inflation target for that period. Then inflationary expectations are formed. Following that, the policymaker picks the actual rate of inflation.

- Derive the policy plans of each of the two types, when in office, in the second period. What is the intuition underlying your answer?
  - Let  $p_2$  be the reputation of policymakers at the beginning of the second period. Find (and motivate) an expression for the public's (rational) expectation of inflation for that period.
  - Derive the policy plans of each of the two types, when in office, in the first period and explain your results intuitively.
  - What is the relationship between second- and first-period reputations in equilibrium? Why?
  - How does the discount factor,  $\beta$ , affect the rates of inflation planned by each of the two types in the first period? Why?
13. Assume that nominal wages are set at the start of each period but that wages are partially indexed against inflation. If  $w^c$  is the contract base nominal wage, the actual nominal wage is  $w = w^c + \kappa(p_t - p_{t-1})$ , where  $\kappa$  is the indexation parameter. Show how indexation affects the equilibrium rate of inflation under pure discretion. What is the effect on average inflation of an increase in  $\kappa$ ? Explain.

14. Based on Beetsma and Jensen (1998). Suppose the social loss function is equal to

$$V^s = \frac{1}{2} \mathbb{E} \left[ \lambda \left( y - y_n - k \right)^2 + \pi^2 \right]$$

and the central bank's loss function is given by

$$V^{cb} = \frac{1}{2} \mathbb{E} \left[ (\lambda - \theta) \left( y - y_n - k \right)^2 + (1 + \theta) \left( \pi - \pi^T \right)^2 \right] + t\pi,$$

where  $\theta$  is a mean zero stochastic shock to the central bank's preferences,  $\pi^T$  is an inflation target assigned by the government, and  $t\pi$  is a linear inflation contract with  $t$  a parameter chosen by the government. Assume that the private sector forms expectations before observing  $\theta$ . Let  $y = y_n + (\pi - \pi^e) + e$ , and  $\pi = \Delta m + v$ . Finally, assume that  $\theta$  and the supply shock  $e$  are uncorrelated.

- Suppose the government only assigns an inflation target (so  $t = 0$ ). What is the optimal value for  $\pi^T$ ?
- Now suppose the government only assigns a linear inflation contract (so  $\pi^T = 0$ ). What is the optimal value for  $t$ ?
- Is the expected social loss lower under the inflation target arrangement or the inflation contract arrangement?

# 7 Nominal Price and Wage Rigidities

## 7.1 Introduction

In this chapter, the focus shifts away from models with flexible wages and prices to models with sticky wages and prices. It begins with a simple example of a model with nominal wage rigidities that last for one period. Then it reviews models that account for the observation that prices and wages may take several periods to adjust to changes in macroeconomic conditions. Time-dependent and state-dependent models of price adjustment are discussed. Time-dependent pricing models assume the probability a firm changes its price is a function only of time, while state-dependent models make this probability a function of the current state of the economy.

The focus here is on models of nominal rigidities. In chapter 8, the new Keynesian Phillips curve developed in section 7.2.4 is incorporated into a general equilibrium framework so that the implications of price and wage rigidities for monetary policy can be studied.

## 7.2 Sticky Prices and Wages

Most macroeconomic models attribute the short-run real effects of monetary disturbances not to imperfect information or limited participation in financial markets, but to the presence of nominal wage and/or price rigidities. These rigidities mean that nominal wages and prices fail to adjust immediately and completely to changes in the nominal quantity of money. In the Lucas imperfect-information model (chapter 5), which assumed flexible prices and wages, a simple calibration exercise suggested that the elasticity of employment with respect to an unexpected change in the money supply would be tiny. In chapter 6, it was noted that a Lucas supply function relating employment to unanticipated monetary policy could be motivated either by imperfect information, as stated by Lucas (1972), or by assuming nominal wages were fixed for one period, as in a simplified version of the model by Fischer (1977). A simple example of one-period wage rigidity is illustrated in

section 7.2.1. In this case, a monetary surprise increases prices and, with nominal wages fixed, lowers the real wage, increases labor demand, and generates a rise in employment and output. Assuming a standard aggregate production function, a monetary surprise has an effect on output about 100 times larger than the value calculated in chapter 5.<sup>1</sup>

While assuming wages are fixed for one period increases the impact of monetary disturbances on real output, it cannot account for persistent real effects of monetary policy. The model of staggered overlapping and multiperiod nominal wage contracts due to (Taylor 1979; 1980) can generate the persistent output responses observed in the data, but Taylor's model was not based on an explicit model of optimizing behavior by workers or firms. The literature in later years has turned to models of monopolistic competition and price stickiness in which the decision problem faced by firms in setting prices can be made explicit. The objective in this section is to review some of the standard models of nominal rigidities. Dynamic stochastic general equilibrium (DSGE) models based on nominal rigidities and their implications for monetary policy analysis is the chief focus of chapter 8.

### 7.2.1 An Example of Nominal Rigidities in General Equilibrium

The first model considered adds a one-period nominal wage rigidity to the money-in-the-utility function (MIU) model of chapter 2. This approach is not based on optimizing behavior by wage setters, but it leads to a reduced-form model that has been widely used in monetary economics. This model played an important role in the analysis of time inconsistency in chapter 6.

#### Wage Rigidity in an MIU Model

One way to introduce nominal price stickiness is to modify a flexible-price model, such as the MIU model, by simply assuming that prices and/or wages are set at the start of each period and are unresponsive to developments within the period. In chapter 2, a linear approximation was used to examine the time series implications of an MIU model. Wages and prices were assumed to adjust to ensure market equilibrium, and consequently the behavior of the money supply mattered only to the extent that anticipated inflation was affected. A positive disturbance to the growth rate of money would, assuming that the growth rate of money was positively serially correlated, raise the expected rate of inflation, leading to a rise in the nominal rate of interest that affects labor supply and output. These last effects depended on the form of the utility function; if utility was separable in money, changes in expected inflation had no effect on labor supply or real output. Introducing wage stickiness into an MIU model illustrates the effect such a modification has on the impact of monetary disturbances.

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1. This result was developed more formally in section 6.2.1 of the third edition. That material can be found at <http://people.ucsc.edu/~walshc/mtp4/>.



Consider the linear approximation to the Sidrauski MIU model. To simplify the model, assume utility is separable in consumption and money holdings ( $b = \Phi$ , or  $\Omega_2 = 0$  in terms of the parameters of the model used in chapter 2). This implies that money and monetary shocks have no effect on output when prices are perfectly flexible.<sup>2</sup> In addition, the capital stock is treated as fixed and investment is zero. This follows McCallum and Nelson (1999), who argued that for most monetary policy and business cycle analyses, fluctuations in the stock of capital do not play a major role. The equations characterizing equilibrium in the resulting MIU model are

$$y_t = (1 - \alpha)n_t + e_t, \quad (7.1)$$

$$y_t = c_t, \quad (7.2)$$

$$y_t - n_t = w_t - p_t, \quad (7.3)$$

$$\Phi E_t(c_{t+1} - c_t) - r_t = 0, \quad (7.4)$$

$$\eta \left( \frac{n^{ss}}{1 - n^{ss}} \right) n_t + \Phi c_t = w_t - p_t, \quad (7.5)$$

$$m_t - p_t = c_t - \left( \frac{1 - i^{ss}}{bi^{ss}} \right) i_t, \quad (7.6)$$

$$i_t = r_t + E_t p_{t+1} - p_t, \quad (7.7)$$

$$m_t = \rho_m m_{t-1} + s_t. \quad (7.8)$$

The system is written in terms of the log price level  $p$  rather than the inflation rate, and, in contrast to the notation of chapter 2,  $m$  represents the nominal stock of money. To briefly review these equations, (7.1) is the economy's production function in which output deviations from the steady state are a linear function of the deviations of labor supply from steady state and a productivity shock. Equation (7.2) is the resource constraint derived from the condition that in the absence of investment or government purchases, output equals consumption. Labor demand is derived from the condition that labor is employed up to the point where the marginal product of labor equals the real wage. With the Cobb-Douglas production function underlying (7.1), this condition, expressed in terms of percentage deviations from the steady state, can be written as (7.3).<sup>3</sup> Equations (7.4)–(7.6) are derived from the representative household's first-order conditions for consumption, leisure, and money holdings. Equation (7.7) is the Fisher equation linking the nominal and real rates of interest. Finally, (7.8) gives the exogenous process for the nominal money supply.<sup>4</sup>

2. From (5.33), money surprises also have no effect on employment and output when  $\Omega_2 = 0$  in Lucas's imperfect-information model.

3. If  $Y = \bar{k}^\alpha N^{1-\alpha}$ , then the marginal product of labor is  $(1 - \alpha)Y/N$ , where  $\bar{K}$  is the fixed stock of capital. In log terms, the real wage is then equal to  $\ln W - \ln P = \ln(1 - \alpha) + \ln Y - \ln N$ , or in terms of deviations from steady state,  $w - p = y - n$ .

4. Alternatively, the nominal interest rate  $i_t$  could be taken as the instrument of monetary policy, with (7.6) then determining  $m_t$ .

When prices are flexible, (7.1)–(7.5) form a system of equations that can be solved for the equilibrium time paths of output, labor, consumption, the real wage, and the real rate of interest. Equations (7.6)–(7.8) then determine the evolution of real money balances, the nominal interest rate, and the price level. Thus, realizations of the monetary disturbance  $s_t$  have no effect on output when prices are flexible. This version of the MIU model displays the *classical dichotomy* (Modigliani 1963; Patinkin 1965): real variables such as output, consumption, investment, and the real interest rate are determined independently of both the money supply process and money demand factors.<sup>5</sup>

Now suppose the nominal wage rate is set prior to the start of the period, and it is set equal to the level *expected* to produce the *real* wage that equates labor supply and labor demand. Since workers and firms are assumed to have a real wage target in mind, the nominal wage will adjust fully to reflect expectations of price level changes held at the time the nominal wage is set. This means that the information available at the time the wage is set, and on which expectations are based, will be important. If unanticipated changes in prices occur, the actual real wage will differ from its expected value. In the standard formulation, firms are assumed to determine employment on the basis of the actual, realized real wage. If prices are unexpectedly low, the actual real wage will exceed the level expected to clear the labor market, and firms will reduce employment.<sup>6</sup>

The equilibrium level of employment and the real wage with flexible prices can be obtained by equating labor supply and labor demand (from (7.5) and (7.3)) and then using the production function (7.1) and the resource constraint (7.2) to obtain

$$n_t^* = \left[ \frac{1 - \Phi}{1 + \bar{\eta} + (1 - \alpha)(\Phi - 1)} \right] e_t = b_0 e_t,$$

$$\omega_t^* = \left[ \frac{\bar{\eta} + \Phi}{1 + \bar{\eta} + (1 - \alpha)(\Phi - 1)} \right] e_t = b_1 e_t,$$

where  $n^*$  is the flexible-wage equilibrium employment,  $\omega^*$  is the flexible-wage equilibrium real wage, and  $\bar{\eta} \equiv \eta n^{ss} / (1 - n^{ss})$ .

The contract nominal wage  $w^c$  satisfies

$$w_t^c = E_{t-1} \omega_t^* + E_{t-1} p_t. \quad (7.9)$$

With firms equating the marginal product of labor to the actual real wage, actual employment equals  $n_t = y_t - (w_t^c - p_t) = y_t - E_{t-1} \omega_t^* + (p_t - E_{t-1} p_t)$ , or using the production

5. This is stronger than the property of monetary superneutrality, in which the real variables are independent of the money supply process. For example, Lucas's model does not display the classical dichotomy as long as  $\Omega_2 \neq 0$  because the production function, the resource constraint, and the labor supply condition cannot be solved for output, consumption, and employment without knowing the real demand for money, since real balances directly affect the marginal utility of consumption.

6. This implies that the real wage falls in response to a positive money shock. Using a VAR approach based on U.S. data, Christiano, Eichenbaum, and Evans (1997) found that an expansionary monetary policy shock actually leads to a slight increase in real wages.

function and noting that  $E_{t-1}\omega_t^* = -\alpha E_{t-1}n_t^* + E_{t-1}e_t$ ,

$$n_t = E_{t-1}n_t^* + \left(\frac{1}{\alpha}\right)(p_t - E_{t-1}p_t) + \left(\frac{1}{\alpha}\right)\varepsilon_t, \quad (7.10)$$

where  $\varepsilon_t = (e_t - E_{t-1}e_t)$ . Equation (7.10) shows that employment deviates from the expected flexible-price equilibrium level in the face of unexpected movements in prices. An unanticipated increase in prices reduces the real value of the contract wage and leads firms to expand employment. An unexpected productivity shock  $\varepsilon_t$  raises the marginal product of labor and leads to an employment increase.

By substituting (7.10) into the production function, one obtains

$$y_t = (1 - \alpha) \left[ E_{t-1}n_t^* + \left(\frac{1}{\alpha}\right)(p_t - E_{t-1}p_t) + \left(\frac{1}{\alpha}\right)\varepsilon_t \right] + e_t,$$

which implies that

$$y_t - E_{t-1}y_t^* = a(p_t - E_{t-1}p_t) + (1 + a)\varepsilon_t, \quad (7.11)$$

where  $E_{t-1}y_t^* = (1 - \alpha)E_{t-1}n_t^* + E_{t-1}e_t$  is expected equilibrium output under flexible wages and  $a = (1 - \alpha)/\alpha$ . Innovations to output are positively related to price innovations. Thus, monetary shocks that produce unanticipated price movements directly affect real output.

The linear approximation to the MIU model, augmented with one-period nominal wage contracts, produces one of the basic frameworks often used to address policy issues. This framework generally assumes serially uncorrelated disturbances, so  $E_{t-1}y_t^* = 0$  and the aggregate supply equation (7.11), often called a *Lucas supply function* (see chapter 6), becomes

$$y_t = a(p_t - E_{t-1}p_t) + (1 + a)\varepsilon_t. \quad (7.12)$$

The demand side often consists of a simple quantity equation of the form

$$m_t - p_t = y_t. \quad (7.13)$$

This model can be obtained from the model of the chapter appendix by letting  $b \rightarrow \infty$ ; this implies that the interest elasticity of money demand goes to zero. According to (7.12), a 1 percent deviation of  $p$  from its expected value causes a  $(1 - \alpha)/\alpha \approx 1.8$  percent deviation of output if the benchmark value of 0.36 is used for  $\alpha$ . To solve the model for equilibrium output and the price level, given the nominal quantity of money, note that (7.13) and (7.8) imply

$$p_t - E_{t-1}p_t = m_t - E_{t-1}m_t - (y_t - E_{t-1}y_t) = s_t - y_t.$$

Substituting this result into (7.12), one obtains

$$y_t = \left(\frac{a}{1 + a}\right)s_t + \left(\frac{1 + a}{1 + a}\right)\varepsilon_t = (1 - \alpha)s_t + \varepsilon_t. \quad (7.14)$$

A 1 percent money surprise increases output by  $1 - \alpha \approx 0.64$  percent. Notice that in (7.12), the coefficient  $a$  on price surprises depends on parameters of the production function. This is in contrast to Lucas's misperceptions model, in which the impact on output of a price surprise depends on the variances of shocks (see section 5.2.2). The model consisting of (7.12) and (7.13) plays an important role in the analysis of monetary policy in (see chapter 6).

When (7.13) is replaced with an interest-sensitive demand for money, the systematic behavior of the money supply can matter for the real effects of money surprises. For example, if money is positively serially correlated ( $\rho_m > 0$ ), a positive realization of  $s_t$  implies that the money supply will be higher in the future as well. This leads to increases in  $E_t p_{t+1}$ , expected inflation, and the nominal rate of interest. The rise in the nominal rate of interest reduces the real demand for money today, causing, for a given shock  $s_t$ , a larger increase in the price level today than occurs when  $\rho_m = 0$ . This means the price surprise today is larger and implies that the real output effect of  $s_t$  will be increasing in  $\rho_m$ .<sup>7</sup>

Benassy (1995) showed how one-period wage contracts affect the time series behavior of output in a model similar to the one used here but in which capital is not ignored. However, the dynamics associated with consumption smoothing and capital accumulation are inadequate on their own to produce anything like the output persistence that is revealed by the data.<sup>8</sup> That is why real business cycle models assume that the productivity disturbance itself is highly serially correlated. Because it is assumed here that nominal wages are fixed for only one period, the estimated effects of a monetary shock on output die out almost completely after one period.<sup>9</sup> This would continue to be the case even if the money shock were serially correlated. While serial correlation in the  $s_t$  shock would affect the behavior of the price level, this will be incorporated into expectations, and the nominal wage set at the start of  $t + 1$  will adjust fully to make the expected real wage (and therefore employment and output) independent of the predictable movement in the price level. Just adding one-period sticky nominal wages will not capture the persistent effects of monetary shocks, but it will significantly influence the effect of a money shock on the economy.

## 7.2.2 Early Models of Intertemporal Nominal Adjustment

Modern models of price and wage rigidities emphasize that both adjust over time in a process that requires several periods for the adjustment to macroeconomic disturbances to be completed. Prices and/or wages adjust gradually over several periods. Two models consistent with such behavior are discussed here.

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7. See problem 1 at the end of this chapter. I thank Henrik Jensen for pointing out this effect of systematic policy.

8. Cogley and Nason (1995) demonstrated this for standard real business cycle models.

9. With Benassy's model and parameters ( $\alpha = 0.40$  and  $\delta = 0.019$ ), equilibrium output (expressed as a deviation from trend) is given by  $y_t \approx 0.6 \times (1 + .006L - .002L^2 \dots)(m_t - m_t^e)$ , so that the effects of a money surprise die out almost immediately (Benassy 1995, eq. 51, p. 313).

### Taylor's Model of Staggered Nominal Adjustment

One of the first models of nominal rigidities that also assumed rational expectations is due to Taylor (1979).<sup>10</sup> Because his model was originally developed in terms of nominal wage-setting behavior, that approach is followed here. Prices are assumed to be a constant markup over wage costs, so the adjustment of wages translates directly into a model of the adjustment of prices.

Assume wages are set for two periods, with one half of all contracts negotiated each period. Let  $x_t$  equal the log contract wage set at time  $t$ . The average wage faced by the firm is equal to  $w_t = (x_t + x_{t-1})/2$  since in period  $t$ , contracts set in the previous period ( $x_{t-1}$ ) are still in effect. Assuming a constant markup, the log price level is given by  $p_t = w_t + \mu$ , where  $\mu$  is the log markup. For convenience, normalize so that  $\mu = 0$ .

For workers covered by the contract set in period  $t$ , the average expected real wage over the life of the contract is  $\frac{1}{2}[(x_t - p_t) + (x_t - E_t p_{t+1})] = x_t - \frac{1}{2}(p_t + E_t p_{t+1})$ .<sup>11</sup> Taylor (1980) assumed the expected average real contract wage to be increasing in the level of economic activity, represented by log output:

$$x_t = \frac{1}{2}(p_t + E_t p_{t+1}) + k y_t. \quad (7.15)$$

With  $p_t = 0.5(x_t + x_{t-1})$ ,

$$\begin{aligned} p_t &= \frac{1}{2} \left[ \frac{1}{2}(p_t + E_t p_{t+1}) + k y_t + \frac{1}{2}(p_{t-1} + E_{t-1} p_t) + k y_{t-1} \right] \\ &= \frac{1}{4} [2p_t + E_t p_{t+1} + p_{t-1} + \eta_t] + \frac{k}{2} (y_t + y_{t-1}), \end{aligned}$$

where  $\eta_t \equiv E_{t-1} p_t - p_t$  is an expectational error term. Rearranging,

$$p_t = \frac{1}{2} p_{t-1} + \frac{1}{2} E_t p_{t+1} + k (y_t + y_{t-1}) + \frac{1}{2} \eta_t. \quad (7.16)$$

The basic Taylor specification leads to inertia in the aggregate price level. The value of  $p_t$  is influenced both by expectations of future prices and by the price level in the previous period.

Expressed in terms of the rate of inflation  $\pi_t = p_t - p_{t-1}$ , (7.16) implies

$$\pi_t = E_t \pi_{t+1} + 2k (y_t + y_{t-1}) + \eta_t. \quad (7.17)$$

The key implication of (7.17) is that while prices display inertia, the inflation rate need not exhibit inertia, that is, it depends on expected future inflation but not on past inflation. This is important, as can be seen by considering the implications of Taylor's model for a

10. See also Taylor (1980; 2016).

11. It would be more appropriate to assume that workers care about the present discounted value of the real wage over the life of the contract. This would lead to a specification of the form  $0.5(1 + \beta)x_t - 0.5(p_t + \beta E_t p_{t+1})$  for  $0 < \beta < 1$ , where  $\beta$  is a discount factor.

policy of disinflation. Suppose that the economy is in an initial, perfect-foresight equilibrium with a constant inflation rate  $\pi_1$ . Now suppose that in period  $t - 1$  the policymaker announces a policy that will lower the inflation rate to  $\pi_2$  in period  $t$  and then maintain inflation at this new lower rate. Using (7.17) and the definition of  $\eta_t$ , it can be shown that this disinflation has no impact on total output. As a consequence, inflation can be costlessly reduced. The price level is sticky in Taylor's specification, but the rate at which it changes, the rate of inflation, is not. The backward-looking aspect of price behavior causes unanticipated reductions in the *level* of the money supply to cause real output declines. Prices set previously are too high relative to the new path for the money supply; only as contracts expire can their real value be reduced to levels consistent with the new lower money supply. However, as Ball (1994) showed, price rigidities based on such backward-looking behavior need not imply that policies to reduce inflation by reducing the *growth rate* of money will cause a recession. Since  $m$  continues to grow, just at a slower rate, the real value of preset prices continues to be eroded, unlike the case of a level reduction in  $m$ .<sup>12</sup>

### Quadratic Costs of Price Changes

Rotemberg (1982) modeled the sluggish adjustment of prices by assuming firms faced quadratic costs of making price changes. Unlike the Taylor model, the Rotemberg model assumed all firms could adjust their price each period, but because of the adjustment costs, they would only close partially any gap between their current price and the "optimal" price.

Suppose, for example, that the desired price of firm  $j$  depends on the aggregate average price level and a measure of real economic activity. As with the sticky-information model of chapter 5, assume the firm's desired price in log terms is given by

$$p_t^*(j) = p_t + \alpha x_t. \quad (7.18)$$

Furthermore, assume profits are a decreasing quadratic function of the deviation of the firm's actual log price from  $p_t^*(j)$ :

$$\Pi_t(j) = -\delta [p_t(j) - p_t^*(j)]^2 = -\delta [p_t(j) - p_t - \alpha x_t]^2.$$

The costs of adjusting price are also quadratic and equal to

$$c_t(j) = \phi [p_t(j) - p_{t-1}(j)]^2.$$

Each period, firm  $j$  chooses  $p_t(j)$  to maximize

$$\sum_{i=0}^{\infty} \beta^i \mathbb{E}_t [\Pi_{t+i}(j) - c_{t+i}(j)].$$

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12. For example, when the policy to reduce inflation from  $\pi_1$  to  $\pi_2$  is announced in period  $t - 1$ ,  $\mathbb{E}_{t-1}\pi_t$  falls. For a given level of output, this decline would reduce  $\pi_{t-1}$ . If the policymaker acts to keep inflation at the time of the announcement (i.e.,  $\pi_{t-1}$ ) unchanged, output must rise.

The first-order condition for the firm's problem is

$$-\delta [p_t(j) - p_t^*(j)] - \phi [p_t(j) - p_{t-1}(j)] + \beta\phi [E_t p_{t+1}(j) - p_t(j)] = 0.$$

Since all firms are identical,  $p_t(j) = p_t(s) = p_t$ , and one can rewrite this first-order condition in terms of inflation as

$$\pi_t = \beta E_t \pi_{t+1} + \left( \frac{\alpha\delta}{\phi} \right) x_t. \quad (7.19)$$

Actual inflation depends on the real activity variable  $x_t$  and expected future inflation. Because firms are concerned with their price relative to other firms' prices, and they recognize that future price changes are costly, the price they set at time  $t$  is higher if they anticipate higher inflation in the future. The expression for inflation given by (7.19) is very similar to those obtained from other models of price stickiness, particularly in the role given to expected future inflation.<sup>13</sup> Ireland (1997a; 2001) estimated general equilibrium models of inflation and output based on quadratic costs of adjusting prices.<sup>14</sup>

However, while the assumption that firms face quadratic costs of adjusting prices provides a very tractable specification that leads to a simple expression for inflation, the quadratic cost formulation has not been as widely used as the models discussed in section 7.2.4. The more common approach has been to imbed sticky prices into an explicit model of monopolistic competition, to assume not all firms adjust prices each period, and consequently, to allow prices to differ across firms. In contrast, the quadratic cost model, in its basic form, assumes all firms adjust prices every period and so set the same price. The microeconomic evidence discussed in section 7.3.1 is not consistent with models in which all firms adjust prices every period.

### 7.2.3 Imperfect Competition

A common argument is that nominal rigidities arise because of small *menu costs*, essentially fixed costs, associated with changing wages or prices. As economic conditions change, a firm's *optimal* price will also change, but if there are fixed costs of changing prices, it may not be optimal for the firm to adjust its price continuously to economic changes. Only if the firm's actual price diverges sufficiently from the equilibrium price will it be worthwhile to bear the fixed cost and adjust prices. The macroeconomic implications of menu cost models were first explored by Akerlof and Yellen (1985) and Mankiw (1985) and were surveyed by Romer (2012). Ball and Romer (1991) showed how small menu costs can interact with imperfect competition in either goods or labor markets to amplify

13. This is a point made by Roberts (1995).

14. Ireland (2001) also introduced quadratic costs of changing the inflation rate to capture the idea that inflation might be sticky. His empirical results supported the hypothesis that prices are sticky but inflation is not. However, in his model, the persistence of inflation observed in the data is attributed to persistence of the exogenous shocks rather than to large costs of adjustment.

the impact of monetary disturbances, create strategic complementarities, and lead potentially to multiple equilibria. While menu costs rationalize sluggish price-setting behavior, such costs may seem implausible as the reason monetary disturbances have significant real effects. After all, adjusting production is also costly, and it is difficult to see why shutting down an assembly line is less costly than reprinting price catalogs. And computers have lowered the cost of changing prices for most retail establishments, though it seems unlikely that this has had an important effect on the ability of monetary authorities to have short-run real effects on the economy. Money seems to matter in important ways because of nominal rigidities, but there is no completely satisfactory integration of microeconomic models of nominal adjustment with monetary models of macroeconomic equilibrium.

A problem with simply introducing nominal price or wage rigidity into an otherwise competitive model is that any sort of nominal rigidity naturally raises the question of who is setting wages and prices, a question the perfectly competitive model begs. To address the issue of price setting, one must examine models that incorporate some aspect of imperfect competition, such as monopolistic competition.

### A Basic Model of Monopolistic Competition

To explore the implications of nominal rigidities, a basic model that incorporates monopolistic competition among intermediate goods producers is developed. Examples of similar models include Blanchard and Kiyotaki (1987), Ball and Romer (1991), Beaudry and Devreux (1995), and King and Watson (1996). Imperfect competition can lead to aggregate demand externalities (Blanchard and Kiyotaki 1987), equilibria in which output is inefficiently low, and multiple equilibria (Ball and Romer 1991; Rotemberg and Woodford 1995), but imperfect competition alone does not lead to monetary non-neutrality. If prices are free to adjust, one-time permanent changes in the level of the money supply induce proportional changes in all prices, leaving the real equilibrium unaffected. Price stickiness remains critical to generating significant real effects of money. The present example follows Chari, Kehoe, and McGrattan (2000), and in the following section, price stickiness is added by assuming that intermediate goods producers engage in multiperiod staggered price setting.

Let  $Y_t$  be the output of the final good; it is produced using inputs of the intermediate goods according to

$$Y_t = \left[ \int Y_t(i)^q di \right]^{\frac{1}{q}}, \quad 0 < q \leq 1, \quad (7.20)$$

where  $Y_t(i)$  is the input of intermediate good  $i$ . Firms producing final goods operate in competitive output markets and maximize profits given by  $P_t Y_t - \int P_t(i) Y_t(i) di$ , where  $P_t$  is the price of final output and  $P_t(i)$  is the price of input  $i$ . The first-order conditions



for profit maximization by final goods producers yield the following demand function for intermediate good  $i$ :

$$Y_t^d(i) = \left[ \frac{P_t}{P_t(i)} \right]^{\frac{1}{1-q}} Y_t. \quad (7.21)$$

Final goods firms earn zero profit as long as

$$P_t = \left[ \int P_t(i)^{\frac{q}{q-1}} di \right]^{\frac{q-1}{q}}.$$

Each intermediate good is produced according to a constant returns to scale, Cobb-Douglas production function:

$$Y_t(i) = K_t(i)^a L_t(i)^{1-a}, \quad (7.22)$$

where  $K$  and  $L$  denote capital and labor inputs purchased in competitive factor markets at prices  $r$  and  $W$ . The producer of good  $Y(i)$  chooses  $P(i)$ ,  $K(i)$ , and  $L(i)$  to maximize profits subject to the demand function (7.21) and the production function (7.22). Intermediate profits are equal to

$$\begin{aligned} \Pi_t(i) &= P_t(i)Y_t(i) - r_t K_t(i) - W_t L_t(i) \\ &= [P_t(i) - P_t V_t] \left[ \frac{P_t}{P_t(i)} \right]^{\frac{1}{1-q}} Y_t, \end{aligned} \quad (7.23)$$

where  $V_t$  is equal to minimized unit costs of production (so  $P_t V_t$  is nominal unit cost). The first-order condition for the value of  $P_t(i)$  that maximizes profits for the intermediate goods producing firm is

$$\left[ \frac{P_t}{P_t(i)} \right]^{\frac{1}{1-q}} Y_t - \frac{1}{1-q} [P_t(i) - P_t V_t] \left[ \frac{P_t}{P_t(i)} \right]^{\frac{2-q}{1-q}} \left( \frac{1}{P_t} \right) Y_t = 0.$$

After some rearranging, this yields

$$P_t(i) = \frac{P_t V_t}{q}. \quad (7.24)$$

Thus, the price of intermediate good  $i$  is set as a constant markup  $1/q$  over unit nominal costs  $PV$ .

For the intermediate goods producers, labor demand involves setting

$$\frac{W_t}{P_t(i)} = q \left[ \frac{(1-\alpha)Y_t(i)}{L_t(i)} \right], \quad (7.25)$$

where  $W_t$  is the nominal wage rate and  $(1 - \alpha)Y_t(i)/L_t(i)$  is the marginal product of labor. In a symmetric equilibrium, all intermediate firms charge the same relative price, employ the same labor and capital inputs, and produce at the same level, so  $P_t(i) = P_t(j) = P_t$ , and (7.25) implies

$$L_t = \frac{q(1 - \alpha)Y_t}{W_t/P_t}. \quad (7.26)$$

Firms will be concerned with their relative price, not the absolute price level, so money remains neutral. As (7.25) and (7.26) show, proportional changes in all nominal prices (i.e.,  $P(i)$ ,  $P$ , and  $W$ ) leave firm  $i$ 's optimal relative price and aggregate labor demand unaffected. If the household's decision problem is not altered from the earlier analysis, consumption, labor supply, and investment decisions would not be altered by proportional changes in all nominal prices and the nominal stock of money.<sup>15</sup>

To complete the specification of the model, the aggregate demand for labor given by (7.26) must be equated to the aggregate labor supply derived from the outcome of household choices. In the flexible-price models examined so far, labor market equilibrium with competitive factor markets require that the marginal rate of substitution between leisure and consumption be equal to the real wage, which in turn is equal to the marginal product of labor. With imperfect competition, (7.26) shows that  $q$  drives a wedge between the real wage and the marginal product of labor.<sup>16</sup> Thus, labor market equilibrium requires that

$$\frac{U_l}{U_c} = \frac{W}{P} = q\text{MPL} \leq \text{MPL}. \quad (7.27)$$

If we now linearize the model around the steady state,  $q$  drops out of the labor market equilibrium condition because of the way in which it enters multiplicatively.

The example of a model of monopolistic competition assumes flexible prices (and wages). The basic structure of this example is now used to explore alternative models of nominal rigidities.

## 7.2.4 Time-Dependent Pricing (TDP) Models

An important class of models treats the adjustment of prices (and wages) as depending on time but not on the state of the economy. That is, they assume the probability a firm adjusts its price does not depend on whether there have been big shocks since the

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15. The household's budget constraint is altered, since real profits of the intermediate goods producers must be paid out to households. However, as (7.23) shows, nominal profits are homogeneous of degree 1 in prices, so their real value will be homogeneous of degree 0. Thus, proportional changes in the nominal money stock and all prices leave the household's budget constraint unaffected.

16. In their calibrations, Chari, Kehoe, and McGrattan used a value of 0.9 for  $q$ .

price was last changed or whether inflation has been high or low since the last adjustment. Instead, this probability may depend simply on how many periods since the firm last adjusted its price, or the probability of adjustment might be the same, regardless of how long it has been since a price change or how economic conditions may have changed. The Taylor model discussed earlier is an example of a time-dependent pricing (TDP) model. Time-dependent models of price adjustment are more tractable than models in which the decision to change price depends on the state of the economy. As a consequence, time-dependent models are very popular and are the basis of most models employed for policy analysis.<sup>17</sup>

### The Taylor Model Revisited

Taylor (1979; 1980) argued that the presence of multiperiod nominal contracts, with only a fraction of wages or prices negotiated each period, could generate the type of real output persistence in response to monetary shocks observed in the data. When setting a price during period  $t$  that will remain in effect for several periods, a firm bases its decisions on its expectations of conditions in future periods. But the aggregate price level also depends on those prices set in earlier periods that are still in effect. This imparts both forward-looking and backward-looking aspects to the aggregate price level and, as Taylor showed, provides a framework capable of replicating aggregate dynamics.

To develop a simple example based on Chari, Kehoe, and McGrattan (2000) and their model of monopolistic competition (see section 7.2.3), suppose that each intermediate goods producing firm sets its price  $P(i)$  for two periods, with half of all firms adjusting in each period.<sup>18</sup> Thus, if  $i \in [0, 0.5)$ , assume that  $P(i)$  is set in period  $t, t + 2, t + 4$ , and so on. If  $i \in [0.5, 1]$ , the firm sets prices in periods  $t + 1, t + 3$ , and so on. Since only symmetric equilibria are considered in which all firms setting prices at time  $t$  pick the same price, one can drop the index  $i$  and let  $\bar{P}_{t+j}$  denote the intermediate goods price set in period  $t + j$  for periods  $t + j$  and  $t + j + 1$ .

Consider a firm  $i$  setting its price in period  $t$ . This price will be in effect for periods  $t$  and  $t + 1$ . Thus, if  $R_t$  is the gross interest rate,  $\bar{P}_t$  will be chosen to maximize

$$E_t \left[ \left( \bar{P}_t - P_t V_t \right) \left( \frac{P_t}{\bar{P}_t} \right)^{\frac{1}{1-q}} Y_t + R_t^{-1} \left( \bar{P}_t - P_{t+1} V_{t+1} \right) \left( \frac{P_{t+1}}{\bar{P}_t} \right)^{\frac{1}{1-q}} Y_{t+1} \right],$$

17. The focus of this section is on models of price adjustment; similar models have been applied to explain the adjustment of nominal wages. Chapter 8 examines the implications of incorporating both sticky prices and sticky wages into a general equilibrium model.

18. This is a form of time-dependent pricing; prices are set for a fixed length of time regardless of economic conditions.

which represents the expected discounted profits over periods  $t$  and  $t + 1$ .<sup>19</sup> After some manipulation of the first-order condition, one obtains

$$\bar{P}_t = \frac{E_t \left( P_t^\theta V_t Y_t + R_t^{-1} P_{t+1}^\theta V_{t+1} Y_{t+1} \right)}{q E_t \left( P_t^{1-q} Y_t + R_t^{-1} P_{t+1}^{1-q} Y_{t+1} \right)}, \quad (7.28)$$

where  $\theta = (2 - q)/(1 - q)$ . If prices are set for only one period, the terms involving  $t + 1$  drop out, and one obtains the earlier pricing equation (7.24).

What does (7.28) imply about aggregate price adjustment? Let  $\bar{p}$ ,  $p$ , and  $v$  denote percentage deviations of  $\bar{P}$ ,  $P$ , and  $V$  around a zero inflation steady state. If discounting is ignored for simplicity, (7.28) can be approximated in terms of percentage deviations around the steady state as

$$\bar{p}_t = \frac{1}{2} (p_t + E_t p_{t+1}) + \frac{1}{2} (v_t + E_t v_{t+1}). \quad (7.29)$$

The average price of the final good, expressed in terms of deviations from the steady state, is  $p_t = \frac{1}{2} (\bar{p}_{t-1} + \bar{p}_t)$ , where  $\bar{p}_{t-1}$  is the price of intermediate goods set at time  $t - 1$  and  $\bar{p}_t$  is the price set in period  $t$ . Similarly,  $E_t p_{t+1} = \frac{1}{2} (\bar{p}_t + \bar{p}_{t+1})$ . Substituting these expressions into the equation for  $\bar{p}_t$  yields

$$\bar{p}_t = \frac{1}{2} \bar{p}_{t-1} + \frac{1}{2} E_t \bar{p}_{t+1} + (v_t + E_t v_{t+1}).$$

This reveals the backward-looking (via the presence of  $\bar{p}_{t-1}$ ) and forward-looking (via the presence of  $E_t \bar{p}_{t+1}$  and  $E_t v_{t+1}$ ) nature of price adjustment.

The variable  $v_t$  is the deviation of minimized unit costs from its steady state. Suppose this is proportional to output:  $v_t = \gamma y_t$ .<sup>20</sup> If one further assumes a simple money demand equation of the form  $m_t - p_t = y_t$ , then

$$\begin{aligned} \bar{p}_t &= \frac{1}{2} \bar{p}_{t-1} + \frac{1}{2} E_t \bar{p}_{t+1} + \gamma (y_t + E_t y_{t+1}) \\ &= \frac{1}{2} \bar{p}_{t-1} + \frac{1}{2} E_t \bar{p}_{t+1} + \gamma (m_t - p_t + E_t m_{t+1} - E_t p_{t+1}) \\ &= \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (\bar{p}_{t-1} + E_t \bar{p}_{t+1}) + \left( \frac{\gamma}{1 + \gamma} \right) (m_t + E_t m_{t+1}). \end{aligned} \quad (7.30)$$

19. Chari, Kehoe, and McGrattan (2000) considered situations in which a fraction  $1/N$  of all firms set prices each period for  $N$  periods. They could then vary  $N$  to examine its role in affecting aggregate dynamics. They altered the interpretation of the time period so that  $N$  always corresponds to one year; thus, varying  $N$  alters the degree of staggering. They concluded that  $N$  has little effect.

20. The coefficient  $\gamma$  will depend on the elasticity of labor supply with respect to the real wage. See Chari, Kehoe, and McGrattan (2000).

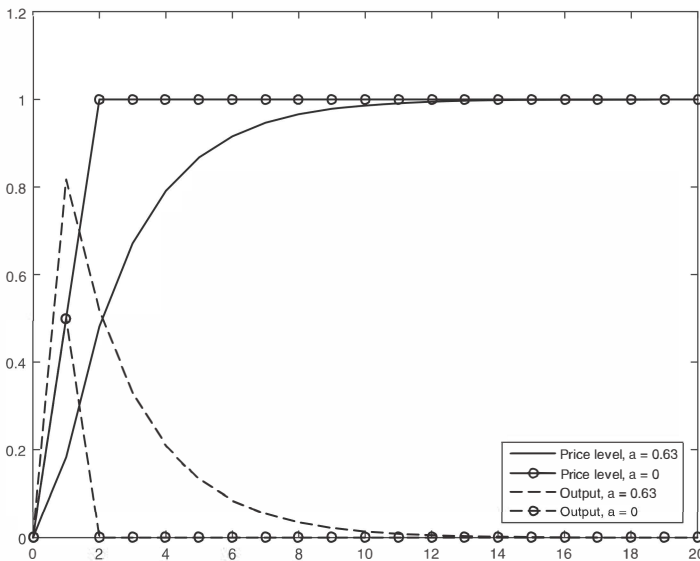
This is a difference equation in  $\bar{p}$ . It implies that the behavior of prices set during period  $t$  will depend on prices set during the previous period, on prices expected to be set during the next period, and on the path of the nominal money supply over the two periods during which  $\bar{p}_t$  will be in effect. For the case in which  $m_t$  follows a random walk (so that  $E_t m_{t+1} = m_t$ ), the solution for  $\bar{p}_t$  is

$$\bar{p}_t = a\bar{p}_{t-1} + (1-a)m_t, \quad (7.31)$$

where  $a = (1 - \sqrt{\gamma}) / (1 + \sqrt{\gamma})$  is the root less than 1 of  $a^2 - 2(1 + \gamma)a / (1 - \gamma) + 1 = 0$ .<sup>21</sup> Since the aggregate price level is an average of prices set at  $t$  and  $t - 1$ ,

$$p_t = ap_{t-1} + \frac{1}{2}(1-a)(m_t + m_{t-1}). \quad (7.32)$$

Taylor (1979; 1980) demonstrated that a price adjustment equation of the form given by (7.32) is capable of mimicking the dynamic response of U.S. prices.<sup>22</sup> The response, however, depends critically on the value of  $a$  (which, in turn, depends on  $\gamma$ ). Figure 7.1 shows the response of the price level and output for  $\gamma = 1$  ( $a = 0$ ) and  $\gamma = 0.05$  ( $a = 0.63$ ).



**Figure 7.1**

Effects of a money shock with staggered price adjustment.

21. See problem 6 at the end of this chapter.

22. Taylor's actual model was based on nominal wage adjustment rather than on price adjustment as presented here.

This latter value is the one Taylor found matches U.S. data, and, as the figure shows, an unexpected permanent increase in the nominal money supply produces a rise in output with a slow adjustment back to the baseline, mirrored by a gradual rise in the price level. Though the model assumes that prices are set for only two periods, the money shock leads to a persistent, long-lasting effect on output with this value of  $\gamma$ .

Chari, Kehoe, and McGrattan (2000) assumed that employment must be consistent with household labor supply choices, and they showed that  $\gamma$  is a function of the parameters of the representative agent's utility function. They argued that a very high labor supply elasticity is required to obtain a value of  $\gamma$  on the order of 0.05. With a low labor supply elasticity, as seems more plausible,  $\gamma$  will be greater than or equal to 1. If  $\gamma = 1$ ,  $a = 0$ , and for this value the figure suggests the Taylor model is not capable of capturing realistic adjustment to monetary shocks. Ascari (2000) reached similar conclusions in a model that is similar to the framework in Chari, Kehoe, and McGrattan but that follows Taylor's original work in making wages sticky rather than prices. However, rather than drawing the implication that staggered price (or wage) adjustment is unimportant for price dynamics, the assumption that observed employment is consistent with the labor supply behavior implied by the model of the household can be questioned. Models that interpret observed employment as tracing out a labor supply function typically have difficulty matching other aspects of labor market behavior (Christiano and Eichenbaum 1992a).

### Calvo's Model

An alternative model of staggered price adjustment is due to Calvo (1983). He assumed that firms adjust their prices infrequently, and that opportunities to adjust arrive as an exogenous Poisson process. Each period, there is a constant probability  $1 - \omega$  that the firm can adjust its price; the expected time between price adjustments is  $1/(1 - \omega)$ . Because these adjustment opportunities occur randomly, the interval between price changes for an individual firm is a random variable.

The popularity of the Calvo specification is due, in part, to its tractability. This arises from two aspects of the model. First, all firms that adjust their price at time  $t$  set the same prices. And since the firms that do not adjust represent a random sample of all firms, the average price of the firms that do not adjust is simply  $P_{t-1}$ , last period's average price across all firms. Thus, rather than needing to keep track of the prices of firms that do not adjust, one only needs to know the average price level in the previous period.

When firm  $i$  has an opportunity to reset its price, it will do so to maximize the expected present discounted value of profits,

$$E_t \sum_{j=0}^{\infty} \beta^j \pi_{t+j}(i) = E_t \sum_{j=0}^{\infty} \beta^j [P_{t+j}(i) - P_{t+j} V_{t+j}] \left[ \frac{P_{t+j}}{P_{t+j}(i)} \right]^{\frac{1}{1-q}} Y_{t+j}, \quad (7.33)$$

where  $V_t$  is the real marginal cost of production, and the demand curve faced by the individual firm, (7.21), has been used. All adjusting firms are the same, so each chooses the

same price to maximize profits subject to the assumed process for determining when the firm will next be able to adjust. Let  $P_t^*$  denote the optimal price. If only the terms in (7.33) involving the price set at time  $t$  are written out, they are

$$\begin{aligned} & [P_t^* - P_t V_t] \left[ \frac{P_t}{P_t^*} \right]^{\frac{1}{1-q}} Y_t + \omega \beta E_t [P_t^* - P_{t+1} V_{t+1}] \left[ \frac{P_{t+1}}{P_t^*} \right]^{\frac{1}{1-q}} Y_{t+1} \\ & + \omega^2 \beta^2 E_t [P_t^* - P_{t+2} V_{t+2}] \left[ \frac{P_{t+2}}{P_t^*} \right]^{\frac{1}{1-q}} Y_{t+2} + \dots, \end{aligned}$$

or

$$\sum_{j=0}^{\infty} \omega^j \beta^j E_t [P_t^* - P_{t+j} V_{t+j}] \left[ \frac{P_{t+j}}{P_t^*} \right]^{\frac{1}{1-q}} Y_{t+j},$$

since  $\omega^j$  is the probability that the firm has not adjusted after  $j$  periods so that the price set at  $t$  still holds in  $t+j$ . Thus, the first-order condition for the optimal choice of  $P_t^*$  requires that

$$\left( \frac{q}{1-q} \right) E_t \sum_{j=0}^{\infty} \omega^j \beta^j \left[ \left( \frac{P_t^*}{P_{t+j}} \right) - \left( \frac{1}{q} \right) V_{t+j} \right] \left( \frac{1}{P_t^*} \right) \left( \frac{P_{t+j}}{P_t^*} \right)^{\frac{1}{1-q}} Y_{t+j} = 0, \quad (7.34)$$

which can be rearranged to yield

$$\left( \frac{P_t^*}{P_t} \right) = \left( \frac{1}{q} \right) \frac{E_t \sum_{j=0}^{\infty} \omega^j \beta^j V_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\frac{1}{1-q}} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \omega^j \beta^j \left( \frac{P_{t+j}}{P_t} \right)^{\frac{q}{1-q}} Y_{t+j}}. \quad (7.35)$$

To interpret (7.35), note that if prices are perfectly flexible ( $\omega = 0$ ), then

$$\left( \frac{P_t^*}{P_t} \right) = \left( \frac{1}{q} \right) V_t,$$

and the firm desires to set its real price as a constant markup over real marginal cost. Since all firms set the same price,  $P_t^* = P_t$  in an equilibrium with flexible prices, and real marginal cost is equal to  $q$ . When  $\omega > 0$  so that not all firms adjust each period, a firm that can adjust will take into account expected future marginal costs when setting its price. The more rigid prices are (the larger is  $\omega$ ), the more pricing decisions are based on expected future marginal costs, since firms expect more time to pass before having another opportunity to adjust.<sup>23</sup>

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23. For a firm that can adjust its price, the expected number of periods it must wait before adjusting again is one with probability  $1 - \omega$ , two with probability  $\omega(1 - \omega)$ , three with probability  $\omega^2(1 - \omega)$ , etc. Hence, the expected duration between price changes is

$$(1 - \omega) + 2\omega(1 - \omega) + 3\omega^2(1 - \omega) + \dots = \frac{1}{1 - \omega}.$$

With a large number of firms, a fraction  $1 - \omega$  will actually adjust their price each period, and the aggregate average price level can be expressed as a weighted average of the prices set by those firms that adjust and the average price of the firms that do not adjust. The latter, as previously noted, is  $P_{t-1}$ .

The Calvo model of sticky prices is commonly employed in the new Keynesian models that have come to dominate monetary policy analysis (see chapter 8). The chapter 8 appendix shows that the Calvo model, when approximated around a zero average inflation, steady-state equilibrium, yields an expression for aggregate inflation of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{v}_t, \quad (7.36)$$

where

$$\kappa = \frac{(1 - \omega)(1 - \beta\omega)}{\omega} \quad (7.37)$$

is an increasing function of the fraction of firms able to adjust each period, and  $\hat{v}_t$  is real marginal cost expressed as a percentage deviation around its steady-state value. Equation (7.36) is called the new Keynesian Phillips curve.

Comparing this to the inflation equation from Taylor's model, (7.17), shows them to be quite similar. Current inflation depends on expectations of future inflation and on current output. One difference is that in deriving an inflation equation based on Calvo's specification, expected future inflation has a coefficient equal to the discount factor  $\beta < 1$ . In deriving an expression for inflation using Taylor's specification, however, discounting was ignored in (7.15), the equation giving the value of the contract wage. A further difference between the Taylor model and the Calvo model was highlighted by Kiley (2002a). He showed that Taylor-type staggered adjustment models display less persistence than the Calvo-type partial adjustment model when both are calibrated to produce the same average frequency of price changes. Under the Taylor model, for example, suppose contracts are negotiated every two periods. The average frequency of wage changes is one-half—half of all wages adjust each period—and no wage remains fixed for more than two periods. In contrast, suppose  $\omega = 1/2$  in the Calvo model. The expected time between price changes is two periods, so on average prices are adjusted every two periods. However, many prices will remain fixed for more than two periods. For instance,  $\omega^3 = 0.125$  of all prices remain fixed for at least three periods. In general, the Calvo model implies that there is a tail of the distribution of prices that consists of prices that have remained fixed for many periods, while the Taylor model implies that no wages remain fixed for longer than the duration of the longest contract.

One attractive aspect of Calvo's model is that it shows how the coefficient on output in the inflation equation depends on the frequency with which prices are adjusted. A rise in  $\omega$ , which means that the average time between price changes for an individual firm



increases, causes  $\kappa$  in (7.36) to decrease. Output movements have a smaller impact on current inflation, holding expected future inflation constant. Because opportunities to adjust prices occur less often, current demand conditions become less important.

### 7.2.5 State-Dependent Pricing (SDP) Models

The Taylor and Calvo models assumed that pricing decisions were time-dependent. Recent research on nominal price adjustment has stressed the implications of state-dependent (STP) models of price adjustment. In contrast to the TDP models of Taylor or Calvo, the firms that adjust prices in a given period are not a random sample of all firms. Instead, the firms that choose to adjust are those for whom adjustment is most profitable. The implications of this important difference can be illustrated through a simple example.

Suppose half of all firms happen to have a price of 1 and half have a price of 3. The aggregate average price is 2. Assume also that the money supply is 2. Now suppose the money supply doubles to 4. Assume that conditional on adjusting, firms would choose a price of 4. If firms are chosen at random to adjust, half the firms with prices equal to 1 and 3 will set their price at 4, while the other half will not adjust. The aggregate price level will now be  $0.25 \times 1 + 0.25 \times 3 + 0.5 \times 4 = 3$ . Real money balances rise to  $4/3 = 1.33$ . Now instead of randomly choosing the firms that adjust, suppose it is the firms furthest from the optimal price (4, in this example) that adjust. If there is a small fixed cost of adjusting, all firms with a price of 1 might find it optimal to adjust to 4, while none of the firms with a price of 3 would adjust. The aggregate average price would then be  $0.5 \times 3 + 0.5 \times 4 = 3.5$ , and real money balances only rise to  $4/3.5 = 1.14$ . While simple, this example emphasizes how the effects of a change in the nominal money supply on the real supply of money can depend critically on whether firms adjust at random, as in the Calvo specification, or based on how far the firm's current price is from the optimal price. The fact that the firms that adjust are more likely to be those furthest from their desired price is called the *selection effect* by Golosov and Lucas Jr. (2007). This effect acts to make the aggregate price level more flexible than might be suggested by simply looking at the fraction of firms that change price.<sup>24</sup>

SDP models allow price behavior to be influenced by an intensive and an extensive margin: after a large shock, those firms that adjust will make, on average, bigger adjustments (this is the intensive margin), *and* more firms will adjust (this is the extensive margin).

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24. Caplin and Spulber (1987) were one of the first to demonstrate how the dynamic response of output to money would differ under SDP compared to TDP. They showed that SDP could restore monetary neutrality even in the presence of menu costs. Their model setup was similar to models used in transportation economics to address the following question: If adding traffic to a road increases wear and tear on the road, does average road quality decline with an increase in traffic? If road repairs are done on a fixed schedule (a time-dependent strategy), the answer is yes. If repair work is state-dependent, then an increase in traffic leads to more frequent scheduling of repair work and average quality may remain unchanged.

Beginning in the 1970s, a number of researchers examined the implications of state-dependent pricing models, but the focus here will be on the recent generation of SDP models.<sup>25</sup>

As noted earlier, SDP models are generally less tractable than TDP models, thus accounting for their less frequent use. And prior to the availability of microeconomic data on price changes (see section 7.3.1), TDP models were seen as adequate for modeling aggregate phenomena. SDP models are closely related to Ss models of inventory behavior; as long as the firm's price remains in a region close to the optimal price, no adjustment occurs, but whenever the price hits an upper (S) or lower (s) boundary of this region, the firm changes its price. Ss models have proven difficult to aggregate, so SDP models generally impose assumptions on the distribution of adjustment costs or the distribution of shocks to obtain tractable solutions.

### Dotsey, King, and Wolman Model

Dotsey, King, and Wolman (1999) assumed that firms face a fixed cost of price adjustment, that is, a cost independent of the size of the price change. They assumed, however, that this cost is stochastic and differs across firms and time. Each period firms receive a new realization of the cost, but expected future costs are the same for all firms, so each firm that does decide to adjust its price will choose the same price. The DKW model defines a vintage  $j$  firm as a firm that last adjusted its price  $j$  periods ago. Let  $\theta_{j,t}$  be the fraction of firms of vintage  $j$ . Since all vintage  $j$  firms adjust at the same time, they all have the same price. Among the firms of vintage  $j$ , there will be a critical fixed cost such that all firms with smaller fixed costs adjust and those with larger fixed costs do not. Let  $\alpha_{j,t}$  be the fraction of vintage  $j$  firms that adjust their price. Then, in period  $t + 1$ , the fraction of firms that become vintage  $j + 1$  is equal to  $1 - \alpha_{j,t}$  times the fraction who were of vintage  $j$  in period  $t$ :

$$\theta_{j+1,t+1} = (1 - \alpha_{j,t})\theta_{j,t} \equiv \omega_{j,t},$$

while the fraction of all firms who do adjust in period  $t$  is equal to

$$\omega_{0,t} = \sum_{j=1}^J \alpha_{j,t}\theta_{j,t},$$

where  $J$  is the maximum number of periods any firm has not adjusted its price.<sup>26</sup> Prices of each vintage  $j$  are weighted by  $\omega_{j,t}$  in forming the aggregate average price level.

25. Dotsey and King (2005) provided an overview of the aggregate implications of some of the earlier SDP models. Caballero and Engel (2007) argued that it is the presence of an extensive margin, not the selection effect, that accounts for the greater flexibility of the aggregate price level in SDP models.

26. In the Calvo model,  $\alpha_{j,t} = 1 - \omega$  for all  $j$  and  $t$ , and  $J = \infty$ . In the Taylor model,  $J$  is equal to the length of the longest contract.

Let  $v_{j,t}$  be the value function for a firm of vintage  $j$  at time  $t$ . Then, the value function for firms that do adjust their price at time  $t$ ,  $v_{0,t}$ , takes the form

$$v_{0,t} = \max_{P_t^*} \left\{ [P_t^* - P_t V_t] \left[ \frac{P_t}{P_t^*} \right]^{\frac{1}{1-q}} Y_t + \beta E_t (1 - \alpha_{1,t+1}) v_{1,t+1} \right. \\ \left. + \beta E_t \alpha_{1,t+1} v_{0,t+1} - \beta E_t \Xi_{1,t+1} \right\},$$

where current period profit is written as in (7.33), and  $E_t \Xi_{1,t+1}$  is the present value of next period's adjustment costs. Notice that with probability  $1 - \alpha_{1,t+1}$  the firm does not adjust at  $t + 1$  and so becomes a vintage 1 firm, while with probability  $\alpha_{1,t+1}$  the firm does adjust at  $t + 1$  and remains a vintage 0 firm.

For nonadjusting firms of vintage  $j$ ,

$$v_{j,t} = \left\{ [P_{t-j}^* - P_t V_t] \left[ \frac{P_t}{P_{t-j}^*} \right]^{\frac{1}{1-q}} Y_t + \beta E_t (1 - \alpha_{j+1,t+1}) v_{j+1,t+1} \right. \\ \left. + \beta E_t \alpha_{j+1,t+1} v_{0,t+1} - \beta E_t \Xi_{j+1,t+1} \right\},$$

since such firms optimally set their price in period  $t - j$ .

Suppose  $w_t \xi$  is the randomly distributed fixed cost of changing price expressed in terms of labor costs, where  $w_t$  is the wage. Then a vintage  $j$  firm will change its price if

$$v_{0,t} - v_{jt} \geq w_t \xi.$$

If  $G$  is the distribution function of the costs, then the fraction of vintage  $j$  firms that changes price is just the fraction of firms whose fixed cost realization is less than  $(v_{0,t} - v_{jt}) / w_t$ . Hence,

$$\alpha_{jt} = G \left( \frac{v_{0,t} - v_{jt}}{w_t} \right).$$

If the value of adjusting as measured by  $v_{0,t} - v_{jt}$  is high, more firms of the same vintage will pay the fixed cost to adjust. The expected adjustment costs next period for a vintage  $j$  firm are equal to the expected value of  $w_{t+1} \xi$ , conditional on  $\xi$  being less than or equal to  $(v_{0,t+1} - v_{j+1,t+1}) / w_{t+1}$ , so that firm finds it optimal to adjust. Thus,

$$\Xi_{j,t+1} = E_t \left( w_{t+1} \int_0^{G^{-1}(\alpha_{j+1,t+1})} \xi g(\xi) dx \right),$$

where  $g(\xi) = G'(\xi)$  is the density function of  $\xi$ .

Let current profits for firms that are adjusting at time  $t$  be denoted by  $\Pi_{0,t}$ ; then the first-order condition for optimal pricing, conditional on adjusting, is

$$\frac{\partial \Pi_{0,t}}{\partial P_t^*} + \beta E_t \left[ \frac{\partial (1 - \alpha_{1,t+1}) v_{1,t+1}}{\partial P_t^*} \right] = 0.$$

The impact of  $P_t^*$  on current profits is balanced against the effect on future profits, weighed by the probability the firm does not adjust next period. This probability is no longer fixed, as in the Calvo model, but is endogenous.

Dotsey and King (2005) compare the response of the price level and inflation to a monetary shock in the DKW model and in a model with fixed probabilities of adjustment. Interestingly, the two variants display similar responses for the first several periods after the shock. However, as the price level adjusts, more firms now find themselves with prices that are far from the optimal level. In the SDP model of DKW, this leads more firms to change their price, whereas in a Calvo model, the fraction of adjusting firms remains constant.

### Firm-Specific Shocks

Most models of price adjustment developed for use in macroeconomics have assumed that firms only face aggregate shocks. This generally implies that all firms that do adjust their price choose the same new price, as they all face the same (aggregate) shock. The DKW model features firm-specific shocks to the menu cost, but these shocks only influence whether a firm adjusts, not how much it changes prices. In contrast, Golosov and Lucas Jr. (2007) and Gertler and Leahy (2008) emphasized the role of idiosyncratic shocks in influencing which firms adjust prices and in generating a distribution of prices across firms.

### Gertler and Leahy's Ss Model

Gertler and Leahy (2008) developed an Ss model with monopolistically competitive firms located on separate islands, of which there exists a continuum of mass unity. Each island has a continuum of households that can supply labor only on the island on which they live. An island receives a productivity shock with probability  $1 - \alpha$ . These shocks affect all firms on the island but are independent across islands. There is, however, perfect consumption insurance, so consumption is the same across all islands, and firm profits are distributed to households via lump-sum transfers.

Suppose island  $z$  is hit by such a shock. Then, a randomly chosen fraction  $1 - \tau$  of the firms on the island disappear. These firms are replaced by new entrants to maintain a constant number of firms on the island. New entrants can optimally set their price. The surviving old firms (there are a fraction  $\tau$  of such firms) experience independent and identically distributed productivity shocks. These shocks are uniformly distributed.

Gertler and Leahy incorporated two types of fixed costs of adjusting. First, there is a decision cost. If the firm pays this cost, it can then decide whether to adjust its price. The cost can be thought of as capturing the time and effort necessary to evaluate the firm's pricing strategy. If this cost is paid, and the firm decides to adjust its price, then there is a fixed menu cost associated with changing price. Optimal pricing policy takes the form of an Ss rule.

An exact solution is not possible, but an approximate analytical solution can be obtained as a local expansion around a zero inflation steady state. Obtaining a log-linear approximation is made difficult because there is a discontinuity in the adjustment of firms near the Ss boundary. For a firm near the top that does not receive an idiosyncratic shock, an aggregate shock of one sign would push it to the barrier and result in an adjustment, while the same size aggregate shock with opposite sign would move the firm into the interior of the region of inaction. The role of the decision cost is to deal with this problem. Given the necessity of paying this cost, and if aggregate shocks are also small relative to idiosyncratic shocks, the  $\alpha$  fraction of firms that do not receive an idiosyncratic shock do not adjust price. This leads to smooth behavior around the boundaries of the Ss region.

All firms that change their price choose the same markup over real marginal cost, and they do so to ensure the expected (log) markup is equal to the steady-state markup:

$$E_t \sum_{i=0}^{\infty} (\alpha\beta)^i \ln \mu_{t+i} = \ln \bar{\mu},$$

where  $\bar{\mu}$  is the steady-state markup and  $\mu_{t+i}$  is the actual markup in period  $t+i$ .<sup>27</sup> Notice, that as in the Calvo model, the future expected markups are discounted by  $\beta$  and the probability of not adjusting  $\alpha$ , since only if the firm has not adjusted will future markups be influenced by the current pricing choice.

Aggregating across islands, Gertler and Leahy showed that the economywide inflation rate is given by

$$\pi_t = \beta E_t \pi_{t+1} + \bar{\kappa} \hat{v}_t, \quad (7.38)$$

where  $\hat{v}_t$  is, as before, the log deviation of real marginal cost from its steady-state level. The elasticity of inflation with respect to real marginal cost is

$$\bar{\kappa} = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left[ \frac{\mu-1}{(1+\varphi)\mu-1} \right], \quad (7.39)$$

where  $\varphi$  is the inverse of the wage elasticity of labor supply. Comparing (7.38) and (7.39) to (7.36) and (7.37) reveals their close parallel. The first term in (7.39) is identical in form to the marginal cost elasticity of inflation in the Calvo model, with  $\alpha$ , the probability an island (and its firms) do not receive a productivity shock, replacing  $\omega$ , the probability a firm does not receive an opportunity to adjust price. The second term in (7.39) arises because of the assumption of local, island-specific labor markets in Gertler and Leahy's model; if a similar assumption about the labor market were incorporated into the Calvo model, a similar term would appear in (7.37).

To see how  $\alpha$  is related to the probability a firm adjusts its price, suppose  $\ln \mu^H$  and  $\ln \mu^L$  are the upper and lower values of the log markup that trigger price adjustment. Assume

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27. In terms of the notation employed in the discussion of TDP models,  $\mu = 1/q$ .

also that the firm-specific idiosyncratic shocks are uniformly distributed with mean zero and density  $1/\phi$ . Then, Gertler and Leahy showed,

$$1 - \omega = (1 - \alpha) \left[ (1 - \tau) + \tau \left( 1 - \frac{\ln \mu^H + \ln \mu^L}{\phi} \right) \right]. \quad (7.40)$$

To understand this equation, note that  $1 - \alpha$  is the probability an island has a productivity shock. Under the assumptions of the model, no firms on the remaining islands change their price. On the islands receiving productivity shocks, a fraction  $1 - \tau$  firms disappear and are replaced. All these new firms set new prices. The remaining  $\tau$  fraction of firms only adjust if their current markup is above or below the Ss limits given by  $\ln \mu^H$  and  $\ln \mu^L$ . The probability this occurs is 1 minus the probability the firm's idiosyncratic shock leaves its markup in the range  $\ln \mu^L < \ln \mu_t < \ln \mu^H$ . Given the uniform distribution, the probability of this occurring is  $(\ln \mu^H + \ln \mu^L) / \phi$ . So a fraction

$$\left( 1 - \frac{\ln \mu^H + \ln \mu^L}{\phi} \right)$$

of the  $\tau$  surviving firms choose to change price.

Notice that (7.40) can be rewritten as

$$1 - \omega = (1 - \alpha) \left[ 1 - \tau \left( \frac{\ln \mu^H + \ln \mu^L}{\phi} \right) \right] < 1 - \alpha,$$

which implies  $\omega > \alpha$ . This means that  $\bar{\kappa} > \kappa$ . Inflation will be more sensitive to real marginal cost with state-dependent pricing than implied by Calvo's time-dependent pricing model. In addition, the degree of nominal rigidity in this SDP model is directly related to  $\alpha$ , the fraction of islands that do not receive productivity shocks. The variance of aggregate productivity is  $(1 - \alpha)^2$  times the variance of the firm-specific idiosyncratic shocks. Thus, if more sectors in the economy experience aggregate productivity shocks—a decline in  $\alpha$ —the degree of nominal rigidity will fall.

### 7.2.6 Frictions in the Timing of Price Adjustment or in the Adjustment of Prices?

The workhorse model of price adjustment, particularly in the types of empirical general equilibrium models used for policy analysis, has remained the TDP model of Calvo because of its simplicity and ease of aggregation. Much of this tractability comes from fixing the frequency of price adjustment and focusing solely on the firm's decision on what price to set, conditional on adjusting.

An undesirable aspect of the Calvo model is its assumption of a fixed frequency of price adjustment. Alvarez and Gonzalez-Rozada (2011) examined a number of historical periods across several countries and documented the rising frequency of price adjustment as inflation rises. Levin and Yun (2007) developed a model in which firms decide optimally on the frequency with which they will adjust prices. As average inflation rises, firms chose

to change prices more frequently. For fluctuations of inflation within fairly narrow ranges, the assumption that each period a fixed fraction of firms adjust prices seems consistent with the microeconomic evidence (see section 7.3.1). However, it would be inappropriate to treat  $\omega$  as a structural parameter that will remain invariant if average inflation changes significantly.

Recently a number of tractable SDP models have been developed, and this is an area of active research. These models endogenize the frequency of adjusting, often emphasizing the selection effect, as it is not a random subset of firms that will choose to adjust as in the Calvo model, but the decision whether to adjust depends on the firm's own situation, as in the models of Golosov and Lucas Jr. (2007) or Dotsey, King, and Wolman (2013).

SDP models allow for time variation in the size of price changes, conditional on the firm changing its price (the intensive margin), variation in the number of firms that change their price in a given period (the extensive margin), and variation in the composition of firms that adjust (the selection effect). Costain and Nakov (2011) developed a generalized SDP model that nests both the TDP model of Calvo and the menu cost model of Golosov and Lucas. They do so by assuming the probability a firm adjusts is a nondecreasing function of the value of adjusting. If this probability is constant, they obtain the Calvo model; if the probability is zero when the value of adjusting is zero or negative and 1 when the value is positive, they obtain the Golosov-Lucas model. They found the best fit to the microeconomic data is obtained when the degree of state dependence is low.

In both the SDP and the Calvo models, no frictions inhibit the choice of the optimal price when the firm does adjust. That is, the friction arises in the timing of adjustment, not in the actual adjustment of prices. In a model such as the quadratic adjustment cost model of Rotemberg (1982), firms could change their price every period, but costs were increasing in the size of the price change they made. Costain and Nakov (2015) provided a model that focuses on frictions that arise when the choice of which price to set is subject to decision errors. They assume the decision-making process is costly, with more precise decisions incurring greater managerial costs. Saving on managerial costs of decision making makes it more likely the firm will make errors in its decision; by incurring these costs, the firm is more likely to get close to the optimal price. In addition to saving on managerial decision costs, a firm might leave its price unchanged as a precautionary measure if the firm fears making a wrong decision. Costain and Nakov argued that their model can match both microeconomic and macroeconomic evidence on price adjustment.<sup>28</sup>

### 7.3 Assessing Alternatives

In this section, the microeconomic and aggregate time series evidence on price adjustment and the behavior of inflation is briefly reviewed.

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28. See also Costain and Nakov (2014).

### 7.3.1 Microeconomic Evidence

One important consequence of the popularity of macroeconomic models based on sticky prices is that new research employs microeconomic data on prices and wages to better understand the behavior of prices. In turn, this evidence provides grounds for evaluating alternative models of price adjustment.

Bils and Klenow (2004) investigated price behavior for the United States for a large fraction of the goods and services that households purchase. They reported that the median duration between price changes is 4.3 months.<sup>29</sup> This median figure masks wide variation in the typical frequency with which prices of different categories of goods and services adjust. At one end, gasoline prices adjust with high frequency, remaining unchanged for less than a month on average. In contrast, more than a year separated price changes for driver's licenses, vehicle inspections, and coin-operated laundry and dry cleaning.

Based on an analysis of the U.S. data used in the consumer and producer price indexes, Nakamura and Steinsson (2008) presented five facts that they argue characterize price adjustment. First, sales have a significant effect on estimates of the median duration between price changes for items in the U.S. CPI. Excluding price changes associated with sales roughly doubles the estimated median duration between price changes from around 4.5 months when sales are included to 10 months when they are excluded. Standard TDP and STP models focus on explaining aggregate inflation and ignore the role of sales. Second, one-third of nonsale price changes are price decreases. Third, the frequency of price increases is positively correlated with the inflation rate, while the frequency of price decreases and the size of price changes is not. In fact, Nakamura and Steinsson concluded that most of the variation in the aggregate inflation rate can be accounted for by variations in the frequency of price increases. Fourth, the frequency of price changes follows a seasonal pattern. Price changes are more common during the first quarter of the year. Fifth, the probability the price of an item changes (the hazard function) declines during the first few months after a change in price. This last fact is inconsistent with the Calvo model, which implies the probability a firm changes its price is constant, independent of the time since the price was last changed. Nakamura and Steinsson concluded that while the first three facts are consistent with a standard menu cost model of price adjustment, the fourth and fifth are not.

Klenow and Kryvtsov (2008) found, based on U.S. CPI data, somewhat greater frequency of price change (about seven months when sales are excluded) than did Nakamura and Steinsson. They also found that despite that tendency for price changes to be large on average, a significant fraction of the changes are small. This finding is inconsistent with

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29. Bils and Klenow focused on the nonshelter component of the consumer price index and weighed individual price durations by the good's expenditure share to obtain this median figure.



a basic menu cost model with fixed cost of adjustment. Nakamura and Steinsson reported that the distribution of price change frequency is not symmetric; the average frequency is much higher than the median, suggesting that while many prices change frequently, some prices remain unchanged for sizable periods of time. Klenow and Kryvtsov also found that variations in the size of price changes, rather than variation in the fraction of prices that change, can account for most of the variance of aggregate inflation. As Nakamura and Steinsson argued, this result is consistent with their finding that the variance of aggregate inflation is attributable to changes in the frequency of price increases, since the average size of price changes is a weighted average of the sizes of price increases and decreases with weights equal to the frequency of each type of change.

Klenow and Kryvtsov (2008) compared the ability of the Calvo and Taylor TDP models and the DKW and Golosov-Lucas SDP models to match the empirical evidence from the CPI microeconomic data. Of six microeconomic facts they consider, the Golosov-Lucas model was able to match all except the presence of many small price changes. The DKW model was able to match this fact because it allows for a stochastic menu cost that varies across firms. Thus, some firms will have small costs and therefore adjust price even when they are already close to the optimal price. However, this model is not consistent with three of the other facts (flat hazard rates, size of price change does not increase with duration since last change, and intensive margin accounts for most of the variance of inflation). The Taylor model cannot match the flat hazard rates nor does it imply the size of price changes does not increase with the duration since the last change. This last fact is also not captured by the Calvo model. Surprisingly, this is the only one of the six facts with which the Calvo model, augmented with idiosyncratic firm shocks, is inconsistent. SDP models with idiosyncratic shocks and small menu costs, such as the models of Golosov-Lucas, Gertler-Leahy, or DKW augmented with idiosyncratic shocks as in Dotsey, King, and Wolman (2006) seem most promising for matching the stylized facts found in the microeconomic evidence.<sup>30</sup>

Hobijn, Ravenna, and Tambalotti (2006) provided a direct test of models that assume price stickiness is attributable to menu costs by using the natural experiment provided by the switch to the euro in January 2002. They found that firms concentrated price changes around the time of the currency switch, and prior to the changeover, prices did not fully reflect increased marginal costs expected to occur after the adoption of the euro. They showed that a menu cost model augmented to allow for a state-dependent decision on when to adopt the euro successfully captures the behavior of restaurant prices.

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30. Alvarez et al. (2005) summarized microeconomic evidence from the European Inflation Persistence Network (IPN) project of the European Central Bank. Angeloni et al. (2006) compared this evidence with several models of price adjustment and concluded that "a basic Calvo model (possibly extended to allow for sectors with different degrees of price stickiness) may not be a bad approximation." See also Altissimo, Ehrmann, and Smets (2006). See Dhyne et al. (2006) for a comparison of microeconomic evidence from the euro area and the United States.

Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2015) distinguished between what the former called reference prices and Kehoe and Midrigan called regular prices. These prices change infrequently, while actual prices fluctuate frequently around their reference level. Eichenbaum, Jaimovich, and Rebelo examined weekly price and cost data from a major U.S. retailer. Prices and costs change often, with the median duration between changes only three weeks. This suggests prices are reasonably flexible, and this high degree of flexibility suggests monetary policy should have, at best, small and short-lived real effects. Reference prices, which they define as the most often quoted price within a quarter for an item, display inertial behavior and have a median duration between changes of almost a year. If these prices are the relevant ones for aggregate fluctuations, then prices are very sticky. They found that a model calibrated to match the weekly frequency of price changes cannot match the evidence on the real effects of monetary shocks, while one calibrated to match the frequency of reference price changes can. They suggest a model in which firms choose pricing plans that allow for the actual price to vary among a small set of values. Price changes within a given pricing plan do not incur menu costs. Changing the price plan is costly, though, and so it is done infrequently. Finally, they argue the existing price adjustment models are not consistent with the finding in their data that prices are more volatile than marginal costs.

In chapter 8, the welfare costs of inflation are related to the dispersion of relative prices that can arise when prices are sticky and not all firms adjust their prices at the same time. The Calvo model, for example, predicts that price dispersion will increase with average inflation if the frequency with which firms adjust remains constant. State-dependent pricing models predict that more firms will decide to adjust each period as average inflation rises. By constructing a new data set on prices that goes back to 1977 and so includes some years of high U.S. inflation, Nakamura et al. (2016) found no evidence that dispersion varies positively with inflation.

While the work examining microeconomic evidence on pricing behavior has helped in assessing alternative models, it is not yet clear what aspects of the microeconomic evidence is of greatest relevance for understanding macroeconomic phenomena such as the impact of monetary policy on aggregate inflation and real output. The development of microeconomic data sets has, however, greatly expanded our knowledge about the behavior of individual prices.

### **7.3.2 Evidence on the New Keynesian Phillips Curve**

A large body of research has used time series methods to estimate the basic new Keynesian Phillips curve based on the Calvo model of price adjustment. This literature originated with the work of Galí and Gertler (1999) and is surveyed in Galí (2015). Three issues have been the focus of this work: measuring real marginal cost; reconciling time series estimates of the frequency of price adjustment with the microeconomic evidence; and accounting for persistence in the rate of inflation.

### Measuring Marginal Cost

Initial attempts to estimate the new Keynesian Phillips equation (NKPC) using aggregate time series data for the United States were not very successful (Galí and Gertler 1999; Sbordone 2002a). In fact, when  $\hat{v}_t$  was proxied by detrended real GDP, the estimated coefficient on the output gap was small and often negative in quarterly data, although Roberts (1995) found a small positive coefficient using annual data. Galí and Gertler (1999) and Sbordone (2002a) argued that detrended output was not the correct measure to enter in the NKPC. According to the basic theory, the appropriate variable is real marginal cost. Hence, one interpretation for the poor results using a standard output gap measure is that it is simply a poor proxy for real marginal cost.

To deal with measuring real marginal cost, Galí and Gertler (1999) noted that in the baseline model, real marginal cost is equal to the real wage divided by the marginal product of labor. With a Cobb-Douglas production function, the marginal product of labor is proportional to its average product. Thus, real marginal cost can be written as

$$MC_t = \frac{W_t/P_t}{MPL_t} \propto \frac{W_t/P_t}{Y_t/L_t} = \frac{W_t L_t}{P_t Y_t}.$$

Hence, real marginal cost is proportional to labor's share of total income. Expressed in terms of percent deviations around the steady state,  $\hat{v}_t = ls_t$ , where  $ls$  is the measure of labor's share. Galí and Gertler (1999) and Sbordone (2002a) reported evidence in favor of the new Keynesian Phillips curve when labor's share, rather than a standard output gap variable, is used to proxy for real marginal cost. Sbordone also reported evidence in favor of the implied dependence of inflation on expected future inflation and real marginal cost, as did Neiss and Nelson (2005).

These results suggest that it is the link between marginal cost and output that is the problem, not the link between marginal cost and inflation. This is perhaps not surprising. To go from marginal cost to an output gap measure, real wages were replaced by the marginal rate of substitution between leisure and consumption. This procedure assumed that while prices were sticky, nominal wages were perfectly flexible so that the real wage could adjust to maintain workers on their labor supply curve. If nominal wages are also sticky, a gap can open between the real wage and the marginal rate of substitution between leisure and consumption. The implications of nominal wage stickiness are discussed in chapter 8.

Rudd and Whelan (2005), however, argued that evidence for using labor's share in an inflation equation is weak. In particular, the basic new Keynesian Phillips curve given in (8.23) can be solved forward to yield

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{v}_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t \hat{v}_{t+i}, \quad (7.41)$$

showing that current inflation is proportional to the expected present discounted value of current and future real marginal cost. This means that current inflation should forecast

future movements in real marginal cost, as, for example, a rise in future real marginal cost that can be forecast should immediately raise current inflation. Rudd and Whelan found that VAR-generated expected discounted future labor share is only very weakly correlated with inflation.

### Persistence

While the new Keynesian Phillips curve was derived under the assumption that prices are sticky, the inflation rate is a purely forward-looking variable and is allowed to jump in response to any change in either current or expected future real marginal cost (see 7.41). Thus, as noted in discussing Chari, Kehoe, and McGrattan (2000), the NKPC is unable to match the persistence inflation displays in actual data (Nelson 1998; Estrella and Fuhrer 2002).<sup>31</sup> For example, suppose real marginal cost follows an exogenous AR(1) process:  $\hat{v}_t = \rho\hat{v}_{t-1} + e_t$ . To solve for the equilibrium process for inflation using (7.41), assume that  $\pi_t = A\hat{v}_t$ , where  $A$  is an unknown parameter. Then  $E_t\pi_{t+1} = AE_t\hat{v}_{t+1} = A\rho\hat{v}_t$ , and

$$\pi_t = A\hat{v}_t = \beta A\rho\hat{v}_t + \kappa\hat{v}_t \Rightarrow A = \frac{\kappa}{1 - \beta\rho}.$$

If  $\pi_t$  is multiplied by  $(1 - \rho L)$ , where  $L$  is the lag operator,  $(1 - \rho L)\pi_t = (1 - \rho L)A\hat{v}_t = Ae_t$ , so  $\pi_t = \rho\pi_{t-1} + Ae_t$ . The dynamics characterizing inflation depend solely on the serial correlation in  $\hat{v}_t$  in the form of the parameter  $\rho$ . The fact that prices are sticky makes no additional contribution to the resulting dynamic behavior of inflation. In addition,  $e_t$ , the innovation to  $\hat{v}_t$ , has its maximum impact on inflation immediately, with inflation then reverting to its steady-state value at a rate governed by  $\rho$ .

In order to capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation, yielding an equation of the form

$$\pi_t = (1 - \phi)\beta E_t\pi_{t+1} + \kappa\hat{v}_t + \phi\pi_{t-1}. \quad (7.42)$$

In this formulation, the parameter  $\phi$  is often described as a measure of the degree of backward-looking behavior in price setting. Fuhrer (1997) found little role for future inflation once lagged inflation is added to the inflation adjustment equation. Rudebusch (2002a) estimated (7.42) using U.S. data and argued that  $\phi$  is on the order of 0.7, suggesting that inflation is predominantly backward-looking.<sup>32</sup>

31. An entire chapter in the 2011 *Handbook of Monetary Economics* was devoted to the issue of inflation persistence; see Fuhrer (2011).

32. Both Rudebusch and Fuhrer employed a statistically based measure of the output gap, namely, detrended real GDP. Linde (2005) also questioned the empirical robustness of the new Keynesian Phillips curve. Galí, Gertler, and López-Salido (2005) responded to the criticisms of both Rudd and Whelan (2005) and Linde (2005), arguing that forward-looking behavior plays the dominant role in inflation determination.

Galí and Gertler (1999) modified the basic Calvo model of sticky prices to introduce lagged inflation into the Phillips curve. They assume that a fraction  $\lambda$  of the firms that are allowed to adjust each period simply set  $p_{jt} = \bar{\pi} p_{t-1}^*$ , where  $\bar{\pi}$  is the average inflation rate and  $p_{t-1}^*$  is the price chosen by optimizing firms in the previous period. They showed that the inflation adjustment equation then becomes

$$\pi_t = \left( \frac{1}{\delta} \right) [\beta \omega E_t \pi_{t+1} + (1 - \lambda) \bar{\kappa} \hat{v}_t + \lambda \pi_{t-1}] + \varepsilon_t, \quad (7.43)$$

where  $\bar{\kappa} = (1 - \omega)(1 - \omega\beta)$  and  $\delta = \omega + \lambda[1 - \omega(1 - \beta)]$ . Based on U.S. data, their estimate of the coefficient on  $\pi_{t-1}$  is in the range 0.25–0.4, suggesting that the higher weight on lagged inflation obtained when the output gap is used reflects the fact that the gap may be a poor proxy for real marginal cost.

Christiano, Eichenbaum, and Evans (2005) distinguished between firms that reoptimize in setting their price and those that do not. This might capture the idea the costs of changing prices are those associated with optimization and decision making rather than from actual menu costs. In their formulation, each period a fraction  $1 - \omega$  of all firms optimally set their price. The remaining firms either adjust their price based on the average rate of inflation, so that  $p_{jt} = \bar{\pi} p_{jt-1}$  where  $\bar{\pi}$  is the average inflation rate, or they adjust based on the most recently observed rate of inflation, so that  $p_{jt} = \pi_{t-1} p_{jt-1}$ . The first specification leads to (7.41) when the steady-state inflation rate is zero. The second specification results in an inflation adjustment equation of the form

$$\pi_t = \left( \frac{\beta}{1 + \beta} \right) E_t \pi_{t+1} + \left( \frac{1}{1 + \beta} \right) \pi_{t-1} + \left( \frac{\tilde{\kappa}}{1 + \beta} \right) \hat{v}_t. \quad (7.44)$$

The presence of lagged inflation in this equation introduces inertia into the inflation process. Since  $\beta \approx 0.99$  in quarterly data, the weights on expected future inflation and lagged inflation in the Christiano, Eichenbaum, and Evans formulation are both about 0.5. Such a value is within the range of estimates obtained by Rudebusch (2002a) and Galí, Gertler, and López-Salido (2001).

Woodford (2003a) introduced partial indexation to lagged inflation so that the nonoptimizing firms set  $p_{jt} = \lambda \pi_{t-1} p_{jt-1}$ , for  $0 \leq \lambda \leq 1$ . The  $\lambda = 1$  case corresponds to the model of Christiano, Eichenbaum, and Evans (2005). Woodford then showed that the inflation equation takes the form

$$\pi_t - \lambda \pi_{t-1} = \beta (E_t \pi_{t+1} - \lambda \pi_t) + \kappa \hat{v}_t.$$

It has become standard to assume some form of indexation of either prices or wages in empirical new Keynesian models of inflation. For example, indexation is included in many DSGE models that are estimated using quarterly data. Examples are found in Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003; 2007), Levin et al.

(2006), Adolfson et al. (2007), and Justiniano, Primiceri, and Tambalotti (2013). However, the microeconomic evidence on firm level pricing behavior offers no support for indexation.<sup>33</sup>

It is important to note that the standard derivation of the new Keynesian Phillips curve given by (7.41) is based on a linear approximation around a steady state that is characterized by zero inflation. Ascari (2004) showed that the behavior of inflation implied by models with staggered price setting, such as the Calvo model, is significantly affected when trend inflation differs from zero. For example, because of the staggered nature of price adjustment in a Calvo-type model, higher trend inflation leads to a dispersion of relative prices. Since firms have different prices, output levels also differ across firms, and households consume different amounts of the final goods. These differences are inefficient, causing steady-state output to decline as steady-state inflation rises.<sup>34</sup> The dynamic behavior of inflation in response to shocks is also influenced (Ascari and Ropele 2007). However, indexation of the type discussed here eliminates the effects of trend inflation by allowing those firms that do not adjust optimally to still reset their prices to reflect the average rate of inflation. Fuhrer (2011) provided an extensive discussion focused on the issue of explaining inflation persistence, and Ascari and Sbordone (2014) provided a comprehensive discussion of issues raised by nonzero trend inflation.

Cogley and Sbordone (2008) combined indexation by nonoptimizing firms with a time-varying trend rate of inflation in a Calvo-type model and showed that the resulting Phillips curve is given by

$$\hat{\pi}_t = \alpha_{1t} (\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}}) + \alpha_{2t} \hat{v}_t + \alpha_{3t} E_t \hat{\pi}_{t+1} + \alpha_{4t} E_t \sum_{j=2}^{\infty} \phi_{1t}^{j-1} \hat{\pi}_{t+j}, \quad (7.45)$$

where  $\hat{\pi}_t$  is log inflation relative to the current trend level,  $\hat{g}_t^{\bar{\pi}}$  is the growth rate of the inflation trend, and  $\hat{v}_t$  is log real marginal cost relative to its steady-state value. Relative to the basic NKPC with a zero trend inflation rate, the coefficients on expectations of future inflation are time-varying, and expectations of inflation more than one period into the future affect current inflation. The time variation of the coefficients occurs because all of them are functions of the (time-varying) trend rate of inflation. Estimating (7.45) using U.S. data, Cogley and Sbordone argued that a purely forward-looking version of their model (i.e., a version without indexation, so that  $\alpha_{1t} = 0$  and lagged inflation does not appear) can capture short-run inflation dynamics. This success arises, in part, from the high volatility of trend inflation that they estimated. Sbordone (2007) finds that if (7.45) with  $\alpha_{1t} = 0$  is the true model of inflation but a fixed-coefficient model of the form given by (7.42) is

33. Indexation also has implications for optimal monetary policy. See Steinsson (2003).

34. The role of the dispersion of relative prices in affecting the welfare of fluctuations is discussed in section 8.4.1.

estimated instead, one is likely to conclude, incorrectly, that there is a backward-looking component to inflation.<sup>35</sup>

A final explanation for inflation persistence emphasizes deviations from rational expectations. For example, following Roberts (1997), suppose

$$\pi_t = \beta\pi_{t,t+1}^e + \kappa\hat{v}_t,$$

where  $\pi_{t,t+1}^e$  is the public's average expectation of  $\pi_{t+1}$  formed at time  $t$ . Suppose further that this expectation is a mixture of rational and backward-looking:

$$\pi_{t,t+1}^e = \alpha E_t\pi_{t+1} + (1 - \alpha)\pi_{t-1}.$$

Then

$$\pi_t = \beta\alpha E_t\pi_{t+1} + \beta(1 - \alpha)\pi_{t-1} + \kappa\hat{v}_t.$$

In this case, the presence of lagged inflation arises because expectations are not fully rational. The deviation from rational expectations is ad hoc, but another possibility is that backward-looking expectations arise because of adaptive learning on the part of the public.

### The Degree of Nominal Price Rigidity

A final problem uncovered by structural estimates of the NKPC is that values for  $\omega$ , the probability a firm does not adjust its price, were much larger than found in the microeconomic evidence (see section 7.3.1). Dennis (2006) reported that estimates of  $\omega$  have generally been in the range of 0.758–0.911. These values are similar to those reported by Eichenbaum and Fisher (2007) and would imply very long durations between price changes. For example, a value of 0.8, which is in the middle of this range, would imply firms leave prices unchanged for, on average, five quarters, or over one year. As we saw earlier, the microeconomic evidence for the United States suggests median durations between price changes closer to two quarters or less, implying  $\omega < 0.5$ .

The large values of  $\omega$  obtained in time series estimates of the Phillips curve arise because inflation does not appear to respond strongly to real marginal cost. Since the elasticity of inflation with respect to marginal cost is given by

$$\kappa = \frac{(1 - \omega)(1 - \beta\omega)}{\omega},$$

and  $\beta$  is roughly 0.99 in quarterly data, a small  $\kappa$  can only be made consistent with the theory if  $\omega$  is large.

To understand the modifications that have been made to the basic model in an attempt to reconcile time series estimates with values of price change frequency from the microeconomic evidence, some further elements need to be incorporated into the model. Chapter 8

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35. Rotemberg (2007) demonstrated that inflation persistence can arise with the forward-looking new Keynesian Phillips curve if real marginal cost consists of two unobserved components of differing persistence. Rotemberg showed that inflation will, in this situation, actually be more persistent than total marginal cost.

embeds the new Keynesian Phillips curve into a general equilibrium setting, so the discussion here is brief.

In section 8.3, it is shown that real marginal cost in the basic new Keynesian model can be expressed as

$$\hat{v}_t = (\eta + \sigma) \left( \hat{y}_t - \hat{y}_t^{\text{flex}} \right),$$

where  $\eta$  is the inverse of the wage elasticity of labor supply,  $\sigma$  is the coefficient of relative risk aversion, and  $\hat{y}_t^{\text{flex}}$  is the equilibrium output under flexible prices, expressed as a percent deviation around steady-state output. The NKPC can therefore be written in terms of an output gap as

$$\pi_t = \beta E_t \pi_{t+1} + \bar{\kappa} \left( \hat{y}_t - \hat{y}_t^{\text{flex}} \right),$$

with  $\bar{\kappa} \equiv \kappa (\eta + \sigma)$ . Notice that the output gap is defined relative to a flexible-price output and need not correspond closely to a standard output gap defined as detrended output. This derivation suggests a small effect of the output gap on inflation (a small  $\bar{\kappa}$ ) can be reconciled with a value of  $\omega$  around 0.5 if the wage elasticity is large. As output rises, firms need to hire more workers to expand production. This increase in labor demand pushes up real wages and marginal costs, causing inflation to rise. However, if labor supply is highly elastic, then the rise in real wages will be small, marginal cost will not rise significantly, and inflation will not move a lot in response to changes in the output gap. This elasticity is, however, normally thought to be small.

This intuition does suggest, though, that the aggregate evidence could be reconciled with a small  $\omega$  if a richer production technology were introduced that dampens the impact of output on marginal cost. Three modifications have been explored.

First, variable capital utilization was introduced by Christiano, Eichenbaum, and Evans (2005). So far, capital has been ignored, and only labor was used to produce output. Once capital is introduced and its rate of utilization can vary, then output can increase by utilizing capital more intensely rather than solely by employing more labor. By essentially allowing firms more margins along which to adjust, the effects of output variations on marginal cost are muted.

Second, Sbordone (2002b) and Eichenbaum and Fisher (2007) argued that more plausible estimates of  $\omega$  can be obtained with the introduction of firm-specific capital. To understand the role played by firm-specific capital, consider the situation in the basic Calvo model. Each individual firm takes the aggregate real wage as given. The same would be true for the rental cost of capital if there were an economywide rental market for capital. Consequently, no individual firm takes into account the effect of its output choice on aggregate real factor prices. However, when capital is firm-specific, the firm faces diminishing returns; each firm knows that its marginal costs will rise if it expands production. Faced with an opportunity to adjust price, a firm that would like to raise its price knows that



doing so will reduce the demand for its product. The firm will recognize that lower demand, and therefore a lower level of production, will lower its future marginal costs. This mutes the firm's desired price increase, since price depends on both current and expected future marginal costs. Conversely, a firm considering a cut in price will recognize that this will lead to an increase in the demand it faces, which in turn will require an increase in production and an increase in marginal costs. This anticipated rise in marginal costs will dampen the desired price reduction.

To illustrate this mechanism, suppose the production function for firm  $i$  is

$$Y_t(i) = A_t \bar{K}^{1-a} N_t(i)^a.$$

Real marginal cost for the firm is the real wage relative to the marginal product of labor:

$$MC_t(i) = \frac{W_t}{a A_t \bar{K}^{1-a} N_t(i)^{a-1}} = \frac{W_t N_t(i)}{a Y_t(i)}.$$

From the production function,  $N_t(i) = [Y_t(i)/A_t \bar{K}^{1-a}]^{1/a}$ , so

$$MC_t(i) = \frac{W_t N_t(i)}{a Y_t(i)} = \left[ \frac{W_t Y_t(i)^{\frac{1-a}{a}}}{a A_t \bar{K}^{\frac{1-a}{a}}} \right].$$

Thus, unlike the basic model, marginal cost now depends on the firm's output and so varies across firms. Marginal cost at firm  $i$  relative to aggregate marginal cost can be written, using the demand curve given by (7.21), as

$$\frac{MC_t(i)}{MC_t} = \left( \frac{Y_t(i)}{Y_t} \right)^{\frac{1-a}{\sigma}} = \left( \frac{P_t(i)}{P_t} \right)^{\frac{a-1}{a(1-q)}}$$

where  $1/(1-q)$  is the price elasticity of demand. Hence, in terms of deviations around the steady state,

$$mc_t(i) = mc_t - \left[ \frac{1-a}{a(1-q)} \right] [p_t(i) - p_t] = mc_t - A [p_t(i) - p_t],$$

where  $A = (1-a)/[a(1-q)] > 0$ . The inflation adjustment equation becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (1+A)^{-1} mc_t.$$

and the impact of a change in marginal cost on inflation is dampened, since  $\kappa(1+A)^{-1} < \kappa$ .

A third modification of the basic model involves relaxing the standard assumption that firms face a constant elasticity demand curve. If the demand for the firm's output becomes more elastic in response to a price increase, the increase in the firm's desired price when marginal costs rise will be less. Facing a more elastic demand, the firm's optimal relative price declines, so this mutes the degree to which the firm will raise its price.

Eichenbaum and Fisher (2007) argued that by adding indexation and firm-specific capital, and dropping the assumption that firms face a constant elasticity of demand, estimated values of  $\omega$ , the frequency of price changes, are lower than the high values found in the basic Calvo model. In fact, they concluded that for some specifications the estimated value of  $\omega$  is consistent with firms reoptimizing prices every two quarters, a value more in line with microeconomic evidence. However, that evidence referred to the average duration between price changes. Eichenbaum and Fisher proposed that all firms change prices every period (because of the indexation assumption). Thus, it is not clear how the price change frequency found in the microeconomic data and the reoptimization frequency are related.

Standard models of price adjustment assume the frequency of price adjustment is the same across all firms in the economy. de Carvalho (2006) and Nakamura and Steinsson (2008) considered heterogeneity in this frequency by studying multisector economies. When prices adjust at different rates across the sectors, Carvalho showed, in response to a monetary shock, most price changes initially occur in sectors characterized by a low degree of nominal rigidity (i.e., in sectors in which prices adjust frequently). As time passes, the speed of adjustment slows, as firms in the sectors with greater nominal rigidity are now the primary adjusters. In addition, if strategic complementarities lead each firm's optimal price to be a function of the prices of other firms, then the price changes of the firms in sectors that adjust rapidly are affected by the presence of more slowly adjusting sectors of the economy. In response to a positive monetary shock, the existence of slow adjusters will cause the early adjusters to limit their price increases. As a result, monetary shocks have longer-lived impacts on the real economy when price adjustment frequencies differ across the economy, relative to an economy in which this frequency is the same for all firms. Carvalho reported that to generate the same dynamic responses to monetary shocks, a model with identical firms needs a frequency of price change that is as much as three times lower than the average frequency in a model with heterogeneous firms.

### 7.3.3 Sticky Prices versus Sticky Information

Several authors have attempted to test the sticky-information Phillips curve (SIPC) model of inflation (see section 5.2) against the sticky-price new Keynesian Phillips curve. For example, Kiley (2007a) estimated the NKPC as well as hybrid versions that incorporate lagged inflation, as in (7.43) or (7.44). For the period 1983–2002, he found that a simple sticky-price model with one lag of inflation performs reasonably well, as does a sticky-information model augmented with one lag of inflation. However, the sticky-price model does better than the sticky-information version when the number of lags is increased. Thus, both models require ad hoc augmentation to fully account for the behavior of inflation. Kiley argued that the addition of such long lags in inflation might reflect the type of behavior Galí and Gertler (1999) showed led to (7.43). Galí and Gertler assumed that a fraction

of firms that could adjust their price simply employed a rule of thumb that called for setting a new price based on lagged information about the optimal price. If this lagged information is assumed to include older information on past optimal prices, one might justify the best-fitting model as one that incorporates sticky prices together with sticky information. Trabandt (2003) showed either Mankiw-Reis sticky information or Calvo with indexing can do about the same in matching inflation dynamics.

Most macroeconomic models impose the assumption that conditional on the available information set, expectations are rational. Thus, both the SIPC and the NKPC are based on rational expectations, but they differ in terms of the information that is assumed to be available to agents. Coibion (2010) used historical survey measures of inflation forecasts to avoid imposing rational expectations. He found that when the structural parameters of the SIPC are estimated, little evidence of informational stickiness is uncovered. He also found that conditional on the survey forecasts, the SIPC is rejected in favor of the sticky-price NKPC.

Klenow and Willis (2007) proposed a reconciliation between microeconomic flexibility and macroeconomic rigidity and set up a model in which firms have price adjustment costs (which will lead to price stickiness) and information costs (information stickiness). The former are introduced to account for the fact that in a given month most prices do not change. Information updating about the aggregate economy occurs every  $N$  periods, as in a Taylor adjustment model. This differs from Mankiw and Reis's original sticky-information model in which the probability of updating information each period was constant. Klenow and Willis also introduced idiosyncratic firm shocks about which the firm always has full information. Expectations about inflation are assumed to be based on a simple forecasting rule. They found that in microeconomic data from the U.S. CPI, price changes appear to depend on old information in a manner consistent with theories of sticky information.

## 7.4 Summary

Monetary economists generally agree that the models discussed in chapters 2–4, while useful for examining issues such as the welfare cost of inflation and the optimal inflation tax, need to be modified to account for the short-run effects of monetary factors on the economy. Chapters 5 and 7 reviewed three such modifications: informational frictions, portfolio adjustment frictions, and nominal price adjustment frictions. Most monetary models designed to address short-run monetary issues assume that wages and/or prices do not adjust instantaneously in response to changes in economic conditions. This chapter examined some standard models of price adjustment, including both time-dependent and state-dependent models. It also briefly discussed some of the microeconomic evidence that has provided new facts against which to judge models of nominal stickiness as well as time series evidence on sticky-price and sticky-information models.

In chapter 8, the basic Calvo model of price adjustment is embedded in a general equilibrium framework in which households optimally choose consumption and labor supply. The resulting dynamic stochastic general equilibrium model has been widely used to address issues in monetary policy.

## 7.5 Appendix: A Sticky-Wage MIU Model

In section 7.2.1, an MIU model was modified to include one-period nominal wage contracts. Equations (7.1)–(7.8) characterized equilibrium in the flexible-price MIU model. Output was shown to equal

$$y_t - E_{t-1}y_t^* = a(p_t - E_{t-1}p_t) + (1 + a)\varepsilon_t, \quad (7.46)$$

where  $E_{t-1}y_t^* = (1 - \alpha)E_{t-1}n_t^* + E_{t-1}e_t$  is the expected equilibrium output under flexible prices,  $a = (1 - \alpha)/\alpha$ , and

$$y_t^* = \left[ \frac{1 + \bar{\eta}}{1 + \bar{\eta} + (1 - \alpha)(\Phi - 1)} \right] e_t = b_2 e_t.$$

The aggregate demand side of this economy consists of (7.4) and (7.6)–(7.8). Making use of the economy's resource constraint, (7.4) can be written as

$$y_t = E_t y_{t+1} - \left( \frac{1}{\Phi} \right) r_t. \quad (7.47)$$

Use the Fisher equation, (7.7), together with (7.47) to get the money demand condition

$$\begin{aligned} m_t - p_t &= y_t - \left( \frac{1 - i^{ss}}{bi^{ss}} \right) [r_t + E_t p_{t+1} - p_t] \\ &= y_t - \frac{\Phi}{bi^{ss}} [E_t y_{t+1} - y_t] - \left( \frac{1 - i^{ss}}{bi^{ss}} \right) (E_t p_{t+1} - p_t). \end{aligned}$$

Notice that expected future income affects the demand for money. Higher expected income raises the expected real interest rate for a given level of current output, and this implies lower money demand.

The equations of the model can now be collected.

Aggregate supply:  $y_t = b_2 E_{t-1} e_t + a(p_t - E_{t-1} p_t) + (1 + a)\varepsilon_t,$

Aggregate demand:  $y_t = E_t y_{t+1} - \left( \frac{1}{\Phi} \right) r_t,$

Money demand:  $m_t - p_t = y_t - \frac{\Phi}{bi^{ss}} [E_t y_{t+1} - y_t] - \left( \frac{1 - i^{ss}}{bi^{ss}} \right) (E_t p_{t+1} - p_t),$

Fisher equation:  $i_t = r_t + E_t p_{t+1} - p_t.$

To complete the solution to the model, assume that the productivity shock  $e_t$  and the money supply shock  $s_t$  are both serially and mutually uncorrelated. Then  $E_{t-1}e_t = E_{t-1}y_t^* = 0$ . The model reduces to

$$y_t = a(p_t - E_{t-1}p_t) + (1+a)\varepsilon_t,$$

$$m_t - p_t = \left(1 + \frac{\Phi}{bi^{ss}}\right)y_t - \left(\frac{1-i^{ss}}{bi^{ss}}\right)(E_t p_{t+1} - p_t),$$

$$m_t = m_{t-1} + s_t.$$

Define  $B \equiv [1 + (b-1)i^{ss} + a(bi^{ss} + \Phi)]$ . Combining the first and second equations,

$$Bp_t = bi^{ss}m_t + a(bi^{ss} + \Phi)E_{t-1}p_t - (1+a)(bi^{ss} + \Phi)\varepsilon_t + (1-i^{ss})E_t p_{t+1}. \quad (7.48)$$

Guess a solution of the form  $p_t = \gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 \varepsilon_t$ . Then  $E_{t-1}p_t = \gamma_1 m_{t-1}$  and  $E_t p_{t+1} = \gamma_1 m_t = \gamma_1 m_{t-1} + \gamma_1 s_t$ . Substituting these expressions into (7.48),

$$\begin{aligned} B(\gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 \varepsilon_t) &= bi^{ss}(m_{t-1} + s_t) + a(bi^{ss} + \Phi)\gamma_1 m_{t-1} \\ &\quad - (1+a)(bi^{ss} + \Phi)\varepsilon_t + (1-i^{ss})(\gamma_1 m_{t-1} + \gamma_1 s_t). \end{aligned}$$

Equating the coefficients on either side,  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  must satisfy

$$B\gamma_1 = bi^{ss} + a(bi^{ss} + \Phi)\gamma_1 + (1-i^{ss})\gamma_1 \Rightarrow \gamma_1 = 1,$$

$$B\gamma_2 = bi^{ss} + (1-i^{ss})\gamma_1 \Rightarrow \gamma_2 = \frac{1 + (b-1)i^{ss}}{B},$$

$$B\gamma_3 = -(1+a)(bi^{ss} + \Phi) \Rightarrow \gamma_3 = -\left[\frac{(1+a)(bi^{ss} + \Phi)}{B}\right].$$

To determine the impact of a money shock  $s_t$  on output, note that  $p_t - E_{t-1}p_t = \gamma_2 s_t + \gamma_3 \varepsilon_t$ , so

$$\begin{aligned} y_t &= a(p_t - E_{t-1}p_t) + (1+a)\varepsilon_t \\ &= a\gamma_2 s_t + (a\gamma_3 + 1+a)\varepsilon_t. \end{aligned}$$

From the definitions of  $\gamma_2$  and  $\gamma_3$ ,

$$y_t = \left[\frac{1 + (b-1)i^{ss}}{B}\right][as_t + (1+a)\varepsilon_t].$$

Using the parameter values from table 2.1 ( $\alpha = 0.36$ ,  $b = 9$ ,  $\Phi = 2$ ) and a steady-state gross nominal interest rate of 1.011 (since average money growth and hence inflation equal zero), the coefficient on  $s_t$  is equal to 0.40. Letting  $b \rightarrow \infty$  yields (7.14).

## 7.6 Problems

1. An increase in average inflation lowers the real demand for money. Demonstrate this by using the steady-state version of the model given by (7.1)–(7.7), assuming that the nominal money supply grows at a constant trend rate  $\mu$  so that  $m_t = \mu t$ , to show that real money balances  $m_t - p_t$  are decreasing in  $\mu$ .
2. Suppose that the nominal money supply evolves according to  $m_t = \mu + \rho_m m_{t-1} + s_t$  for  $0 < \rho_m < 1$  and  $s_t$ , a white noise control error. If the rest of the economy is characterized by (7.1)–(7.7), solve for the equilibrium expressions for the price level, output, and the nominal rate of interest. What is the effect of a positive money shock ( $s_t > 0$ ) on the nominal rate? How does this result compare to the  $\rho_m = 1$  case discussed in the appendix? Explain.
3. Assume that nominal wages are set for one period but that they can be indexed to the price level:

$$w_t^c = w_t^0 + b(p_t - E_{t-1}p_t),$$

where  $w^0$  is a base wage and  $b$  is the indexation parameter ( $0 \leq b \leq 1$ ).

- a. How does this change modify the aggregate supply equation given by (7.11)?
  - b. Suppose the demand side of the economy is represented by a simple quantity equation,  $m_t - p_t = y_t$ , and assume  $m_t = v_t$ , where  $v_t$  is a mean zero shock. Assume the indexation parameter is set to minimize  $E_{t-1}(n_t - E_{t-1}n_t^*)^2$ . Show that the optimal degree of wage indexation is increasing in the variance of  $v$  and decreasing in the variance of  $e$  (Gray 1978).
4. Show how (7.17) can be derived from (7.16) in the Taylor model, where  $\pi_t = p_t - p_{t-1}$ .
  5. Equation (7.29) was obtained from equation (7.28) by assuming that  $R = 1$ . Show that in general, if  $R$  is constant but  $R^{ss} > 1$ ,

$$\bar{p}_t = \left( \frac{R^{ss}}{1 + R^{ss}} \right) \left[ p_t + \frac{1}{R^{ss}} E_t p_{t+1} \right] + \left( \frac{R^{ss}}{1 + R^{ss}} \right) \left[ v_t + \frac{1}{R^{ss}} E_t v_{t+1} \right].$$

6. The Chari, Kehoe, and McGrattan (2000) model of price adjustment led to (7.31). Using (7.30), show that the parameter  $a$  in (7.31) equals  $(1 - \sqrt{\gamma})/(1 + \sqrt{\gamma})$ .
7. The basic Taylor model of price level adjustment was derived under the assumption that the nominal wage set in period  $t$  remained unchanged for periods  $t$  and  $t + 1$ . Suppose instead that each period  $t$  contract specifies a nominal wage  $x_t^1$  for period  $t$  and  $x_t^2$  for period  $t + 1$ . Assume these are given by  $x_t^1 = p_t + \kappa y_t$  and  $x_t^2 = E_t p_{t+1} + \kappa E_t y_{t+1}$ . The aggregate price level at time  $t$  is equal to  $p_t = \frac{1}{2}(x_t^1 + x_{t-1}^2)$ . If aggregate demand is given by  $y_t = m_t - p_t$  and  $m_t = m_0 + \omega_t$ , what is the effect of a money shock  $\omega_t$  on  $p_t$  and  $y_t$ ? Explain why output shows no persistence after a money shock.

8. Following Rotemberg (1988), suppose the representative firm  $i$  sets its price to minimize a quadratic loss function that depends on the difference between the firm's actual log price in period  $t$ ,  $p_{i,t}$ , and its optimal log price,  $p_t^*$ . If the firm can adjust at time  $t$ , it will set its price to minimize

$$\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left( p_{i,t+j} - p_{t+j}^* \right)^2, \quad (7.49)$$

subject to the assumed process for determining when the firm will next be able to adjust.

- a. If the probability of resetting prices each period is  $1 - \omega$ , as in the Calvo model, and  $\hat{p}_t$  denotes the optimal price chosen by all firms that can adjust at time  $t$ , show that  $\hat{p}_t$  minimize

$$\frac{1}{2} \sum_{j=0}^{\infty} \omega^j \beta^j E_t \left( p_{i,t} - p_{t+j}^* \right)^2.$$

- b. Derive the first-order condition for the optimal choice of  $\hat{p}_t$ .  
c. Using your result from (b), show that

$$\hat{p}_t = (1 - \omega\beta) \sum_{j=0}^{\infty} \omega^j \beta^j E_t p_{t+j}^*. \quad (7.50)$$

Explain intuitively why the weights on future optimal prices  $p_{t+j}^*$  depend on  $\omega$ .

9. Suppose the representative firm  $i$  sets its price to minimize a quadratic loss function that depends on the difference between the firm's actual log price in period  $t$ ,  $p_{i,t}$ , and its optimal log price,  $p_t^*$ . The probability of resetting prices each period is  $1 - \omega$ , as in the Calvo model. If the firm can adjust at time  $t$ , it will set its price to minimize

$$\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left( p_{i,t+j} - p_{t+j}^* \right)^2,$$

subject to the assumed process for determining when the firm will next be able to adjust.

- a. If  $\hat{p}_t$  is the log price chosen by adjusting firms, show that

$$\hat{p}_t = (1 - \omega\beta) p_t^* + \omega\beta E_t \hat{p}_{t+1}.$$

- b. Assume the log price target  $p^*$  depends on the aggregate log price level and output:  $p_t^* = p_t + \gamma y_t + \varepsilon_t$ , where  $\varepsilon$  is a random disturbance to capture other determinants of  $p^*$ . The log aggregate price level is  $p_t = (1 - \omega)\hat{p}_t + \omega p_{t-1}$ . Using this definition and the result in part (a), obtain an expression for aggregate inflation as a function of expected future inflation, output, and  $\varepsilon$ .

- c. Is the impact of output on inflation increasing or decreasing in  $\omega$ , the measure of the degree of nominal rigidity? Explain.
10. The basic Calvo model assumes that each period a fraction  $\omega$  of all firms do not change price. Suppose instead that these firms index their price to last period's inflation so that for such firms, their log price is given by  $p_{i,t} = p_{i,t-1} + \pi_{t-1}$ .
- How should the quadratic objective function given by (7.49) be modified under this alternative assumption about the behavior of the firms that do not optimally adjust their price?
  - What is the first-order condition for  $\hat{p}_t$ , the price chosen by firms that do adjust optimally in period  $t$ ?
  - The log aggregate price level becomes

$$p_t = (1 - \omega)\hat{p}_t + \omega(p_{t-1} + \pi_{t-1}).$$

Use this equation with the first-order condition for  $\hat{p}_t$ , obtained in part (b), to find an expression for the aggregate inflation rate. How is current inflation affected by lagged inflation?



# 8 New Keynesian Monetary Economics

## 8.1 Introduction

For the past 20 years, the most common framework employed in monetary economics and monetary policy analysis has incorporated nominal wage and/or price rigidity into a dynamic stochastic general equilibrium (DSGE) framework that is based on optimizing behavior by the agents in the model. These DSGE models with nominal frictions are commonly labeled new Keynesian (NK) models because, like older models in the Keynesian tradition, aggregate demand plays a central role in determining output in the short run, and there is a presumption that some fluctuations both can be and should be dampened by countercyclical monetary and/or fiscal policy.<sup>1</sup> Early examples of models with these properties include those of Rotemberg and Woodford (1995; 1997), Yun (1996), Goodfriend and King (1997), and McCallum and Nelson (1999). Book-length treatments of the new Keynesian model are provided by Woodford (2003a) and Galí (2015).

The first section of this chapter shows how a basic money-in-the-utility function (MIU) model, combined with the assumption of monopolistically competitive goods markets and price stickiness, can form the basis for a simple linear new Keynesian model.<sup>2</sup> The model is a consistent general equilibrium model in which all agents face well-defined decision problems and behave optimally, given the environment in which they find themselves. To obtain a canonical new Keynesian model, three key modifications are made to the MIU model of chapter 2. First, endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who show that, at least for the United States, there is little relationship between the capital stock and output at business cycle frequencies. Endogenous capital stock dynamics play a key role in equilibrium business cycle models in the real business cycle tradition, but as Cogley and Nason (1995) showed, the response

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1. Goodfriend and King (1997) proposed the name “the new neoclassical synthesis” to emphasize the connection with neoclassical, rather than Keynesian traditions.

2. See chapter 2 for a discussion of MIU models.

of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by such models. For simplicity, then, the capital stock is ignored.<sup>3</sup>

Second, the single final good in the MIU model is replaced by a continuum of differentiated goods produced by monopolistically competitive firms. These firms face constraints on their ability to adjust prices, thus introducing nominal price stickiness into the model. In the basic model, nominal wages are allowed to fluctuate freely. Section 8.5.1 explores the implications of assuming both prices and wages are sticky.

Third, monetary policy is represented by a rule for setting the nominal rate of interest. Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. The nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate. Chapter 11 takes up the issues that arise when the use of an interest rate instrument is constrained by the zero lower bound on nominal rates.<sup>4</sup> Even absent the zero lower bound, important issues are involved in choosing between money supply policy procedures and interest rate procedures; some of these are discussed in chapter 12.

These three modifications yield a new Keynesian framework that is consistent with optimizing behavior by private agents and incorporates nominal rigidities yet is simple enough to use for exploring a number of policy issues. It can be linked directly to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the workhorses for monetary policy analysis and is still common in most undergraduate texts. Once the basic framework has been developed, section 8.4 considers optimal policy as well as a variety of policy rules and policy frameworks, including inflation targeting. Section 8.5 discusses the role of sticky wages in the new Keynesian model and the integration of modern theories of unemployment into the basic model.

## 8.2 The Basic Model

The model consists of households, firms, and a central bank. Households supply labor, purchase goods for consumption, and hold money and bonds, while firms hire labor and produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977). The model of price stickiness is taken from Calvo (1983).<sup>5</sup> Each firm sets the price of the good

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3. However, Dotsey and King (2006) and Christiano, Eichenbaum, and Evans (2005) emphasized the importance of variable capital utilization for understanding the behavior of inflation. New Keynesian models that are taken to the data incorporate investment and capital stock dynamics (e.g., Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007; Altig et al. 2011).

4. It is perhaps better to speak of an effective lower bound on nominal interest rate, as policy rates of the Bank of Japan, the European Central Bank, the Swedish Riksbank, and the Danmarks Nationalbank were all below zero by 2015.

5. See section 7.2.4.

it produces, but not all firms reset their price in each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. Initially the central bank, in contrast to households and firms, is assumed to follow a simple rule; optimal policy is explored in section 8.4.

### 8.2.1 Households

The preferences of the representative household are defined over a composite consumption good  $C_t$ , real money balances  $M_t/P_t$ , and the time devoted to market employment  $N_t$ . Households maximize the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \quad (8.1)$$

The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers (firms). There is a continuum of such firms of measure 1, and firm  $j$  produces good  $c_j$ . The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \quad (8.2)$$

The household's decision problem can be dealt with in two stages. First, regardless of the level of  $C_t$  the household decides on, it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of  $C_t$ , the household chooses  $C_t$ ,  $N_t$ , and  $M_t$  optimally.

Dealing first with the problem of minimizing the cost of buying  $C_t$ , the household's decision problem is

$$\begin{aligned} & \min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj \\ & \text{subject to} \\ & \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t, \end{aligned} \quad (8.3)$$

where  $p_{jt}$  is the price of good  $j$ . Letting  $\psi_t$  be the Lagrangian multiplier on the constraint, the first-order condition for good  $j$  is

$$p_{jt} - \psi_t \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$

Rearranging,  $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$ . From the definition of the composite level of consumption (8.2), this implies

$$C_t = \left[ \int_0^1 \left[ \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left( \frac{1}{\psi_t} \right)^{-\theta} \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t.$$

Solving for  $\psi_t$ ,

$$\psi_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t. \quad (8.4)$$

The Lagrangian multiplier is the appropriately aggregated price index for consumption, as it gives the marginal cost of an additional unit of the consumption basket  $C_t$ . The demand for good  $j$  can then be written as

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t. \quad (8.5)$$

The price elasticity of demand for good  $j$  is equal to  $\theta$ . As  $\theta \rightarrow \infty$ , the individual goods become closer and closer substitutes, and consequently individual firms will have less market power.

Given the definition of the aggregate price index in (8.4), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t, \quad (8.6)$$

where  $M_t$  ( $B_t$ ) is the household's nominal holdings of money (one-period bonds). Bonds pay a nominal rate of interest  $i_t$ . Real profits received from firms are equal to  $\Pi_t$ .

In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (8.1), subject to (8.6). This leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium:

$$C_t^{-\sigma} = \beta(1 + i_t)E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}, \quad (8.7)$$

$$\frac{\gamma \left( \frac{M_t}{P_t} \right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1 + i_t}, \quad (8.8)$$

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \quad (8.9)$$

These conditions represent the Euler condition for the optimal intertemporal allocation of consumption, the intratemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage.<sup>6</sup>

### 8.2.2 Firms

Firms maximize profits, subject to three constraints. The first is the production function summarizing the available technology. For simplicity, capital is ignored, so output is a function solely of labor input  $N_{jt}$  and an aggregate productivity disturbance  $Z_t$  displaying constant returns to scale:

$$c_{jt} = Z_t N_{jt}, \quad E(Z_t) = 1. \quad (8.10)$$

The second constraint is given by the demand curve each firm faces; this is given by (8.5). The third constraint is that in each period some firms are not able to adjust their price. The specific model of price stickiness used here is due to Calvo (1983) (see section 7.2.4). Each period, the firms that adjust their price are randomly selected, and a fraction  $1 - \omega$  of all firms adjust while the remaining  $\omega$  fraction do not adjust. The parameter  $\omega$  is a measure of the degree of nominal rigidity; a larger  $\omega$  implies that fewer firms adjust each period and that the expected time between price changes is longer. Those firms that do adjust their price at time  $t$  do so to maximize the expected discounted value of current and future profits. Profits at some future date  $t + s$  are affected by the choice of price at time  $t$  only if the firm has not received another opportunity to adjust between  $t$  and  $t + s$ . The probability of this is  $\omega^s$ .<sup>7</sup>

Before analyzing the firm's pricing decision, consider its cost minimization problem, which involves minimizing  $W_t N_{jt}$ , subject to producing  $c_{jt} = Z_t N_{jt}$ . This problem can be written, in real terms, as

$$\min_{N_{jt}} \left( \frac{W_t}{P_t} \right) N_{jt} + \varphi_t (c_{jt} - Z_t N_{jt}),$$

where  $\varphi_t$  is equal to the firm's real marginal cost.<sup>8</sup> The first-order condition implies

$$\varphi_t = \frac{W_t/P_t}{Z_t}. \quad (8.11)$$

6. See chapter 2 for further discussion of these first-order conditions in a basic MIU model.

7. In this formulation, the degree of nominal rigidity as measured by  $\omega$  is constant, and the probability that a firm has adjusted its price is a function of time but not of the current state.

8. The Lagrangian multiplier  $\varphi_t$  gives the effect on costs if the firm produces an additional unit of output.

To produce one extra unit of  $c_j$  the firm must hire  $1/Z_t$  units of labor at a real cost of  $(W_t/P_t)/Z_t$ .

The firm's pricing decision problem then involves picking  $p_{jt}$  to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right],$$

where the discount factor  $\Omega_{i,t+i}$  is given by  $\beta^i (C_{t+i}/C_t)^{-\sigma}$ .<sup>9</sup> Using the demand curve (8.5) to eliminate  $c_{jt}$ , this objective function can be written as

$$E_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}.$$

While individual firms produce differentiated products, they all have the same production technology and face demand curves with constant and equal demand elasticities. In other words, they are essentially identical except that they may have set their current price at different dates in the past. However, all firms adjusting in period  $t$  face the same problem, so all adjusting firms will set the same price. Let  $p_t^*$  be the optimal price chosen by all firms adjusting at time  $t$ . The first-order condition for the optimal choice of  $p_t^*$  is

$$E_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[ (1-\theta) \left( \frac{p_t^*}{P_{t+i}} \right) + \theta \varphi_{t+i} \right] \left( \frac{1}{p_t^*} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0. \quad (8.12)$$

Using the definition of  $\Omega_{i,t+i}$ , (8.12) can be rearranged to yield

$$\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta-1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \quad (8.13)$$

Consider the case of flexible prices so that all firms are able to adjust their prices every period ( $\omega = 0$ ). When  $\omega = 0$ , (8.13) reduces to

$$\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta-1} \right) \varphi_t = \mu \varphi_t. \quad (8.14)$$

Each firm sets its price  $p_t^*$  equal to a markup  $\mu > 1$  over its nominal marginal cost  $P_t \varphi_t$ . This is the standard result in a model of monopolistic competition. Because price exceeds marginal cost, output will be inefficiently low. When prices are flexible, all firms charge

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9. Given that firms are owned by households in this model, the appropriate discount factor for firms to use is based on the representative household's discounted marginal utility of consumption in the future relative to the marginal utility of consumption today.

the same price. In this case,  $p_t^* = P_t$  and  $\varphi_t = 1/\mu$ . Using the definition of real marginal cost, this means  $W_t/P_t = Z_t/\mu < Z_t$  in a flexible-price equilibrium. However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. This condition implies, from (8.9), that

$$\frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t} = \frac{Z_t}{\mu}. \quad (8.15)$$

With flexible prices, goods market clearing and the production function imply that  $C_t = Y_t$  and  $N_t = Y_t/Z_t$ . Using these conditions in (8.15), and letting  $Y_t^f$  denote equilibrium output under flexible prices,  $Y_t^f$  is given by

$$Y_t^f = \left( \frac{1}{\chi \mu} \right)^{1/(\sigma+\eta)} Z_t^{(1+\eta)/(\sigma+\eta)}. \quad (8.16)$$

When prices are flexible, output is a function of the aggregate productivity shock, reflecting the fact that, in the absence of sticky prices, the new Keynesian model reduces to a real business cycle model.

When prices are sticky ( $\omega > 0$ ), output can differ from the flexible-price equilibrium level. Because a firm will not adjust its price every period, it takes into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust its price. Equation (8.13) gives the optimal price to set, conditional on the current aggregate price level  $P_t$ . This aggregate price index, defined by (8.4), is an average of the price charged by the fraction  $1 - \omega$  of firms setting their price in period  $t$  and the average of the remaining fraction  $\omega$  of all firms that do not change their price in period  $t$ . However, because the adjusting firms were selected randomly from among all firms, the average price of the nonadjusters is just the average price of all firms that prevailed in period  $t - 1$ . Thus, from (8.4), the average price in period  $t$  satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \quad (8.17)$$

Thus, (8.13) and (8.17) jointly describe the evolution of the price level.

### 8.2.3 Market Clearing

In addition to affecting the evolution of the price level over time, price rigidity also affects the aggregate market-clearing condition for goods. Let  $y_{jt}$  denote the output produced by firm  $j$ . Then for each good  $j$ , market clearing requires  $y_{jt} = c_{jt}$ , where  $c_{jt}$  is the demand for good  $j$ . Defining aggregate output as

$$Y_t = \left[ \int y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (8.18)$$

and using the definition of  $C_t$  from (8.2), aggregate goods market clearing implies

$$Y_t = C_t. \quad (8.19)$$

Because the production function for firm  $j$  is  $y_{jt} = Z_t N_{jt}$ , aggregate employment is, using (8.5),

$$N_t \equiv \int N_{jt} dj = \int \left( \frac{y_{jt}}{Z_t} \right) dj = \left( \frac{Y_t}{Z_t} \right) \int \left( \frac{p_{jt}}{P_t} \right)^{-\theta} dj = \left( \frac{C_t}{Z_t} \right) \Delta_t, \quad (8.20)$$

where

$$\Delta_t = \int \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} dj \geq 1 \quad (8.21)$$

is a measure of price dispersion across the individual firms. If all firms set the same price,  $\Delta_t = 1$ , and the total employment necessary to produce  $C_t$  is simply  $C_t/Z_t = Y_t/Z_t$ , as was assumed in deriving an expression for  $Y_t^f$ . With sticky prices, however,  $\Delta_t \geq 1$ , but this means that aggregate employment is  $N_t = (C_t/Z_t) \Delta_t \geq (C_t/Z_t)$ . Price dispersion implies that more labor is required to produce a given overall consumption basket  $C_t$  than would be the case if all firms charged the same price. When firms are charging different prices, given that they all share the same technology, households purchase a combination of goods (more of the cheaper ones, less of the more expensive ones) that is socially inefficient. Suppose, for example, that good  $j$  is more expensive than good  $s$ . To maintain total consumption  $C_t$  constant, for every unit of  $c_{jt}$  the household fails to purchase because of its high price, it must purchase  $(c_{jt}/c_{st})^{-1/\theta}$  extra units of good  $s$ .<sup>10</sup> For  $c_{jt} < c_{st}$ ,  $\theta > 1$  implies  $(c_{jt}/c_{st})^{-1/\theta} > 1$ . The labor freed up from producing less  $c_{jt}$  is not sufficient to produce enough  $c_{sj}$  to maintain the same level of  $C_t$ . To keep  $C_t$  constant requires more labor input. Because working generates disutility, price dispersion is costly in terms of the welfare of households. This inefficiency is shown in section 8.4.1 to account for the costs of inflation variability in the new Keynesian model.<sup>11</sup>

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10. This follows because

$$dC_t = C_t^{1/\theta} \left[ c_{jt}^{-1/\theta} dc_{jt} + c_{sj}^{-1/\theta} dc_{sj} \right] = 0$$

implies

$$dc_{sj} = - \left( \frac{c_{jt}}{c_{sj}} \right)^{-1/\theta} dc_{jt}.$$

Of course, the household adjusts its consumption of all the individual goods, not just  $j$  and  $s$ , but focusing on just two of the goods helps illustrate the basic intuition.

11. Note that  $C_t = Y_t = (Z_t/\Delta_t)N_t$ , illustrating how price dispersion acts like a negative productivity shock to the aggregate production function. See problem 1 at the end of this chapter for an alternative derivation of the distortion generated by price dispersion.



It is useful to note that (8.21) implies<sup>12</sup>

$$\Delta_t = (1 - \omega) \left( \frac{P_t^*}{P_t} \right)^{-\theta} + \omega \left( \frac{P_t}{P_{t-1}} \right)^\theta \Delta_{t-1}. \quad (8.22)$$

### 8.3 A Linearized New Keynesian Model

Equations (8.7)–(8.9), (8.11), (8.13), (8.17), (8.19)–(8.20), and (8.22) provide the equilibrium conditions characterizing private sector behavior for the basic new Keynesian model. They represent a system in  $C_t$ ,  $N_t$ ,  $M/P_t$ ,  $Y_t$ ,  $\varphi_t$ ,  $P_t$ ,  $p_t^*$ ,  $W_t/P_t$ ,  $\Delta_t$ , and  $i_t$  that can be combined with a specification of monetary policy to determine the economy's equilibrium. These equations are nonlinear, but one reason for the popularity of the new Keynesian model is that it allows for a simple linear representation of private sector behavior in terms of an inflation adjustment equation, or Phillips curve, and an output and real interest rate relationship that corresponds to the investment-saving (IS) curve of undergraduate macroeconomics. To derive this linearized version, the nonlinear equilibrium conditions of the model are linearized around a steady state in which the inflation rate is zero. In what follows, let  $\hat{x}_t$  denote the percentage deviation of a variable  $X_t$  around its steady state, and let the superscript  $f$  denote the flexible-price equilibrium.

#### 8.3.1 The Linearized Phillips Curve

Equations (8.13) and (8.17) can be approximated around a zero average inflation steady-state equilibrium to obtain an expression for aggregate inflation (see section 8.7.1) of the form

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\varphi}_t, \quad (8.23)$$

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12. A fraction  $1 - \omega$  of firms all set their price equal to  $p_t^*$ . Therefore

$$\Delta_t = \int \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} dj = (1 - \omega) \left( \frac{P_t^*}{P_t} \right)^{-\theta} + \omega \int_{j \in NA} \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} dj,$$

where the notation  $j \in NA$  indicates the second integral is over firms in the set of nonadjusting (NA) firms, of which there are a measure  $\omega$ . Because for these firms  $p_{j,t} = p_{j,t-1}$ , and these firms are a random sample of all firms,

$$\begin{aligned} \int_{j \in NA} \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} dj &= \int_{j \in NA} \left( \frac{P_{t-1} p_{j,t-1}}{P_t P_{t-1}} \right)^{-\theta} dj \\ &= \left( \frac{P_{t-1}}{P_t} \right)^{-\theta} \Delta_{t-1}. \end{aligned}$$

Thus,

$$\Delta_t = (1 - \omega) \left( \frac{P_t^*}{P_t} \right)^\theta + \omega \left( \frac{P_t}{P_{t-1}} \right)^\theta \Delta_{t-1}.$$

where  $\hat{\varphi}_t$  is real marginal cost, expressed as a percentage deviation around its steady-state value, and

$$\tilde{\kappa} = \frac{(1 - \omega)(1 - \beta\omega)}{\omega}$$

is an increasing function of the fraction of firms able to adjust each period.<sup>13</sup>

Equation (8.23) is often referred to as the *new Keynesian Phillips curve*. It implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking, with current inflation a function of expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods. Solving (8.23) forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \hat{\varphi}_{t+i},$$

which shows that inflation is a function of the present discounted value of current and future real marginal costs.

The new Keynesian Phillips curve also differs from traditional Phillips curves in having been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices). This derivation reveals how  $\tilde{\kappa}$ , the impact of real marginal cost on inflation, depends on the structural parameters  $\beta$  and  $\omega$ . An increase in  $\beta$  means that the firm gives more weight to future expected profits. As a consequence,  $\tilde{\kappa}$  declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in  $\omega$ ) reduces  $\tilde{\kappa}$ ; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price, and fewer firms adjust each period.

Equation (8.23) implies that inflation depends on real marginal cost, not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.<sup>14</sup> However, real marginal costs can be related to an output gap measure. The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor (see 8.11). In a flexible-price equilibrium, all firms set the same price, so (8.14) implies that real marginal cost will equal its steady-state value of  $1/\mu$ . Because nominal wages have been assumed to be completely flexible, the real wage must, according to

13. See also Sections 7.2.4 and 7.3.2. Ascari (2004) showed that the behavior of inflation in the Calvo model can be significantly affected if steady-state inflation is not zero. See section 7.3.2.

14. See Ravenna and Walsh (2008), Blanchard and Galí (2010), and Galí (2011) for models of labor market frictions that relate inflation to unemployment. Incorporating unemployment into the NK model is discussed in section 8.5.

(8.9), equal the marginal rate of substitution between leisure and consumption. Expressed in terms of percentage deviations around the steady state, (8.11) implies  $\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t = \eta \hat{n}_t + \sigma \hat{c}_t - \hat{z}_t$ . From the goods-clearing condition,  $C_t = Y_t$ , so  $\hat{c}_t = \hat{y}_t$ . From (8.20),  $N_t = Y_t \Delta_t / Z_t$ . To first order, this becomes  $\hat{n}_t = \hat{y}_t - \hat{z}_t$ .<sup>15</sup> Hence, the percentage deviation of real marginal cost around its steady-state value is

$$\begin{aligned}\hat{\varphi}_t &= \eta (\hat{y}_t - \hat{z}_t) + \sigma \hat{y}_t - \hat{z}_t \\ &= (\sigma + \eta) \left[ \hat{y}_t - \left( \frac{1 + \eta}{\sigma + \eta} \right) \hat{z}_t \right].\end{aligned}$$

To interpret the term involving  $\hat{z}_t$ , linearize (8.16) giving flexible-price output to obtain

$$\hat{y}_t^f = \left( \frac{1 + \eta}{\sigma + \eta} \right) \hat{z}_t. \quad (8.24)$$

Thus, (8.24) can be used to express real marginal cost as

$$\hat{\varphi}_t = (\sigma + \eta) (\hat{y}_t - \hat{y}_t^f). \quad (8.25)$$

Defining  $x_t \equiv \hat{y}_t - \hat{y}_t^f$  as the gap between actual output and flexible-price equilibrium output and using (8.25), the inflation adjustment equation (8.23) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \quad (8.26)$$

where

$$\kappa = (\sigma + \eta) \tilde{\kappa} = (\sigma + \eta) \left[ \frac{(1 - \omega)(1 - \beta\omega)}{\omega} \right]. \quad (8.27)$$

The preceding assumed firms face constant returns to scale. If instead each firm's production function is  $c_{jt} = Z_t N_{jt}^a$ , where  $0 < a \leq 1$ , then the results must be modified slightly. When  $a < 1$ , firms with different production levels will face different marginal costs, and real marginal cost for firm  $j$  will equal

$$\varphi_{jt} = \frac{W_t/P_t}{a Z_t N_{jt}^{a-1}} = \frac{W_t/P_t}{a c_{jt}/N_{jt}}.$$

Linearizing this expression for firm  $j$ 's real marginal cost and using the production function yields

$$\hat{\varphi}_{jt} = (\hat{w}_t - \hat{p}_t) - (\hat{c}_{jt} - \hat{n}_{jt}) = (\hat{w}_t - \hat{p}_t) - \left( \frac{a-1}{a} \right) \hat{c}_{jt} - \left( \frac{1}{a} \right) \hat{z}_t. \quad (8.28)$$

15. When linearized, the last term becomes

$$\hat{\Delta}_t = -\theta \int (\hat{p}_{jt} - \hat{p}_t) dj,$$

but to a first-order approximation,  $\int \hat{p}_{jt} dj = \hat{p}_t$ , so the deviation of the price dispersion term around the steady state is approximately (to first order) equal to zero.

Marginal cost for the individual firm can be related to average marginal cost,  $\varphi_t = (W_t/P_t)/(aC_t/N_t)$ , where

$$N_t = \int_0^1 N_{jt} dj = \int_0^1 \left( \frac{c_{jt}}{Z_t} \right)^{\frac{1}{a}} dj = \left( \frac{C_t}{Z_t} \right)^{\frac{1}{a}} \int_0^1 \left( \frac{p_{jt}}{P_t} \right)^{-\frac{\theta}{a}} dj.$$

When this last expression is linearized around a zero inflation steady state, the final term involving the dispersion of relative prices is of second order, so to first order one obtains

$$\hat{n}_t = \left( \frac{1}{a} \right) (\hat{c}_t - \hat{z}_t),$$

and therefore

$$\hat{\varphi}_t = (\hat{w}_t - \hat{p}_t) - (\hat{c}_t - \hat{n}_t) = (\hat{w}_t - \hat{p}_t) - \left( \frac{a-1}{a} \right) \hat{c}_t - \left( \frac{1}{a} \right) \hat{z}_t. \quad (8.29)$$

Subtracting (8.29) from (8.28) gives

$$\hat{\varphi}_{jt} - \hat{\varphi}_t = - \left( \frac{a-1}{a} \right) (\hat{c}_{jt} - \hat{c}_t).$$

Finally, employing the demand relationship (8.5) to express  $\hat{c}_{jt} - \hat{c}_t$  in terms of relative prices,

$$\hat{\varphi}_{jt} = \hat{\varphi}_t - \left[ \frac{\theta(1-a)}{a} \right] (\hat{p}_{jt} - \hat{p}_t).$$

Firms with relatively high prices (and therefore low output) have relatively low real marginal costs. In the case of constant returns to scale ( $a = 1$ ), all firms face the same marginal cost. When  $a < 1$ , Sbordone (2002b) and Galí, Gertler, and López-Salido (2001) showed the new Keynesian inflation adjustment equation<sup>16</sup> becomes

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \left[ \frac{a}{a + \theta(1-a)} \right] \hat{\varphi}_t.$$

In addition, the labor market equilibrium condition under flexible prices becomes

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^\eta}{C_t^{-\sigma}},$$

which implies flexible-price output is

$$\hat{y}_t^f = \left[ \frac{1 + \eta}{1 + \eta + a(\sigma - 1)} \right] \hat{z}_t.$$

When  $a = 1$ , this reduces to (8.24).

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16. See the chapter appendix for further details on the derivation.

### 8.3.2 The Linearized IS Curve

The new Keynesian Phillips curve given by (8.26) is one of the two key components of the new Keynesian model. The other component is a linearized version of the household's Euler condition, (8.7). Because consumption is equal to output in this model (there is no government or investment because capital has been ignored), (8.7) can be approximated around the zero inflation steady state<sup>17</sup> as

$$\hat{y}_t = E_t \hat{y}_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r), \quad (8.30)$$

where  $i_t$  is the nominal interest rate, and  $r$  is the steady-state real interest rate. Expressing this in terms of the output gap  $x_t = \hat{y}_t - \hat{y}_t^f$ ,

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r) + u_t, \quad (8.31)$$

where  $u_t \equiv E_t \hat{y}_{t+1}^f - \hat{y}_t^f$  depends only on the exogenous productivity disturbance (see 8.24).

Combining (8.31) with (8.26) gives a simple two-equation, forward-looking rational-expectations model for inflation and the output gap, once the behavior of the nominal rate of interest is specified.<sup>18</sup> This two-equation model consists of the equilibrium conditions for a well-specified general equilibrium model. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. Equation (8.31) represents the demand side of the economy (an expectational, forward-looking IS curve), while the new Keynesian Phillips curve (8.26) corresponds to the supply side. In fact, both equations are derived from optimization problems, with (8.31) based on the Euler condition for the representative household's decision problem and (8.26) derived from the optimal pricing decisions of individual firms.

There is a long tradition of using two-equation, aggregate demand–aggregate supply (AD-AS) models in intermediate-level macroeconomic and monetary policy analysis. Models in the AD-AS tradition are often criticized as “starting from curves” rather than starting from the primitive tastes and technology from which behavioral relationships can be derived, given maximizing behavior and a market structure (Sargent 1982). This criticism does not apply to (8.31) and (8.26). The parameters appearing in these two equations

17. In the steady state with constant consumption, (8.7) implies  $1 = \beta(1+r)$ , where  $r = (1+i)/(1+\pi)$  is the steady-state real interest rate. Hence, one can write (8.7) as

$$(1 - \sigma \hat{c}_t) = E_t \left[ \frac{1 + i_t}{(1+r)(1+\pi_{t+1})} \right] (1 - \sigma \hat{c}_{t+1}),$$

following the approach used in chapter 2 (see the appendix to that chapter) and noting that  $\hat{c}_t = \hat{y}_t$  yields (8.30). Previous editions expressed (8.30) in terms of  $\hat{i}_t \equiv i_t - r$ .

18. With the nominal interest rate treated as the monetary policy instrument, (8.8) simply determines the real quantity of money in equilibrium.

are explicit functions of the underlying structural parameters of the production and utility functions and the assumed process for price adjustment.<sup>19</sup> Equations (8.31) and (8.26) contain expectations of future variables; the absence of this type of forward-looking behavior is a critical shortcoming of older AD-AS frameworks. The importance of incorporating a role for future income was emphasized by Kerr and King (1996).

Equations (8.31) and (8.26) contain three variables: the output gap, inflation, and the nominal interest rate. The model can be closed by a monetary policy rule describing the central bank's behavior in setting the nominal interest rate.<sup>20</sup> Alternatively, if the central bank implements monetary policy by setting a path for the nominal supply of money, (8.26) and (8.31), together with the linearized version of (8.8), determine  $x_t$ ,  $\pi_t$ , and  $i_t$ .<sup>21</sup>

### 8.3.3 Local Uniqueness of the Equilibrium

Suppose monetary policy is represented by the following purely exogenous process for  $i_t$ :

$$\dot{i}_t = r + v_t, \quad (8.32)$$

where  $v_t$  is a stationary stochastic process. Combining (8.32) with (8.31) and (8.26), the resulting system of equations can be written as

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}.$$

Premultiplying both sides by the inverse of the matrix on the left produces

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \quad (8.33)$$

where

$$M = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Blanchard and Kahn (1980) showed that systems such as (8.33) have a unique stationary solution if and only if the number of eigenvalues of  $M$  outside the unit circle is equal to the number of forward-looking variables, in this case, two.<sup>22</sup> However, only the largest

19. The process for price adjustment, however, has not been derived from the underlying structure of the economic environment.

20. Important issues of price level determinacy rather than inflation determinacy arise under interest rate–setting policies, (see chapter 10.)

21. An alternative approach, discussed in section 8.3.4, specifies an objective function for the monetary authority and then derives the policymaker's decision rule for setting the nominal interest rate.

22. See the chapter 2 appendix.

eigenvalue of this matrix is outside the unit circle, implying that multiple bounded equilibria exist and that the equilibrium is locally indeterminate. Stationary sunspot equilibria are possible.

This example illustrates that an exogenous policy rule—one that does not respond to the endogenous variables  $x$  and  $\pi$ —introduces the possibility of multiple equilibria. To understand why, consider what would happen if expected inflation were to rise. Because (8.32) does not allow for any endogenous feedback from this rise in expected inflation to the nominal interest rate, the real interest rate must fall. This decline in the real interest rate is expansionary, and the output gap increases. The rise in output increases actual inflation, according to (8.26). Thus, a change in expected inflation, even if due to factors unrelated to the fundamentals of inflation, can set off a self-fulfilling change in actual inflation.

This discussion suggests that a policy that raises the nominal interest rate when inflation rises, and raises  $i_t$  enough to increase the real interest rate so that the output gap falls, would be sufficient to ensure a unique equilibrium. For example, suppose the nominal interest rate responds to inflation according to the rule

$$i_t = r + \delta\pi_t + v_t. \quad (8.34)$$

Combining (8.34) with (8.26) and (8.31),  $i_t$  can be eliminated and the resulting system written as

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \quad (8.35)$$

where

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{\beta\delta-1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Bullard and Mitra (2002) showed that a unique stationary equilibrium exists as long as  $\delta > 1$ .<sup>23</sup> Setting  $\delta > 1$  is referred to as satisfying the *Taylor principle*, because Taylor was the first to stress the importance of interest rate rules that called for responding more than one-to-one to changes in inflation.

Suppose that instead of reacting solely to inflation, as in (8.34), the central bank responds to both inflation and the output gap according to

$$i_t = r + \delta_\pi \pi_t + \delta_x x_t + v_t.$$

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23. If the nominal interest rate is adjusted in response to expected future inflation (rather than current inflation), multiple solutions again become possible if  $i_t$  responds too strongly to  $E_t \pi_{t+1}$ . See Clarida, Gali, and Gertler (2000).

This type of policy rule is called a *Taylor rule* (Taylor 1993a), and variants of it have been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida, Galí, and Gertler 2000).<sup>24</sup> With this policy rule, Bullard and Mitra (2002) showed that the condition necessary to ensure that the economy has a unique stationary equilibrium becomes

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0. \quad (8.36)$$

Determinacy now depends on both the policy parameters  $\delta_\pi$  and  $\delta_x$ . A policy that fails to raise the nominal interest rate sufficiently when inflation rises would lead to a rise in aggregate demand and output. This rise in  $x$  could produce a rise in the real interest rate that serves to contract spending if  $\delta_x$  were large. Thus, a policy rule with  $\delta_\pi < 1$  could still be consistent with a unique stationary equilibrium. At a quarterly frequency, however,  $\beta$  is about 0.99, so  $\delta_x$  would need to be very large to offset a value of  $\delta_\pi$  much below 1.

The Taylor principle is an important policy lesson that has emerged from the new Keynesian model. It has been argued that the failure of central banks such as the Federal Reserve to respond sufficiently strongly to inflation during the 1970s provides an explanation for the rise in inflation experiences at the time (see Lubik and Schorfheide 2005). Further, Orphanides (2001) argued that estimated Taylor rules for the Federal Reserve are sensitive to whether real-time data are used, and he found a much weaker response to inflation in the 1987–1999 period based on real-time data.<sup>25</sup> Because the Taylor principle is based on the mapping from policy response coefficients to eigenvalues in the state space representation of the model, one would expect the exact restrictions the policy responses must satisfy to ensure determinacy will depend on the specification of the model.

Two aspects of the model have been explored that lead to significant modifications of the Taylor principle. First, Hornstein and Wolman (2005), Ascari and Ropele (2007), and Kiley (2007b) found that the Taylor rule can be insufficient to ensure determinacy when trend inflation is positive rather than zero, as assumed when obtaining the standard linearized new Keynesian inflation equation. For example, Coibion and Gorodnichenko (2011) showed, in a calibrated model, that the central bank's response to inflation in a rule such as (8.34) would need to be over 8 to ensure determinacy if steady-state inflation exceeded 6 percent. However, many models assume some form of indexation (see chapter 7), and for these models, the standard Taylor principle ( $\delta_\pi > 1$ ) would continue to hold even in the face of a positive steady-state rate of inflation. In this context, it is

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24. Sometimes the term *Taylor rule* is reserved for the case in which  $\delta_\pi = 1.5$  and  $\delta_x = 0.5$  when inflation and the interest rate are expressed at annual rates. These are the values Taylor (1993a) found matched the behavior of the federal funds rate during the Greenspan period.

25. Other papers employing real-time data to estimate policy rules include Rudebusch (2006) for the United States and Papell, Molodtsova, and Nikolsko-Rzhevskyy (2008) for the United States and Germany.



important to note that (8.26) was obtained by linearizing around a zero inflation steady state. If steady-state inflation is nonzero, then the linearized Calvo model takes a much more complex form, as shown by Ascari (2004).<sup>26</sup> For a survey on the implications of trend inflation, see Ascari and Sbordone (2014).

Second, the Taylor principle can be significantly affected when interest rates have direct effects on real marginal cost. Such an effect, usually referred to as the cost channel of monetary policy, is common in models in which firms need to finance wage payments, as in the models of Christiano, Eichenbaum, and Evans (2005), Ravenna and Walsh (2006), or Christiano, Eichenbaum, and Trabandt (2014), or in which search frictions in the labor market introduce an intertemporal aspect to the firm's labor demand condition (Ravenna and Walsh 2008). For example, see Llosa and Tuesta (2009) for a model with a cost channel and Kurozumi and Van Zandweghe (2010) for a model with search and matching frictions in which satisfying the standard Taylor principle of responding more than one-to-one to inflation may not ensure determinacy.

Note that if  $v_t$  and  $u_t$  are zero for all  $t$ , the solution to (8.35) would be  $\pi_t = x_t = 0$  for all  $t$ . In this case, the parameter  $\delta$  in the policy rule (8.34) could not be identified, yet the fact that it exceeds 1 is necessary to ensure  $\pi_t = x_t = 0$  is the unique equilibrium. As Cochrane (2011a) emphasized, determinacy relies on assumptions about how the central bank would respond to movements of inflation out of equilibrium. Estimated Taylor rules may not reveal how policy would react in circumstances that are not observed. Cochrane also argued that determinacy requires the central bank to act in a manner that introduces an explosive root into the dynamic system; he characterized this as requiring the central bank to “blow up the world” to ensure determinacy.<sup>27</sup>

Finally, Benhabib, Schmitt-Grohé, and Uribe (2001b) emphasized that the equilibrium singled out when the Taylor principle is satisfied is only locally unique. By introducing an explosive root, equilibria in which  $\pi_t > 0$  are ruled out as stationary solutions to (8.35) because they lead to explosive inflations. However, if  $\pi_t < 0$ , the explosive root leads to falling inflation and a falling nominal interest rate. But if the nominal interest rate is restricted to be non-negative, then it cannot keep falling. Instead, once  $i_t = 0$ , the economy reaches a second stationary equilibrium with the nominal interest rate at zero. Thus, the standard equilibrium of (8.35) is locally but not globally unique. This issue is discussed further in chapter 11, where the focus is on equilibria at the effective lower bound for nominal interest rates.

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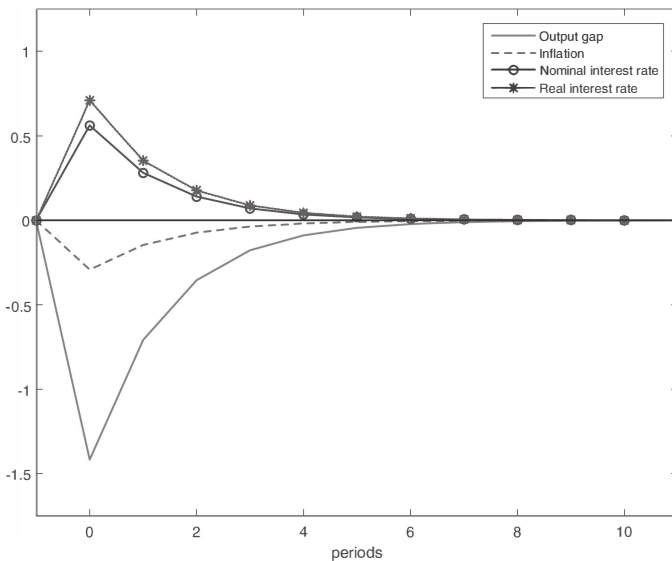
26. Ascari and Ropele (2007) considered the implications of trend inflation for optimal monetary policy, and Lago Alves (2014) showed that the divine coincidence (that monetary policy can achieve a zero inflation and a zero output gap in the absence of cost shocks) no longer holds when trend inflation is nonzero. Cogley and Sbordone (2008) estimated a linearized Calvo model accounting for positive trend inflation.

27. Recall that the basic model with  $i_t = r + v_t$  had only one eigenvalue outside the unit circles, but two were needed to ensure a unique equilibrium.

### 8.3.4 The Monetary Transmission Mechanism

The model consisting of (8.26) and (8.31) assumes the impact of monetary policy on output and inflation operates through the real rate of interest. As long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. Changes in the real interest rate alter the optimal time path of consumption. An increase in the real rate of interest, for instance, leads households to attempt to postpone consumption. Current consumption falls relative to future consumption.<sup>28</sup> With sticky prices, the fall in current aggregate demand causes a fall in output.

Figure 8.1 illustrates the impact of a monetary policy shock (an increase in the nominal interest rate) in the model consisting of (8.26), (8.31), and the policy rule (8.34).<sup>29</sup> The parameter values used in constructing the figure are  $\beta = 0.99$ ,  $\sigma = \eta = 1$ ,  $\delta = 1.5$ , and  $\omega = 0.8$ . In addition, the policy shock  $v_t$  in the policy rule is assumed to follow an AR(1) process given by  $v_t = \rho_v v_{t-1} + \varepsilon_t$ , with  $\rho_v = 0.5$ . The rise in the nominal rate causes inflation and the output gap to fall immediately. This reflects the forward-looking nature of



**Figure 8.1**

Output, inflation, and real interest rate responses to a policy shock in the new Keynesian model.

28. Estrella and Fuhrer (2002) noted that the forward-looking Euler equation implies counterfactual dynamics; (8.31) implies that  $E_t \hat{c}_{t+1} - \hat{c}_t = \sigma^{-1}(i_t - E_t \pi_{t+1} - r) - u_t$ , so that a rise in the real interest rate means that consumption must be expected to increase from  $t$  to  $t + 1$ ;  $\hat{c}_t$  falls to ensure this is true.

29. The programs used to obtain figures in this chapter are available at <http://people.ucsc.edu/~walshc/mtp4e/>.

both variables. In fact, all the persistence displayed by the responses arises from the serial correlation introduced into the process for the monetary shock  $v_t$ . If  $\rho_v = 0$ , all variables return to their steady-state values in the period after the shock.<sup>30</sup>

To emphasize the interest rate as the primary channel through which monetary influences affect output, it is convenient to express the output gap as a function of an *interest rate gap*, the gap between the current interest rate and the interest rate consistent with the flexible-price equilibrium. For example, let  $r_t \equiv i_t - E_t \pi_{t+1}$  be the real interest rate, and write (8.31) as

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (r_t - \tilde{r}_t), \quad (8.37)$$

where  $\tilde{r}_t \equiv r + \sigma u_t$ . Woodford (2003a) labeled  $\tilde{r}_t$  the *Wicksellian real interest rate*. It is the interest rate consistent with output equaling the flexible-price equilibrium level. If  $r_t = \tilde{r}_t$  for all  $t$ , then  $x_t = 0$  and output is kept equal to the level that would arise in the absence of nominal rigidities. The interest rate gap  $r_t - \tilde{r}_t$  then summarizes the effects on the actual equilibrium that are due to nominal rigidities.<sup>31</sup>

The presence of expected future output in (8.37) implies that the future path of the one-period real interest rate matters for current demand. To see this, recursively solve (8.37) forward to yield

$$x_t = - \left( \frac{1}{\sigma} \right) \sum_{i=0}^{\infty} E_t (r_{t+i} - \tilde{r}_{t+i}).$$

Changes in the one-period rate that are persistent will influence expectations of future interest rates. Therefore, persistent changes should have stronger effects on  $x_t$  than more temporary changes in real interest rates.

The basic interest rate transmission mechanism for monetary policy could be extended to include effects on investment spending if capital were reintroduced into the model (Christiano, Eichenbaum, and Evans 2005; Dotsey and King 2006). Increases in the real interest rate would reduce the demand for capital and lead to a fall in investment spending. In the case of both investment and consumption, monetary policy effects are transmitted through interest rates.

In addition to these interest rate channels, monetary policy is often thought to affect the economy either indirectly through credit channels or directly through the quantity of money. Real money holdings represent part of household wealth; an increase in real balances should induce an increase in consumption spending through a wealth effect.

30. See Galí (2003) for a discussion of the monetary transmission mechanism incorporated into the basic new Keynesian model.

31. Neiss and Nelson (2003) used a structural model to estimate the real interest rate gap  $r_t - \tilde{r}_t$  and found that it has value as a predictor of inflation.

This channel is often called the *Pigou effect* and was viewed as generating a channel through which price level declines during a depression would eventually increase real balances and household wealth sufficiently to restore consumption spending. During the Keynesian/monetarist debates of the 1960s and early 1970s, some monetarists argued for a direct wealth effect that linked changes in the money supply directly to aggregate demand (Patinkin 1965). The effect of money on aggregate demand operating through interest rates was viewed as a Keynesian interpretation of the transmission mechanism, whereas most monetarists argued that changes in monetary policy lead to substitution effects over a broader range of assets than Keynesians normally considered. Because wealth effects are likely to be small at business cycle frequencies, most simple models used for policy analysis ignore them.<sup>32</sup>

Direct effects of the quantity of money are not present in the model used here; the quantity of money appears in neither (8.26) nor (8.31). The underlying model is derived from an MIU model, and the absence of money in (8.31) and (8.26) results from the assumption that the utility function is separable (see 8.1). If utility is not separable, then changes in the real quantity of money alter the marginal utility of consumption and/or leisure. This would affect the model specification in two ways. First, the real money stock would appear in the household's Euler condition and therefore in (8.31). Second, to replace real marginal cost with a measure of the output gap in (8.26), the real wage was equated to the marginal rate of substitution between leisure and consumption, and this would also involve real money balances if utility were nonseparable (see problem 10 at the end of this chapter). Thus, the absence of money constitutes a special case. However, McCallum and Nelson (1999) and Woodford (2003a) argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability. Ireland (2004) found little evidence for nonseparable preferences in a model estimated on U.S. data.

The quantity of money is not totally absent from the underlying model, because (8.8) must also hold in equilibrium. Linearizing this equation around the steady state<sup>33</sup> yields

$$\hat{m}_t - \hat{p}_t = \left( \frac{1}{b i^{ss}} \right) [\sigma \hat{y}_t - (i_t - i^{ss})]. \quad (8.38)$$

Given the nominal interest rate chosen by the monetary policy authority, this equation determines the nominal quantity of money. Alternatively, if the policymaker sets the nominal quantity of money, then (8.26), (8.31), and (8.38) must all be used to solve jointly for  $x_t$ ,  $\pi_t$ , and  $i_t$ .

Chapter 10 discusses the role of credit channels in the monetary transmission process.

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32. For an analysis of the real balance effect, see Ireland (2005). Kaplan, Moll, and Violante (2016) developed a model with heterogeneous households and incomplete markets and argued that the direct interest rate impact on consumption is small. Canzoneri, Cumby, and Diba (2007) argued that Euler equations that form the basis of the new Keynesian aggregate demand relationship are poor matches to the data.

33. See the chapter 2 appendix.

### 8.3.5 Adding Economic Disturbances

As the model consisting of (8.26) and (8.31) stands, there are no underlying nonpolicy disturbances that might generate movements in either the output gap or inflation other than the productivity disturbance that affects the flexible-price output level. It is common, however, to include in these equations stochastic disturbances arising from other sources.

Suppose the representative household's utility from consumption is subject to random shocks that alter the marginal utility of consumption and the marginal disutility of work. Specifically, let the utility function in (8.1) be modified to include stochastic taste shocks  $\psi_t$  and  $\chi_t$ :

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{(\psi_{t+i} C_{t+i})^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi_t \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \quad (8.39)$$

The Euler condition (8.7) becomes

$$\psi_t^{1-\sigma} C_t^{-\sigma} = \beta(1+i_t) E_t (P_t/P_{t+1}) (\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}),$$

which, when linearized around the zero inflation steady state, yields

$$\hat{c}_t = E_t \hat{c}_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r) + \left( \frac{\sigma-1}{\sigma} \right) (E_t \hat{\psi}_{t+1} - \hat{\psi}_t). \quad (8.40)$$

If, in addition to consumption by households, the government purchases final output  $G_t$ , the goods market equilibrium condition becomes  $Y_t = C_t + G_t$ . When this is expressed in terms of percentage deviations around the steady state, one obtains

$$\hat{y}_t = \left( \frac{C}{Y} \right)^{ss} \hat{c}_t + \left( \frac{G}{Y} \right)^{ss} \hat{g}_t.$$

Using this equation to eliminate  $\hat{c}_t$  from (8.40) and then replacing  $\hat{y}_t$  with  $\hat{y}_t^f + x_t$  yields an expression for the output gap:

$$x_t = E_t x_{t+1} - \left( \frac{1}{\tilde{\sigma}} \right) (i_t - E_t \pi_{t+1} - r) + \xi_t, \quad (8.41)$$

where  $\tilde{\sigma}^{-1} = \sigma^{-1} (C/Y)^{ss}$  and

$$\xi_t \equiv \left( \frac{\sigma-1}{\sigma} \right) \left( \frac{C}{Y} \right)^{ss} (E_t \hat{\psi}_{t+1} - \hat{\psi}_t) - \left( \frac{G}{Y} \right)^{ss} (E_t \hat{g}_{t+1} - \hat{g}_t) + (E_t \hat{y}_{t+1}^f - \hat{y}_t^f).$$

Equation (8.41) represents the Euler condition consistent with the representative household's intertemporal optimality condition linking consumption levels over time. It is also consistent with the resource constraint  $Y_t = C_t + G_t$ . The disturbance term arises from taste shocks that alter the marginal utility of consumption, shifts in government purchases, and

shifts in the flexible-price equilibrium output.<sup>34</sup> In each case, it is expected changes in  $\psi$ ,  $g$ , and  $\hat{y}^f$  that matter. For example, an expected rise in government purchases implies that future consumption must fall. This reduces current consumption as forward-looking households respond immediately to the expected fall in future consumption.

Defining  $r_t \equiv r + \tilde{\sigma} \xi_t$ , (8.41) can be written in a convenient form as

$$x_t = E_t x_{t+1} - \left( \frac{1}{\tilde{\sigma}} \right) (i_t - E_t \pi_{t+1} - r_t). \quad (8.42)$$

Written in this form it shows that  $r_t$  is the equilibrium real interest rate consistent with a zero output gap. That is, if  $x_t = 0$  for all  $t$ , then the actual real interest rate  $i_t - E_t \pi_{t+1}$  must equal  $r_t$ .

Two commonly assumed objectives of monetary policy are to maintain a low and stable average rate of inflation and to stabilize output around full employment. If the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (8.26) implies the central bank can always achieve a zero output gap (i.e., keep output at its flexible-price equilibrium level) and simultaneously keep inflation equal to zero. Solving (8.26) forward yields

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i}.$$

By keeping current and expected future output equal to the flexible-price equilibrium level,  $E_t x_{t+i} = 0$  for all  $i$ , and inflation remains equal to zero. Blanchard and Galí (2007) describe this as the “divine coincidence.” This result holds even with the addition of a taste shock  $\chi_t$  that affects the marginal rate of substitution between work and consumption and so affects the flexible-price output level (see problem 4 at the end of the chapter).

However, if an error term is added to (8.26) so that

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (8.43)$$

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+i}.$$

As long as  $\sum_{i=0}^{\infty} \beta^i E_t e_{t+i} \neq 0$ , maintaining  $\sum_{i=0}^{\infty} \beta^i E_t x_{t+i} = 0$  is not sufficient to ensure that inflation always remains equal to zero. A trade-off between stabilizing output and stabilizing inflation can arise. Disturbance terms in the inflation adjustment equation are often called *cost shocks* or *inflation shocks*. Because these shocks ultimately affect only the price level, they are also called *price shocks*.

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34. These three terms are not independent, as  $\psi_t$  and  $\hat{g}_t$  affect flexible-price output  $\hat{y}_t^f$ .

Benigno and Woodford (2005) showed that a cost shock arises in the presence of stochastic variation in the gap between the welfare-maximizing level of output and the flexible-price equilibrium level of output. In the model developed so far, only two distortions are present, one due to monopolistic competition and one due to nominal price stickiness. The first distortion implies the flexible-price output level is below the efficient output level even when prices are flexible. However, this wedge, measured by the markup due to imperfect competition, is constant, so when the model is linearized, percent deviations of the flexible-price output and the efficient output around their respective steady-state values are equal. If there are time-varying distortions such as would arise with stochastic variation in markups in product or labor markets or in distortionary taxes, then fluctuations in the two output concepts will differ. In this case, if  $x_t^w$  is the percent deviation of the welfare-maximizing output level around its steady state (the welfare gap),

$$x_t = x_t^w + \delta_t,$$

where  $\delta_t$  represents these stochastic distortions. Because policymakers would be concerned with stabilizing fluctuations in  $x_t^w$ , the relevant constraint the policymaker faces is obtained by rewriting the Phillips curve (8.26) as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t = \beta E_t \pi_{t+1} + \kappa x_t^w + \kappa \delta_t. \quad (8.44)$$

In this formulation,  $\kappa \delta_t$  acts as a cost shock; stabilizing inflation in the face of nonzero realizations of  $\delta_t$  cannot be achieved without creating volatility in the welfare gap  $x_t^w$ . One implication of (8.44) is that the variance of the cost shock depends on  $\kappa^2$ . Thus, if the degree of price rigidity is high, implying that  $\kappa$  is small, cost shocks will also be less volatile (see Walsh (2005a)).

New Keynesian models, particularly those designed to be taken to the data, introduce a disturbance in the inflation equation by assuming that individual firms face random variation in the price elasticity of demand. That is, the parameter  $\theta$  in household preferences that determines the demand elasticity (see 8.5) is assumed to be time-varying. This modification leads to stochastic variation in markups, generating a wedge between flexible-price output and efficient output, and giving rise to cost shocks when the inflation equation is expressed in terms of the welfare gap, as in (8.44).

#### 8.4 Monetary Policy Analysis in New Keynesian Models

During the ten years after its first introduction, the new Keynesian model discussed in section 8.3 became the standard framework for monetary policy analysis. Clarida, Galí, and Gertler (1999), McCallum and Nelson (1999), Woodford (2003a), and Svensson and Woodford (2005), among others, popularized this simple model for use in monetary policy analysis. Galí and Gertler (2007) and Galí (2008) discussed some of the model's

implications for monetary policy, while Galí (2015) provided an excellent treatment of the model and its implications for policy.

As seen in section 8.3, the basic new Keynesian model takes the form

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t), \quad (8.45)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (8.46)$$

where  $x_t$  is the output gap, defined as output relative to the equilibrium level of output under flexible prices,  $i_t$  is the nominal rate of interest, and  $\pi_t$  is the inflation rate. The demand disturbance  $r_t$  can arise from taste shocks to the preferences of the representative household, fluctuations in the flexible-price equilibrium output level, or shocks to government purchases of goods and services. The  $e_t$  shock is a cost shock that reflects exogenous stochastic variations in the markup. In this section, (8.45) and (8.46) are used to address issues of monetary policy design.

### 8.4.1 Policy Objectives

Given the economic environment that leads to (8.45) and (8.46), what are the appropriate objectives of the central bank? There is a long history in monetary policy analysis of assuming that the central bank is concerned with minimizing a quadratic loss function that depends on output and inflation. Models that make this assumption were discussed in chapter 6. Although such an assumption is plausible, it is ultimately ad hoc. In the new Keynesian model, the description of the economy is based on an approximation to a fully specified general equilibrium model. One can therefore develop a policy objective function that can be interpreted as an approximation to the utility of the representative household. The general equilibrium foundations of (8.45) and (8.46) can then provide insights into the basic objectives central banks should pursue. Woodford (2003a), building on the earlier work by Rotemberg and Woodford (1997), provided the most detailed analysis of the link between a welfare criterion derived as an approximation to the utility of the representative agent and the types of quadratic loss functions common in the older literature.

Much of the literature that derives policy objectives based on the utility of the representative household follows Woodford (2003a) in restricting attention to the case of a cashless economy, so real money balances do not appear in the utility function. Thus, assume the representative household seeks to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right], \quad (8.47)$$

where the consumption aggregate  $C_t$  is defined as in (8.2). Woodford demonstrated that deviations of the expected discounted utility of the representative agent around the level of



steady-state utility can be approximated by

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right] + \text{t.i.p.}, \quad (8.48)$$

where t.i.p. indicates terms independent of monetary policy. A detailed derivation of (8.48) and the values of  $\Omega$  and  $\lambda$  are given in section 8.7.2. In (8.48),  $x_t$  is the gap between output and the output level that would arise under flexible prices, and  $x^*$  is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the actual steady-state level of output.

Equation (8.48) looks like the standard quadratic loss function employed in chapter 6 to represent the objectives of the monetary policy authority. There are, however, two critical differences. First, the output gap is measured relative to output under flexible prices. In the traditional literature the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output, which in turn was often defined as output in the absence of price surprises (see section 6.2.2).

A second difference between (8.48) and the quadratic loss function of chapter 6 arises from the reason inflation variability enters the loss function. When prices are sticky, and firms do not all adjust simultaneously, inflation results in an inefficient dispersion of relative prices and production among individual producers. Households respond to relative price dispersion by buying more of the relatively cheaper goods and less of the relatively more expensive goods. In turn, (8.20) showed that price dispersion means that more labor is required to produce an overall consumption basket  $C_t$  than would be the case if all firms charged the same price. Because working generates disutility, price dispersion is inefficient and reduces welfare. When each firm does not adjust its price every period, price dispersion is caused by inflation. These welfare costs can be eliminated under a zero inflation policy.

In chapter 6, the efficiency distortion represented by  $x^* > 0$  was used to motivate an overly ambitious output target in the central bank's objective function. The presence of  $x^* > 0$  implies a central bank acting under discretion to maximize (8.48) would produce a positive average inflation bias. However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, it is common now to simply assume  $x^* = 0$ . In this case, the central bank is concerned with stabilizing the output gap  $x_t$ , and no average inflation bias arises.<sup>35</sup> If tax subsidies can be used to offset the distortions associated with monopolistic competition, then one could assign fiscal policy the task of ensuring  $x^* = 0$ . In this case, the central bank has no incentive to create inflationary expansions, and average inflation will be zero under discretion. Dixit and Lambertini (2003) showed that when both the monetary and fiscal authorities are acting optimally, the fiscal authority will

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35. In addition, the inflation equation was derived by linearizing around a zero inflation steady state. It would thus be inappropriate to use it to study situations in which average inflation is positive.

use its tax instruments to set  $x^* = 0$ , and the central bank then ensures that inflation remains equal to zero.<sup>36</sup>

Expanding the period loss function in (8.48),

$$\pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 = \pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda (x^*)^2.$$

Employing a first-order approximation for the structural equations is adequate for evaluating the  $\pi_{t+i}^2$  and  $x_{t+i}^2$  terms because any higher-order terms in the structural equations would become of order greater than 2 when squared. However, this is not the case for the  $2\lambda x^* x_{t+i}$  term, which is linear in  $x_{t+i}$ . Hence, to approximate this correctly to the required degree of accuracy would require second-order approximations to the structural equations rather than the linear approximations represented by (8.45) and (8.46). However, if the fiscal authority employs a subsidy to undo the distortion arising from imperfect competition so that  $x^* = 0$ , the linear approximations to the structural equations allow one to correctly evaluate the second-order approximation to welfare. See Benigno and Woodford (2005) for a discussion of optimal policy in the presence of a distorted steady state.

The parameter  $\lambda$  appearing in (8.48) plays a critical role in the evaluation of monetary policy, as it governs the trade-off implied by the preferences of the representative household between volatility in inflation and volatility in real economic activity. The chapter appendix shows that

$$\lambda = \left[ \frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \left( \frac{\sigma + \eta}{\theta} \right) = \frac{\kappa}{\theta}, \quad (8.49)$$

where  $\kappa$  is defined in (8.27) and is the elasticity of inflation with respect to the output gap. Recall that  $\omega$  is the fraction of firms that do not adjust price each period. An increase in  $\omega$  represents an increase in the degree of price stickiness and reduces the weight placed on output gap volatility in the welfare function. With more rigid prices, inflation variability generates more relative price dispersion, leading to larger welfare losses. It therefore becomes more important to stabilize inflation. The welfare costs of inflation also depend on  $\theta$ , the price elasticity of demand faced by individual firms. An increase in  $\theta$  implies households respond more to changes in relative prices; thus, a given level of relative price dispersion generates larger distortions as households shift their expenditures from high-price to low-price firms. In this case, avoiding price dispersion by stabilizing inflation becomes more important, so  $\lambda$  falls.

### 8.4.2 Policy Trade-offs

The basic new Keynesian inflation adjustment equation given by (8.26) did not include a disturbance term, such as the  $e_t$  that was added to (8.46). The absence of  $e_t$  implied

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36. See also Benigno and Woodford (2004) and Angeletos (2004).

there was no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. If  $x_{t+i} = 0$  for all  $i \geq 0$ , then  $\pi_{t+i} = 0$ . A central bank that wants to maximize the expected utility of the representative household, assuming  $x^* = 0$ , will ensure that output is kept equal to the flexible-price equilibrium level of output. This also guarantees inflation equals zero, thereby eliminating the costly dispersion of relative prices that arises with inflation. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant. Thus, a key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.<sup>37</sup>

The optimality of zero inflation conflicts with the Friedman rule for optimal inflation. M. Friedman (1969) concluded that the optimal inflation rate must be negative to make the nominal rate of interest zero (see chapter 4). The reason a different conclusion is reached here is the absence of any explicit role for money; (8.48) was derived from the utility function (8.47), in which money did not appear. In general, zero inflation still generates a monetary distortion. With zero inflation, the nominal rate of interest is positive, and the private opportunity cost of holding money exceeds the social cost of producing it. Khan, King, and Wolman (2003) and Adao, Correia, and Teles (2003) considered models that integrate nominal rigidities and the Friedman distortion. Khan, King and Wolman introduced money into a sticky-price model by assuming the presence of cash and credit goods, with money required to purchase cash goods. If prices are flexible, it is optimal to have a rate of deflation such that the nominal interest rate is zero. If prices are sticky, price stability is optimal in the absence of the cash-in-advance constraint. With both sticky prices and the monetary inefficiency associated with a positive nominal interest rate, the optimal rate of inflation is less than zero but greater than the rate that yields a zero nominal interest rate. Khan, King, and Wolman conducted simulations in a calibrated version of their model and found that the relative price distortion dominates the Friedman monetary inefficiency. Thus, the optimal policy is close to the policy that maintains price stability.

In the baseline model with no monetary distortion and with  $x^* = 0$ , the optimality of price stability is a reflection of the presence of only one nominal rigidity. The welfare costs of a single nominal rigidity can be eliminated using the single instrument provided by monetary policy. Erceg, Henderson, and Levin (2000) introduced nominal wage stickiness into the basic new Keynesian framework as a second nominal rigidity. With two distortions—sticky prices and sticky wages—the single instrument of monetary policy cannot simultaneously offset both distortions. Sticky wages and other labor market distortions are discussed in section 8.5.1.

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37. Notice that the conclusion that price stability is optimal is independent of the degree of nominal rigidity, a point made by Adao, Correia, and Teles (2004).

### 8.4.3 Optimal Commitment and Discretion

Suppose the central bank attempts to minimize a quadratic loss function such as (8.48), defined in terms of inflation and output relative to the flexible-price equilibrium.<sup>38</sup> Assume the steady-state gap between output and its efficient value is zero (i.e.,  $x^* = 0$ ). In this case, the central bank's loss function takes the form

$$L_t = \left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right). \quad (8.50)$$

Two alternative policy regimes can be considered. In a discretionary regime, the central bank behaves optimally in each period, taking as given the current state of the economy and private sector expectations. Given that the public knows the central bank optimizes each period, any promises the central bank makes about future inflation will not be credible—the public knows that whatever may have been promised in the past, the central bank will do what is optimal at the time it sets policy. The alternative regime is one of commitment. In a commitment regime, the central bank can make credible promises about what it will do in the future. By promising to take certain actions in the future, the central bank can influence the public's expectations about future inflation.

#### Commitment

A central bank able to precommit chooses a path for current and future inflation and the output gap to minimize the loss function (8.50) subject to the expectational IS curve (8.45) and the inflation adjustment equation (8.46). Let  $\theta_{t+i}$  and  $\psi_{t+i}$  denote the Lagrangian multipliers associated with the period  $t+i$  constraints (8.45) and (8.46). The central bank's objective is to pick  $i_{t+i}$ ,  $\pi_{t+i}$ , and  $x_{t+i}$  to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{1}{2}\right) \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right) + \theta_{t+i} \left[ x_{t+i} - x_{t+i+1} + \sigma^{-1} (i_{t+i} - \pi_{t+i+1} - r_{t+i}) \right] \right. \\ \left. + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right\}.$$

The first-order conditions for  $i_{t+i}$  take the form

$$\sigma^{-1} E_t \theta_{t+i} = 0, \quad i \geq 0.$$

Hence,  $\theta_t = E_t \theta_{t+i} = 0$  for all  $i > 0$ . This result implies that (8.45) imposes no real constraint on the central bank as long as there are no restrictions on, or costs associated with,

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38. Svensson (1999a;1999b) argued that there is widespread agreement among policymakers and academics that inflation stability and output gap stability are the appropriate objectives of monetary policy.

varying the nominal interest rate. Given the central bank's optimal choices for the output gap and inflation, (8.45) will simply determine the setting for  $i_t$  necessary to achieve the desired value of  $x_t$ . For that reason, it is often more convenient to treat  $x_t$  as if it were the central bank's policy instrument and drop (8.45) as an explicit constraint.

Setting  $\theta_t = E_t \theta_{t+i} = 0$ , the first-order conditions for  $\pi_{t+i}$  and  $x_{t+i}$  can be written as

$$\pi_t + \psi_t = 0, \quad (8.51)$$

$$E_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0, \quad i \geq 1, \quad (8.52)$$

$$E_t (\lambda x_{t+i} - \kappa \psi_{t+i}) = 0, \quad i \geq 0. \quad (8.53)$$

Equations (8.51) and (8.52) reveal the dynamic inconsistency that characterizes the optimal commitment policy. At time  $t$ , the central bank sets  $\pi_t = -\psi_t$  and promises to set  $\pi_{t+1} = -(\psi_{t+1} - \psi_t)$  in the future. But when period  $t + 1$  arrives, a central bank that reoptimizes will again obtain  $\pi_{t+1} = -\psi_{t+1}$  as its optimal setting for inflation. That is, the first-order condition (8.51) updated to  $t + 1$  will reappear.

An alternative definition of an optimal commitment policy requires that the central bank implement conditions (8.52) and (8.53) for all periods, including the current period. Woodford (1999) labeled this the *timeless perspective* approach to precommitment.<sup>39</sup> One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (8.52) and (8.53). McCallum and Nelson (2004) provided further discussion of the timeless perspective and argued that this approach agrees with the one commonly used in many studies of precommitment policies.

Combining (8.52) and (8.53), under the timeless perspective optimal commitment policy inflation and the output gap satisfy

$$\pi_{t+i} = - \left( \frac{\lambda}{\kappa} \right) (x_{t+i} - x_{t+i-1}) \quad (8.54)$$

for all  $i \geq 0$ . Using this equation to eliminate inflation from (8.46) and rearranging, one obtains

$$\left( 1 + \beta + \frac{\kappa^2}{\lambda} \right) x_t = \beta E_t x_{t+1} + x_{t-1} - \frac{\kappa}{\lambda} e_t. \quad (8.55)$$

The solution to this expectational difference equation for  $x_t$  is of the form  $x_t = a_x x_{t-1} + b_x e_t$ . To determine the coefficients  $a_x$  and  $b_x$ , note that if  $e_t = \rho e_{t-1} + \varepsilon_t$ , the proposed solution implies  $E_t x_{t+1} = a_x x_t + b_x \rho e_t = a_x^2 x_{t-1} + (a_x + \rho) b_x e_t$ . Substituting this into (8.55)

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39. See also Woodford (2000).

and equating coefficients, the parameter  $a_x$  is the solution less than 1 of the quadratic equation

$$\beta a_x^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right) a_x + 1 = 0,$$

and  $b_x$  is given by

$$b_x = - \left\{ \frac{\kappa}{\lambda [1 + \beta (1 - \rho - a_x)] + \kappa^2} \right\}.$$

From (8.54), equilibrium inflation under the timeless perspective policy is

$$\pi_t = \left(\frac{\lambda}{\kappa}\right) (1 - a_x)x_{t-1} + \left[ \frac{\lambda}{\lambda [1 + \beta (1 - \rho - a_x)] + \kappa^2} \right] e_t. \quad (8.56)$$

Woodford (1999) stressed that even if  $\rho = 0$ , so that there is no natural source of persistence in the model itself,  $a_x > 0$ , and the precommitment policy introduces inertia into the output gap and inflation processes. Because the central bank responds to the lagged output gap (see 8.54), past movements in the gap continue to affect current inflation. This commitment to inertia implies that the central bank's actions at date  $t$  allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap. Equation (8.46) implies that the inflation impact of a positive cost shock, for example, can be stabilized at a lower output cost if the central bank can induce a fall in expected future inflation. Such a fall in expected inflation is achieved when the central bank follows (8.54).

A condition for policy such as (8.54) that is derived from the central bank's first-order conditions and only involves variables that appear in the objective function (in this case, inflation and the output gap) is generally called a *targeting rule or criterion* (e.g., Svensson and Woodford 2005). It represents a relationship among the targeted variables that the central bank should maintain because doing so is consistent with the first-order conditions from its policy problem.

Because the timeless perspective commitment policy is not the solution to the policy problem under optimal commitment (it ignores the different form of the first-order condition (8.51) in the initial period), the policy rule given by (8.54) may be dominated by other policy rules. For instance, it may be dominated by the optimal discretion policy (see next section). Under the timeless perspective, inflation as given by (8.54) is the same function each period of the current and lagged output gap; the policy displays the property of continuation in the sense that the policy implemented in any period continues the plan it was optimal to commit to in an earlier period. Blake (2002), Damjanovic, Damjanovic, and Nolan (2008), and Jensen and McCallum (2010) considered *optimal* continuation policies that require the policy instrument, in this case  $x_t$ , to be a time-invariant function, as under the timeless perspective, but rather than ignore the first period conditions, as is done under

the timeless perspective, they focused on the optimal unconditional continuation policy to which the central bank should commit. This policy minimizes the unconditional expectation of the objective function, so that the Lagrangian for the policy problem becomes

$$\tilde{\mathcal{L}} = \tilde{\mathbb{E}} \left\{ E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{2} (\pi_{t+i}^2 + \lambda x_{t+i}^2) + \psi_{t+i} (\pi_{t+i} - \beta E_t \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right] \right\},$$

where  $\tilde{\mathbb{E}}$  denotes the unconditional expectations operator. Because

$$\tilde{\mathbb{E}} (E_t \psi_{t+i} \pi_{t+i+1}) = \tilde{\mathbb{E}} (\psi_{t-1} \pi_t),$$

the unconditional Lagrangian can be expressed as

$$\tilde{\mathcal{L}} = \left( \frac{1}{1-\beta} \right) \tilde{\mathbb{E}} \left\{ \left[ \frac{1}{2} (\pi_t^2 + \lambda x_t^2) + \psi_t \pi_t - \beta \psi_{t-1} \pi_t - \kappa \psi_t x_t - \psi_t e_t \right] \right\}.$$

The first-order conditions then become

$$\pi_t + \psi_t - \beta \psi_{t-1} = 0, \tag{8.57}$$

$$\lambda x_t - \kappa \psi_t = 0.$$

Combining these to eliminate the Lagrangian multiplier yields the optimal unconditional continuation policy:

$$\pi_t = - \left( \frac{\lambda}{\kappa} \right) (x_t - \beta x_{t-1}). \tag{8.58}$$

Comparing this to (8.54) shows that rather than give full weight to past output gaps, the optimal unconditional continuation policy discounts the past slightly (at a quarterly frequency,  $\beta \approx 0.99$ ).

Notice that neither (8.55) nor (8.56) involve the aggregate productivity shock that affect the economy's flexible-price equilibrium output. By definition, actual output is  $\hat{y}_t = \hat{y}_t^f + x_t$ . Thus, under the optimal commitment policy, monetary policy prevents a positive productivity shock from affecting the output gap, allowing output to move as it would if prices were flexible. The response to a positive productivity shock involves an increase firms' labor demand at the initial real wage. The efficient response requires a rise in the real wage to ensure labor supply and demand balance. The real wage is free to adjust appropriately because only prices have been assumed to be sticky. The flexibility of the nominal wage ensures the real wage and output can adjust as they would if prices had been flexible.

### Discretion

When the central bank operates with discretion, it acts each period to minimize the loss function (8.50) subject to the inflation adjustment equation (8.46). Because the decisions of the central bank at date  $t$  do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation. Thus, the decision problem

of the central bank becomes the single-period problem of minimizing  $\pi_t^2 + \lambda x_t^2$  subject to the inflation adjustment equation (8.46).

The first-order condition for this problem is

$$\kappa \pi_t + \lambda x_t = 0. \quad (8.59)$$

Equation (8.59) is the optimal targeting rule under discretion. Notice that by combining (8.51) with (8.53) evaluated at time  $t$ , one obtains (8.59); thus, the central bank's first-order condition relating inflation and the output gap at time  $t$  is the same under discretion or under the fully optimal precommitment policy (but not under the timeless perspective policy). The differences appear in subsequent periods. For  $t + 1$ , under discretion  $\kappa \pi_{t+1} + \lambda x_{t+1} = 0$ , whereas under commitment (from 8.52 and 8.53),  $\kappa \pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$ .

The equilibrium expressions for inflation and the output gap under discretion can be obtained by using (8.59) to eliminate inflation from the inflation adjustment equation. This yields

$$\left(1 + \frac{\kappa^2}{\lambda}\right) x_t = \beta E_t x_{t+1} - \left(\frac{\kappa}{\lambda}\right) e_t. \quad (8.60)$$

Guessing a solution of the form  $x_t = \delta e_t$ , so that  $E_t x_{t+1} = \delta \rho e_t$ , one obtains

$$\delta = - \left[ \frac{\kappa}{\lambda(1 - \beta\rho) + \kappa^2} \right].$$

Equation (8.59) implies that equilibrium inflation under optimal discretion is

$$\pi_t = - \left(\frac{\lambda}{\kappa}\right) x_t = \left[ \frac{\lambda}{\lambda(1 - \beta\rho) + \kappa^2} \right] e_t. \quad (8.61)$$

Policy does not introduce inertia as it did under commitment. According to (8.61) the unconditional expected value of inflation is zero; there is no average inflation bias under discretion. However, when forward-looking expectations play a role, as in (8.46), discretion leads to what is known as a *stabilization bias* in that the response of inflation to a cost shock under discretion differs from the response under commitment. This can be seen by comparing (8.61) to (8.56).<sup>40</sup>

### Discretion versus Commitment

The impact of a cost shock on inflation and the output gap under the timeless perspective optimal precommitment policy and optimal discretionary policy can be obtained by calibrating the model and numerically solving for the equilibrium under the alternative

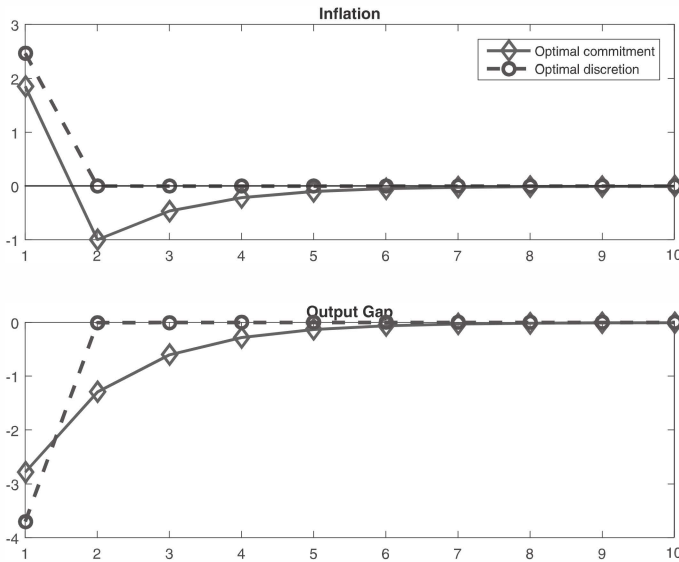
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40. In models containing an endogenous state variable, such as the stock of capital or government debt, issues of determinacy, discussed earlier with respect to instrument rules, can also arise under optimal discretion. See Blake and Kirsanova (2012) and Dennis and Kirsanova (2013).



policies. Four unknown parameters appear in the model:  $\beta$ ,  $\kappa$ ,  $\lambda$ , and  $\rho$ . The discount factor,  $\beta$ , is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of  $\lambda = 0.25$  is used. This value is also used by Jensen (2002), McCallum and Nelson (2004), and Debortoli et al. (2015) to represent the Fed’s dual mandate of price stability and maximum sustainable employment.<sup>41</sup> The parameter  $\kappa$  captures both the impact of a change in real marginal cost on inflation and the co-movement of real marginal cost and the output gap and is set equal to 0.05. McCallum and Nelson (2004) reported that empirical evidence is consistent with a value of  $\kappa$  in the range [0.01, 0.05]. Roberts (1995) reported higher values; his estimate of the coefficient on the output gap is about 0.3 when inflation is measured at an annual rate, so this translates into a value for  $\kappa$  of 0.075 for inflation at quarterly rates. Jensen (2002) used a baseline value of  $\kappa = 0.1$ , while Walsh (2003b) used 0.05.

The solid lines in figure 8.2 show the response of the output gap and inflation to a transitory, one standard deviation cost push shock under the optimal precommitment policy. Although the shock itself has no persistence, the output gap displays strong positive serial correlation. By keeping output below potential (a negative output gap) for several periods



**Figure 8.2**

Responses to a cost shock under commitment and discretion. Upper: Inflation. Lower: Output gap.

41. If (8.50) is interpreted as an approximation to the welfare of the representative agent, the implied value of  $\lambda$  would be much smaller.

into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in  $E_t\pi_{t+1}$  at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

Outcomes under optimal discretion are shown by the dashed lines in figure 8.2. There is no inertia under discretion; both the output gap and inflation return to their steady-state values in the period after the shock occurs. The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion. The intuition behind the suboptimality of discretion can be seen by considering the inflation adjustment equation given by (8.46). Under discretion, the central bank's only tool for offsetting the effects on inflation of a cost shock is the output gap. In the face of a positive realization of  $e_t$ ,  $x_t$  must fall to help stabilize inflation. Under commitment, however, the central bank has two instruments; it can affect both  $x_t$  and  $E_t\pi_{t+1}$ . By creating expectations of a deflation at  $t + 1$ , the reduction in the output gap does not need to be as large. Of course, under commitment a promise of future deflation must be honored, so actually inflation falls below the baseline beginning in period  $t + 1$  (see the upper panel of figure 8.2). Consistent with producing a deflation, the output gap remains negative for several periods.<sup>42</sup>

The analysis so far has focused on the goal variables, inflation and the output gap. Using (8.45), the associated behavior of the interest rate can be derived. For example, under optimal discretion, the output gap is given by

$$x_t = - \left[ \frac{\kappa}{\lambda(1 - \beta\rho) + \kappa^2} \right] e_t,$$

while inflation is given by (8.61). Using these to evaluate  $E_t x_{t+1}$  and  $E_t \pi_{t+1}$  and then solving for  $i_t$  from (8.45) yields

$$\begin{aligned} i_t &= r_t + E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t) \\ &= r_t + \left[ \frac{\lambda\rho + (1 - \rho)\sigma\kappa}{\lambda(1 - \beta\rho) + \kappa^2} \right] e_t. \end{aligned} \quad (8.62)$$

Equation (8.62) is the reduced-form solution for the nominal rate of interest. The nominal interest rate is adjusted to offset completely the impact of the demand disturbance  $r_t$  on the output gap. As a result,  $r_t$  affects neither inflation nor the output gap. Section 8.3.3 illustrated how a policy that commits to a rule that calls for responding to the exogenous shocks renders the new Keynesian model's equilibrium indeterminate. Thus, it is important to recognize that (8.62) describes the *equilibrium* behavior of the nominal interest rate under optimal discretion; (8.62) is not an instrument rule (see Svensson and Woodford 2005).

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42. While it is not obvious from the figure, the unconditional expectation of  $\pi_t^2 + \lambda x_t^2$  is  $0.9901\sigma_e^2$  under discretion and  $0.9134\sigma_e^2$  under commitment, using the same calibration as in the figure. This represents a 7.74 percent improvement under commitment.

### 8.4.4 Commitment to a Rule

In the Barro-Gordon model popular in the 1980s and 1990s (see chapter 6), optimal commitment was interpreted as commitment to a policy that was a (linear) function of the state variables. In the present model, consisting of (8.45) and (8.46), the only state variable is the current realization of the cost shock  $e_t$ . Suppose then that the central bank can commit to a rule of the form<sup>43</sup>

$$x_t = b_x e_t. \tag{8.63}$$

What is the optimal value of  $b_x$ ? With  $x_t$  given by (8.63), inflation satisfies

$$\pi_t = \beta E_t \pi_{t+1} + \kappa b_x e_t + e_t,$$

and the solution to this expectational difference equation<sup>44</sup> is

$$\pi_t = b_\pi e_t, \quad b_\pi = \frac{1 + \kappa b_x}{1 - \beta\rho}. \tag{8.64}$$

Using (8.63) and (8.64), the loss function can be written as

$$\left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) = \left(\frac{1}{2}\right) \sum_{i=0}^{\infty} \beta^i \left[ \left(\frac{1 + \kappa b_x}{1 - \beta\rho}\right)^2 + \lambda b_x^2 \right] E_t e_{t+i}^2.$$

This is minimized when

$$b_x = - \left[ \frac{\kappa}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right].$$

Using this solution for  $b_x$  in (8.64), equilibrium inflation is given by

$$\pi_t = \left( \frac{1 + \kappa b_x}{1 - \beta\rho} \right) e_t = \left[ \frac{\lambda(1 - \beta\rho)}{\lambda(1 - \beta\rho)^2 + \kappa^2} \right] e_t. \tag{8.65}$$

Comparing the solution for inflation under optimal discretion, given by (8.61), and the solution under commitment to a simple rule, given by (8.65), note that they are identical if the cost shock is serially uncorrelated ( $\rho = 0$ ). If  $0 < \rho < 1$ , there is a stabilization bias under discretion relative to the case of committing to a simple rule.

Clarida, Galí, and Gertler (1999) argued that this stabilization bias provides a rationale for appointing a Rogoff-conservative central banker—a central banker who puts more

43. This commitment does not raise the same uniqueness of equilibrium problem that would arise under a commitment to an instrument rule of the form  $i_t = r_t + b_i e_t$ . See problem 2 at the end of this chapter.

44. To verify this is the solution, note that

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa b_x e_t + e_t = \beta b_\pi \rho e_t + \kappa b_x e_t + e_t \\ &= [\beta b_\pi \rho + \kappa b_x + 1] e_t, \end{aligned}$$

so that  $b_\pi = \beta b_\pi \rho + \kappa b_x + 1 = (\kappa b_x + 1)/(1 - \beta\rho)$ .

weight on inflation objectives than is reflected in the social loss function—when  $\rho > 0$ , even though in the present context there is no average inflation bias.<sup>45</sup> A Rogoff conservative central banker places a weight  $\hat{\lambda} < \lambda$  on output gap fluctuations (see section 6.3.2). In a discretionary environment with such a central banker, (8.61) implies inflation will equal

$$\pi_t = \left[ \frac{\hat{\lambda}}{\hat{\lambda}(1 - \beta\rho) + \kappa^2} \right] e_t.$$

Comparing this with (8.65) reveals that if a central banker is appointed for whom  $\hat{\lambda} = \lambda(1 - \beta\rho) < \lambda$ , the discretionary solution will coincide with the outcome under commitment to the optimal simple rule. Such a central banker stabilizes inflation more under discretion than would be the case if the relative weight placed on output gap and inflation stability were equal to the weight in the social loss function,  $\lambda$ . Because the public knows inflation will respond less to a cost shock, future expected inflation rises less in the face of a positive  $e_t$  shock. As a consequence, current inflation can be stabilized with a smaller fall in the output gap. The inflation-output trade-off is improved.

Recall, however, that the notion of commitment used here is actually suboptimal. As seen earlier, fully optimal commitment leads to inertial behavior in that future inflation depends not on the output gap but on the change in the gap.

#### 8.4.5 Endogenous Persistence

The empirical research on inflation (see section 7.3.2) has generally found that when lagged inflation is added to (8.46), its coefficient is statistically and economically significant. If lagged inflation affects current inflation, then even under discretion the central bank faces a dynamic optimization problem; decisions that affect current inflation also affect future inflation, and this intertemporal link must be taken into account by the central bank when setting current policy. Svensson (1999c) and Vestin (2006) illustrated how the linear-quadratic structure of the problem allows one to solve for the optimal discretionary policy in the face of endogenous persistence.

To analyze the effects introduced when inflation depends on both expected future inflation and lagged inflation, suppose (8.46) is replaced by

$$\pi_t = (1 - \phi)\beta E_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t. \quad (8.66)$$

The coefficient  $\phi$  measures the degree of backward-looking behavior exhibited by inflation.<sup>46</sup> If the central bank's objective is to minimize the loss function given by (8.50), the

45. Rogoff (1985) proposed appointing a conservative central banker as a way to solve the average inflation bias that can arise under discretionary policies, (see chapter 6).

46. Galí and Gertler (1999), Woodford (2003a), and Christiano, Eichenbaum, and Evans (2005) developed inflation adjustment equations in which lagged inflation appears by assuming that some fraction of firms do not reset

policy problem under discretion can be written in terms of the value function defined by

$$V(\pi_{t-1}, e_t) = \min_{\pi_t, x_t} \left\{ \left( \frac{1}{2} \right) (\pi_t^2 + \lambda x_t^2) + \beta E_t V(\pi_t, e_{t+1}) \right. \\ \left. + \psi_t [\pi_t - (1 - \phi)\beta E_t \pi_{t+1} - \phi \pi_{t-1} - \kappa x_t - e_t] \right\}. \quad (8.67)$$

The value function depends on  $\pi_{t-1}$  because lagged inflation is an endogenous state variable.

Because the objective function is quadratic and the constraints are linear, the value function is quadratic, and one can hypothesize that it takes the form

$$V(\pi_{t-1}, e_t) = a_0 + a_1 e_t + \frac{1}{2} a_2 e_t^2 + a_3 e_t \pi_{t-1} + a_4 \pi_{t-1} + \frac{1}{2} a_5 \pi_{t-1}^2. \quad (8.68)$$

As Vestin demonstrated, this guess is only needed to evaluate  $E_t V_\pi(\pi_t, e_{t+1})$ , where  $E_t V_\pi(\pi_t, e_{t+1}) = a_3 E_t e_{t+1} + a_4 + a_5 \pi_t$ . If one assumes the cost shock is serially uncorrelated,  $E_t e_{t+1} = 0$ , and as a consequence the only unknown coefficients in (8.68) that play a role are  $a_4$  and  $a_5$ .

The solution for inflation takes the form

$$\pi_t = b_1 e_t + b_2 \pi_{t-1}. \quad (8.69)$$

Using this proposed solution, one obtains  $E_t \pi_{t+1} = b_2 \pi_t$ . This expression for expected future inflation can be substituted into (8.66) to yield

$$\pi_t = \frac{\kappa x_t + \phi \pi_{t-1} + e_t}{1 - (1 - \phi)\beta b_2}, \quad (8.70)$$

which implies  $\partial \pi_t / \partial x_t = \kappa / [1 - (1 - \phi)\beta b_2]$ .

Collecting these results, the first-order condition for the optimal choice of  $x_t$  by a central bank whose decision problem is given by (8.67) is

$$\left[ \frac{\kappa}{1 - (1 - \phi)\beta b_2} \right] [\pi_t + \beta E_t V_\pi(\pi_t, e_{t+1})] + \lambda x_t = 0. \quad (8.71)$$

Using (8.70) to eliminate  $x_t$  from (8.71) and recalling that  $E_t V_\pi(\pi_t, e_{t+1}) = a_4 + a_5 \pi_t$ , one obtains

$$\pi_t = \left[ \frac{\Psi}{\kappa^2(1 + \beta a_5) + \lambda \Psi^2} \right] \left[ \lambda \phi \pi_{t-1} + \lambda e_t - \left( \frac{\beta \kappa^2}{\Psi} \right) a_4 \right], \quad (8.72)$$

where  $\Psi \equiv 1 - (1 - \phi)\beta b_2$ .

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their prices optimally (see section 7.3). See also Eichenbaum and Fisher (2007). Lagged inflation also appears when firms index prices to past inflation.

From the envelope theorem and (8.71),

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= a_3 e_t + a_4 + a_5 \pi_{t-1} \\ &= \left[ \frac{\phi}{1 - (1 - \phi)\beta b_2} \right] [\pi_t + E_t V_\pi(\pi_t, e_{t+1})] = - \left( \frac{\lambda \phi}{\kappa} \right) x_t. \end{aligned}$$

Again using (8.70) to eliminate  $x_t$ ,

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= - \left( \frac{\lambda \phi}{\kappa} \right) \left[ \frac{\Psi \pi_t - \phi \pi_{t-1} - e_t}{\kappa} \right] \\ &= - \left( \frac{\lambda \phi}{\kappa} \right) \left[ \frac{(\Psi b_2 - \phi) \pi_{t-1} + (\Psi b_1 - 1) e_t}{\kappa} \right]. \end{aligned} \quad (8.73)$$

However, (8.68) implies that  $V_\pi(\pi_{t-1}, e_t) = a_3 e_t + a_4 + a_5 \pi_{t-1}$ . Comparing this with (8.73) reveals that  $a_4 = 0$ ,

$$a_3 = \lambda \phi \left( \frac{1 - \Psi b_1}{\kappa^2} \right),$$

and

$$a_5 = \lambda \phi \left( \frac{\phi - \Psi b_2}{\kappa^2} \right).$$

Finally, substitute these results into (8.72) to obtain

$$\pi_t = \left[ \frac{\Psi}{\kappa^2 + \beta \lambda \phi (\phi - \Psi b_2) + \lambda \Psi^2} \right] [\lambda \phi \pi_{t-1} + \lambda e_t].$$

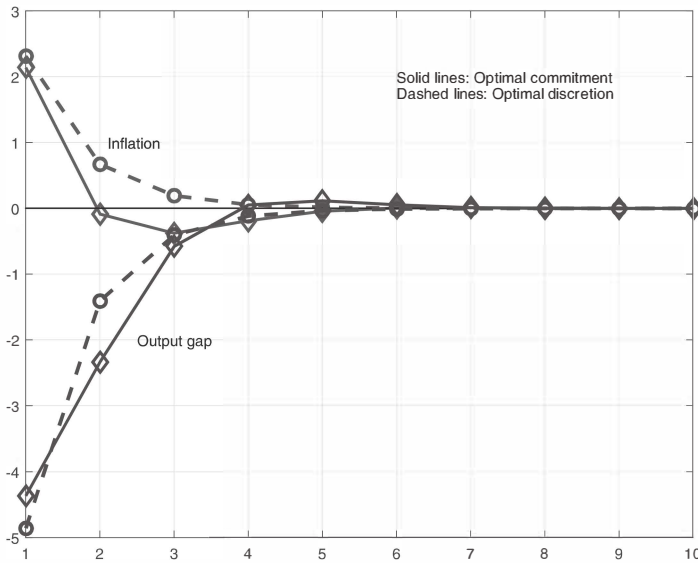
Equating coefficients with (8.69),

$$b_1 = \left[ \frac{\lambda \Psi}{\kappa^2 + \beta \lambda \phi (\phi - \Psi b_2) + \lambda \Psi^2} \right],$$

$$b_2 = \left[ \frac{\lambda \Psi \phi}{\kappa^2 + \beta \lambda \phi (\phi - \Psi b_2) + \lambda \Psi^2} \right]. \quad (8.74)$$

Because  $\Psi$  also depends on the unknown parameter  $b_2$ , (8.74) does not yield a convenient analytic solution. To gain insights into the effects of backward-looking aspects of inflation, it is useful to employ numerical techniques. This is done to generate figure 8.3, which shows the response of the output gap and inflation to an i.i.d. cost shock under optimal discretion when  $\phi = 0.5$ . Also shown for comparison are the responses under the optimal commitment policy. Both the output gap and inflation display more persistence under discretion than when  $\phi = 0$  (see figure 8.2).

It is insightful to consider explicitly the first-order conditions for the optimal policy problem under commitment when lagged inflation affects current inflation. Adopting the



**Figure 8.3**  
Responses to a cost shock with endogenous persistence;  $\phi = 0.5$ .

timeless perspective, maximizing (8.50) subject to (8.66) leads to the following first-order conditions:

$$\pi_t = (1 - \phi)\beta E_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t,$$

$$\pi_t + \psi_t - (1 - \phi)\psi_{t-1} - \beta \phi E_t \psi_{t+1} = 0,$$

$$\lambda x_t - \kappa \psi_t = 0,$$

where  $\psi_t$  is the Lagrangian multiplier associated with (8.66). Eliminating this multiplier, the optimal targeting criterion becomes

$$\pi_t = - \left( \frac{\lambda}{\kappa} \right) [x_t - (1 - \phi)x_{t-1} - \beta \phi E_t x_{t+1}]. \tag{8.75}$$

As noted earlier, the presence of forward-looking expectations leads optimal policy to be backward-looking by introducing inertia through the appearance of  $x_{t-1}$  in the optimal targeting rule. The presence of lagged inflation in the inflation adjustment equation when  $\phi > 0$  leads policy to be forward-looking through the role of  $E_t x_{t+1}$  in the targeting rule. This illustrates a key aspect of policy design; when policy affects the economy with a lag, policymakers must be forward-looking.

### 8.4.6 Targeting Regimes and Instrument Rules

The analysis of optimal policy in section 8.4.3 specified an objective function for the central bank. The central bank was assumed to behave optimally, given its objective function and the constraints imposed on its choices by the structure of the economy. A policy regime in which the central bank is assigned an objective is commonly described as a *targeting regime*. A targeting regime is defined by (1) the variables in the central bank's loss function (the objectives), and (2) the weights assigned to these objectives, with policy implemented under discretion to minimize the expected discounted value of the loss function.<sup>47</sup> Targeting rules were also discussed in section 6.3.5, in the context of solving the inflation bias that can arise under discretion.

The most widely analyzed targeting regime is inflation targeting.<sup>48</sup> In 1990, New Zealand became the first country to adopt formal targets for inflation, while now almost 30 countries are formal inflation targeters (see Roger 2010 and Rose 2014). Experiences with inflation targeting are analyzed by Ammer and Freeman (1995), Bernanke et al. (1998), Mishkin and Schmidt-Hebbel (2002; 2007), Amato and Gerlach (2002), and the contributors to Lowe (1997) and Leiderman and Svensson (1995). Some of the lessons from inflation targeting are discussed in Walsh (2009; 2011).

This section also briefly discusses instrument rules. These constitute an alternative approach to policy that assumes the central bank can commit to a simple feedback rule for its policy instrument. The best known of such rules is the Taylor rule.<sup>49</sup>

#### Inflation Targeting

The announcement of a formal target for inflation is a key component of any inflation-targeting regime, and this is often accompanied by publication of the central bank's inflation forecasts. An inflation-targeting regime can be viewed as the assignment to the central bank of an objective function of the form

$$L_t^{\text{IT}} = \left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i \left[ (\pi_{t+i} - \pi^T)^2 + \lambda_{\text{IT}} x_{t+i}^2 \right], \quad (8.76)$$

where  $\pi^T$  is the target inflation rate and  $\lambda_{\text{IT}}$  is the weight assigned to achieving the output gap objective relative to the inflation objective.  $\lambda_{\text{IT}}$  may differ from the weight

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47. This definition of a targeting regime is consistent with that of Svensson (1999b). An alternative interpretation of a targeting regime is that it is a rule for adjusting the policy instrument in the face of deviations between the current (or expected) value of the targeted variable and its target level (see McCallum 1990b and the references he cites). Jensen (2002) and Rudebusch (2002a) illustrated these two alternative interpretations of targeting.

48. Early contributions to the literature on inflation targeting were made by Bernanke and Mishkin (1997), Svensson (1997a; 1997b; 1999a; 1999b), and Svensson and Woodford (2005).

49. Walsh (2015) compares a regime such as inflation targeting, in which the central bank is assigned a goal (e.g., achieve 2 percent inflation), to a regime in which the central bank is assigned an instrument rule (e.g., follow the Taylor rule).



placed on output gap stabilization in the social loss function (8.50). As long as  $\lambda_{IT} > 0$ , specifying inflation targeting in terms of the loss function (8.76) assumes that the central bank is concerned with output stabilization as well as inflation stabilization. An inflation targeting regime in which  $\lambda_{IT} > 0$  is described as a flexible inflation-targeting regime.<sup>50</sup>

In the policy problems analyzed so far, the central bank's choice of its instrument  $i_t$  allows it to affect both output and inflation immediately. This absence of any lag between the time a policy action is taken and the time it affects output and inflation is unrealistic. If policy decisions taken in period  $t$  only affect future output and inflation, then the central bank must rely on forecasts of future output and inflation when making its policy choices. In analyzing the case of such policy lags, Svensson (1997b) and Svensson and Woodford (2005) emphasized the role of inflation forecast targeting. To illustrate the role of forecasts in the policy process, suppose the central bank must set  $i_t$  prior to observing any time  $t$  information. This assumption implies that the central bank cannot respond to time  $t$  shocks contemporaneously; information about shocks occurring in period  $t$  will affect the central bank's choice of  $i_{t+1}$  and, as a consequence,  $x_{t+1}$  and  $\pi_{t+1}$  can be affected. Assume that the demand shock in (8.45) is serially uncorrelated. The central bank's objective is to choose  $i_t$  to minimize

$$\left(\frac{1}{2}\right) E_{t-1} \sum_{i=0}^{\infty} \beta^i \left[ (\pi_{t+i} - \pi^T)^2 + \lambda_{IT} x_{t+i}^2 \right],$$

where the subscript on the expectations operator is now  $t-1$  to reflect the information available to the central bank when it sets policy. The choice of  $i_t$  is subject to the constraints represented by (8.45) and (8.46).<sup>51</sup> Taking expectations based on the central bank's information, these two equations can be written as

$$E_{t-1} x_t = E_{t-1} x_{t+1} - \left(\frac{1}{\sigma}\right) (i_t - E_{t-1} \pi_{t+1} - E_{t-1} r_t), \quad (8.77)$$

$$E_{t-1} \pi_t = \beta E_{t-1} \pi_{t+1} + \kappa E_{t-1} x_t + \rho e_{t-1}, \quad (8.78)$$

where the cost shock follows an AR(1) process:  $e_t = \rho e_{t-1} + \varepsilon_t$ . Under discretion, the first-order condition for the central bank's choice of  $i_t$  implies that

$$E_{t-1} \left[ \kappa (\pi_t - \pi^T) + \lambda x_t \right] = 0. \quad (8.79)$$

50. This terminology is used in section 6.3.5.

51. Because (8.46) was obtained by linearizing around a zero inflation steady state, one should set  $\pi^T = 0$  for consistency. A common assumption in empirical models is that firms not optimally adjusting price link price changes to the central bank's target for inflation. In this case, (8.46) would be replaced with

$$\pi_t - \pi^T = \beta E_t (\pi_{t+1} - \pi^T) + \kappa x_t + e_t.$$

Rearranging this first-order condition yields

$$E_{t-1}x_t = -\left(\frac{\kappa}{\lambda}\right)E_{t-1}(\pi_t - \pi^T).$$

Thus, if the central bank forecasts that period  $t$  inflation will exceed its target rate of inflation, it should adjust policy to ensure that the forecast of the output gap is negative.

Svensson (1997b) and Svensson and Woodford (2005) provided detailed discussions of inflation forecast targeting, focusing on the implications for the determinacy of equilibrium under different specifications of the policy decision process. The possibility of multiple equilibria becomes particularly relevant if the central bank bases its own forecasts on private sector forecasts, which are in turn based on expectations about the central bank's actions.

### Other Targeting Regimes

Inflation targeting is just one example of a policy targeting regime. A number of alternative targeting regimes have been analyzed in the literature. These include price level targeting (Dittmar, Gavin, and Kydland 1999; Svensson 1999c; Vestin 2006; Dib, Mendicino, and Zhang 2013; Kryvtsov, Shukayev, and Ueberfeldt 2008; Cateau et al. 2009; Billi 2015), nominal income growth targeting (Jensen 2002), hybrid price level inflation targeting (Batini and Yates 2003); average inflation targeting (Nessen and Vestin 2005), and regimes based on the change in the output gap or its quasi-difference (Jensen and McCallum 2002; Walsh 2003b). In each case, it is assumed that given the assigned loss function, the central bank chooses policy under discretion. The optimal values for the parameters in the assigned loss function, for example, the value of  $\lambda_{IT}$  in (8.76), are chosen to minimize the unconditional expectation of the social loss function (8.50).

The importance of forward-looking expectations in affecting policy choice is well illustrated by work on price level targeting. The traditional view argued that attempts to stabilize the price level rather than the inflation rate would generate undesirable levels of output variability. A positive cost shock that raised the price level would require a deflation to bring the price level back on target, and this deflation would be costly. However, as figure 8.2 shows, an optimal commitment policy that focuses on output and inflation stability also induces a deflation after a positive cost shock. By reducing  $E_t\pi_{t+1}$ , such a policy achieves a better trade-off between inflation variability and output variability. The deflation generated under a discretionary policy concerned with *price level* stability might actually come closer to the commitment policy outcomes than discretionary inflation targeting would. Using a basic new Keynesian model, Vestin (2006) showed that this intuition is correct. In fact, when inflation is given by (8.46) and the cost shock is serially uncorrelated, price level targeting can replicate the timeless precommitment solution exactly if the central bank is assigned the loss function  $(p_t - \bar{p})^2 + \lambda_{PL}x_t^2$ , where  $\lambda_{PL}$  differs appropriately from the weight  $\lambda$  in the social loss function.

Jensen (2002) showed that a nominal income growth targeting regime can also dominate inflation targeting. Walsh (2003b) added lagged inflation to the inflation adjustment equation and showed that the advantages of price level targeting over inflation targeting decline as the weight on lagged inflation increases. Walsh analyzed discretionary outcomes when the central bank targets inflation and the change in the output gap (a *speed limit policy*). Introducing the change in the gap induces inertial behavior similar to that obtained under precommitment. For empirically relevant values of the weight on lagged inflation ( $\phi$  in the range 0.3–0.7), speed limit policies dominate price level targeting, inflation targeting, and nominal income growth targeting. For  $\phi$  below 0.3, price level targeting does best. Svensson and Woodford (2005) considered interest rate–smoothing objectives as a means of introducing into discretionary policy the inertia that is optimal under commitment.

### Instrument Rules

The approach to policy analysis adopted in the preceding sections starts with a specification of the central bank's objective function and then derives the optimal setting for the policy instrument. An alternative approach specifies an instrument rule directly. The best known of such instrument rules is the Taylor rule. Taylor (1993a) showed that the behavior of the federal funds interest rate in the United States from the mid-1980s through 1992 (when Taylor was writing) could be fairly well matched by a simple rule of the form

$$i_t = r^* + \pi^T + 0.5x_t + 1.5(\pi_t - \pi^T),$$

where  $\pi^T$  was the target level of average inflation (Taylor assumed it to be 2 percent) and  $r^*$  was the equilibrium real rate of interest (Taylor assumed this, too, was equal to 2 percent). The Taylor rule for general coefficients is often written

$$i_t = r^* + \pi^T + \alpha_x x_t + \alpha_\pi (\pi_t - \pi^T). \quad (8.80)$$

The nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to a rise in the nominal rate, as does a deviation of actual inflation above target. With Taylor's original coefficients,  $\alpha_\pi = 1.5$ , so that the nominal rate is changed more than one-to-one with deviations of inflation from target. Thus, the rule satisfies the Taylor principle (see section 8.3.3); a greater than one-to-one reaction of  $i_t$  ensures that the economy has a unique stationary rational-expectations equilibrium. Lansing and Trehan (2003) explored conditions under which the Taylor rule emerges as the fully optimal instrument rule under discretionary policy.

A large literature has estimated Taylor rules, or similar simple rules, for a variety of countries and time periods. For example, Clarida, Galí, and Gertler (1998) did so for the central banks of Germany, France, Italy, Japan, the United Kingdom, and the United States. In their specification, however, actual inflation is replaced by expected future inflation, so that the central bank is assumed to be forward-looking in setting policy. Estimates for the

United States under different Federal Reserve chairs were reported by Judd and Rudebusch (1997). In general, the basic Taylor rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate. However, Orphanides (2000) found that when estimated using the data on the output gap and inflation actually available at the time policy actions were taken (i.e., using real-time data), the Taylor rule does much more poorly in matching the U.S. funds rate. Clarida, Galí, and Gertler (1998) found the Fed moved the funds rate less than one-to-one during the period 1960–1979, thereby violating the Taylor principle and failing to ensure a determinant equilibrium. Coibion and Gorodnichenko (2011) showed that when average inflation is positive, assessing determinacy depends on the level of inflation and the policy responses to output as well as the policy response to inflation. In a further example of the importance of using real-time data, however, Perez (2001) found that when the Fed's reaction function is reestimated for this earlier period using real-time data, the coefficient on inflation is greater than 1. Lubik and Schorfheide (2004) estimated a complete DSGE new Keynesian model of the U.S. economy and found evidence that Federal Reserve policy has been consistent with determinacy since 1982. However, their estimates suggested policy was not consistent with determinacy prior to 1979. Questioning these results, Cochrane (2011a) argued that the Taylor principle applies to beliefs about how the central bank would respond to off-the-equilibrium path behavior. Because such behavior is not observed, the relevant response coefficient is unidentified.

When a policy interest rate such as the federal funds rate in the United States is regressed on inflation and output gap variables, the lagged value of the interest rate normally enters with a statistically significant and large coefficient. The interpretation of this coefficient on the lagged interest rate has been the subject of debate. One interpretation is that it reflects inertial behavior of the sort that arises under an optimal precommitment policy (see Woodford 2003b). It has also been interpreted to mean that central banks adjust gradually toward a desired interest rate level. For example, suppose that  $i_t^*$  is the central bank's desired value for its policy instrument but it wants to avoid large changes in interest rates. Such an interest-smoothing objective might arise from a desire for financial market stability. If the central bank adjusts  $i_t$  gradually toward  $i_t^*$ , then the behavior of  $i_t$  may be captured by a partial adjustment model of the form

$$i_t = i_{t-1} + \theta (i_t^* - i_{t-1}) = (1 - \theta)i_{t-1} + \theta i_t^*. \quad (8.81)$$

The estimated coefficient on  $i_{t-1}$  provides an estimate of  $1 - \theta$ . Values close to 1 imply that  $\theta$  is small; each period the central bank closes only a small fraction of the gap between its policy rate and its desired value.

The view that central banks adjust slowly has been criticized. Sack (2000) and Rudebusch (2002a) argued that the presence of a lagged interest rate in estimated instrument rules is not evidence that the Fed acts gradually. Sack attributed the Fed's behavior

to parameter uncertainty that leads the Fed to adjust the funds rate less aggressively than would be optimal in the absence of parameter uncertainty. Rudebusch argued that imperfect information about the degree of persistence in economic disturbances induces behavior by the Fed that appears to reflect gradual adjustment. He noted that if the Fed followed a rule such as (8.81), future changes in the funds rate would be predictable, but evidence from forward interest rates does not support the presence of predictable changes. Similarly, Lansing (2002) showed that the appearance of interest rate smoothing can arise if the Fed uses real-time data to update its estimate of trend output each period. When final data are used to estimate a policy instrument rule, the serial correlation present in the Fed's real-time errors in measuring trend output will be correlated with lagged interest rates, creating the illusion of interest rate–smoothing behavior by the Fed.

#### 8.4.7 Model Uncertainty

Up to this point, the analysis has assumed that the central bank knows the true model of the economy with certainty. Fluctuations in output and inflation arose only from disturbances that took the form of additive errors. In this case, the linear-quadratic framework results in certainty equivalence holding; the central bank's actions depend on its expectations of future variables but not on the uncertainty associated with those expectations. When error terms enter multiplicatively, as occurs, for example, when the model's parameters are not known with certainty, equivalence will not hold. Brainard (1967) provided the classic analysis of multiplicative uncertainty. He showed that when there is uncertainty about the impact a policy instrument has on the economy, it is optimal to respond more cautiously than would be the case in the absence of uncertainty.

Brainard's basic conclusion can be illustrated with a simple example. Suppose the inflation adjustment equation given by (8.46) is modified to take the following form:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_t x_t + e_t, \quad (8.82)$$

where  $\kappa_t = \bar{\kappa} + v_t$ , and  $v_t$  is a white noise stochastic process. In this formulation, the central bank is uncertain about the true impact of the gap  $x_t$  on inflation. For example, the central bank may have an estimate of the coefficient on  $x_t$  in the inflation equation, but there is some uncertainty associated with this estimate. The central bank's best guess of this coefficient is  $\bar{\kappa}$ , while its actual realization is  $\kappa_t$ . The central bank must choose its policy before observing the actual realization of  $v_t$ .

To analyze the impact of uncertainty about the coefficient on optimal policy, assume that the central bank's loss function is

$$L = \frac{1}{2} E_t \left( \pi_t^2 + \lambda x_t^2 \right),$$

and assume that policy is conducted with discretion. In addition, assume that the cost shock  $e_t$  is serially uncorrelated.

Under discretion, the central bank takes  $E_t \pi_{t+1}$  as given, and the first-order condition for the optimal choice of  $x_t$  is

$$E_t (\pi_t \kappa_t + \lambda x_t) = 0.$$

Because all stochastic disturbances are assumed to be serially uncorrelated, expected inflation is zero, so from (8.82),  $\pi_t = \kappa_t x_t + e_t$ . Using this to rewrite the first-order condition yields

$$E_t [(\kappa_t x_t + e_t) \kappa_t + \lambda x_t] = (\bar{\kappa}^2 + \sigma_v^2) x_t + \bar{\kappa} e_t + \lambda x_t = 0.$$

Solving for  $x_t$ , one obtains

$$x_t = - \left( \frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2 + \sigma_v^2} \right) e_t. \quad (8.83)$$

Equation (8.83) can be compared to the optimal discretionary response to the cost shock when there is no parameter uncertainty. In this case,  $\sigma_v^2 = 0$  and

$$x_t = - \left( \frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2} \right) e_t.$$

The presence of multiplicative parameter uncertainty ( $\sigma_v^2 > 0$ ) reduces the impact of  $e_t$  on  $x_t$ . As uncertainty increases, it becomes optimal to respond less to  $e_t$ , that is, to behave more cautiously in setting policy.

Using (8.83) in the inflation adjustment equation (8.82),

$$\pi_t = \kappa_t x_t + e_t = \left( \frac{\lambda + \sigma_v^2 - \bar{\kappa} (\kappa_t - \bar{\kappa})}{\lambda + \bar{\kappa}^2 + \sigma_v^2} \right) e_t = \left( \frac{\lambda + \sigma_v^2 - \bar{\kappa} v_t}{\lambda + \bar{\kappa}^2 + \sigma_v^2} \right) e_t.$$

Because the two disturbances  $v_t$  and  $e_t$  are uncorrelated, the unconditional variance of inflation is increasing in  $\sigma_v^2$ . In the presence of multiplicative uncertainty of the type modeled here, equilibrium output is stabilized more and inflation less in the face of cost shocks. The reason for this result is straightforward. With a quadratic loss function, the additional inflation variability induced by the variance in  $\kappa_t$  is proportional to  $x_t$ . Reducing the variability of  $x_t$  helps to offset the impact of  $v_t$  on the variance of inflation. It is optimal to respond more cautiously, thereby reducing the variance of  $x_t$  but at the cost of greater inflation variability.

Brainard's basic result—multiplicative uncertainty leads to caution—is intuitively appealing, but it is not a general result. For example, Söderström (2002) examined a model in which there are lagged variables whose coefficients are subject to random shocks. He showed that in this case optimal policy reacts more aggressively. For example, suppose current inflation depends on lagged inflation, but the impact of  $\pi_{t-1}$  on  $\pi_t$  is uncertain. The effect of this coefficient uncertainty on the variance of  $\pi_t$  depends on the variability of  $\pi_{t-1}$ . If the central bank fails to stabilize current inflation, it increases the variance of inflation in the following period. It can be optimal to respond more aggressively to stabilize

inflation, thereby reducing the impact of the coefficient uncertainty on the unconditional variance of inflation.

Some studies have combined the notion of parameter uncertainty with models of learning to examine the implications for monetary policy (see Sargent 1999; Evans and Honkapohja 2009). Wieland (2000a; 2000b) examined the trade-off between control and estimation that can arise under model uncertainty. A central bank may find it optimal to experiment, changing policy to generate observations that can help it learn about the true structure of the economy.

Another aspect of model uncertainty is measurement error or the inability to observe some relevant variables. For example, the flexible-price equilibrium level of output is needed to measure the gap variable  $x_t$ , but it is not directly observable. Svensson and Woodford (2003; 2004) provided a general treatment of optimal policy when the central bank's problem involves both an estimation problem (determining the true state of the economy, such as the value of the output gap) and a control policy (setting the nominal interest rate to affect the output gap and inflation). In a linear-quadratic framework in which private agents and the central bank have the same information, these two problems can be dealt with separately.<sup>52</sup> Svensson and Williams (2008) developed a general approach for dealing with a variety of sources of model and data uncertainty.

Finally, the approach adopted in section 8.4.1 derived welfare-based policy objectives from an approximation to the welfare of the representative agent. The nature of this approximation, however, depends on the underlying model structure. For example, Steinsson (2003) showed that in the Galí and Gertler (1999) hybrid inflation model, in which lagged inflation appears in the inflation adjustment equation, the loss function also includes a term in the squared change in inflation. Woodford (2003a) found that if price adjustment is characterized by partial indexation to lagged inflation so that the inflation adjustment equation involves  $\pi_t - \gamma\pi_{t-1}$  and  $E_t(\pi_{t+1} - \gamma\pi_t)$  (see section 7.3.2), the period loss function includes  $(\pi_t - \gamma\pi_{t-1})^2$  rather than  $\pi_t^2$ . Thus, uncertainty about the underlying model also translates into uncertainty about the appropriate objectives of monetary policy because policy objectives cannot be defined independently of the model that defines the costs of economic fluctuations (Walsh 2005a).

## 8.5 Labor Market Frictions and Unemployment

In this section, the basic new Keynesian model is extended in two ways. First, sticky wages are introduced into the model. The resulting framework with sticky prices and sticky wages forms the core foundation of most empirical DSGE models, early examples of

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52. As an example of the policy problems that arise when the true state of the economy is unobservable, Orphanides (2000) emphasized the role of the productivity slowdown during the 1970s in causing the Fed to overestimate potential output. See also Levin, Wieland, and Williams (1999), Ehrmann and Smets (2003), Orphanides and Williams (2002), and Levin et al. (2006).

which include work by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Second, the assumption of the basic model that all labor adjustment occurs through fluctuations in hours per worker is dropped, and instead adjustment in the number of workers employed is introduced. This change allows unemployment and variations in the fraction of the labor force that is employed to be incorporated. The model of unemployment is based on the search and matching framework of Mortensen and Pissarides (1994) and integrates modern theories of unemployment into a general equilibrium setting with nominal rigidities following Walsh (2003a; 2005b).<sup>53</sup>

### 8.5.1 Sticky Wages and Prices

The discussion so far has employed a basic new Keynesian model in which prices are sticky but wages are flexible. The underlying labor market in the model featured fluctuations in employment as output fluctuated, but the wage always adjusted to ensure households were able to work their desired number of hours. With sticky prices but flexible wages, a key relative price—the real wage—was able to adjust. It was for this reason that in the face of a productivity shock, actual output could move with the economy's flexible-price output level, keeping the output gap at zero, if the central bank maintained price stability.<sup>54</sup> For example, a positive productivity shock would increase the marginal product of labor; monetary policy could ensure aggregate demand rises in line with flexible-price output, and the real wage would rise to maintain labor market equilibrium while prices could remain unchanged. If, however, both wages and prices are sticky, the real wage becomes sticky. Monetary policy would only be able to keep the output gap at zero if it allowed inflation (or deflation) to achieve the required adjustment in the real wage.

Erceg, Henderson, and Levin (2000) employed the Calvo specification to incorporate sticky wages *and* sticky prices into an optimizing framework.<sup>55</sup> The goods market side of their model is identical in structure to the one developed in section 8.3.2. In the labor market, they assumed individual households supply differentiated labor services. Firms combine these labor services to produce output. Output is given by a standard production function,  $F(N_t)$ , but the labor aggregate is a composite function of the individual types of labor services:

$$N_t = \left[ \int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \quad \gamma > 1, \quad (8.84)$$

53. A good introduction to the search and matching framework and its implications is found in Hornstein, Krusell, and Violante (2005). Rogerson and Shimer (2011) provided an extensive discussion of the framework and its role in accounting for macroeconomic facts.

54. If the central bank does not maintain price stability, the real wage will still adjust to equilibrate the labor market, but it will do so at an inefficient level of output.

55. Other models incorporating both wage and price stickiness include those of Ravenna (2000), Sbordone (2002a), and Christiano, Eichenbaum, and Evans (2005). This is standard in models taken to the data.



where  $n_{jt}$  is the labor from household  $j$ . With this specification, households face a demand for their labor services that depends on the wage they set relative to the aggregate wage rate:

$$n_{jt} = \left( \frac{W_{jt}}{W_t} \right)^{-\gamma} N_t, \quad (8.85)$$

where  $W_{jt}$  is the nominal rate set by household  $j$ , and  $W_t$  is the aggregate average nominal wage. Erceg, Henderson, and Levin assumed that a randomly drawn fraction of households optimally set their wage each period, just as the Calvo model of price stickiness assumed only a fraction of firms adjust their price each period.

The model of inflation adjustment based on the Calvo specification implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equals the gap between the real wage ( $\omega_t$ ) and the marginal product of labor (mpl). Similarly, wage inflation (when linearized around a zero inflation steady state) responds to a gap variable, but this time the appropriate gap depends on a comparison between the real wage and the household's marginal rate of substitution between leisure and consumption. With flexible wages, workers are always on their labor supply curves; nominal wages adjust to ensure the real wage equals the marginal rate of substitution (mrs) between leisure and consumption. When nominal wages are also sticky, however,  $\omega_t$  and  $mrs_t$  can differ. If  $\omega_t < (>)$   $mrs_t$ , workers want to raise (lower) their nominal wage when the opportunity to adjust arises.<sup>56</sup> Letting  $\pi_t^w$  denote the rate of nominal wage inflation, Erceg, Henderson, and Levin showed that

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w (mrs_t - \omega_t). \quad (8.86)$$

From the definition of the real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \quad (8.87)$$

Equations (8.86) and (8.87), when combined with the new Keynesian Phillips curve in which inflation depends on  $\omega_t - \text{mpl}_t$ , constitute the inflation and wage adjustment block of an optimizing model with both wage and price rigidities.

### Policy Implications

The dispersion of relative wages that arises when not all workers can adjust wages every period generates a welfare loss, just as a dispersion of relative prices in the goods market did. To see this, let  $N_t^s$  denote the total hours of work supplied by households, defined as

$$N_t^s \equiv \int_0^1 n_{jt} dj.$$

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56. The variables mpl, mrs, and  $\omega$  refer to the percent deviation of the marginal productivity of labor, the marginal rate of substitution between leisure and consumption, and the real wage around their steady-state values, respectively.

The demand for labor supplied by household  $j$  is given by (8.85), Thus,

$$N_t^s = \int_0^1 n_{jt} dj = \left[ \int_0^1 \left( \frac{W_{jt}}{W_t} \right)^{-\gamma} dj \right] N_t = \Delta_{w,t} N_t \geq N_t,$$

where  $\Delta_{w,t} \geq 1$  is a measure of relative wage dispersion (compare with (8.21)). Output is  $F(N_t) = F(\Delta_{w,t}^{-1} N_t^s) \leq F(N_t^s)$ . The effective amount of labor,  $\Delta_{w,t}^{-1} N_t^s$ , is less than the total hours workers supply when relative wages differ across workers. Wage dispersion causes the hours of different labor types to be combined in production inefficiently.

Erceg, Henderson and Levin showed that the second-order approximation to the welfare of the representative household no longer is given by (8.48) but now depends on the volatility of price inflation, the output gap, and wage inflation. The welfare approximation takes the form

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 + \lambda_w (\pi_{t+i}^w)^2 \right] + \text{t.i.p.}$$

The parameter  $\lambda_w$  is increasing in the degree of wage rigidity, and like  $\lambda$  it is decreasing in the degree of price rigidity.

When wages are sticky, they adjust to the gap between the real wage and the marginal rate of substitution between leisure and consumption. When prices are sticky, they adjust to the gap between the marginal product of labor and the real wage. Galí, Gertler, and López-Salido (2007) defined the *inefficiency gap* as the sum of these two gaps, the gap between the household's marginal rate of substitution between leisure and consumption ( $mrs_t$ ) and the marginal product of labor ( $mpl_t$ ). This inefficiency gap can be divided into its two parts, the wedge between the real wage and the marginal rate of substitution, labeled the *wage markup*, and the wedge between the real wage and the marginal product of labor, labeled the *price markup*. Based on U.S. data, they concluded that the wage markup accounts for most of the time series variation in the inefficiency gap.<sup>57</sup>

Levin et al. (2006) estimated a new Keynesian general equilibrium model with both price and wage stickiness. They found that the welfare costs of nominal rigidity is primarily generated by wage stickiness rather than by price stickiness. This finding is consistent with those of Christiano, Eichenbaum, and Evans (2005), who concluded that a model with flexible prices and sticky wages does better at fitting impulse responses estimated on U.S. data than a sticky price–flexible wage version of their model. Sbordone (2002a) also suggested that nominal wage rigidity is more important empirically than price rigidity, while Huang and Liu (2002) argued that wage stickiness is more important than price stickiness for generating output persistence. In contrast, Goodfriend and King (2001) argued that the

57. Karabarbounis (2014) examined many countries and found similarly that the gap between  $\omega_t$  and  $mrs_t$  was the major source of fluctuations in the inefficiency gap.

long-term nature of employment relationships reduces the effects of nominal wage rigidity on real resource allocations. Models that incorporate the intertemporal nature of employment relationships based on search and matching models of unemployment are discussed in section 8.5.2.

The wage markup identified by Galí, Gertler, and López-Salido could arise from fluctuations in markups in labor markets or from the presence of wage rigidities, both of which reflect welfare-reducing inefficiencies. However, Chari, Kehoe, and McGrattan (2009) cautioned that this wedge could also reflect time variations in preferences, which do not reflect any distortion or inefficiency. For example, suppose the marginal rate of substitution between leisure and consumption is given by  $\chi_t N_t^\eta / C_t^{-\sigma}$  where  $\chi_t$  is a taste shock. If  $\mu_t^w$  equals a time-varying markup due to imperfect competition in labor markets, then equilibrium with flexible wages will entail  $\mu_t^w \chi_t N_t^\eta / C_t^{-\sigma} = W_t / P_t$ . Expressing this condition in terms of percent deviations around the steady state yields

$$\omega_t - (\eta \hat{n}_t + \sigma \hat{c}_t) = \hat{\mu}_t^w + \hat{\chi}_t.$$

The left side of this equation depends on observable variables (the real wage, employment, and consumption), so conditional on estimates of  $\eta$  and  $\sigma$ , one can obtain a measure of the labor wedge as  $\omega_t - (\eta \hat{n}_t + \sigma \hat{c}_t)$ . However this measure alone does not allow one to infer whether fluctuations in  $\omega_t - (\eta \hat{n}_t + \sigma \hat{c}_t)$  reflect distortionary shocks (the markup shocks) or nondistortionary shocks (the taste shocks). A decline in employment resulting from decreased market competition in labor markets (a positive markup shock) is welfare-reducing; a fall in employment because households desire more leisure (a positive leisure taste shock) is not. For policy purposes, it is important to be able to identify which shock is affecting employment. Because the two shocks enter additively in the measured wedge  $\omega_t - (\eta \hat{n}_t + \sigma \hat{c}_t)$ , some additional identifying assumption is required, and various approaches have been explored. For example, Galí, Smets, and Wouters (2012) used the unemployment rate as an additional observable in an estimated DSGE model in which movements in the unemployment rate only reflect  $\hat{\mu}_t^w$  shocks, while in the baseline estimated DSGE model of Justiniano, Primiceri, and Tambalotti (2013), low-frequency movements in the labor wedge are attributed to taste shocks and much of the high-frequency movement of wages is attributed to measurement error in the wage series. Thus, there is little role left for inefficient markup shocks. Sala, Söderstrom, and Trigari (2010) showed how assuming that  $\hat{\chi}_t$  follows an AR(1) process and  $\hat{\mu}_t^w$  is white noise leads to very different estimates of the magnitude of inefficient fluctuations compared with the case of assuming  $\hat{\chi}_t$  is white noise and  $\hat{\mu}_t^w$  follows an AR(1) process.

### 8.5.2 Unemployment

The basic new Keynesian model adds imperfect competition and nominal rigidities to what is otherwise an equilibrium real business cycle model. In common with many real business cycle models, all labor adjustment occurs along the hours margin, with the measure

of employment in the model defined as the fraction of time the representative household spends engaged in market work. When output in the model declines, hours per worker fall but all workers remain employed; there is no adjustment in the fraction of workers who are employed. Yet for an economy such as the U.S. economy, most of the fluctuation in total hours over a business cycle results from movements in employment rather than in hours per employee. Log total hours is equal to the log of average hours per employee plus the log of employment, and over the 1960–2015 period, the standard deviation of log total hours is 1.87, that of log average hours is 0.50, and that of log employment is 1.57.<sup>58</sup> The 2008–2009 Great Recession in the United States was associated with a fall in total output in the nonfarm business sector of 7.60 percent, a fall in total labor hours of 7.22 percent, a fall in employment of 5.54 percent and a fall in average hours of 1.68 percent. Thus, most the labor adjustment occurs on the employment or extensive margin, and much less on the average hours or intensive margin.

In this section, the new Keynesian model is modified to incorporate fluctuations in the fraction of workers who have jobs, not just fluctuations of hours worked per employee. This modification allows unemployment to be introduced explicitly into the model. Extending the model to include unemployment also allows one to address the welfare effects of fluctuations in unemployment and the possible role that the unemployment rate, as distinct from the output gap, should play in the design of monetary policy.

The standard macroeconomic model of unemployment is provided by the search and matching Diamond-Mortensen-Pissarides (DMP) model (Mortensen and Pissarides 1994). Walsh (2003a; 2005b) was the first to integrate the DMP model into a new Keynesian model with sticky prices. He assumed all labor adjustment occurred along the extensive margin, with hours per employee fixed. He argued that in a model with habit persistence in consumption, inertia in the policy instrument rule, and search and matching frictions in the labor market, persistent effects of monetary policy shocks could be captured with a lower and more realistic degree of price stickiness than occurs in standard new Keynesian models.<sup>59</sup> In a paper contemporaneous with Walsh (2005b), Trigari (2009) developed a similar model and estimated it using U.S. data. Other contributors who added unemployment variation to a new Keynesian model include Blanchard and Galí (2007; 2010), Krause, López-Salido, and Lubik (2008), Thomas (2008; 2011); Ravenna and Walsh (2008; 2011; 2012a; 2012b), Sala, Söderstrom, and Trigari (2008; 2012), Gertler and Trigari (2009); Lago Alves (2012), and Galí (2011).

### **A Sticky-Price New Keynesian Model with Unemployment**

To illustrate the implications of search and matching frictions in a monetary policy model, the discussion here follows Ravenna and Walsh (2011). Given that most employment

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58. This is for the nonfarm business sector. All variables are HP-filtered.

59. Heer and Maussner (2010) found that this result depends on the assumption in Walsh (2005b) of a fixed capital stock.

volatility occurs on the extensive margin, the model assumes hours per worker are fixed and all labor adjustment consists of changes in the fraction of workers who are employed. This reverses the standard new Keynesian specification, in which only hours per worker vary. Because the model contains two frictions (sticky prices and search frictions in the labor market), it is convenient to follow Walsh (2003a) and introduce two types of firms, one with sticky prices and the other facing labor market frictions.<sup>60</sup>

Specifically, assume there is a measure one of retail firms who sell differentiated goods to households and whose prices are sticky. These retail firms do not employ labor but instead buy a homogeneous intermediate good that they use to produce their final output. Price adjustment by retail firms follows the Calvo model, so price inflation of the consumption goods purchased by households depends on expected future inflation and the real marginal cost of retail firms. Their real marginal cost is simply  $P_t^I/P_t$ , where  $P_t^I$  is the price of intermediate inputs and  $P_t$  is the consumer price index. The other type of firms hire labor and produce the intermediate good. This good is sold in a competitive market to the retail firms, and the price of the intermediate good is flexible.

Rather than assume each household equates the marginal rate of substitution between leisure and consumption to the real wage, workers are assumed either to be employed, in which case they work a fixed number of hours, or unemployed and searching for a new job. Employment is an endogenous state variable, and a new equation is added to keep track of its evolution. Employment will decrease if the flow of workers from employment to unemployment exceeds the flow of unemployed workers into jobs. For simplicity, the flow of workers who separate from jobs and become unemployed is taken to occur at a constant rate  $s$  per period.<sup>61</sup> Assume in period  $t$  a fraction  $q_t^u$  of the unemployed job seekers find jobs. If employment is denoted by  $e_t$  and the number of job seekers in period  $t$  is denoted by  $u_t$ ,

$$u_t = 1 - (1 - s)e_{t-1}, \quad (8.88)$$

and employment is

$$e_t = (1 - s)e_{t-1} + q_t^u u_t. \quad (8.89)$$

With this particular specification,  $u_t$  is predetermined and equals the fraction of the labor force that is seeking jobs during period  $t$ .<sup>62</sup>

The key innovation of Mortensen and Pissarides (1994) was to introduce a matching function to capture the process by which unemployed workers and vacant positions at

60. The decision problem of a firm simultaneously facing both price adjustment frictions and labor market search frictions was analyzed by Thomas (2008; 2011) and Lago Alves (2012).

61. For a search and matching model with endogenous separations, see den Haan, Ramey, and Watson (2000). For endogenous separations in NK models, see Walsh (2005b) or Ravenna and Walsh (2012a).

62. The choice of timing allows workers who find jobs in period  $t$  to produce within the period. With hours per employee fixed, this ensures in a sticky-price new Keynesian model that in response to an aggregate demand shock, output can expand within the same period.

firms lead to actual employment matches. Let  $v_t$  denote the number of job vacancies. Then the number of new job matches  $m_t$  is given by

$$m_t = m(u_t, v_t), \quad (8.90)$$

where  $m$  is increasing in both  $u$  and  $v$ . With random search, the job-finding rate for an unemployed worker is  $m_t/u_t$  and the job-filling rate for a firm with a vacancy is  $m_t/v_t$ . It is common (and consistent with empirical evidence) to assume the matching function  $m$  displays constant returns to scale, and a Cobb-Douglas function form is often assumed. In this case, one can write

$$m_t = m_0 u_t^a v_t^{1-a} = m_0 \theta_t^{1-a} u_t, \quad 0 < a < 1,$$

where  $\theta_t \equiv v_t/u_t$  is a measure of labor market tightness. The rate at which unemployed workers find jobs,  $m_t/u_t = q_t^u = m_0 \theta_t^{1-a}$ , is an increasing function of labor market tightness. The job-filling rate  $m_t/v_t = q_t^v = m_0 \theta_t^{1-a} u_t/v_t = m_0 \theta_t^{-a}$  is decreasing in labor market tightness.

Since a new variable, job vacancies, has been added, a theory of job creation is needed. Assume a firm faces a cost  $k$  per period to post a job vacancy.<sup>63</sup> If  $V_t^V$  is the value to a firm of having an open vacancy, then

$$V_t^V = -k + q_t^v V_t^J + (1 - q_t^v) \beta E_t \Omega_{t,t+1} V_{t+1}^V, \quad (8.91)$$

where  $V_t^J$  is the value to the firm of having a worker in a job,  $q_t^v$  is the probability of filling the vacancy, and  $\Omega_{t,t+1}$  is the stochastic factor for discounting time  $t + 1$  valuations back to period  $t$ . If there is free entry to job posting, firms will create job openings until the expected value of a vacancy is driven to zero. Setting  $V_t^V$  and  $V_{t+1}^V$  equal to zero in the previous equation implies

$$q_t^u V_t^J = m_0 \theta_t^{-a} V_t^J = k, \quad (8.92)$$

where  $m_0 \theta_t^{-a}$  is the number of hires the firm expects to make if it posts a job opening for a period, and  $V_t^J$  is the value of having a job filled.

Assume a constant returns to scale production technology with labor as the only variable input, so that output at firm  $j$  is  $y_{j,t} = Z_t N_{j,t}$ . The value of a worker to the firm, expressed in terms of final consumption goods, is therefore

$$V_t^J = \left( \frac{P_t^I}{P_t} \right) Z_t - \omega_t + (1 - s) \beta E_t \Omega_{t,t+1} V_{t+1}^J. \quad (8.93)$$

To understand this expression, define  $\mu_t \equiv P_t/P_t^I$  as the price of retail goods relative to the intermediate good (the retail price markup). The net profit from the hire is the value of

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63. Petrosky-Nadeau and Zhang (2013a; 2013b) discussed the case of nonfixed costs of posting job vacancies.

output produced net of the real wage,  $(P_t^I/P_t)Z_t - \omega_t = \mu_t^{-1}Z_t - \omega_t$ . Because with probability  $1 - s$  the worker does not separate, the current value of the worker to the firm also includes the expected future value of the worker,  $V_{t+1}^J$ .

Recall from (8.15) that in the basic new Keynesian model with flexible wages (as is the case here), firms set  $Z_t/\mu_t = \omega_t$ . In the presence of search and matching equations (8.92) and (8.93) imply

$$\frac{Z_t}{\mu_t} = \omega_t + \left(\frac{k}{m_0}\right) [\theta_t^a - (1-s)\beta E_t \Omega_{t,t+1} \theta_{t+1}^a]. \quad (8.94)$$

The left side of this equation is the real marginal value of a worker; the right side is the cost of labor. It includes the wage plus the search cost of hiring the worker,<sup>64</sup> but it is reduced by the expected savings in search costs in  $t+1$  from having a worker in place in time  $t$ . Importantly, given the current real wage, the firm's labor costs are increasing in current labor market tightness, as a rise in  $\theta_t$  implies it takes longer to fill a job, but they are decreasing in expected future labor market tightness, as a rise in  $\theta_{t+1}$  increases the value to the firm of its existing workers.

To close the model, a specification of wage determination must be added. Because of the search frictions present in the labor market, the value to the firm of having a worker in place is greater than the alternative of having an unfilled vacancy as long as  $V_t^J > 0$ . Similarly for a worker, being employed is worth more to the worker than being unemployed. Thus, there is a surplus to both parties to a job match. The surplus to the worker is the difference between having a job and not having one. Let  $V_t^E$  denote the value to the worker of having a job. It is given by the wage net of the disutility of working, plus the expected discounted value of still being employed (which occurs with probability  $1 - s$ ) and the value of not being employed. The latter, which occurs with probability  $s$ , consists of the expected value of finding a new job,  $q_{t+1}^u V_{t+1}^E$  plus the expected value of continuing to be unemployed,  $(1 - q_{t+1}^u) V_{t+1}^U$ . Thus,<sup>65</sup>

$$V_t^E = \omega_t + \beta E_t \Omega_{t,t+1} \{ (1-s) V_{t+1}^E + s [q_{t+1}^u V_{t+1}^E + (1 - q_{t+1}^u) V_{t+1}^U] \}.$$

The value of being unemployed arises from any unemployment benefit or home production when unemployed,  $\omega_t^u$ , plus the expected gain if a new job is found:

$$V_t^U = \omega_t^u + \beta E_t \Omega_{t,t+1} [q_{t+1}^u V_{t+1}^E + (1 - q_{t+1}^u) V_{t+1}^U].$$

Noting that  $q_{t+1}^u = m_0 \theta_{t+1}^{1-a}$ , the worker's surplus from being employed is therefore

$$V_t^E - V_t^U = \omega_t - \omega_t^u + (1-s)\beta E_t \Omega_{t,t+1} (1 - m_0 \theta_{t+1}^{1-a}) (V_{t+1}^E - V_{t+1}^U). \quad (8.95)$$

64. If the probability of filling a vacancy is  $m_0 \theta_t^{-a}$ , the expected time it takes to hire a worker is  $(1/m_0 \theta_t^{-a})$  at a cost  $k$  per unit of time.

65. For simplicity, this assumes there is no disutility from working; if there is, the wage should be interpreted as net of such costs.

Any wage such that  $V_t^J \geq 0$  and  $V_t^E - V_t^U \geq 0$  is compatible with the worker and firm each finding it individually rational to continue the employment match. To determine the actual wage, the standard approach in the literature has been to assume Nash bargaining with fixed bargaining weights.<sup>66</sup> Under Nash bargaining, both the worker and the firm have an incentive to maximize the joint surplus from the match and then to divide this maximized surplus with a fixed share  $\alpha$  going to the worker and  $1 - \alpha$  going to the firm. Using (8.93) and (8.95), the joint surplus,  $V_t^E - V_t^U + V_t^J$ , does not depend directly on the wage. The role of the wage is to ensure the appropriate division of the surplus between worker and firm. Thus, the wage ensures  $V_t^E - V_t^U = a(V_t^E - V_t^U + V_t^J)$ . Using (8.93), (8.92), and (8.95),

$$\omega_t = \omega_t^u + \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{\kappa}{m_0} \right) \left[ \theta_t^a - (1 - s) \beta E_t \Omega_{t,t+1} \left( 1 - m_0 \theta_{t+1}^{1-a} \right) \theta_{t+1}^a \right]. \quad (8.96)$$

The new Keynesian model with search and matching frictions in the labor market consists then of the standard household Euler condition, the model of price adjustment by retail firms, and the specification of the labor market. Equations (8.7), (8.13) with real marginal cost given by  $1/\mu_t$ , and (8.17) give the equilibrium conditions for consumption, optimal price setting by retail firms, and the definition of the aggregate price level as a function of the prices set by adjusting firms and the lagged price level. Goods clearing implies

$$C_t + kv_t = Y_t, \quad (8.97)$$

where  $Y_t$  is retail output that is used for consumption and the costs of posting vacancies. From (8.20), output of the intermediate goods sector used to produce retail goods is

$$Y_t^I = Y_t \Delta_t, \quad (8.98)$$

where  $\Delta_t$  is the measure of retail price dispersion given by (8.21). The aggregate production function in the intermediate sector is

$$Y_t^I = Z_t e_t, \quad (8.99)$$

and employment evolves according to

$$e_t = (1 - s) e_{t-1} + m_0 \theta_t^{1-a} [1 - (1 - s) e_{t-1}]. \quad (8.100)$$

This last equation is obtained from (8.88), (8.89), and the definition of  $q_t^u$ . From the definition of  $\theta_t$ ,

$$v_t = \frac{u_t}{\theta_t} = \frac{1 - (1 - s) e_t}{\theta_t}. \quad (8.101)$$

66. Petrosky-Nadeau and Zhang (2013b) and Christiano, Eichenbaum, and Trabandt (2013) adopted the alternating offer bargaining model of Hall and Milgrom (2008).



Collecting the equilibrium conditions, one has (8.7), (8.13), (8.17), (8.21), (8.94), (8.96), and (8.97)–(8.101). These eleven equations, plus the specification of monetary policy, determine the equilibrium values of consumption, the nominal interest rate, the optimal price chosen by adjusting firms, the retail price index, the measure of relative price dispersion, the retail price markup, the real wage, output in the retail sector, output in the intermediate goods sector, employment, vacancies, and labor market tightness.

### Implications for Monetary Policy

A number of authors have investigated the role of monetary policy in models with nominal rigidities and search and matching frictions in the labor market. The focus in Walsh (2003a; 2005b) was on the dynamic effects of labor market frictions. Faia (2008) considered monetary policy rules, and Kurozumi and Van Zandweghe (2010) studied how search and matching frictions affect the conditions on policy required to ensure determinacy. Thomas (2008), Faia (2009), and Ravenna and Walsh (2011) studied optimal policy.<sup>67</sup>

In a standard labor market framework, efficiency requires that workers' marginal rate of substitution between leisure and consumption equals their marginal productivity. When search and matching frictions characterize the labor market, however, congestion externalities are present. For example, when a firm is deciding whether to open a new vacancy, the firm takes labor market tightness as given. But by posting a new vacancy, the firm increases labor market tightness and makes it more difficult for other firms to hire. If firms capture a large share of a match surplus, they have an incentive to post many job vacancies, and the labor market can be inefficiently tight. If firms receive only a small share of the surplus, they will post too few vacancies, and the labor market will be inefficiently slack. Hosios (1990) showed that efficiency is achieved if labor's share of the joint match surplus equals the elasticity of matches with respect to employment, or  $\alpha = a$ . Thus, there is an optimal wage that divides the joint surplus of a match in a manner that ensures efficient job creation. Even with flexible prices and wages, wage fluctuations around this optimal level will generate distortions that reduce welfare; these distortions are measured by the fluctuations in labor market tightness around its efficient level. These labor market distortions are in addition to those arising from price stickiness.

In section 8.4.1, optimal policy in a basic NK model was studied using a second-order approximation to the welfare of the representative household. Ravenna and Walsh (2011) derived the second-order approximation to the welfare of the representative household in an NK model that includes a search and matching model of the labor market. They assumed wages were flexible, set by Nash bargaining, but allowed the bargaining share to vary stochastically. Shocks to the bargaining share operate like inefficient markup shocks in a basic NK model. Their results illustrated directly the role that labor market variables play in a welfare-based policy objective.

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67. See also Blanchard and Gali (2010).

Assume inflation in the steady-state is zero and the steady-state output level is efficient, so that  $x^*$  in (8.48) is zero. As in the basic new Keynesian model, this requires a fiscal subsidy to offset the steady-state distortions arising from imperfect competition. It also requires that labor's steady-state share of the joint match surplus equals the elasticity of matches with respect to employment ( $\alpha = a$ ), so that Hosios' condition for efficiency is satisfied. Ravenna and Walsh then showed that the welfare approximation is given by

$$E_t \sum_{i=0}^{\infty} \beta^i (V_{t+i} - \bar{V}) \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda_x \tilde{c}_{t+i}^2 + \lambda_\theta \tilde{\theta}_{t+i}^2 \right] + \text{t.i.p.}, \quad (8.102)$$

where  $\bar{V}$  is steady-state welfare, and  $\tilde{c}_t$  and  $\tilde{\theta}_t$  are the log gaps between consumption and labor market tightness and their efficient values, respectively. The first two terms, including the parameter  $\lambda_x$  on  $\tilde{c}_t^2$ , are identical to the squared inflation and output gap terms in (8.48) because in the basic model without capital, consumption and output are equal (to first order).<sup>68</sup>

The new term,  $\tilde{\theta}_t^2$ , reflects inefficient labor market fluctuations. The weight on this term is

$$\lambda_\theta = \alpha \left( \frac{\lambda_x}{\sigma} \right) \frac{\kappa \bar{v}}{\bar{C}},$$

where  $\kappa \bar{v} / \bar{C}$  equals steady-state vacancy posting costs relative to consumption. Recall that output in this model is used for consumption or for job posting. If job posting costs are large relative to consumption, then it becomes more important to stabilize the labor market at its efficient level of tightness. For their baseline calibration, however, Ravenna and Walsh reported that  $\lambda_\theta$  is small, reflecting both the finding in the basic NK model that  $\lambda_x$  is small but also the assumption that vacancy posting costs are a small share of output. In fact, Blanchard and Galí (2010) assumed such costs were small in deriving a welfare measure and so ended up with only inflation and an output gap appearing in their policy objective function.

Ravenna and Walsh (2012a) found that when wages are set by Nash bargaining but the Hosios condition does not hold, the cost of labor search inefficiencies can be large, but the associated welfare cost is primarily a steady-state cost, so there is little scope for cyclical monetary policy to correct it. Price stability remains close to optimal.

### Sticky Wages in Search and Matching Models

The model of the previous section took prices to be sticky but treated wages as flexible. As Shimer (2005) demonstrated, the basic DMP model with flexible wages is unable to match the volatility of unemployment, implying too little volatility in unemployment and

68. The value of  $\lambda_x$  in (8.48) depended on the inverse wage elasticity of labor supply,  $\eta$ , but in the model of this section, labor is supplied inelastically, so  $\eta = \infty$ .

too much in wages to be consistent with the macroeconomic evidence.<sup>69</sup> The standard response has been to follow Hall (2005) by introducing wage stickiness; doing so increases the volatility of unemployment. For example, Gertler and Trigari (2009) adopted a Calvo formulation in which a fraction of matches renegotiate wages each period.

In the search and matching approach to labor markets, Pissarides (2009) explained why it is the wage of newly hired workers that is relevant for the firm's job-posting decisions, and the microeconomic evidence suggests these wages are much more flexible than wages of existing workers.<sup>70</sup> The Shimer puzzle has been studied primarily in models in which productivity shocks are the only source of aggregate fluctuations and both prices and wages are flexible. However, Andrés, Domenech, and Ferri (2006) showed that in a rich general equilibrium environment, price stickiness plays an important role in increasing the volatility of unemployment and vacancies closer to that observed in the data. Lago Alves (2012) showed that even if all wages are flexible, introducing a nonzero trend rate of inflation when prices adjust according to the Calvo model increases the volatility of unemployment sufficiently to solve the Shimer puzzle.

The monetary policy implications of sticky wages in a search and matching framework are similar to those seen earlier. When combined with sticky prices, the presence of multiple sources of nominal frictions forces the policymaker to make trade-offs in attempting to stabilize inflation, wage inflation, the output gap, and the labor tightness gap. In the extreme case of fixed nominal wages, labor market inefficiencies are large and volatile over the business cycle. However, Ravenna and Walsh (2012a) found that monetary policy is not an efficient instrument for correcting these distortions in labor markets, as large and costly deviations from price stability would be required.

## 8.6 Summary

This chapter has reviewed the basic new Keynesian model that has come to dominate the analysis of monetary policy issues. The basic model is a DSGE model based on optimizing households, with firms operating in an environment of monopolistic competition and facing limited ability to adjust their prices. The staggered overlapping process of price adjustment apparent in the microeconomic evidence (see chapter 7) is captured through the use of the Calvo mechanism. The details would differ slightly if an alternative model of price stickiness were employed, but the basic model structure would not change. This structure

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69. Shimer adopted standard values to calibrate the model. Hagedorn and Manovskii (2008) showed a better match to the data if the value of the worker's outside option of unemployment is close to the value of employment. However, Costain and Reiter (2008) showed that adopting the Hagedorn-Manovskii calibration implies the model is inconsistent with evidence on the effects of labor market policies.

70. See Haefke, Sonntag, and van Rens (2013). Kudlyak (2014) estimated the cyclical nature of the user cost of labor and found it to be very procyclical.

consists of two basic parts. The first is an expectational IS curve derived from the Euler condition describing the first-order condition implied by intertemporal optimization on the part of the representative household. The second is a Phillips curve relationship linking inflation to an output gap measure. These two equilibrium relationships are then combined with a specification of monetary policy.

The model provides insights into the costs of nominal price and wage rigidities, with inflation generating an inefficient dispersion of relative prices and wage inflation generating an inefficient dispersion of relative wages. In the basic new Keynesian model with sticky prices but flexible wages, a model-consistent objective function for policy, derived as a second-order approximation to the welfare of the representative agent, calls for stabilizing inflation volatility and volatility in the gap between output and the output level that would arise under flexible prices. Additional objectives appear in the approximation to welfare as more friction are included in the model. If nominal wages are sticky, stabilizing wage inflation also becomes a valid objective of monetary policy, but this objective must be balanced against the objectives of stabilizing price inflation and output gap volatility. In the presence of labor market frictions, such as those arising due to search and matching frictions, the gap between labor market tightness and its efficient level also affects welfare.

The new Keynesian approach emphasizes the role of forward-looking expectations. The presence of forward-looking expectations implies that expectations about future policy actions play an important role, and a central bank that can influence these expectations, as assumed under a policy regime of commitment, can do better than one that sets policy in a discretionary manner.

## 8.7 Appendix

This appendix provides details on the derivation of the linear new Keynesian Phillips curve and on the approximation to the welfare of the representative household.

### 8.7.1 The New Keynesian Phillips Curve

In this section, (8.13) and (8.17) are used to obtain an expression for the deviations of the inflation rate around its steady-state level. The steady state is assumed to involve a zero rate of inflation. Let  $Q_t = p_t^*/P_t$  be the relative price chosen by all firms that adjust their price in period  $t$ . The steady-state value of  $Q_t$  is  $Q = 1$ ; this is also the value  $Q_t$  equals when all firms are able to adjust every period. Dividing (8.17) by  $P_t^{1-\theta}$ , one obtains  $1 = (1 - \omega)Q_t^{1-\theta} + \omega(P_{t-1}/P_t)^{1-\theta}$ . Expressed in terms of percentage deviations around the zero inflation steady state, this becomes

$$0 = (1 - \omega)\hat{q}_t - \omega\pi_t \Rightarrow \hat{q}_t = \left( \frac{\omega}{1 - \omega} \right) \pi_t. \quad (8.103)$$

To obtain an approximation to (8.13), note that it can be written as

$$\left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} \right] Q_t = \mu \left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} \right]. \quad (8.104)$$

In the flexible-price equilibrium with zero inflation,  $Q_t = \mu \varphi_t = 1$ . The left side of (8.104) is approximated by

$$\left( \frac{C^{1-\sigma}}{1-\omega\beta} \right) + \left( \frac{C^{1-\sigma}}{1-\omega\beta} \right) \hat{q}_t + C^{1-\sigma} \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma) E_t \hat{c}_{t+i} + (\theta-1) (E_t \hat{p}_{t+i} - \hat{p}_t)].$$

The right side is approximated by

$$\mu \left\{ \left( \frac{C^{1-\sigma}}{1-\omega\beta} \right) \varphi + \varphi C^{1-\sigma} \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} + (1-\sigma) E_t \hat{c}_{t+i} + \theta (E_t \hat{p}_{t+i} - \hat{p}_t)] \right\}.$$

Setting these two expressions equal and noting that  $\mu\varphi = 1$  yields

$$\begin{aligned} & \left( \frac{1}{1-\omega\beta} \right) \hat{q}_t + \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\sigma) E_t \hat{c}_{t+i} + (\theta-1) (E_t \hat{p}_{t+i} - \hat{p}_t)] \\ & = \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} + (1-\sigma) E_t \hat{c}_{t+i} + \theta (E_t \hat{p}_{t+i} - \hat{p}_t)]. \end{aligned}$$

Canceling the terms that appear on both sides of this equation leaves

$$\begin{aligned} \left( \frac{1}{1-\omega\beta} \right) \hat{q}_t & = \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i} - \hat{p}_t), \quad \text{or} \\ \left( \frac{1}{1-\omega\beta} \right) \hat{q}_t & = \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}) - \left( \frac{1}{1-\omega\beta} \right) \hat{p}_t. \end{aligned}$$

Multiplying by  $1-\omega\beta$  and adding  $\hat{p}_t$  to both sides yields

$$\hat{q}_t + \hat{p}_t = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}).$$

The left side is the optimal nominal price  $\hat{p}_t^* = \hat{q}_t + \hat{p}_t$ , and this is set equal to the expected discounted value of future nominal marginal costs. This equation can be rewritten as  $\hat{q}_t + \hat{p}_t = (1-\omega\beta) (\hat{\varphi}_t + \hat{p}_t) + \omega\beta (E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1})$ . Rearranging this expression yields

$$\begin{aligned} \hat{q}_t & = (1-\omega\beta) \hat{\varphi}_t + \omega\beta (E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1} - \hat{p}_t) \\ & = (1-\omega\beta) \hat{\varphi}_t + \omega\beta (E_t \hat{q}_{t+1} + E_t \pi_{t+1}). \end{aligned}$$

Now using (8.103) to eliminate  $\hat{q}_t$ , one obtains

$$\begin{aligned} \left(\frac{\omega}{1-\omega}\right)\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta \left[ \left(\frac{\omega}{1-\omega}\right)E_t\pi_{t+1} + E_t\pi_{t+1} \right] \\ &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta \left(\frac{1}{1-\omega}\right)E_t\pi_{t+1}. \end{aligned}$$

Multiplying both sides by  $(1-\omega)/\omega$  produces the forward-looking new Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\varphi}_t,$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

When production is subject to diminishing returns to scale, firm-specific marginal cost may differ from average marginal cost. Let  $A = \theta(1-a)/a$ . All firms adjusting at time  $t$  set their relative price such that

$$\begin{aligned} \hat{q}_t + \hat{p}_t &= (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}) \\ &= (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} - A(\hat{q}_t + \hat{p}_t - E_t \hat{p}_{t+i}) + E_t \hat{p}_{t+i}]. \end{aligned}$$

This equation can be rewritten as

$$\begin{aligned} \hat{q}_t + \hat{p}_t &= (1-\omega\beta) (\hat{\varphi}_t - A\hat{q}_t + \hat{p}_t) \\ &\quad + \omega\beta(1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+1+i} - A(\hat{q}_t + \hat{p}_t - E_t \hat{p}_{t+1+i}) + E_t \hat{p}_{t+1+i}]. \end{aligned}$$

By rearranging this equation, and recalling that  $\hat{q}_t = \omega\pi_t/(1-\omega)$ , one obtains

$$\begin{aligned} \left(\frac{\omega}{1-\omega}\right)(1+A)\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A) \left[ \left(\frac{\omega}{1-\omega}\right)E_t\pi_{t+1} + E_t\pi_{t+1} \right] \\ &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A) \left(\frac{1}{1-\omega}\right)E_t\pi_{t+1}. \end{aligned}$$

Multiplying both sides by  $(1-\omega)/\omega(1+A)$  produces

$$\pi_t = \beta E_t \pi_{t+1} + \left(\frac{\tilde{\kappa}}{1+A}\right) \hat{\varphi}_t.$$

### 8.7.2 Approximating Utility

The details of the welfare approximation that lead to (8.48) are provided. In addition to the discussion provided in Woodford (2003a), see Galí (2015, ch. 4, app. A).

To derive an approximation to the representative agent's utility, it is necessary to first introduce some additional notation. For any variable  $X_t$ , let  $\bar{X}$  be its steady-state value, let  $X_t^*$  be its efficient level (if relevant), and let  $\tilde{X}_t = X_t - \bar{X}$  be the deviation of  $X_t$  around the steady state. Let  $\hat{x}_t = \log(X_t/\bar{X})$  be the log deviation of  $X_t$  around its steady-state value. Using a second-order Taylor approximation, the variables  $\tilde{X}_t$  and  $\hat{X}_t$  can be related as

$$\tilde{X}_t = X_t - \bar{X} = \bar{X} \left( \frac{X_t}{\bar{X}} - 1 \right) \approx \bar{X} \left( \hat{x}_t + \frac{1}{2} \hat{x}_t^2 \right). \quad (8.105)$$

Employing this notation, one can develop a second-order approximation to the utility of the representative household given in (8.47) as

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} = E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right].$$

Start by approximating each term in the utility function

$$V_t = U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}.$$

In general, if utility from consumption is  $U(C_t)$ , a second-order Taylor expansion around steady-state consumption  $\bar{C}$  yields

$$\begin{aligned} U(C_t) &\approx U(\bar{C}) + U_c(\bar{C})(C_t - \bar{C}) + \frac{1}{2} U_{cc}(\bar{C})(C_t - \bar{C})^2 \\ &= U(\bar{C}) + U_c(\bar{C})\bar{C}_t + \frac{1}{2} U_{cc}(\bar{C})\bar{C}_t^2 \\ &= U(\bar{C}) + U_c(\bar{C})\bar{C} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + \frac{1}{2} U_{cc}(\bar{C})\bar{C}^2 \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right)^2 \\ &= U(\bar{C}) + U_c(\bar{C})\bar{C} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + \frac{1}{2} U_{cc}(\bar{C})\bar{C}^2 \left( \hat{c}_t^2 + \frac{1}{2} \hat{c}_t^3 + \frac{1}{4} \hat{c}_t^4 \right) \\ &\approx U(\bar{C}) + U_c(\bar{C})\bar{C}\hat{c}_t + \frac{1}{2} U_{cc}(\bar{C})\bar{C}\hat{c}_t^2 + \frac{1}{2} U_{cc}(\bar{C})\bar{C}^2 \hat{c}_t^2, \end{aligned}$$

where  $U_c$  and  $U_{cc}$  denote the first and second derivatives of  $U$  and terms of order 3 or higher such as  $\hat{c}_t^3$  and  $\hat{c}_t^4$  have been ignored. When  $U(C_t) = C_t^{1-\sigma}/(1-\sigma)$ , the utility

from consumption can then be approximated around the steady state as

$$\begin{aligned} \frac{C_t^{1-\sigma}}{1-\sigma} &\approx \frac{\bar{C}^{1-\sigma}}{1-\sigma} + \bar{C}^{1-\sigma} \hat{c}_t + \frac{1}{2} \bar{C}^{1-\sigma} \hat{c}_t^2 - \frac{1}{2} \sigma \bar{C}^{1-\sigma} \hat{c}_t^2 \\ &\approx \frac{\bar{C}^{1-\sigma}}{1-\sigma} + \bar{C}^{1-\sigma} \left[ \hat{c}_t + \frac{1}{2} (1-\sigma) \hat{c}_t^2 \right]. \end{aligned} \quad (8.106)$$

Next, one can analyze the term arising from the disutility of work:

$$\begin{aligned} \chi \frac{N_t^{1+\eta}}{1+\eta} &\approx \chi \frac{\bar{N}^{1+\eta}}{1+\eta} + \chi \left[ \bar{N}^\eta \tilde{N}_t + \frac{1}{2} \eta \bar{N}^{\eta-1} \tilde{N}_t^2 \right] \\ &\approx \chi \frac{\bar{N}^{1+\eta}}{1+\eta} + \chi \bar{N}^\eta \left[ \bar{N} \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} \eta \bar{N} \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right)^2 \right] \\ &= \chi \frac{\bar{N}^{1+\eta}}{1+\eta} + \chi \bar{N}^{1+\eta} \left[ \hat{n}_t + \frac{1}{2} \hat{n}_t^2 + \frac{1}{2} \eta \left( \hat{n}_t^2 + \frac{1}{2} \hat{n}_t^3 + \frac{1}{4} \hat{n}_t^4 \right) \right] \\ &\approx \chi \frac{\bar{N}^{1+\eta}}{1+\eta} + \chi \bar{N}^{1+\eta} \left[ \hat{n}_t + \frac{1}{2} (1+\eta) \hat{n}_t^2 \right]. \end{aligned} \quad (8.107)$$

Hence, the second-order approximation of  $V_t$  yields

$$V_t - \bar{V} \approx \bar{C}^{1-\sigma} \left[ \hat{c}_t + \frac{1}{2} (1-\sigma) \hat{c}_t^2 \right] - \chi \bar{N}^{1+\eta} \left[ \hat{n}_t + \frac{1}{2} (1+\eta) \hat{n}_t^2 \right]. \quad (8.108)$$

From the goods market clearing condition,  $Y_t = C_t$ , and from (8.20),  $\Delta_t C_t = \Delta_t Y_t = Z_t N_t$ , where

$$\Delta_t = \int \left( \frac{p_{jt}}{P_t} \right)^{-\theta} dj$$

is a measure of price dispersion. Hence,

$$\hat{y}_t = \hat{n}_t + \hat{z}_t - \hat{\Delta}_t. \quad (8.109)$$

A second-order approximation for  $\Delta_t$  is obtained by first noting that if  $x_{jt} \equiv p_{jt}/P_t$ ,

$$\begin{aligned} \left( \frac{p_{jt}}{P_t} \right)^{-\theta} &= x_{jt}^{-\theta} \approx 1 - \theta \bar{x}^{-\theta-1} \tilde{x}_{jt} + \frac{1}{2} \theta (\theta+1) \bar{x}^{-\theta-2} \tilde{x}_{jt}^2 \\ &= 1 - \theta \tilde{x}_{jt} + \frac{1}{2} \theta (\theta+1) \tilde{x}_{jt}^2 \\ &= 1 - \theta \hat{x}_{jt} + \frac{1}{2} \theta^2 \hat{x}_{jt}^2. \end{aligned}$$



Furthermore,

$$\begin{aligned}
 \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} &= x_{jt}^{1-\theta} \approx 1 + (1-\theta)\bar{x}^{-\theta}\tilde{x}_{jt} - \frac{1}{2}\theta(1-\theta)\bar{x}^{-\theta-1}\tilde{x}_{jt}^2 \\
 &= 1 + (1-\theta)\tilde{x}_{jt} - \frac{1}{2}\theta(1-\theta)\tilde{x}_{jt}^2 \\
 &= 1 + (1-\theta)\left(\hat{x}_{jt} + \frac{1}{2}\hat{x}_{jt}^2\right) - \frac{1}{2}\theta(1-\theta)\hat{x}_{jt}^2 \\
 &= 1 + (1-\theta)\hat{x}_{jt} + \frac{1}{2}(1-\theta)^2\hat{x}_{jt}^2.
 \end{aligned}$$

Integrating over  $j$ ,

$$\int \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} dj = 1 + (1-\theta) \int \hat{x}_{jt} dj + \frac{1}{2}(1-\theta)^2 \int \hat{x}_{jt}^2 dj. \quad (8.110)$$

But from the definition of

$$P_t = \left[ \int p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

it follows that

$$\int \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} dj = 1,$$

so combined with (8.110) this implies

$$\int \hat{x}_{jt} dj = -\frac{1}{2}(1-\theta) \int \hat{x}_{jt}^2 dj.$$

Hence,

$$\begin{aligned}
 \Delta_t &= \int \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj \approx 1 - \theta \int \hat{x}_{jt} dj + \frac{1}{2}\theta^2 \int \hat{x}_{jt}^2 dj \\
 &= 1 + \frac{1}{2}\theta(1-\theta) \int \hat{x}_{jt}^2 dj + \frac{1}{2}\theta^2 \int \hat{x}_{jt}^2 dj \\
 &= 1 + \frac{1}{2}\theta \int \hat{x}_{jt}^2 dj. \\
 &= 1 + \frac{1}{2}\theta \text{var}_j (\ln p_{jt} - \ln P_t) \\
 &= 1 + \frac{1}{2}\theta \text{var}_j \hat{x}_{jt}.
 \end{aligned}$$

Then

$$\hat{\Delta}_t \approx \frac{1}{2} \theta \text{var}_j (\ln p_{jt} - \ln P_t).$$

Notice that because  $\bar{\Delta} = 1$  and  $\hat{\Delta}_t = \frac{1}{2} \theta \int \hat{x}_{jt}^2 dj$ ,  $\hat{\Delta}_t \approx 0$  to a first-order approximation. This in turn means the first-order approximation to  $\Delta_t Y_t = Z_t N_t$  is

$$\hat{y}_t = \hat{z}_t + \hat{n}_t. \quad (8.111)$$

Using  $\hat{c}_t = \hat{y}_t$  and (8.109) in (8.108),

$$\begin{aligned} V_t - \bar{V} &\approx \bar{C}^{1-\sigma} \left[ \hat{y}_t + \frac{1}{2} (1-\sigma) \hat{y}_t^2 \right] \\ &\quad - \chi \bar{N}^{1+\eta} \left[ (\hat{y}_t - \hat{z}_t + \hat{\Delta}_t) + \frac{1}{2} (1+\eta) (\hat{y}_t - \hat{z}_t + \hat{\Delta}_t)^2 \right] \\ &\approx \bar{C}^{1-\sigma} \left[ \hat{y}_t + \frac{1}{2} (1-\sigma) \hat{y}_t^2 \right] - \chi \bar{N}^{1+\eta} \hat{\Delta}_t \\ &\quad - \chi \bar{N}^{1+\eta} \left[ \hat{y}_t - \hat{z}_t + \frac{1}{2} (1+\eta) (\hat{y}_t^2 - 2\hat{y}_t \hat{z}_t + \hat{z}_t^2 + 2(\hat{y}_t - \hat{z}_t) \hat{\Delta}_t + \hat{\Delta}_t^2) \right] \\ &= (\bar{C}^{1-\sigma} - \chi \bar{N}^{1+\eta}) \hat{y}_t + \frac{1}{2} \left[ (1-\sigma) \bar{C}^{1-\sigma} - (1+\eta) \chi \bar{N}^{1+\eta} \right] \hat{y}_t^2 - \chi \bar{N}^{1+\eta} \hat{\Delta}_t \\ &\quad + (1+\eta) \chi \bar{N}^{1+\eta} \hat{z}_t \hat{y}_t + \chi \bar{N}^{1+\eta} \left[ \hat{z}_t - \frac{1}{2} (1+\eta) \hat{z}_t^2 \right], \end{aligned}$$

because  $(\hat{y}_t - \hat{z}_t) \hat{\Delta}_t$  and  $\hat{\Delta}_t^2$  are of order 3 and 4, respectively.

In the steady state, equilibrium in the labor market implies

$$\frac{\chi \bar{N}^\eta}{\bar{C}^{-\sigma}} = \omega = \frac{1}{\mu},$$

where  $\omega$  is the real wage, and  $\mu$  is the steady-state markup in the goods market. In addition, goods market clearing and the aggregate production function imply  $\bar{C} = \bar{Y} = \bar{N}$ . Define  $1 - \Phi \equiv 1/\mu$ . Then these results imply

$$\begin{aligned} \bar{C}^{1-\sigma} - \chi \bar{N}^{1+\eta} &= \bar{C}^{1-\sigma} \left( 1 - \frac{\chi \bar{N}^{1+\eta}}{\bar{C}^{1-\sigma}} \right) = \bar{C}^{1-\sigma} \left( 1 - \frac{\bar{N}}{\bar{C}} \frac{\chi \bar{N}^\eta}{\bar{C}^{-\sigma}} \right) \\ &= \bar{C}^{1-\sigma} \left( 1 - \frac{\chi \bar{N}^\eta}{\bar{C}^{-\sigma}} \right) = \bar{C}^{1-\sigma} \Phi \end{aligned}$$

and

$$\frac{\chi \bar{N}^{1+\eta}}{\bar{C}^{1-\sigma}} = \frac{\chi \bar{N}^\eta}{\bar{C}^{-\sigma}} = \frac{1}{\mu} = 1 - \Phi.$$

This now allows the approximation to  $V_t - \bar{V}$  to be written as

$$V_t - \bar{V} \approx \bar{C}^{1-\sigma} \Phi \hat{y}_t + \frac{1}{2} \bar{C}^{1-\sigma} [(1-\sigma) - (1+\eta)(1-\Phi)] \hat{y}_t^2 - \bar{C}^{1-\sigma} (1-\Phi) \hat{\Delta}_t \\ + (1+\eta) \bar{C}^{1-\sigma} (1-\Phi) \hat{z}_t \hat{y}_t + \bar{C}^{1-\sigma} (1-\Phi) \left[ \hat{z}_t - \frac{1}{2} (1+\eta) \hat{z}_t^2 \right].$$

The term  $\Phi$  is a measure of the inefficiency generated from imperfect competition; if the steady-state markups were equal to 1,  $\Phi = 0$ . Assume that  $\Phi$  is small (of first order) so that terms such as  $\Phi \hat{y}_t \hat{z}_t$  and  $\Phi \hat{\Delta}_t$  are of third order. Then terms in the approximation of  $V_t - \bar{V}$  that involve  $\hat{y}_t$  can be written as

$$\left\{ \begin{array}{l} \bar{C}^{1-\sigma} \Phi \hat{y}_t + \frac{1}{2} \bar{C}^{1-\sigma} [(1-\sigma) - (1+\eta)(1-\Phi)] \hat{y}_t^2 \\ + \bar{C}^{1-\sigma} (1+\eta)(1-\Phi) \hat{z}_t \hat{y}_t \end{array} \right\} \\ \approx \bar{C}^{1-\sigma} \Phi \hat{y}_t - \frac{1}{2} \bar{C}^{1-\sigma} (\sigma + \eta) \hat{y}_t^2 + \bar{C}^{1-\sigma} (1+\eta) \hat{z}_t \hat{y}_t \\ = \bar{C}^{1-\sigma} \Phi \hat{y}_t - \frac{1}{2} \bar{C}^{1-\sigma} [(\sigma + \eta) \hat{y}_t^2 - 2(1+\eta) \hat{z}_t \hat{y}_t],$$

which can be written as

$$\bar{C}^{1-\sigma} \Phi \hat{y}_t - \frac{1}{2} \bar{C}^{1-\sigma} [(\sigma + \eta) \hat{y}_t^2 - 2(1+\eta) \hat{z}_t \hat{y}_t] \\ = -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \hat{y}_t^2 - 2 \left( \frac{1+\eta}{\sigma + \eta} \right) \hat{z}_t \hat{y}_t - 2 \left( \frac{1}{\sigma + \eta} \right) \Phi \hat{y}_t \right].$$

Now subtracting and adding

$$\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \left( \frac{1+\eta}{\sigma + \eta} \right)^2 \hat{z}_t^2 + \left( \frac{1}{\sigma + \eta} \right)^2 \Phi^2 + 2 \left( \frac{1}{\sigma + \eta} \right) \Phi \left( \frac{1+\eta}{\sigma + \eta} \right) \hat{z}_t \right],$$

one obtains

$$-\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \hat{y}_t^2 \\ -2 \left( \frac{1+\eta}{\sigma + \eta} \right) \hat{z}_t \hat{y}_t - 2 \left( \frac{1}{\sigma + \eta} \right) \Phi \hat{y}_t \end{array} \right] \\ = -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \hat{y}_t^2 \\ -2 \left( \frac{1+\eta}{\sigma + \eta} \right) \hat{z}_t \hat{y}_t \\ -2 \left( \frac{1}{\sigma + \eta} \right) \Phi \hat{y}_t \end{array} \right] \\ -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \left( \frac{1+\eta}{\sigma + \eta} \right)^2 \hat{z}_t^2 + \left( \frac{1}{\sigma + \eta} \right)^2 \Phi^2 \\ + 2 \left( \frac{1}{\sigma + \eta} \right) \Phi \left( \frac{1+\eta}{\sigma + \eta} \right) \hat{z}_t \end{array} \right]$$

$$\begin{aligned}
& + \frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \left( \frac{1+\eta}{\sigma+\eta} \right)^2 \hat{z}_t^2 + \left( \frac{1}{\sigma+\eta} \right)^2 \Phi^2 \\ + 2 \left( \frac{1}{\sigma+\eta} \right) \Phi \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \end{array} \right] \\
& = -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \hat{y}_t^2 - 2 \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \hat{y}_t \\ + \left( \frac{1+\eta}{\sigma+\eta} \right)^2 \hat{z}_t^2 - 2 \left( \frac{1}{\sigma+\eta} \right) \Phi \hat{y}_t \\ + 2 \left( \frac{1}{\sigma+\eta} \right) \Phi \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \\ + \left( \frac{1}{\sigma+\eta} \right)^2 \Phi^2 \end{array} \right] \\
& = -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left[ \begin{array}{c} \left( \hat{y}_t - \hat{y}_t^f \right)^2 \\ - 2 \left( \hat{y}_t - \hat{y}_t^f \right) x^* \\ + \left( \hat{y}_t^f \right)^2 + \left( x^* \right)^2 \end{array} \right] + \text{t.i.p.} \\
& = -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left( \hat{x}_t - x^* \right)^2 + \text{t.i.p.}
\end{aligned}$$

where

$$\hat{y}_t^f \equiv \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t$$

is the economy's flexible-price equilibrium output (expressed as a log deviation from steady state),

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^f$$

is the output gap, and

$$x^* \equiv \left( \frac{1}{\sigma+\eta} \right) \Phi$$

is the steady-state gap between the economy's flexible-price output and efficient output.

These results imply

$$V_t - \bar{V} \approx -\frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \left( \hat{x}_t - x^* \right)^2 - \bar{C}^{1-\sigma} \hat{\Delta}_t + \text{t.i.p.}$$

is the economy's flexible-price equilibrium output (expressed as a log deviation from steady state). Thus, the second-order approximation to the discounted value of the welfare of the representative household is

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \equiv \left( \frac{1}{1-\beta} \right) \bar{V} - E_t \bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{2} (\sigma + \eta) \left( \hat{x}_{t+i} - x^* \right)^2 + \hat{\Delta}_t \right] + \text{t.i.p.}$$

(8.112)

The last step is to relate the price dispersion term,  $\hat{\Delta}_t$ , to the average inflation rate across all firms. Earlier, it was shown that  $\hat{\Delta}_t$  is related to the cross-sectional variance of prices across firms:

$$\hat{\Delta}_t \simeq \frac{1}{2}\theta \text{var}_j (\ln p_{jt} - \ln P_t).$$

Recall that the price-adjustment mechanism involves a randomly chosen fraction  $1 - \omega$  of all firms optimally adjusting price each period. Define  $\bar{P}_t \equiv E_j \log p_{jt}$ . Then, because  $\text{var}_j \bar{P}_{t-1} = 0$ , one can write

$$\begin{aligned} \text{var}_j (\log p_{jt} - \bar{P}_{t-1}) &= E_j (\log p_{jt} - \bar{P}_{t-1})^2 - (E_j \log p_{jt} - \bar{P}_{t-1})^2 \\ &= \omega E_j (\log p_{jt-1} - \bar{P}_{t-1})^2 + (1 - \omega) (\log p_t^* - \bar{P}_{t-1})^2 \\ &\quad - (\bar{P}_t - \bar{P}_{t-1})^2, \end{aligned}$$

where  $p_t^*$  is the price set at time  $t$  by the fraction  $1 - \omega$  of firms that reset their price. Given that  $\bar{P}_t = (1 - \omega) \log p_t^* + \omega \bar{P}_{t-1}$ ,

$$\log p_t^* - \bar{P}_{t-1} = \left( \frac{1}{1 - \omega} \right) (\bar{P}_t - \bar{P}_{t-1}).$$

Using this result,

$$\begin{aligned} \hat{\Delta}_t &= \frac{1}{2}\theta \left[ \omega \Delta_{t-1} + \left( \frac{\omega}{1 - \omega} \right) (\bar{P}_t - \bar{P}_{t-1})^2 \right] \\ &\approx \frac{1}{2}\theta \omega \Delta_{t-1} + \frac{1}{2}\theta \left( \frac{\omega}{1 - \omega} \right) \pi_t^2. \end{aligned}$$

This implies

$$E_t \sum_{i=0}^{\infty} \beta^i \hat{\Delta}_{t+i} = \frac{1}{2}\theta \left[ \frac{\omega}{(1 - \omega)(1 - \omega\beta)} \right] E_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 + \text{t.i.p.},$$

where the terms independent of policy also include the initial degree of price dispersion.

Combining this with (8.112) and ignoring terms independent of policy, the present discounted value of the utility of the representative household can be approximated by

$$\begin{aligned} E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} &\equiv \left( \frac{1}{1 - \beta} \right) \bar{V} - E_t \frac{1}{2} (\sigma + \eta) \bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i (\hat{x}_{t+i} - x^*)^2 - E_t \bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i \hat{\Delta}_{t+i} \\ &= \left( \frac{1}{1 - \beta} \right) \bar{V} - E_t \frac{1}{2} \bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i [(\sigma + \eta) (\hat{x}_{t+i} - x^*)^2 \\ &\quad + \theta \left[ \frac{\omega}{(1 - \omega)(1 - \omega\beta)} \right] \pi_{t+i}^2] \\ &= \left( \frac{1}{1 - \beta} \right) \bar{V} - E_t \frac{1}{2} \Omega \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda (\hat{x}_{t+i} - x^*)^2], \end{aligned}$$

where

$$\Omega = \frac{1}{2} \bar{C}^{1-\sigma} \left[ \frac{\omega}{(1-\omega)(1-\omega\beta)} \right] \theta,$$

$$\lambda = \left[ \frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \left( \frac{\sigma + \eta}{\theta} \right).$$

If fiscal tax and subsidy policies are used to offset the steady-state markups in the goods and labor markets, the steady-state output under flexible prices will be efficient. In this case, which corresponds to ensuring  $\Phi = 0$ ,  $x^* = 0$  and the welfare of the representative household is (again ignoring terms independent of policy) given by

$$E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} = \left( \frac{1}{1-\beta} \right) \bar{V} - E_t \frac{1}{2} \Omega \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda \hat{x}_{t+i}^2).$$

## 8.8 Problems

1. Suppose rather than the definition in (8.18), aggregate output is defined as

$$Y_t = \int c_{jt} dj.$$

- a. Using the demand equation (8.5), show that goods market clearing implies  $Y_t = \Delta_t C_t$ , where  $\Delta_t$  is the measure of price dispersion defined in (8.21).
- b. If each firm faces the production function  $c_{jt} = Z_t N_{jt}$ , show that aggregate employment  $N_t = \int N_{jt} dj$  is equal to  $Y_t / Z_t$ .
- c. Use the results in parts (a) and (b) and the definition of  $C_t$  given by (8.2) to show

$$C_t = \Delta_t^{-1} (Z_t N_t) = Z_t \tilde{N}_t,$$

where  $\tilde{N}_t \equiv \Delta_t^{-1} N_t \leq N_t$ . Explain how price dispersion reduces the effective amount of labor relative to actual employment  $N_t$ .

2. Consider a simple forward-looking model of the form

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + u_t,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$

Suppose policy reacts to the output gap:

$$i_t = \delta x_t.$$

Write this system in the form given by (8.33). Are there values of  $\delta$  that ensure a unique stationary equilibrium? Are there values that do not?

3. Consider the model given by

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Suppose policy sets the nominal interest rate according to a policy rule of the form

$$i_t = \phi_1 E_t \pi_{t+1}$$

for the nominal rate of interest.

- Write this system in the form  $E_t z_{t+1} = M z_t + \eta_t$ , where  $z_t = [x_t, \pi_t]'$ .
  - For  $\beta = 0.99$ ,  $\kappa = 0.05$ , and  $\sigma = 1.5$ , plot the absolute values of the two eigenvalues of  $M$  as a function of  $\phi_1 > 0$ .
  - Are there values of  $\phi_1$  for which the economy does not have a unique stationary equilibrium?
4. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi_t N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is  $Y_t = Z_t N_t$ . The notation is:  $C$  is consumption,  $\chi$  is a stochastic shock to tastes,  $N$  is time spent working,  $Y$  is output,  $Z$  is an aggregate productivity disturbance, and  $\sigma$  and  $\eta$  are constants. The stochastic variable  $\chi$  has a mean of 1.

- Derive the household's first-order condition for labor supply. Show how labor supply depends on the taste shock and explain how a positive realization of  $\chi$  would affect labor supply.
- Derive an expression for the flexible-price equilibrium output  $\hat{y}_t^f$  for this economy.
- Does the taste shock affect the flexible-price equilibrium? If it does, explain how and why.
- The household's Euler condition for optimal consumption choice (expressed in terms of the output gap and in percent deviations around the steady-state) can be written as

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t^n).$$

How does  $r^n$  depend on the behavior of the flexible-price equilibrium output? Does it depend on the taste shock  $\chi$ ? Explain intuitively whether a positive realization of  $\chi$  raises, lowers, or leaves unchanged the flexible-price equilibrium real interest rate.

5. Assume the utility of the representative agent is

$$\frac{\psi_t C_t^{1-\sigma}}{1-\sigma} - \frac{\chi_t N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is  $Y_t = Z_t N_t$ .  $C$  is consumption,  $\psi$  and  $\chi$  are a stochastic shocks to tastes,  $N$  is time spent working,  $Y$  is output,  $Z$  is an aggregate productivity disturbance, and  $\sigma$  and  $\eta$  are constants. The stochastic taste shocks have means of 1.

- Derive the household's first-order condition for labor supply. Show how labor supply depends on the taste shocks, and explain how a positive realization of  $\psi$  affects labor supply.
  - Derive an expression for the flexible-price equilibrium output  $\hat{y}_t^f$  for this economy. How is it affected by  $\psi$ ?
  - In the basic new Keynesian model, inflation depends on real marginal cost. Show that the linearized inflation equation (8.23) can still be written in the form given by (8.26) even with the introduction of taste shocks.
6. The appendix derived the second-order approximation to the welfare of the representative agent based on the utility function given by (8.47). Suppose instead that utility of the representative agent is

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{\psi_{t+i} C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi_{t+i} N_{t+i}^{1+\eta}}{1+\eta} \right].$$

How is the quadratic loss function affected by the presence of the stochastic preference shocks  $\psi_t$  and  $\chi_t$ ?

7. Suppose the economy is characterized by

$$x_t = \mathbb{E}_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - \mathbb{E}_t \pi_{t+1} - r_t^n),$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t.$$

What problems might arise if the central bank decides to set its interest rate instrument according to the rule  $i_t = r_t^n$ ?

8. Suppose the economy is described by the basic new Keynesian model consisting of

$$x_t = \mathbb{E}_t x_{t+1} - \sigma^{-1} (i_t - \mathbb{E}_t \pi_{t+1}),$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t,$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t.$$



- a. If  $\phi_x = 0$ , explain intuitively why  $\phi_\pi > 1$  is needed to ensure that the equilibrium will be unique.
- b. If both  $\phi_\pi$  and  $\phi_x$  are non-negative, the condition given by (8.36) implies that the economy can still have a unique stable equilibrium even when

$$1 - \frac{(1 - \beta)\phi_x}{\kappa} < \phi_\pi < 1.$$

Explain intuitively why some values of  $\phi_\pi < 1$  are still consistent with uniqueness when  $\phi_x > 0$ .

9. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(1 + \xi_t)N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is  $Y_t = Z_t N_t$ .  $C$  is consumption,  $\xi$  is a stochastic shock to tastes,  $N$  is time spent working,  $Y$  is output,  $Z_t = (1 + z_t)$  is a stochastic aggregate productivity disturbance, and  $\sigma$  and  $\eta$  are constants. Both  $\xi$  and  $z$  have zero means. Assume a standard model of monopolistic competition with Calvo pricing.

- a. Assuming a zero steady-state rate of inflation, the inflation adjustment equation can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \mu_t,$$

where  $\mu_t$  is real marginal cost (expressed as a percent deviation around the steady state). Derive an expression for  $\mu_t$  in terms of an output gap.

- b. Does the taste shock affect the output gap? Does it affect inflation? Explain.

10. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma} \left(\frac{M_t}{P_t}\right)^{1-b}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is  $Y_t = Z_t N_t^a$ .

- a. Show that the household's first-order condition for labor supply takes the form

$$\eta \hat{n}_t + \sigma \hat{c}_t - \mu_t^w = \hat{w}_t - \hat{p}_t,$$

$$\text{where } \mu_t^w = (1 - b) (\hat{m}_t - \hat{p}_t).$$

- b. Derive an expression for the flexible-price equilibrium output  $\hat{y}_t^f$  and the output gap  $x_t = \hat{y}_t - \hat{y}_t^f$ .
- c. Does money affect the flexible-price equilibrium? Does the nominal interest rate? Explain.

11. Suppose the economy is characterized by (8.45) and (8.46), and let the cost shock be given by  $e_t = \rho e_{t-1} + \varepsilon_t$ . The central bank's loss function is (8.50). Assume that the central bank can commit to a policy rule of the form  $\pi_t = \gamma e_t$ .
- What is the optimal value of  $\gamma$ ?
  - Find the expression for equilibrium output gap under this policy.
12. In section 8.4.4, the case of commitment to a rule of the form  $x_t = b_x e_t$  was analyzed. Does a unique stationary rational-expectations equilibrium exist under such a commitment? Suppose instead that the central bank commits to the rule  $i_t = b_i e_t$  for some constant  $b_i$ . Does a unique stationary rational-expectations equilibrium exist under such a commitment? Explain why the two cases differ.
13. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero inflation steady state):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (8.113)$$

where  $x_t$  is the gap between output and the flexible-price equilibrium output level, and  $e_t$  is a cost shock. Assume that

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where  $\varepsilon$  is a white noise process. The central bank sets the nominal interest rate  $i_t$  to minimize

$$\frac{1}{2} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right].$$

- Derive the first-order conditions linking inflation and the output gap for the *fully* optimal commitment policy.
  - Explain why the first-order conditions for time  $t$  differ from the first-order conditions for  $t+i$  for  $i > 0$ .
  - What is meant by a commitment policy that is optimal from a timeless perspective? (Explain in words.)
  - What is the first-order condition linking inflation and the output gap that the central bank follows under an optimal commitment policy from a timeless perspective?
  - Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock (assume the cost shock is serially uncorrelated).
14. Explain why inflation is costly in a new Keynesian model.
15. Suppose the economy is described by the following log-linearized system:

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) + E_t (z_{t+1} - z_t) + u_t,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t,$$

where  $u_t$  is a demand shock,  $z_t$  is a productivity shock, and  $e_t$  is a cost shock. Assume that

$$u_t = \rho_u u_{t-1} + \xi_t,$$

$$z_t = \rho_z z_{t-1} + \psi_t,$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where  $\xi$ ,  $\psi$ , and  $\varepsilon$  are white noise processes. The central bank sets the nominal interest rate  $i_t$  to minimize

$$\left(\frac{1}{2}\right) E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right].$$

- a. Derive the optimal time-consistent policy for the discretionary central banker. Write down the first-order conditions and the reduced-form solutions for  $x_t$  and  $\pi_t$ .
  - b. Derive the interest rate feedback rule implied by the optimal discretionary policy.
  - c. Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation.
  - d. How will  $x_t$  and  $\pi_t$  move in response to a demand shock? to a productivity shock?
16. Suppose the central bank cares about inflation variability, output gap variability, and interest rate variability. The objective of the central bank is to minimize

$$\left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i (i_{t+i} - i^*)^2 \right].$$

The structure of the economy is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t,$$

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right) (i_t - E_t \pi_{t+1} - r_t),$$

where  $e$  and  $r$  are exogenous stochastic shocks. Let  $\psi_t$  denote the Lagrangian multiplier on the Phillips curve, and let  $\theta_t$  be the multiplier on the IS curve.

- a. Derive the first-order conditions for the optimal policy of the central bank under discretion.
- b. Show that  $\theta$  is nonzero if  $\lambda_i > 0$ . Explain the economics behind this result.
- c. Derive the first-order conditions for the fully optimal commitment policy. How do these differ from the conditions you found in part (a)?
- d. Derive the first-order conditions for the optimal commitment policy from a timeless perspective. How do these differ from the conditions you found in part (c)?

- e. Eliminate any Lagrangian multipliers from the first-order conditions after adopting the timeless perspective. Write the result in the form of an interest rate rule. How many lagged values of the interest rate appear in the rule?
17. Consider a basic new Keynesian model with Calvo adjustment of prices and flexible nominal wages.
- In this model, inflation volatility reduces the welfare of the representative agent. Explain why.
  - In the absence of cost shocks, optimal policy would ensure that inflation and the output gap both remain equal to zero. What does this imply for the behavior of output? Why can output fluctuate efficiently despite sticky prices?
  - Suppose both prices and nominal wages are sticky (assume a Calvo model for wages). Will volatility in the rate of wage inflation be welfare-reducing? Explain.
  - Is zero inflation and a zero output gap still feasible? Explain.
18. A key issue in the analysis of policy trade-offs is the source of the stochastic shocks in the model. Consider these two examples. (1) The utility function takes the form

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta_t}}{1+\eta_t},$$

where  $\eta_t$  is stochastic. (2) There is a labor tax  $\tau_t$  such that the after-tax wage is  $(1 - \tau_t)W_t$ . Assume a standard model of monopolistic competition.

- Derive the condition for labor market equilibrium under flexible prices for each of the two cases.
  - Linearize the conditions found in part (a) and, for each case, derive the flexible-price equilibrium output in terms of percent deviations from the steady state. Clearly state any assumptions you need to make on the  $\eta$  and  $\tau$  processes or about other aspects of the model.
  - Assume sticky prices as Calvo does. Express real marginal cost in terms of an output gap.
  - Does either  $\eta_t$  or  $\tau_t$  appear as a cost shock?
  - Do you think either  $\eta_t$  or  $\tau_t$  causes a wedge between the flexible-price output level and the efficient output level?
19. Suppose inflation adjustment is given by (8.66). The central bank's objective is to minimize

$$\left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2),$$

subject to (8.66). Use Dynare to answer this question.

- a. Calculate the response of the output gap and inflation to a serially uncorrelated, positive cost shock for  $\phi = 0, 0.25, 0.5, 0.75,$  and  $1$  under the optimal discretionary policy.
  - b. Now do the same for the optimal commitment policy.
  - c. Discuss how the differences between commitment and discretion depend on  $\phi$ , the weight on lagged inflation in the inflation adjustment equation.
20. Suppose

$$\pi_t - \gamma\pi_{t-1} = \beta(E_t\pi_{t+1} - \gamma\pi_t) + \kappa x_t + e_t,$$

$$e_t = 0.25e_{t-1} + \varepsilon_t,$$

and the period loss function is

$$L_0 = \left(\frac{1}{2}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left[ (\pi_t - \gamma\pi_{t-1})^2 + 0.25x_t^2 \right].$$

- a. Analytically find the optimal targeting rule under discretion.
  - b. Analytically find the optimal targeting rule under commitment (timeless perspective).
  - c. Assume  $\beta = 0.99, \kappa = 0.0603, \rho = 0.25,$  and  $\lambda = 0.25.$  Set  $\sigma_\varepsilon^2 = 1.$  Under the targeting rules found in parts (a) and (b), plot the loss  $L$  as a function of  $\gamma = [0 \ 1].$
21. Suppose the inflation equation contains lagged inflation:

$$\pi_t = (1 - \phi)\beta E_t\pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t.$$

- a. Show that the optimal commitment policy from a timeless perspective is
 
$$\pi_t + (\lambda/\kappa) [x_t - (1 - \phi)x_{t-1} - \beta\phi E_t x_{t+1}] = 0.$$
  - b. Show that the unconditional optimal commitment policy takes the form
 
$$\pi_t + (\lambda/\kappa) [x_t - \beta(1 - \phi)x_{t-1} - \phi E_t x_{t+1}] = 0.$$
22. The following model was estimated by Linde (2005), although the values here are from Svensson and Williams (2008):

$$\pi_t = 0.4908E_t\pi_{t+1} + (1 - 0.4908)\pi_{t-1} + 0.0081y_t + \varepsilon_t^\pi,$$

$$y_t = 0.4408E_t y_{t+1} + (1 - 0.4408) [1.1778y_{t-1} + (1 - 1.1778)y_{t-2}]$$

$$- 0.0048(i_t - E_t\pi_{t+1}) + \varepsilon_t^y,$$

$$i_t = (1 - 0.9557 + 0.0673) (1.3474\pi_t + 0.7948y_t) + 0.9557i_{t-1} - 0.0673i_{t-2} + \varepsilon_t^i,$$

with  $\sigma_\pi = 0.5923, \sigma_y = 0.4126,$  and  $\sigma_i = 0.9918.$

- a. Write this system in the form  $E_t z_{t+1} = Mz_t + \eta_t$  for appropriately defined vectors  $z$  and  $\eta.$

- b. Plot the impulse response functions showing how inflation and the output gap respond to each of the three shocks.
- c. How are the impulse responses affected if the coefficient on inflation in the policy rule is reduced from 1.3474 to 1.1?
23. Suppose the firm uses a labor aggregate  $N_t$  to produce output using the technology  $Y_t = F(N_t)$ , where  $F' \geq 0$ ,  $F'' \leq 0$ . The labor aggregate is a composite function of the individual types of labor services and is given by

$$N_t = \left[ \int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \quad \gamma > 1,$$

where  $n_{jt}$  is the labor from household  $j$  that the firm employs. The real wage of labor type  $j$  is  $\omega_{jt}$ . Show that if the firm takes wages as given, its optimal demand for labor type  $j$ , conditional on  $N_t$ , is given by (8.85).

# 9 Monetary Policy in the Open Economy

## 9.1 Introduction

The analysis in earlier chapters was conducted in the context of a closed economy. Useful insights into monetary phenomena can be obtained while still abstracting from the linkages that tie different economies together, but clearly many issues do require an open-economy framework if they are to be addressed adequately. New channels through which monetary factors can influence the economy arise in open economies. Exchange rate movements, for example, play an important role in the transmission process that links monetary disturbances to output and inflation movements. Open economies may be affected by economic disturbances that originate in other countries, and this raises questions of monetary policy design that are absent in a closed-economy environment. Should policy respond to exchange rate movements? Should monetary policy be used to stabilize exchange rates? Should national monetary policies be coordinated?

Chapter 8 developed a new Keynesian closed-economy model based on optimizing households and firms but in which prices were sticky. A large literature has developed open-economy models that share an approach combining optimizing agents and nominal rigidities. Besides the early work of Obstfeld and Rogoff (1995; 1996; 2000), examples include Corsetti and Presenti (2001; 2002), Betts and Devereux (2000), Benigno and Benigno (2008), and Kollman (2001). Lane (2001), Engel (2002), and Corsetti, Dedola, and Leduc (2010) provided surveys of the “new open-economy macroeconomics.”

This chapter begins in section 9.2 with a two-country new Keynesian model based on Clarida, Galí, and Gertler (2002). The two-country model has the advantage of capturing some of the important linkages between economies while still maintaining a degree of simplicity and tractability. Section 9.3 considers the Galí and Monacelli (2005) model of a *small open economy*. In the open-economy literature, a small open economy denotes an economy that is too small to affect world prices, interest rates, or economic activity. Since many countries are small relative to the world economy, the small-open-economy model provides a framework relevant for studying many policy issues.

Sections 9.2 and 9.3 focus on one nominal rigidity, sticky prices. Section 9.4 discusses other sources of nominal rigidity that may be present in open economies. For example, imperfect pass-through and local currency pricing can affect the trade-offs faced by the central bank as well as the nature of welfare-based policy objectives. Optimal monetary policy in a currency union is discussed in section 9.5.

## 9.2 A Two-Country Open-Economy Model

Clarida, Galí, and Gertler (2002) presented a two-country model that is closely related to the new Keynesian model studied in chapter 8. The two countries are denoted as the home country, indicated by the subscript  $h$ , and the foreign country, indicated by the subscript  $f$ . The countries share the same preferences and technologies but may differ in size and may be subject to different shocks. There is a continuum of households of mass 1, with fraction  $1 - \gamma$  residing in the home country and  $\gamma$  in the foreign country. Labor is immobile across countries. Households consume a domestically produced final good and an imported final good, and households in both countries can trade in a complete set of contingent securities. There are two types of firms in each country. Intermediate goods-producing firms hire labor and produce differentiated inputs used by final goods-producing firms. To introduce a nominal rigidity, intermediate goods-producing firms are assumed to be subject to a Calvo process for adjusting prices (see chapter 7), while final goods-producing firms sell a homogeneous consumption good in competitive markets in which prices are flexible. Nominal wages are treated as flexible.

### 9.2.1 Households

The representative household in the home country maximizes the expected discounted value of utility given by

$$E_t \sum_{i=0}^{\infty} \beta^i [U(C_{t+i}) + W(M_{t+i}/P_{t+i}) - V(N_{t+i})], \quad (9.1)$$

subject to a sequence of budget constraints, where  $C_t$  is a consumption aggregate of home and foreign goods defined as

$$C_t \equiv C_{h,t}^{1-\gamma} C_{f,t}^{\gamma}, \quad (9.2)$$

and  $N_t$  equals labor hours supplied. Real money balances are denoted by  $M_t/P_t$ , and  $W(\cdot)$  equals the utility obtained from holding money. The period utility function is assumed to take the form

$$U(C_t) + W(M_t/P_t) - V(N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \phi \frac{(M_t/P_t)^{1-\alpha}}{1-\alpha} - \frac{N_t^{1+\eta}}{1+\eta}.$$



Assume foreign households have symmetric preferences.

Households share consumption risk, resulting in consumption being equalized across households within each country. Each household supplies a homogeneous labor service and faces the same nominal wage  $W_t$ . If  $P_{h,t}$  is the price index for home-produced consumption goods, and  $P_{f,t}$  is the corresponding index for imported consumption goods, the cost minimization problem of purchasing the least-cost combination of home- and foreign-produced goods to yield a given level of  $C_t$  takes the form

$$\min_{C_{h,t}, C_{f,t}} P_{h,t} C_{h,t} + P_{f,t} C_{f,t} + v_t \left( C_t - C_{h,t}^{1-\gamma} C_{f,t}^\gamma \right),$$

where  $v_t$  is the Lagrangian multiplier. The solution to this problem implies that the appropriate definition of the price index for total consumption is

$$P_t = v_t = k^{-1} P_{h,t}^{1-\gamma} P_{f,t}^\gamma, \quad (9.3)$$

where  $k \equiv (1 - \gamma)^{1-\gamma} \gamma^\gamma$ .  $P_t$  corresponds to the consumer price index for the home country. It also follows<sup>1</sup> that

$$P_{h,t} C_{h,t} = (1 - \gamma) P_t C_t, \quad (9.4)$$

$$P_{f,t} C_{f,t} = \gamma P_t C_t. \quad (9.5)$$

Define the terms of trade  $S_t$  as the ratio of the price of foreign-produced consumption goods to home-produced consumption goods:  $S_t \equiv P_{f,t}/P_{h,t}$ . The price index can then be written as

$$P_t = k^{-1} P_{h,t} S_t^\gamma. \quad (9.6)$$

In addition to deciding on consumption and labor supply, households purchase a portfolio of internationally traded financial assets. Assume there is a complete set of state-contingent securities, and denote by  $V_{t,t+1}(s)$  the price at  $t$  of a claim that pays one unit of domestic currency at  $t + 1$  in state  $s$ . The budget constraint of the representative household can now be written as

$$W_t N_t + M_{t-1} + D_t - \mathcal{T}_t + \Gamma_t = P_t C_t + M_t + \sum_{s \in \mathcal{S}} V_{t,t+1}(s) D_{t+1}(s), \quad (9.7)$$

where  $\mathcal{S}$  denotes the set of all states,  $D_t$  is the payoff from the portfolio purchased at  $t - 1$ ,  $\mathcal{T}_t$  represents lump-sum taxes (or transfers if negative), and  $\Gamma_t$  equals profits paid to the households as owners of the country's firms.  $D_{t+1}(s)$  is the number of claims purchased in period  $t$  that pay off in state  $s$  at  $t + 1$ .

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1. See problem 1 at the end of this chapter.

Let  $\tilde{p}_{t+1}(s)$  be the probability of state  $s$  occurring at  $t + 1$ . The first-order conditions for consumption, money holdings, labor supply, and portfolio assets implied by the household's problem of maximizing (9.1) subject to the sequence of budgets (9.7) can be written as

$$U_c(C_t) = C_t^{-\sigma} = \lambda_t, \quad (9.8)$$

$$W_m \left( \frac{M_t}{P_t} \right) = \phi \left( \frac{M_t}{P_t} \right)^{-\alpha} = \lambda_t - \beta E_t \left( \frac{P_t}{P_{t+1}} \right) \lambda_{t+1}, \quad (9.9)$$

$$V_N(N_t) = N_t^\eta = \lambda_t \left( \frac{W_t}{P_t} \right), \quad (9.10)$$

$$\lambda_t \frac{V_{t,t+1}(s)}{P_t} = \beta \tilde{p}_t(s) \left( \frac{1}{P_{t+1}(s)} \right) \lambda_{t+1}(s) \quad \text{for all } s \in S, \quad (9.11)$$

where  $\lambda_t$  is the Lagrangian multiplier on the budget constraint. Noting from the first equation that  $\lambda_t$  is equal to the marginal utility of consumption, the optimal labor supply choice involves the household setting the marginal rate of substitution between leisure and consumption equal to the real wage, or

$$\frac{V_N(N_t)}{U_c(C_t)} = \frac{N_t^\eta}{C_t^{-\sigma}} = \left( \frac{W_t}{P_t} \right). \quad (9.12)$$

From the first-order conditions (9.8) and (9.11),

$$\left( \frac{V_{t,t+1}(s)}{P_t} \right) U_c(C_t) = \beta \tilde{p}_t(s) \left( \frac{1}{P_{t+1}(s)} \right) U_c(C_{t+1}(s)) \quad (9.13)$$

holds for each state  $s$ . Summing this equation over all states  $s \in S$  yields

$$\sum_{s \in S} \left( \frac{V_{t,t+1}(s)}{P_t} \right) U_c(C_t) = \beta \sum_{s \in S} \tilde{p}_t(s) \left( \frac{1}{P_{t+1}(s)} \right) U_c(C_{t+1}(s)),$$

which can be rewritten as

$$U_c(C_t) = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) U_c(C_{t+1}),$$

or, using the assumed function form for utility, as

$$C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}, \quad (9.14)$$

where

$$R_t \equiv \frac{1}{\sum_{s \in S} V_{t,t+1}(s)}$$

is the gross risk-free one-period interest rate. That is, by purchasing a complete set of state-contingent securities at a cost of  $\sum V_{t,t+1}(s)$ , the household can guarantee a nominal payoff of one unit of domestic currency at  $t + 1$ .

### 9.2.2 International Consumption Risk Sharing

Given that a complete set of state-contingent securities is available to both the home and foreign households, international financial markets can be used by residents of each country to ensure against country-specific consumption risk.

Households in the foreign country have identical preferences, face a similar demand for their labor, and solve a similar set of problems. Importantly, since foreign households can also trade in the same securities as households in the home country, it follows that

$$\left( \frac{V_{t,t+1}(s)}{\mathcal{E}_t P_t^*} \right) U_c(C_t^*) = V_{t,t+1}(s) = \beta \tilde{p}_t(s) \left( \frac{1}{\mathcal{E}_{t+1} P_{t+1}^*} \right) U_c(C_{t+1}^*(s)), \quad (9.15)$$

where  $C_t^*$  is consumption by foreign households,  $\mathcal{E}_t$  is the nominal exchange rate, defined as the price of the foreign currency in terms of the home currency, and  $P_t^*$  is the foreign consumption price index expressed in terms of the foreign currency. The cost of the security to a foreign household in terms of the foreign currency is  $V_{t,t+1}(s)/\mathcal{E}_t$ . This amount of foreign currency could have purchased  $1/P_t^*$  units of foreign consumption whose value expressed in terms of utility is  $(1/P_t^*) U_c(C_t^*)$ . Thus, the left side of (9.15) is the utility cost of the security. The right side is the expected payoff, equal to the probability that state  $s$  occurs times the marginal utility of the additional consumption the foreign household can purchase if state  $s$  occurs and the security pays out one unit of the home currency.

Equations (9.13) and (9.15) imply that

$$\left( \frac{P_t}{P_{t+1}(s)} \right) \frac{U_c(C_{t+1}(s))}{U_c(C_t)} = \left( \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+1} P_{t+1}^*(s)} \right) \frac{U_c(C_{t+1}^*(s))}{U_c(C_t^*)} \quad (9.16)$$

for all  $s$ .

Assuming the law of one price holds, so that the price of foreign goods in the home country  $P_{f,t}$  is equal to  $\mathcal{E}_t P_{f,t}^*$ , the CPI price index for the home country is

$$P_t = k^{-1} P_{h,t}^{1-\gamma} \left( \mathcal{E}_t P_{f,t}^* \right)^\gamma.$$

Given the identical preferences of foreign households, the CPI for the foreign country (expressed in terms of the foreign currency) is

$$P_t^* = k^{-1} \left( \frac{P_{h,t}}{\mathcal{E}_t} \right)^{1-\gamma} P_{f,t}^{*\gamma}.$$

These two expressions imply the real exchange rate, the ratio of the two CPI price indexes, is equal to 1:

$$\frac{\mathcal{E}_t P_t^*}{P_t} = 1.$$

This means (9.16) implies, when the assumed functional form for the utility function has been used, that

$$\left( \frac{C_{t+1}(s)}{C_t} \right) = \left( \frac{C_{t+1}^*(s)}{C_t^*} \right).$$

Thus, for all states of nature, consumption growth is equal in the home and foreign countries, reflecting the use by households of the international financial markets to share consumption risk. Hence, consumption moves proportionally in the two countries, allowing one to write  $C_t = \nu C_t^*$ . Normalizing so that  $\nu = 1$ ,

$$C_t = C_t^*. \quad (9.17)$$

Shocks that affect either economy, whether the shock occurs in the home country, the foreign country, or both, cause consumption to move symmetrically in both countries.

### 9.2.3 Firms

In the model of Clarida, Galí, and Gertler (2002), each country is populated by two types of firms. Intermediate goods-producing firms use labor as their sole input to produce differentiated outputs used as inputs by final goods-producing firms. Final goods-producing firms have flexible prices and sell in competitive markets in both the home and the foreign countries (i.e., all final goods are tradeable). Intermediate firms have sticky prices.

Let  $Y_t(h)$  denote the intermediate good of type  $h$  used by a final goods firm in the home country. The production function of the final goods firm is

$$Y_t = \left( \int_0^1 Y_t(h)^{\frac{\xi-1}{\xi}} dh \right)^{\frac{\xi}{\xi-1}}, \quad \xi > 1,$$

where  $Y_t$  and  $Y_t(h)$  are expressed in per capita terms. Each final goods firm takes the price of final output  $P_{h,t}$  and the prices of input types  $P_{h,t}(h)$  as given and maximizes profits. This implies the demand for  $Y_t(h)$  is

$$Y_t(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} Y_t, \quad (9.18)$$

and the domestic price index  $P_{h,t}$  is<sup>2</sup>

$$P_{h,t} = \left( \int_0^1 P_{h,t}(h)^{1-\xi} dh \right)^{\frac{1}{1-\xi}}. \quad (9.19)$$

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2. See problem 2 at the end of this chapter.

Intermediate goods-producing firms maximize profits subject to the demand for their output, a production technology for transforming labor input into output, and a time-dependent process for adjusting their price. These firms are identical to the firms in the closed-economy new Keynesian model of chapter 8. Assume firm  $h$  employs a production technology given by

$$Y_t(h) = A_t N_t(h), \quad (9.20)$$

where  $A_t$  is a stochastic aggregate productivity variable and  $N_t(h)$  is employment at firm  $h$ . If the prices of the intermediate goods firms were flexible, firm  $h$  would set  $P_t(h)$  to maximize profits given by

$$P_{h,t}(h) Y_t(h) - W_t N_t(h) = \left[ P_{h,t}(h) - \frac{W_t}{A_t} \right] \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} Y_t.$$

Profit maximization with flexible prices implies

$$P_{h,t}(h) = \left( \frac{\xi}{\xi - 1} \right) \left( \frac{W_t}{A_t} \right).$$

Each intermediate firm would set its price as a markup  $\xi / (\xi - 1) > 1$  over nominal marginal cost  $W_t/A_t$ . Because the right side of this expression is independent of  $h$ , all intermediate firms would set the same price, implying  $P_{h,t} = P_{h,t}(h) = \mu W_t/A_t$ , where  $\mu \equiv \xi / (\xi - 1)$  is the markup. Thus, in a flexible-price equilibrium, the real product wage ( $W_t/P_{h,t}$ ) is less than the marginal product of labor  $A_t$ :  $W_t/P_{h,t} = A_t/\mu < A_t$ . With flexible prices, each firm's real marginal cost is constant and equal to

$$\text{MC}_t = \frac{W_t/P_{h,t}}{A_t} = \frac{1}{\mu}. \quad (9.21)$$

Rather than assume all prices are flexible, Clarida, Galí, and Gertler (2002) assumed a Calvo adjustment mechanism in which each intermediate goods-producing firm adjust its price with probability  $1 - \omega$  each period. When prices are sticky, real marginal cost can differ from  $\mu^{-1}$ . Using (9.6) and (9.12), real marginal cost is equal to

$$\text{MC}_t = \frac{W_t/P_{h,t}}{A_t} = \frac{N_t^\eta / C_t^{-\sigma}}{A_t} \frac{P_t}{P_{h,t}} = \frac{N_t^\eta / C_t^{-\sigma}}{k A_t} S_t^\gamma.$$

The terms of trade variable appears because in the open-economy model, the real wage relevant for domestic producers,  $W_t/P_{h,t}$ , can differ from the real wage relevant for household labor suppliers,  $W_t/P_t$ , if the consumer price index  $P_t$  varies relative to the price index of domestically produced goods  $P_{h,t}$ . For example, if  $S_t$  increases, the price of foreign-produced consumption goods has risen relative to domestically produced goods. This results in a fall in the real wage from the perspective of households, and to induce the same labor supply, the nominal wage must rise. This increases marginal costs for domestic

producers, so those firms that can adjust their prices will raise them, leading to a rise in inflation of the domestic goods price index.

All firms that adjust face the same decision problem and will choose the same reset price  $P_{h,t}^{\text{opt}}$ . The aggregate domestic producer price index then evolves as

$$P_{h,t}^{1-\xi} = \omega P_{h,t-1}^{1-\xi} + (1-\omega) \left( P_{h,t}^{\text{opt}} \right)^{1-\xi}.$$

When this equation is combined with the first-order condition for the optimal choice of reset price, and both are linearized around a zero inflation steady state, one obtains a standard new Keynesian Phillips curve for the rate of inflation in the domestic producer goods price index:<sup>3</sup>

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa (\omega_t - a_t), \quad (9.22)$$

where  $\omega_t = \eta n_t + \sigma c_t + \gamma s_t$ , and  $\omega_t - a_t$  is the percent deviation of real marginal cost around its steady-state value. Here, lowercase letters denote the percent deviations around the steady state of the corresponding uppercase variables.

### 9.2.4 Equilibrium

Equilibrium requires that the goods markets in each country clear. Domestic producers sell to both domestic and foreign households. Recalling that  $Y_t$  is defined in per capita terms, and the home country's population share is  $1 - \gamma$ , market clearing requires

$$(1 - \gamma) Y_t = (1 - \gamma) C_{h,t} + \gamma C_{h,t}^*.$$

Similarly, for foreign-produced goods,

$$\gamma Y_t^* = \gamma C_{f,t} + (1 - \gamma) C_{f,t}^*.$$

Now using (9.4) and (9.5), the corresponding conditions for the foreign country, the goods-clearing condition, the definitions of the CPI in the home and foreign countries, and  $\mathcal{E}_t P_t^* = P_t$ , one obtains

$$P_t C_t = P_{h,t} Y_t,$$

$$P_t^* C_t^* = P_{f,t}^* Y_t^*.$$

These conditions imply a zero balance of trade. They also imply, using the result that  $P_t^* C_t^* = (P_t / \mathcal{E}_t) C_t$ ,

$$\frac{Y_t}{Y_t^*} = \frac{\mathcal{E}_t P_f^*}{P_{h,t}} = \frac{P_{f,t}}{P_{h,t}} = S_t. \quad (9.23)$$

3. See chapter 8 for details on deriving the first-order condition for the price firms pick when able to reset their price and for the linearization that yields the new Keynesian Phillips curve.

Multiplying both sides of (9.6) by  $C_t$ , one obtains  $P_t C_t = P_{h,t} Y_t = k^{-1} P_{h,t} C_t S_t^\gamma$ , or  $Y_t = k^{-1} C_t S_t^\gamma$ .

Combining these results, consumption of home households can be expressed in terms of home and foreign output as

$$C_t = k Y_t S_t^{-\gamma} = k Y_t^{1-\gamma} (Y_t^*)^\gamma. \quad (9.24)$$

A similar consideration of the foreign economy yields

$$C_t^* = k Y_t^* S_t^{1-\gamma} = k Y_t^{1-\gamma} (Y_t^*)^\gamma = C_t. \quad (9.25)$$

The following equations that come from the first-order conditions for the optimal consumption and money-holding decisions of home and foreign households plus the law of one price and the assumptions on preferences must also be satisfied in equilibrium:

$$C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}, \quad (9.26)$$

$$(C_t^*)^{-\sigma} = \beta R_t^* E_t \left( \frac{P_t^*}{P_{t+1}^*} \right) (C_{t+1}^*)^{-\sigma}, \quad (9.27)$$

$$\phi \left( \frac{M_t}{P_t} \right)^{-\alpha} = C_t^{-\sigma} - \beta E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}, \quad (9.28)$$

$$\phi \left( \frac{M_t^*}{P_t^*} \right)^{-\alpha} = (C_t^*)^{-\sigma} - \beta E_t \left( \frac{P_t^*}{P_{t+1}^*} \right) (C_{t+1}^*)^{-\sigma}, \quad (9.29)$$

$$E_t P_t^* = P_t. \quad (9.30)$$

The remaining equilibrium conditions depend on whether prices are flexible or sticky. However, the model implies connections between nominal interest rates, the exchange rate, and the money supplies in the two economies that are independent of whether prices are flexible or sticky. These implications are discussed, followed by the cases of flexible prices and sticky prices.

### Money, Exchange Rates, and Interest Rates

To understand how nominal interest rates, inflation, and the exchange rate are linked, linearize (9.26) and (9.27) around a zero inflation steady state and use  $C_t = C_t^*$  to obtain the uncovered interest rate parity (UIP) condition

$$i_t - E_t \pi_{t+1} = i_t^* - E_t \pi_{t+1}^*.$$

From the purchasing power parity (PPP) condition (9.30),  $p_t = e_t + p_t^*$ , where  $e_t$  is the log nominal exchange rate. Thus,  $\pi_t = e_t - e_{t-1} + \pi_t^*$ , and the UIP condition can be

written equivalently as

$$i_t = i_t^* + E_t e_{t+1} - e_t.$$

If the exchange rate is expected to depreciate, the domestic nominal interest rate will exceed the foreign interest rate; this is an arbitrage relationship. Investors will only hold domestic currency and foreign currency denominated debt if expected returns are equalized. If the domestic currency is expected to lose value, the domestic nominal rate rises to compensate for this expected fall in the value of the currency.

Suppose the central banks in each country implement policy by following simple rules of the form

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

for the home economy and

$$i_t^* = \rho + \phi_\pi \pi_t^* + v_t^*$$

for the foreign economy, where  $v_t$  and  $v_t^*$  are policy shocks. Define  $\hat{v}_t \equiv v_t - v_t^*$  and define  $\hat{\pi}_t \equiv \pi_t - \pi_t^*$  as the inflation differential. Then the UIP condition implies

$$E_t e_{t+1} - e_t = i_t - i_t^* = \phi_\pi \hat{\pi}_t + \hat{v}_t.$$

The PPP condition, however, implies  $e_t - e_{t-1} = \hat{\pi}_t$ . It follows that

$$E_t \hat{\pi}_{t+1} = \phi_\pi \hat{\pi}_t + \hat{v}_t.$$

As discussed in chapter 8, this will have a locally unique stationary solution for the inflation differential if and only if  $\phi_\pi > 1$ .<sup>4</sup> If this condition is satisfied, and assuming  $\hat{v}_t = \rho \hat{v}_{t-1} + \varepsilon_t$ ,  $0 \leq \rho < 1$ , the equilibrium inflation differential is

$$\hat{\pi}_t = H \hat{v}_t, H = -\frac{1}{\phi_\pi - \rho} < 0.$$

It follows that the nominal exchange rate has a unit root and is given by

$$\hat{\pi}_t = e_t - e_{t-1} = H \hat{v}_t. \quad (9.31)$$

The coefficient  $H$  depends on the  $\hat{v}_t$  process through  $\rho$  and on policy through  $\phi_\pi$ .

A positive realization of  $\hat{v}_t$  represents a rise in the home country nominal interest rate relative to the foreign rate. From (9.31),  $e_t$  falls ( $H$  is negative), reflecting an immediate appreciation when the home country tightens monetary policy. In addition,

$$E_t e_{t+1} - e_t = H E_t \hat{v}_{t+1} = \rho H \hat{v}_t < 0.$$

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4. Responding more than one-to-one to inflation is consistent with the Taylor principle (see chapter 8).



The exchange rate is expected to continue to appreciate as long as  $E_t \hat{v}_{t+i}$  remains positive. This expected appreciation is necessary to maintain UIP. Using (9.31),

$$\lim_{i \rightarrow \infty} E_t e_{t+i} = e_{t-1} + H \hat{v}_t \lim_{i \rightarrow \infty} \sum_{j=0}^i \rho^j = e_{t-1} + \frac{H}{1-\rho} \hat{v}_t < e_{t-1}.$$

Thus, the level of the exchange rate is left permanently lower by the positive shock to  $\hat{v}_t$ . From (9.31), the inflation differential between the home and foreign economies is proportional to  $\hat{v}_t$ . A positive value of  $\hat{v}$  implies home inflation is less than foreign inflation. Hence,  $p_t$  falls relative to  $p_t^*$ ; from  $e_t = p_t - p_t^*$ ,  $e_t$  appreciates to reflect the rise in the value of the home currency as home prices fall relative to foreign prices.

Suppose that instead of following an interest rate rule, both central banks set paths for their money supplies and let nominal interest rates ensure money demand equals money supply. Equation (9.28) can be log-linearized around the steady state and written as

$$m_t - p_t = \delta z_t - \left( \frac{\delta}{1+i} \right) (E_t p_{t+1} - p_t), \quad (9.32)$$

where  $m_t = \log M_t$ ,  $p_t = \log P_t$ ,  $z_t \equiv \sigma [c_t - \beta E_t c_{t+1} / (1 + \pi)]$ ,  $i$  is the steady-state nominal interest rate, and  $\delta \equiv C^{-\sigma} (M/P)^\alpha / \alpha \phi$ . Solving for  $p_t$ ,

$$\left[ 1 + \left( \frac{\delta}{1+i} \right) \right] p_t = m_t + \left( \frac{\delta}{1+i} \right) E_t p_{t+1} - \delta z_t.$$

By renaming parameters, this equation can be written as

$$p_t = d_0 m_t + (1 - d_0) E_t p_{t+1} - d_1 z_t, \quad 0 < d_0 < 1, \quad (9.33)$$

which shows how the equilibrium price level depends on the nominal supply of money, the expected future price level, and the behavior of the real economy as summarized in  $z_t$ . By recursively solving (9.33) forward,

$$p_t = E_t \sum_{i=0}^{\infty} (1 - d_0)^i (d_0 m_{t+i} - d_1 z_{t+i}).$$

The price level depends not on the current nominal supply but on the entire future path of the money supply as well as on the current and expected future values of  $z_t$ . Note that under flexible prices,  $z_t$ , which is a function of  $c_t$  and  $c_{t+1}$ , will be independent of  $m_t$ ; this is not the case if prices are sticky.

Because a similar relationship holds for the foreign economy,

$$p_t^* = E_t \sum_{i=0}^{\infty} (1 - d_0)^i (d_0 m_{t+i}^* - d_1 z_{t+i}^*).$$

With international risk sharing, consumption is equal in the two countries, so if steady-state inflation is the same in both countries,  $z_t = z_t^*$ . In this case, subtracting  $p_t$  from  $p_t^*$ ,

$$e_t = p_t - p_t^* = d_0 E_t \sum_{i=0}^{\infty} (1 - d_0)^i (m_{t+i} - m_{t+i}^*), \quad (9.34)$$

and the nominal exchange rate depends on the relative money supplies in the two countries. An increase in one country's money supply relative to the other leads to a depreciation of that country's exchange rate. A rise in the domestic money supply is associated with an increase in the domestic price level. A rise in the price level means that one unit of the currency buys fewer goods. Similarly, it buys fewer units of the foreign currency, that is, its value falls relative to the other currency.

The equation for the exchange rate can be rewritten as

$$\begin{aligned} e_t &= d_0 (m_t - m_t^*) + d_0 (1 - d_0) E_t \sum_{i=0}^{\infty} (1 - d_0)^i (m_{t+1+i} - m_{t+1+i}^*) \\ &= d_0 (m_t - m_t^*) + (1 - d_0) E_t e_{t+1}. \end{aligned}$$

Rearranging this expression yields

$$E_t e_{t+1} - e_t = -d_0 \left[ (m_t - m_t^*) - d_0 E_t \sum_{i=0}^{\infty} (1 - d_0)^i (m_{t+1+i} - m_{t+1+i}^*) \right].$$

Analogously to Friedman's concept of permanent income, the term

$$d_0 E_t \sum_{i=0}^{\infty} (1 - d_0)^i (m_{t+1+i} - m_{t+1+i}^*)$$

can be interpreted as the permanent nominal money supply differential. Suppose the current value of  $m_t - m_t^*$  is high relative to the permanent value of this differential. If  $e_t$  reflects the permanent money supply differential at time  $t$ , and  $m_t$  is temporarily high relative to  $m_t^*$ , then the permanent differential will be lower beginning in period  $t + 1$ . As a result,  $E_t e_{t+1} - e_t$  falls as the home currency is expected to appreciate.

An explicit solution for the nominal exchange rate can be obtained if specific processes for the nominal money supplies are assumed. To take a simple case, suppose  $m_t$  and  $m_t^*$  follow constant deterministic growth paths given by

$$\begin{aligned} m_t &= m_0 + \mu t, \\ m_t^* &= m_0^* + \mu^* t. \end{aligned}$$

Strictly speaking, (9.33) applies only to deviations around the steady state, not to money supply processes that include deterministic trends.<sup>5</sup> However, it is very common to specify

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5. The parameter  $\delta$  depends on  $M/P$ , whose steady-state value depends on the steady-state rate of inflation.

(9.32), which was used to derive (9.33), in terms of the log levels of the variables, perhaps adding a constant to represent steady-state levels. The advantage of interpreting (9.33) as holding for the log levels of the variables is that one can then use it to analyze shifts in the trend growth paths of the nominal money supplies rather than just deviations around the trend. The limitations of doing so should be kept in mind; the underlying representative agent model implies that the interest rate coefficients in the money demand equations are functions of the steady-state rate of inflation.

Under the assumed processes for the money supplies, (9.34) implies

$$e_t = e_0 + (\mu - \mu^*)t,$$

where  $e_0 = m_0 - m_0^* + (1 - d_0)(\mu - \mu^*)/d_0$ .<sup>6</sup> In this case, the nominal exchange rate has a deterministic trend equal to the difference in the trend of money growth rates in the two economies (also equal to the inflation rate differentials, since  $\pi = \mu$  and  $\pi^* = \mu^*$ ). If domestic money growth exceeds foreign money growth ( $\mu > \mu^*$ ),  $e$  will rise over time to reflect the falling value of the home currency relative to the foreign currency.

This analysis has focused on relationships that exploit the fact  $C_t = C_t^*$ . The behavior of output in each economy and the common level of consumption will depend on whether prices are flexible or sticky.

### Equilibrium with Flexible Prices

Suppose prices in both countries are completely flexible. All firms set price as a markup over marginal cost, which in turn is the real product wage divided by the marginal product of labor. The real wage in terms of consumer prices equals the household's marginal rate of substitution between leisure and consumption. For the home economy,

$$MC_t = \frac{N_t^\eta / C_t^{-\sigma}}{kA_t} S_t^\gamma = \frac{1}{\mu},$$

and for the foreign economy,

$$MC_t^* = \frac{N_t^{*\eta} / C_t^{*-\sigma}}{kA_t} S_t^{\gamma-1} = \frac{1}{\mu^*}.$$

From (9.24) linking consumption, output, and the terms of trade, and from the aggregate production function,  $N_t = Y_t/A_t$ , one obtains

$$\frac{(Y_t/A_t)^\eta / (kY_t S_t^{-\gamma})^{-\sigma}}{kA_t} S_t^\gamma = \frac{Y_t^{\eta+\sigma}}{k^{1-\sigma} A_t^{1+\eta}} S_t^{\gamma(1-\sigma)} = \frac{1}{\mu}.$$

6. This uses the fact that  $\sum_{i=0}^{\infty} ib^i = b/(1-b)^2$  for  $|b| < 1$ . In addition,  $z_t$  and  $z_t^*$  will differ by a constant if steady-state inflation rates are different in the two economies. This difference will only affect the constant term  $e_0$  in the solution, so it is ignored for simplicity.

Using (9.23), this can be written as

$$\frac{Y_t^{\eta+\sigma-\gamma(\sigma-1)} (Y_t^*)^{\gamma(\sigma-1)}}{k^{1-\sigma} A_t^{1+\eta}} = \frac{1}{\mu}. \quad (9.35)$$

For the foreign country, a similar set of steps leads to

$$\frac{(Y_t^*)^{\eta+\sigma-(1-\gamma)(\sigma-1)} Y_t^{(1-\gamma)(\sigma-1)}}{k^{1-\sigma} (A_t^*)^{1+\eta}} = \frac{1}{\mu^*}. \quad (9.36)$$

These two equations determine output in the two countries under flexible prices. Consumption levels are given by (9.24) and (9.25), and the terms of trade are obtained from (9.23). The presence of the markups implies the flexible-price equilibrium is inefficient. If firms receive a subsidy  $\tau$  on revenue, then  $(1 + \tau) / \mu$  and  $(1 + \tau^*) / \mu^*$  would appear in these conditions, and setting the subsidy at  $\tau = \mu - 1$  and  $\tau^* = \mu^* - 1$  would ensure an efficient equilibrium when prices are flexible.

The real variables such as output and consumption in the two economies and the relative price given by the terms of trade are thus independent of nominal variables and monetary policy when prices are flexible. Given the path of consumption, the real rate of interest is determined by the Euler conditions such as (9.14). Hence, the model displays the classical dichotomy discussed in chapter 2 between real and nominal variables. As also discussed in chapter 2, this result depends on the assumption that utility is separable in money balances. Separability is a common assumption, and it is useful in focusing on the nominal variables, as it allows the real variables to be treated as exogenous from the perspective of inflation and exchange rate determination when prices are flexible.

From (9.35), the effect of output in the foreign economy on the home economy depends on the sign of  $\sigma - 1$ . This reflects two channels through which  $Y^*$  affects  $Y$ . A rise in foreign output causes the price of foreign goods to fall as their supply increases, producing a fall in  $S_t = Y_t / Y_t^*$  (see 9.23). The fall in  $P_{f,t}$  increases workers' real wage at home and increases labor supply, causing  $Y_t$  to rise. However, a rise in  $Y_t^*$  increases home consumption through the international risk-sharing channel. This has a wealth effect on home country labor supply that acts to reduce  $Y_t$ . If  $\sigma > 1$ , this latter effect dominates and a rise in  $Y^*$  reduces  $Y_t$ . If  $\sigma = 1$ , the effects cancel and  $Y_t$  is independent of  $Y_t^*$ . It is common to assume log utility, in which case  $\sigma = 1$ , so it is important to recognize that this parameter choice makes home country output under flexible prices independent of output in the foreign country.

Finally, when the dichotomy holds, (9.26)–(9.30) constitute five equations that contain seven nominal variables: the two price levels, the two nominal interest rates, the two nominal money supplies, and the nominal exchange rate. The model is closed by specifying monetary policy in the two countries.

### Equilibrium with Sticky Prices

When prices are sticky, the dichotomy does not hold; the real and monetary aspects of the model are not independent and must be solved jointly to obtain the equilibrium real and nominal variables. While a linearized version of the model is used to investigate the properties of the equilibrium, one nonlinear relationship is important in highlighting the distortions created by inflation. Aggregate employment can be derived using (9.18) and (9.20) as

$$\begin{aligned} N_t &= \int_0^1 N_t(h) dh = \left( \frac{1}{A_t} \right) \left[ \int_0^1 \frac{Y_t(h)}{Y_t} dh \right] Y_t \\ &= \left( \frac{1}{A_t} \right) \left[ \int_0^1 \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} dh \right] Y_t = \left( \frac{1}{A_t} \right) \Delta_{h,t} Y_t, \end{aligned} \quad (9.37)$$

where  $\Delta_{h,t} \equiv \int_0^1 (P_{h,t}(h)/P_{h,t})^{-\xi} dh$  is a measure of domestic price dispersion. Because  $\Delta_{h,t} \geq 1$ ,  $Y_t = A_t \Delta_{h,t}^{-1} N_t < A_t N_t$ , reflecting the distortion arising from the inefficient allocation of labor across firms when there is a dispersion of relative prices. A dispersion of relative prices arises in the presence of inflation. However, the distortion in the use of labor in the home economy depends on the dispersion of domestic prices, not foreign prices. This becomes important when the model is employed to study optimal monetary policy.

The equilibrium conditions of this two-country model can be linearized around a symmetric, zero inflation steady state to obtain a simple representation of the model. This can then be used to study the model's dynamics once the model is closed with a specification of monetary policy in each of the economies.

In the closed-economy new Keynesian model, the goods-clearing condition was simply  $C_t = Y_t$ . This condition was used to express the Euler condition in terms of  $Y_t$ . It was also used, together with the aggregate production function, to express the marginal rate of substitution between leisure and consumption in terms of output and the productivity shock. This allowed real marginal cost to be expressed in terms of the gap between output and flexible-price output. This same strategy can be adopted in simplifying the two-country model, but account must be taken of the fact that trade implies the goods-clearing condition is no longer  $C_t = Y_t$ . Instead, it is given by (9.24).

Linearizing (9.24) yields

$$c_t = (1 - \gamma)y_t + \gamma y_t^* = y_t - \gamma s_t.$$

Using this in the Euler condition, obtained from linearizing (9.14), gives

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) - \gamma (E_t s_{t+1} - s_t).$$

For reasons that will become clear, use (9.6) to note that  $\pi_t = \pi_{h,t} + \gamma (s_t - s_{t-1})$ . Now rewrite the Euler equation by substituting out  $\pi_t$ , and then use (9.23) to obtain

$$y_t = E_t y_{t+1} - \left( \frac{1}{\sigma_0} \right) [i_t - E_t \pi_{h,t+1} - \rho + \gamma (1 - \sigma) (E_t y_{t+1}^* - y_t^*)],$$

where  $\sigma_0 \equiv \sigma [1 + \gamma (1 - \sigma)]$ . Finally, subtract  $E_t y_{t+1}^f - y_t^f$  from both sides, where  $y_t^f$  is the flexible-price equilibrium output. Doing so yields a relationship between domestic aggregate demand and the domestic real interest rate, defined with respect to domestic inflation:

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma_0} \right) (i_t - E_t \pi_{h,t+1} - \tilde{\rho}_t), \quad (9.38)$$

where  $x_t = y_t - y_t^f$  is the output gap, and

$$\tilde{\rho}_t \equiv \rho + \sigma_0 (E_t y_{t+1}^f - y_t^f) - \gamma (1 - \sigma) (E_t y_{t+1}^* - y_t^*).$$

Earlier, it was shown that real marginal cost equals  $\eta n_t + \sigma c_t + \gamma s_t - a_t$  when linearized around the steady state. It was shown in chapter 8 that to a first-order approximation around a zero inflation steady state,  $\Delta_{h,t} = 1$ . Thus, when (9.37) is linearized, one obtains  $y_t = n_t + a_t$  to first order, as the price dispersion term is zero to order 1. Using the linearized versions of (9.23) and (9.24), the expression for real marginal cost becomes

$$mc_t = [\eta + \sigma + \gamma (1 - \sigma)] y_t - \gamma (1 - \sigma) y_t^* - (1 + \eta) a_t. \quad (9.39)$$

As Clarida, Galí, and Gertler (2002) discussed, the effects of foreign output on marginal cost are ambiguous. A rise in foreign output lowers the price of the imported foreign good and reduces the home country CPI. For a given real wage expressed in terms of the CPI, the fall in the cost of living reduces the domestic nominal wage and lowers the real marginal cost of domestic firms. However, a rise in foreign income also increases domestic consumption through international risk sharing. The wealth effect on labor supply, operating through the  $\sigma c_t$  term in marginal cost, increases the real wage and pushes up marginal cost. If the wealth effect is large,  $\sigma > 1$ , this second effect dominates, and a rise in  $y_t^*$  increases domestic marginal cost.

Under flexible prices, real marginal cost is constant (see 9.21), so  $mc_t = 0$ . Letting  $y_t^f$  denote the deviation of the flexible-price equilibrium output around the steady state, (9.35) implies

$$y_t^f = \frac{\gamma (1 - \sigma) y_t^* + (1 + \eta) a_t}{\eta + \sigma + \gamma (1 - \sigma)}. \quad (9.40)$$

When  $\gamma = 0$ ,  $y_t^f = (1 + \eta) a_t / (\eta + \sigma)$ , which is the expression obtained in chapter 8 for the closed economy. When prices are sticky, (9.35) and (9.39) imply real marginal cost can be written in terms of the output gap  $x_t = y_t - y_t^f$  as

$$mc_t = [\eta + \sigma + \gamma (1 - \sigma)] x_t.$$

This means the inflation in domestic goods prices, given in (9.22), becomes

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \bar{\kappa} x_t, \quad (9.41)$$

where  $\bar{\kappa} = \kappa [\eta + \sigma + \gamma (1 - \sigma)]$ . A similar calculation for the foreign economy implies

$$\pi_{f,t}^* = \beta E_t \pi_{f,t+1}^* + \bar{\kappa}^* x_t^*,$$

where  $\pi_{f,t}^*$  is the inflation rate of foreign goods prices in the foreign currency,  $x_t^*$  is the foreign output gap, and  $\bar{\kappa}^* = \kappa [\eta + \sigma + (1 - \gamma)(1 - \sigma)]$ .

After these manipulations, notice that (9.38) and (9.41) look exactly like the two-equation system obtained for the closed economy NK model in chapter 8. While the model still needs to be closed with a specification of monetary policy, it is instructive to highlight the similarities and differences with the corresponding two equations in a closed economy. First, according to (9.38), aggregate demand depends on the expected future output gap and negatively on the gap between the real interest rate and what here has been labeled  $\tilde{\rho}_t$ . Importantly, the real interest rate is defined in terms of expected future domestic price inflation  $E_t \pi_{h,t}$ . The reference real rate  $\tilde{\rho}_t$  depends on both the expected change in the flexible-price output (as it did in the closed economy) and on the expected output change in the foreign country. Thus, expected changes in foreign output act as an additional demand side disturbance to the home country entering through  $\tilde{\rho}_t$  (unless  $\sigma = 1$ ).

The key parameters of the open-economy model also differ from the ones in the closed-economy context. In the closed-economy, the interest elasticity of aggregate demand was equal to  $1/\sigma$ , the intertemporal elasticity of substitution. In the open economy, it is  $1/\sigma_0$ , where  $\sigma_0 = \sigma [1 + \gamma (1 - \sigma)]$  can be greater than or less than  $\sigma$  depending on whether  $\sigma < 1$  or  $\sigma > 1$ . This is related to the earlier discussion about the two channels through which foreign output affects the home country and its dependence on the size of the wealth effect on labor supply. In addition, the elasticity of domestic price inflation to the output gap is  $\bar{\kappa} = \kappa [\eta + \sigma + \gamma (1 - \sigma)]$  in this open-economy model, while it was  $\kappa (\eta + \sigma)$  in the closed-economy model.

Highlighting the similarities in the basic structure of (9.38) and (9.41) with their closed-economy counterparts, Clarida, Gali, and Gertler (2001; 2002) described the open-economy model as isomorphic to the closed-economy NK model. An important consequence of this isomorphism is that if monetary policy follows an instrument rule of the form

$$i_t = \rho + \phi_\pi \pi_{h,t} + \phi_x x_t$$

that depends on domestic price inflation, then all the conclusions from the discussions of the closed-economy NK model apply to the open economy. For instance, ensuring a locally unique stationary rational-expectations equilibrium means the Taylor principle must be satisfied. As shown in section 8.3.3, this requires

$$\bar{\kappa} (\phi_\pi - 1) + (1 - \beta) \phi_x > 0.$$

### 9.2.5 Optimal Policy

The implications of the open-economy NK model of Clarida, Galí, and Gertler (2002) for optimal monetary policy will exactly parallel those derived for the closed-economy NK model if the monetary authority seeks to minimize an objective function involving domestic price inflation of the form

$$L_t = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left( \pi_{h,t+j}^2 + \lambda x_{t+j}^2 \right), \quad (9.42)$$

subject to (9.38) and (9.41). However, for the parallels with the closed economy to hold, it must be that the central bank is concerned with stabilizing  $\pi_{h,t}$ , the inflation rate of domestic goods prices. This section begins with a discussion of the quadratic approximation to welfare in the open-economy model. Then, following Clarida, Galí, and Gertler (2002), the case of optimal policy by the domestic central bank when developments in the foreign economy are taken as given is discussed. The section then turns to optimal policy when the two central banks cooperate in setting policy.

#### Welfare

To assess whether minimizing (9.42) is an appropriate policy objective, it is important to note that just as in the closed-economy NK model, a steady-state distortion arises in this open-economy model because of imperfect competition in the goods market. If fiscal subsidies are used to offset this steady-state distortion, thereby ensuring the flexible-price equilibrium is efficient, a welfare-consistent objective for monetary policy in the closed economy took the form of a quadratic loss function in inflation and the output gap. The basic intuition for such a policy objective would seem to carry over to the open economy: if tax subsidies have dealt with the distortions associated with monopolistic competition, then the role of monetary policy should be to eliminate the distortions created by domestic sticky prices. From (9.37) it was the dispersion of domestic relative prices that distorted employment. If the home central bank takes foreign income and inflation as given, then it can eliminate domestic price dispersion by stabilizing the sticky prices of the domestically produced goods.

However, as Corsetti and Presenti (2001), Clarida, Galí, and Gertler (2002), Benigno and Benigno (2003), Kirsanova, Leith, and Wren-Lewis (2006), and Corsetti, Dedola, and Leduc (2010) have discussed, this intuition is not correct in general. Even if the distortion arising from the markup is offset by fiscal subsidies, there remains an additional factor present in the open-economy context that creates an incentive for the policymaker to deviate from price stability. Because foreign-produced and domestically produced goods are imperfect substitutes, the central bank faces an incentive to affect their relative price, the terms of trade. From (9.23), a policy that leads to a depreciation (a rise in  $\mathcal{E}_t$ ) or a fall in domestic prices increases the relative price of foreign goods and shifts demand toward



domestic production. This is welfare-improving and would cause the optimal policy to deviate from domestic price stability.

Benigno and Benigno (2003) showed that the optimal fiscal subsidy should not eliminate the markup distortion completely. Instead, the markup distortion can be reduced to the point where the incentive it creates for the central bank to engage in a more expansionary policy to raise output closer to its efficient level just offsets the incentive to affect the terms of trade through a surprise deflation. In this case, if the home country takes foreign output as exogenous, Clarida, Galí, and Gertler (2002) showed, based on a second-order approximation to the welfare of the representative domestic household, the central bank should minimize a loss function given by (9.42).<sup>7</sup> Policy should focus on stabilizing domestic goods price inflation, and the relative weight on output gap stabilization,  $\lambda = \bar{\kappa}/\xi$ , is the same function of the model's structural parameters as it was in the close economy context, once  $\sigma$  is replaced by  $\sigma_0$ .

The implication that welfare depends on stabilizing inflation in domestic producer prices stands in contrast to the goals of inflation-targeting countries, which all define their objectives in terms of  $\pi_t$ , inflation as measured by the consumer price index. In the closed economy of chapter 8, there was, by definition, no distinction between the consumer price index and the producer price index, and in the absence of steady-state distortions and distortionary markup shocks, optimal policy involved replicating the flexible-price equilibrium.

### Optimal Domestic Policy

Suppose the domestic central bank attempts to minimize (9.42) subject to (9.41). Since (9.41) does not contain a disturbance, the optimal policy simply ensures complete price stability,  $\pi_{h,t} = 0$ , and also ensures the output gap is maintained at zero,  $x_t = 0$ . Suppose, therefore, that there is a stochastic wedge between the marginal rate of substitution between leisure and consumption on the part of workers and the real wage they receive. As discussed in chapter 8, such a wedge generates an inefficiency and leads to a stochastic disturbance term in the NK Phillips curve. The presence of this disturbance implies that achieving zero domestic price inflation and a zero output gap is no longer feasible.

The problem of the home country central bank under discretion can be written as

$$\min_{\pi_{h,t}, x_t} \frac{1}{2} \left( \pi_{h,t}^2 + \lambda x_t^2 \right) + \theta_t \left[ \pi_{h,t} - \beta \mathbb{E}_t \pi_{h,t+1} - \bar{\kappa} x_t + \mu_t \right],$$

where  $\mu_t$  is the markup shock,  $\theta_t$  is the Lagrangian multiplier,  $\lambda = \bar{\kappa}/\xi$ , and  $\bar{\kappa} = \kappa [\eta + \sigma + \gamma(1 - \sigma)]$ . After eliminating the Lagrangian multiplier, the first-order conditions for  $\pi_{h,t}$  and  $x_t$  imply

$$\bar{\kappa} \pi_{h,t} + \lambda x_t = 0. \tag{9.43}$$

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7. They did not include money in their model, so any welfare costs arising when money enters the utility function (as it does in (9.1)) were ignored.

The central bank's first-order condition has the same form as it did in the closed-economy NK model, but the elasticity of inflation with respect to the output gap,  $\bar{\kappa}$ , is a different function of the underlying structural parameters than it was in the case of the closed economy. One case in which their functional forms coincide occurs with log utility ( $\sigma = 1$ ), so that  $\bar{\kappa} = \kappa(\eta + \sigma) = \kappa(\eta + 1)$ . However, if the loss function is interpreted as a quadratic approximation to the welfare of the representative domestic resident, then  $\lambda = \bar{\kappa}/\xi$ , and the central bank's first-order condition becomes  $\pi_{h,t} + (1/\xi)x_t = 0$ , which is independent of the parameters that distinguish the open economy from the closed economy. Openness still affects the economy's response to shocks, as the Phillips curve (9.41) depends on  $\bar{\kappa}$ .

The domestic price inflation rate and output gap can be solved using (9.41) and (9.43). CPI inflation can be obtained by log differencing (9.6), yielding  $\pi_t = \pi_{h,t} + \gamma(s_t - s_{t-1})$ . From (9.23), the terms of trade are equal to  $s_t = y_t - y_t^*$ , and the nominal exchange rate is  $e_t = s_t + p_{h,t} - p_{f,t}^*$ , where  $p_{h,t} = p_{h,t-1} + \pi_{h,t}$  and  $p_{f,t}^* = p_{f,t-1}^* + \pi_{f,t}^*$ . CPI inflation in the foreign country is  $\pi_t^* = \pi_{f,t}^* - (1 - \gamma)(s_t - s_{t-1})$ . The terms of trade depend on the level of output in each country, not just the output gaps. Thus, the linearized versions of (9.35) and (9.36) are needed to determine the levels of output with flexible prices.

When combined with the NK Phillips curve for the foreign economy and a specification of foreign monetary policy, the model can then be solved for the output gaps and inflation rates in the two countries, CPI inflation rates, the terms of trade, the price levels of domestic- and foreign-produced goods, and the nominal exchange rate. Under optimal monetary policy, the Euler equations are only needed to solve for the interest rate consistent with the equilibrium behavior of output and inflation. If policy is characterized by an instrument rule for the nominal interest rate, then the Euler conditions are needed to solve for the complete set of endogenous variables.

Optimal policy under commitment can influence expectations by making promises about future policies. This leads to policies that are history-dependent but time-inconsistent (see section 8.4.3). In the present model, the optimal commitment policy is implemented by ensuring

$$\bar{\kappa}\pi_{h,t} + \lambda x_t = 0$$

and

$$\bar{\kappa}\pi_{h,t+i} + \lambda(x_{t+i} - x_{t+i-1}) = 0, \text{ for } i > 0.$$

Figure 9.1 shows the response of domestic variables to a domestic cost shock  $\mu_t$  for different assumptions about monetary policy. The parameter values used for this simulation are  $\beta = 0.99$ ,  $\sigma = 1.0$ ,  $\eta = 3$ ,  $\gamma = 0.4$ ,  $\omega = 0.75$ , and  $\xi = 6$ . The markup shock follows an AR(1) process with coefficient 0.7. The two upper panels of the figure illustrate the responses of  $x_t$  and  $\pi_{h,t}$  under optimal discretion and optimal commitment.<sup>8</sup> The positive

8. The foreign central bank is assumed to follow an optimal discretionary policy. The set of linearized equations used for the simulation can be found in the chapter appendix. The Dynare program to solve the model is available at <http://people.ucsc.edu/~walshc/mtp4e/>.

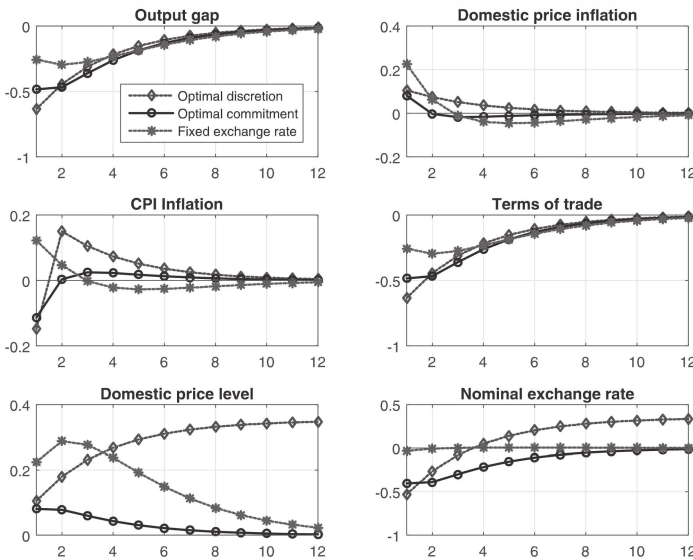


Figure 9.1

Responses to a domestic markup shock under optimal discretion (*diamonds*), optimal commitment (*circles*), and a fixed exchange rate (*stars*).

markup shock increases domestic price inflation, and optimal policy generates a negative output gap to insulate price inflation partially from the shock. The key difference between discretion and commitment is the better trade-off between the output gap and inflation under commitment. Inflation increases by similar amounts under the two policies, but commitment generates a smaller negative output gap. CPI inflation initially fails to rise even though domestic price inflation rises. The two measures of inflation differ when the terms of trade move, and the right middle panel of figure 9.1 shows that  $s_t$  falls as domestic output falls relative to output in the foreign economy. The corresponding appreciation, shown in the right lower panel, causes CPI inflation to fall. Under commitment, domestic price inflation turns negative during the transition back to zero, and as a result, the domestic price level is stationary, as shown in the left lower panel of the figure. Under discretion,  $\pi_{h,t}$  returns to zero, but the price level is left permanently higher. The nominal exchange rate also is nonstationary under discretion, but it is stationary under optimal commitment.

Figure 9.1 also illustrates the response to the markup shock under an instrument rule that places a large weight on the nominal exchange rate. The weight is chosen to ensure this policy is very close to a fixed exchange rate policy, and it is labeled as such in the figure. With the exchange rate fixed rather than experiencing an appreciation, CPI inflation increases along with domestic price inflation, and the terms of trade are stabilized too much relative to either optimal discretion or optimal commitment. By fixing the level of the nominal exchange rate, rather than anchoring an inflation rate as optimal discretion does, the exchange rate policy ensures the price level is stationary (see left lower

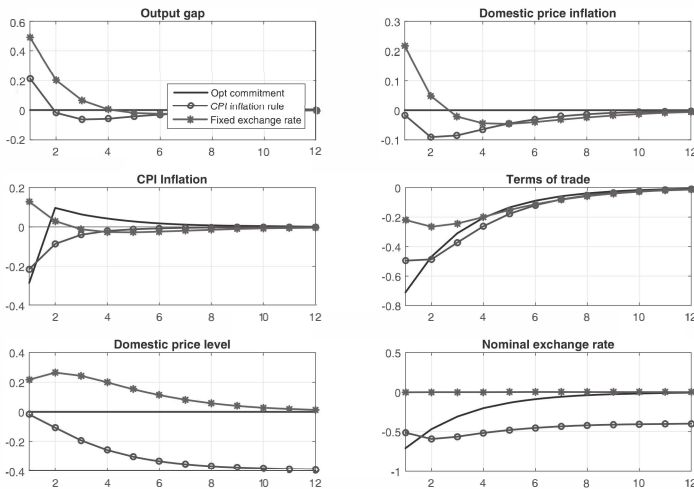
panel), though the price level initially increases much more than it would under optimal commitment.

Figure 9.2 illustrates the impact on the domestic economy of a shock to foreign output under three policies: optimal commitment, CPI inflation targeting, and a fixed exchange rate policy. Under the baseline calibration with  $\sigma = 1$ ,  $y_t^*$  only affects the home economy through the terms of trade. The optimal commitment policy (and optimal discretion) stabilizes  $\pi_{h,t}$  and  $x_t$  completely in the face of this shock. The rise in foreign income causes  $s_t$  to fall as the increased supply of foreign goods lowers their relative price. The fall in  $s_t$  leads to an appreciation and a decline in CPI inflation.

The remaining two policies illustrated in the figure are instrument rules of the form<sup>9</sup>

$$i_t = \phi_\pi \pi_t + \phi_e e_t.$$

The responses labeled CPI inflation are obtained under a policy that sets  $\phi_\pi = 1.5$  and  $\phi_e = 0$ , and the fixed exchange rate rule sets  $\phi_e$  to a large enough value to effectively fix the nominal exchange rate. Notice in the left lower panel, however, that the price level is left permanently lower with the CPI policy rule. If the central bank responds to CPI inflation,



**Figure 9.2**

Effects of a foreign output shock under three different policies: optimal commitment (*solid lines*), CPI targeting (*circles*), and a fixed exchange rate (*stars*).

9. A policy of the form  $i_t = \phi_\pi \pi_{h,t}$  stabilizes domestic price inflation and replicates the optimal policy. If  $\sigma$  were not equal to 1, then the foreign income shock would affect home aggregate demand directly and the  $\pi_{h,t}$  rule would not coincide with an optimal policy.

policy offsets the negative effect on CPI inflation of the fall in the terms of trade by expanding domestic output, leading to a smaller decline in the terms of trade than occurs under the optimal policy. The nominal exchange rate appreciates, and both it and the domestic price level are left permanently lower.

Finally, the rule that responds strongly to the nominal exchange rate leads to large swings in the output gap and domestic price inflation. By preventing the currency appreciation that occurs under the optimal policy, CPI inflation rises, as does domestic price inflation. The decline in the terms of trade is much smaller under this policy than under the others.

### More on Welfare

The conditions under which (9.42) provides the correct welfare-based loss function are special and depend on the assumptions made about preferences. Benigno and Benigno (2003) developed a two-country model similar to the model of Clarida, Galí, and Gertler (2002) but with a more general specification of preferences. They investigated conditions under which optimal monetary policy will try to replicate the flexible-price equilibrium, thereby undoing the effects of sticky domestic prices. They showed that the assumptions made about preferences are important for determining where domestic price stability is optimal. Specifically, assume utility of the households in both countries depends on a consumption index defined not by (9.2) but by

$$C_t = \left[ \alpha_H^{\frac{1}{\theta}} C_{h,t}^{\frac{\theta-1}{\theta}} + (1 - \alpha_H)^{\frac{1}{\theta}} C_{f,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (9.44)$$

where  $C_h$  and  $C_f$  are bundles of home- and foreign-produced final consumption goods.<sup>10</sup> The definition of  $C_t$  in (9.44) yields the special case given by (9.2) when  $\theta = 1$  and  $\alpha_H = \gamma$ . Relative to the model of Clarida, Galí, and Gertler (2002), (9.44) allows the elasticity of substitution between home and foreign goods to differ from the value of 1. The bundles  $C_h$  and  $C_f$  are defined as composites of the individual differentiated goods  $c(h)$  and  $c(f)$  produced in the two countries, with

$$C_{h,t} = \left[ \left( \frac{1}{1-\gamma} \right)^{\frac{1}{\xi}} \int_0^{1-\gamma} c_t(h)^{\frac{\xi-1}{\xi}} dh \right]^{\frac{\xi}{\xi-1}},$$

$$C_{f,t} = \left[ \left( \frac{1}{\gamma} \right)^{\frac{1}{\xi}} \int_{1-\gamma}^1 c_t(f)^{\frac{\xi-1}{\xi}} df \right]^{\frac{\xi}{\xi-1}},$$

with  $\xi > 1$ .

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10. Benigno and Benigno (2003) assumed  $\alpha_H = 1 - \gamma$ , where  $\gamma$  is related to the size of the country, as in Clarida, Galí, and Gertler (2002). The specification here follows Corsetti, Dedola, and Leduc (2010) in allowing the weights on home and foreign goods to differ. The case of a home bias in consumption occurs when  $\alpha_H > 1/2$ .

These preferences imply the demand for home-produced good  $h$  is

$$c_t(h) = \left( \frac{p_t(h)}{\lambda} \right)^{-\xi} C_{h,t}. \quad (9.45)$$

The demands for the home- and foreign-produced consumption bundles are

$$C_{h,t} = \alpha_H \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} C_t, \quad (9.46)$$

$$C_{f,t} = (1 - \alpha_H) \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} C_t,$$

and the consumption-based price index is

$$P_t = \left[ \alpha_H P_{h,t}^{1-\theta} + (1 - \alpha_H) P_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (9.47)$$

where

$$P_{h,t} = \left[ \left( \frac{1}{1-\gamma} \right) \int_0^{1-\gamma} p_t(h)^{1-\xi} dh \right]^{\frac{1}{1-\xi}}$$

and

$$P_{f,t} = \left[ \left( \frac{1}{\gamma} \right) \int_{1-\gamma}^1 p_t(f)^{1-\xi} df \right]^{\frac{1}{1-\xi}}.$$

In these expressions,  $p_t(f)$  is the price of  $c_t(f)$  in units of the home currency. Assume foreign households' preferences are defined similarly, with weights  $\alpha_H^*$  and  $1 - \alpha_H^*$  in (9.44).

Assume all goods are traded and that the law of one price holds. This means that if  $p_t^*(f)$  is the foreign currency price of good  $f$ , then  $p_t(f) = \mathcal{E}_t p_t^*(f)$ , and  $p_t^*(h) = p_t(h)/\mathcal{E}_t$  for all  $h$  and  $f$ , where  $\mathcal{E}_t$  is the nominal exchange rate. If households in both countries consume the same consumption bundle (i.e.,  $\alpha_H = \alpha_H^* = 1/2$ ), it follows that  $P_t = \mathcal{E}_t P_t^*$ ,  $P_{h,t} = \mathcal{E}_t P_{h,t}^*$  and  $P_{f,t} = \mathcal{E}_t P_{f,t}^*$ . In this special case, the real exchange rate given by  $Q_t \equiv \mathcal{E}_t P_t^*/P_t$  equals 1; PPP holds, as it did in the model of Clarida, Galí, and Gertler (2002). If  $\alpha_H, \alpha_H^* \neq 1/2$ , then the real exchange rate can differ from 1. Given (9.47),

$$\begin{aligned} Q_t &\equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{\left[ \alpha_H^* P_{h,t}^{1-\theta} + (1 - \alpha_H^*) P_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[ \alpha_H P_{h,t}^{1-\theta} + (1 - \alpha_H) P_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}} \\ &= \left[ \frac{\alpha_H^* + (1 - \alpha_H^*) S_t^{1-\theta}}{\alpha_H + (1 - \alpha_H) S_t^{1-\theta}} \right]^{\frac{1}{1-\theta}}, \end{aligned}$$

where  $S_t = P_{f,t}/P_{h,t}$  represents the terms of trade. When  $\alpha_H = \alpha_H^* = 1/2$ , this reduces to  $Q_t = 1$ .

From (9.45) and (9.46),

$$c_t(h) = \alpha_H \left( \frac{p_t(h)}{P_{h,t}} \right)^{-\xi} \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} C_t.$$

Similarly, the demand domestic firm  $h$  faces from foreign households is

$$c_t^*(h) = \alpha_H^* \left( \frac{p_t(h)}{P_{h,t}} \right)^{-\xi} \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\theta} C_t^*,$$

where the law of one price has been used in setting  $p_t^*(h)/P_{h,t}^* = p_t(h)/P_{h,t}$ . If the population shares of the home and foreign economy are  $1 - \gamma$  and  $\gamma$ , respectively, total demand for good  $h$  is  $(1 - \gamma)y_t(h) = (1 - \gamma)c_t(h) + \gamma c_t^*(h)$ . Using the definition of the real exchange rate, total demand for domestically produced good  $h$  is

$$y_t(h) = \left( \frac{p_t(h)}{P_{h,t}} \right)^{-\xi} \left[ \alpha_H \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} C_t + \alpha_H^* \left( \frac{\gamma}{1 - \gamma} \right) \left( \frac{P_{h,t}^*}{P_t^*} \right)^{-\theta} C_t^* \right],$$

which can be written as

$$y_t(h) = \left( \frac{p_t(h)}{P_{h,t}} \right)^{-\xi} \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} \left[ \alpha_H C_t + \alpha_H^* \left( \frac{\gamma}{1 - \gamma} \right) Q_t^\theta C_t^* \right]. \quad (9.48)$$

In the Clarida, Galí, and Gertler (2002) model,  $\theta = 1$ , preferences were identical ( $\alpha_H = \alpha_H^*$ ), PPP held ( $Q = 1$ ), and international risk sharing implied  $C_t = C_t^*$ . When PPP does not hold, the risk-sharing condition given by (9.16) implies, using  $Q_t = \mathcal{E}_t P_t^*/P_t$ ,

$$\left( \frac{C_{t+1}(s)}{C_t} \right)^{-\sigma} = \left( \frac{Q_t}{Q_{t+1}(s)} \right) \left( \frac{C_{t+1}^*(s)}{C_t^*} \right)^{-\sigma}.$$

Thus,

$$C_t = \nu Q_t^{\frac{1}{\sigma}} C_t^*$$

for a constant  $\nu$  that will be normalized to equal 1. In this case (9.48) becomes

$$y_t(h) = \left( \frac{p_t(h)}{P_{h,t}} \right)^{-\xi} \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} \left[ \alpha_H + \alpha_H^* \left( \frac{\gamma}{1 - \gamma} \right) Q_t^{\frac{\theta\sigma - 1}{\sigma}} \right] C_t. \quad (9.49)$$

The sign of the exponent on the real exchange rate in (9.49) is determined by  $\theta\sigma - 1$ . As explained by Corsetti, Dedola, and Leduc (2010), if  $\theta\sigma > 1$ , an increase in the consumption of the foreign good decreases the marginal utility of consuming the home good; the two goods are substitutes. If  $\theta\sigma < 1$ , they are complements. When home and foreign goods are substitutes, an increase in the consumption of the foreign good decreases the marginal utility of consuming the home good, and world demand for the home good falls. A rise

in foreign output that causes the price of foreign goods to fall leads to an decrease in the demand for home goods, and home output falls. When the goods are complements, a rise in foreign output increases the demand for home goods, and home output rises.

From (9.49),

$$Y_t = \left[ \left( \frac{1}{1-\gamma} \right) \int_0^{1-\gamma} y_t(h)^{\frac{\xi-1}{\xi}} dh \right]^{\frac{\xi}{\xi-1}} = \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} \left[ \alpha_H(1-\gamma) + \alpha_H^* \gamma Q_t^{\frac{\theta\sigma-1}{\sigma}} \right] C_t.$$

This can be rewritten as

$$P_{h,t} Y_t = \left( \frac{P_{h,t}}{P_t} \right)^{1-\theta} \left[ \alpha_H(1-\gamma) + \alpha_H^* \gamma Q_t^{\frac{\theta\sigma-1}{\sigma}} \right] P_t C_t. \quad (9.50)$$

Under the assumptions made by Clarida, Galí, and Gertler (2002) in deriving the welfare approximation,  $\alpha_H = \alpha_H^* = 1/2$  and  $\theta = \sigma = 1$ . In this case,  $P_{h,t} Y_t = P_t C_t$ . Similarly for the foreign economy,  $P_{f,t}^* Y_t^* = P_t^* C_t^*$ . Together with the law of one price, these results implied  $Y_t/Y^* = S_t$ . Using the definition of the price level in Clarida, Galí, and Gertler,  $C_t = (P_{h,t}/P_t) Y_t = k Y_t^{1-\gamma} (Y_t^*)^\gamma$ . Importantly, consumption moves in proportion to domestic income. This allows domestic welfare to be expressed in terms of fluctuations in domestic output and the dispersion of domestic relative prices, with the latter reflecting distortions in the use of labor across domestic firms. However, when these conditions do not hold, consumption will not move in proportion to domestic output, and the quadratic approximation to welfare will depend on fluctuations of the terms of trade around the efficient level.

For further discussion, see Obstfeld and Rogoff (2002), Corsetti and Presenti (2005), Benigno and Benigno (2003), and Kirsanova, Leith, and Wren-Lewis (2006).

### Policy Coordination

Monetary policy actions by one country will affect other countries, leading to spillover effects that open the possibility of gains from policy coordination. For example, (9.34) showed that the effects on the exchange rate of a change in the domestic money supply depended on  $m_t - m_t^*$ . A rise in  $m_t$  holding  $m_t^*$  fixed will produce a home country depreciation, shifting world demand toward the home country's output. If  $m_t^*$  also rises in line with  $m_t$ , then the exchange rate would be unaffected. When prices are sticky, the exchange rate channel is one way that an expansionary monetary policy affects domestic output. However, if both monetary authorities attempt to generate output expansions, this exchange rate channel will not operate, as both countries cannot depreciate relative to each other. The consequences of a policy action in one country depend on the policy actions in the other country. This dependence raises the issue of whether there are gains from coordinating monetary policy. Hamada (1976) is closely identified with early work that analyzed policy coordination, and Canzoneri and Henderson (1989) provided an extensive discussion of monetary policy coordination issues; a survey of this literature was provided by



Currie and Levine (1991).<sup>11</sup> Policy coordination is also discussed in the survey of Corsetti, Dedola, and Leduc (2010).

In the two-country NK model, the loss function (9.42) was based on the assumption that the home central bank took foreign output and inflation as exogenous. This might be appropriate when analyzing the case of a small open economy (i.e., when  $\gamma$ , the population share of the foreign country is close to 1), but in studying policy in an environment in which both countries are of similar size, the nature of the strategic interaction between the two central banks needs to be considered, and there may be gains from policy coordination.

Consider coordinated monetary policy in the two countries that is designed to maximize the welfare of households in both countries. Clarida, Galí, and Gertler (2002) argued that in this situation, the flexible-price output level defined in (9.35), which was linearized to obtain (9.40), is no longer appropriate because it takes foreign income  $Y_t^*$  as given. Instead, (9.35) and (9.36) must be solved jointly for both home and foreign output levels when prices are flexible in both economies. As is clear from these two equations, in the special case when  $\sigma = 1$  (log utility), flexible-price output in each country is independent of the level of output in the other. In this case,  $y_t^f$  is the same regardless of whether prices are flexible or sticky in the foreign country.

Let  $\tilde{x}_t$  and  $\tilde{x}_t^*$  denote the domestic and foreign output gaps defined with respect to the equilibrium when prices are flexible in both countries, Clarida, Galí, and Gertler (2002) showed that the quadratic approximation to welfare in the two countries is maximized under a cooperative monetary policy that minimizes the present discounted value of a loss function given by

$$\frac{1}{2}E_t \left[ (1 - \gamma) \left( \pi_{h,t}^2 + \lambda \tilde{x}_t^2 \right) + \gamma \left( \pi_{f,t}^* + \lambda^* \tilde{x}_t^{*2} \right) - 2\Phi \tilde{x}_t \tilde{x}_t^* \right], \quad (9.51)$$

where  $\lambda = \bar{\kappa}/\xi$ ,  $\lambda^* = \bar{\kappa}^*/\xi$ , and  $\Phi = \kappa(1 - \sigma)\gamma(1 - \gamma)/\xi$ . When  $\sigma = 1$ ,  $\Phi = 0$  and the loss function reduces to a weighted average of the loss functions for each country. In this case, there is no gain from policy coordination. As can be seen from (9.39) and (9.40), the domestic economy is insulated from foreign variables. Each individual monetary authority can simply minimize its country-specific loss function. When  $\sigma \neq 1$ , there are gains from policy cooperation, but Clarida, Galí, and Gertler (2002) showed that the first-order conditions for the jointly optimal monetary policy take the same form as in the case analyzed previously when the domestic central bank took foreign variables as given. The only difference between that case and the cooperation case is that the reference flexible-price output levels are defined by solving (9.35) and (9.36) together. Furthermore, they show

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11. Earlier editions of this book presented some of the models of policy coordination that were drawn from these earlier approaches. This material can be found at <http://people.ucsc.edu/~walshc/mt4e/>.

that when expressed in terms of the domestic output gap  $x_t$ , the first-order condition in the cooperative case becomes

$$\bar{k} \left( \pi_{h,t} + \chi \pi_{f,t}^* \right) + \lambda x_t = 0,$$

where  $\chi \equiv \gamma(\sigma - 1)/[\eta + \sigma - \gamma(\sigma - 1)]$ . This can be compared to (9.43) and shows how policymakers' actions will depend on domestic and foreign inflation rates. A similar condition characterizes optimal policy for the foreign central bank. When  $\sigma = 1$ ,  $\chi = 0$ , the domestic economy is insulated from foreign income (see 9.35), the output gaps  $x_t$  and  $\tilde{x}_t$  are equal, and the first-order conditions for optimal discretionary policy in the coordination case are exactly the same as when each country acts to maximize welfare in its own country.

Two cases have now been considered. In the first, the domestic central bank took foreign variables as given. If the foreign central bank does the same, the result is a Nash equilibrium. In the second, the jointly optimal, cooperative policy was considered. These clearly are not the only possibilities. One economy may act as a Stackelberg leader, recognizing the impact its choice has on the inflation rate set by the other economy. Reputational considerations along the lines studied in chapter 6 can also be incorporated into the analysis (see Canzoneri and Henderson 1989). Further analyses of policy coordination include those by Benigno and Benigno (2008) and Devereux and Yetman (2014); see also the survey by Corsetti, Dedola, and Leduc (2010).

### 9.3 A Model of the Small Open Economy

Most open economies are small in the sense that developments in the home country do not affect the rest of the world, and domestic policymakers can take foreign output, inflation, and interest rates as given. In this section, the new Keynesian model of a small open economy due to Galí and Monacelli (2005) is discussed. Here, the world economy consists of a continuum of small open economies on the unit interval. Each economy is of measure zero, and so agents and policymakers in each economy take the rest of the world as given. The economies are symmetric in the sense that all have identical preferences, technology, and market structure. There is also a complete set of internationally traded contingent claims. In what follows,  $i$  indexes a country,  $*$  denotes the world aggregate, and the absence of an index indicates the reference (home) economy.

#### 9.3.1 Households

The representative household in the small open economy maximizes

$$E_0 \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{N_{t+i}^{1+\eta}}{1+\eta} \right],$$

where  $N_t$  is labor hours and  $\sigma, \eta > 0$ .  $C_t$  is a constant elasticity of substitution (CES) composite of home and foreign goods, defined as

$$C_t = \left[ (1 - \gamma)^{\frac{1}{\theta}} (C_{h,t})^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} (C_{f,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (9.52)$$

for  $\theta > 1$ .<sup>12</sup>  $C_{h,t}$  is itself a composite of goods produced in the home country and given by

$$C_{h,t} = \left( \int_0^1 C_{h,t}(j)^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}}, \quad \xi > 1,$$

while  $C_{f,t}$  is a index of consumption goods imported from country  $i$  and given by

$$C_{f,t} = \left( \int_0^1 C_{i,t}^{\frac{\alpha-1}{\alpha}} di \right)^{\frac{\alpha}{\alpha-1}}, \quad \alpha > 1.$$

Further, the quantity of goods the home country imports from country  $i$  is a composite of differentiated goods produced in that country:

$$C_{i,t} = \left( \int_0^1 C_{i,t}(j)^{\frac{\xi-1}{\xi}} dj \right)^{\frac{\xi}{\xi-1}}.$$

Let  $P_{h,t}(h)$  and  $P_{i,t}(f)$  be the home currency prices of the individual goods produced at home and in country  $i$ . Households will demand the basket of goods that minimizes the cost of achieving any given  $C_{h,t}$  and  $C_{i,t}$ . This cost minimization problem leads to the following demand functions and definitions of the price indexes for domestic goods and imports from country  $i$ :

$$C_{h,t}(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} C_{h,t}, \quad P_{h,t} \equiv \left[ \int_0^1 P_{h,t}(h)^{1-\xi} dh \right]^{\frac{1}{1-\xi}}, \quad (9.53)$$

$$C_{i,t}(f) = \left( \frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\xi} C_{i,t}, \quad P_{i,t} \equiv \left[ \int_0^1 P_{i,t}(f)^{1-\xi} df \right]^{\frac{1}{1-\xi}}. \quad (9.54)$$

Similarly, the problems of choosing  $C_{i,t}$  to minimize the cost of purchasing  $C_{f,t}$  leads to

$$C_{i,t} = \left( \frac{P_{i,t}}{P_{f,t}} \right)^{-\alpha} C_{f,t}, \quad P_{f,t} \equiv \left[ \int_0^1 P_{i,t}^{1-\alpha} di \right]^{\frac{1}{1-\alpha}}. \quad (9.55)$$

12. For  $\theta = 1$ , one obtains the preferences used for the two-country model of Clarida, Gali, and Gertler (2002). See (9.2).

Minimizing the cost of  $C_t$  by choosing  $C_{h,t}$  and  $C_{f,t}$  implies demand functions

$$C_{h,t} = (1 - \gamma) \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} C_t, \quad (9.56)$$

$$C_{f,t} = \gamma \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} C_t. \quad (9.57)$$

The domestic CPI is defined as

$$P_t \equiv \left[ (1 - \gamma) P_{h,t}^{1-\theta} + \gamma P_{f,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (9.58)$$

Given the price index  $P_t$ , the aggregate budget constraint for the household holding a portfolio  $D_t$  can be written as

$$P_t C_t + E_t \Omega_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t,$$

where  $D_{t+1}$  is the nominal payoff in  $t + 1$  of the portfolio held at end of  $t$ , and  $\Omega_{t,t+1}$  is the stochastic discount factor. The first-order conditions for the household's choice of  $C_t$  and  $N_t$  then take the usual form

$$C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma},$$

where  $R_t$  is the gross nominal interest rate, and

$$\frac{N_t^\varphi}{C_t^{-\sigma}} = \frac{W_t}{P_t}.$$

Thus, when linearized around the nonstochastic steady state, these two equations can be written as

$$c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - \rho), \quad (9.59)$$

$$\varphi n_t + \sigma c_t = w_t - p_t, \quad (9.60)$$

where  $i_t$  is the one-period nominal interest rate,  $\rho = \beta^{-1} - 1$ , and  $\pi_t = p_t - p_{t-1}$  where  $p_t = \log P_t$ . The log CPI can be approximated when  $\theta \neq 1$  as

$$p_t = (1 - \gamma) p_{h,t} + \gamma p_{f,t}.$$

Using the definitions of the various price indexes, the bilateral terms of trade between the home country and country  $i$  can be defined as

$$S_{i,t} = \frac{P_{i,t}}{P_{h,t}}. \quad (9.61)$$

A rise in  $S_{i,t}$  means imports from country  $i$  have become more expensive relative to the goods produced (and exported) by the domestic economy. The effective terms of trade with respect to all trading partners are

$$S_t = \frac{P_{f,t}}{P_{h,t}} = \left( \int_0^1 S_{i,t}^{1-\alpha} di \right)^{\frac{1}{1-\alpha}}.$$

To a first-order log-linear approximation,

$$s_t = p_{f,t} - p_{h,t} = \int_0^1 s_{i,t} di, \quad (9.62)$$

$$p_t = (1 - \gamma) p_{h,t} + \gamma p_{f,t} = p_{h,t} + \gamma s_t. \quad (9.63)$$

Let  $\mathcal{E}_{i,t}$  be the bilateral exchange rate with country  $i$ , defined as the domestic currency price of country  $i$ 's currency. Galí and Monacelli assumed the law of one price holds, and this implies

$$P_{i,t}(f) = \mathcal{E}_{i,t} P_{i,t}^i(f),$$

for all  $i$  and  $j$ , where  $P_{i,t}(f)$  is the price in the home country of good  $f$  imported from country  $i$ , while  $P_{i,t}^i(f)$  is the price of the same good in its country of origin's currency. Using the law of one price and the definitions of  $P_{i,t}$  and  $P_{f,t}$ ,

$$P_{f,t} = \left[ \int_0^1 (\mathcal{E}_{i,t} P_{i,t}^i)^{1-\alpha} di \right]^{\frac{1}{1-\alpha}}.$$

Letting  $p_{i,t}^i$  and  $e_{i,t}$  denote the log domestic price level in country  $i$  and the log nominal exchange rate with country  $i$ , log-linearizing  $P_{f,t}$  around the symmetric steady state yields

$$p_{f,t} = e_t + p_t^*, \quad (9.64)$$

where  $e_t = \int e_{i,t} di$  is the log effective exchange rate and  $p_t^* = \int p_{i,t}^i di$ .

While  $e_t$  is the nominal effective exchange rate, the real effective exchange rate provides a measure of the price of imported goods relative to the domestic consumer price index. The bilateral real exchange rate with country  $i$  is  $Q_{i,t} \equiv \mathcal{E}_{i,t} P_{i,t}^i / P_t$ ; in log terms,  $q_{i,t} = e_{i,t} + p_{i,t}^i - p_t$ . Aggregating over all trading partners, the log effective real exchange rate is

$$q_t = e_t + p_t^* - p_t.$$

The log terms of trade can be written as

$$s_t = e_t + p_t^* - p_{h,t}, \quad (9.65)$$

implying

$$q_t = (1 - \gamma) s_t, \quad (9.66)$$

where use has been made of the fact that  $p_t = (1 - \gamma)p_{h,t} + \gamma p_{f,t} = p_{h,t} + \gamma s_t$ . Finally, one can also link domestic CPI inflation to domestic producer price inflation and the change in the terms of trade:

$$\pi_t = \pi_{h,t} + \gamma \Delta s_t, \quad (9.67)$$

where  $\Delta$  is the first difference operator.

### 9.3.2 International Risk Sharing and Uncovered Interest Parity

As in the model of Clarida, Galí, and Gertler (2002) (see section 9.2), it is assumed that there exists a complete set of internationally traded contingent claims. This allows for international risk sharing. The main difference is that from the perspective of a resident of country  $i$ , instead of the expectation of (9.15), one has

$$(C_t^i)^{-\sigma} = \beta R_t E_t \left( \frac{\mathcal{E}_{i,t} P_t^i}{\mathcal{E}_{i,t+1} P_{t+1}^i} \right) (C_{t+1}^i)^{-\sigma},$$

which takes into account that there is a bilateral exchange rate between the home country and each of the other countries. Using the definition of the bilateral real exchange rate, this becomes

$$(C_t^i)^{-\sigma} = \beta R_t E_t \left( \frac{Q_{i,t} P_t}{Q_{i,t+1} P_{t+1}} \right) (C_{t+1}^i)^{-\sigma}. \quad (9.68)$$

Hence,

$$\beta E_t \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} = \frac{1}{R_t} = \beta E_t \left( \frac{Q_{i,t} P_t}{Q_{i,t+1} P_{t+1}} \right) \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma}.$$

With a complete set of state-contingent securities, this holds not just in expectations but for every possible state realization. For this to be the case,  $C_t^\sigma$  and  $Q_{i,t} (C_t^i)^\sigma$  must move proportionately, or  $C_t = v_i Q_{i,t}^{\frac{1}{\sigma}} C_t^i$  for some constant  $v_i > 0$ . Assuming an initial symmetric equilibrium with zero net asset positions and ex ante identical environments,  $v_i$  can be normalized to 1 for all  $i$ . In this case,

$$C_t = Q_{i,t}^{\frac{1}{\sigma}} C_t^i.$$

Taking logs and integrating over  $i$  yields

$$c_t = \left( \frac{1}{\sigma} \right) q_t + c_t^* = \left( \frac{1 - \gamma}{\sigma} \right) s_t + c_t^*. \quad (9.69)$$

Equation (9.69) can be compared to (9.17), the corresponding international risk-sharing condition in the two-country model of section 9.2. In that model, preferences were assumed to be identical in the two countries. With the law of one price holding for all goods, the

price levels in the two countries were identical. Hence, the real exchange rate was equal to 1. In the present model, preferences differ, and the consumer price indexes differ across countries in how they combine the prices of domestically produced and imported final goods unless  $\gamma = 1$  (see 9.58). Suppose  $q_t$  rises, implying the home country CPI has fallen relative to the effective CPI in the rest of the world. In this case, an efficient response is to shift world consumption away from where it is expensive and toward countries where it is relatively less expensive. Hence,  $c_t$  rises relative to  $c_t^*$ .

A relationship similar to the home country Euler equation (9.59) also holds in country  $i$ , so

$$c_t^i = E_t c_{t+1}^i - \left( \frac{1}{\sigma} \right) (i_t^i - E_t \pi_{t+1}^i - \rho).$$

Let  $r_t = i_t - E_t \pi_{t+1}$  and  $r_t^i = i_t^i - E_t \pi_{t+1}^i$  denote the real interest rates at home and in country  $i$ . Then the linearized Euler condition for country  $i$  can be subtracted from the home country Euler condition to obtain

$$c_t - c_t^i = E_t (c_{t+1} - c_{t+1}^i) - \left( \frac{1}{\sigma} \right) (r_t - r_t^i).$$

From the risk-sharing condition,  $c_t = (1/\sigma) q_t^i + c_t^i$ , implying

$$r_t = r_t^i + E_t (q_{t+1}^i - q_t). \quad (9.70)$$

This is the uncovered interest parity condition expressed in real terms; the real interest rate in the home country is equal to the real interest rate in country  $i$  plus the expected change in the real bilateral exchange rate. Using the definition of the real exchange rate, this interest parity condition expressed in nominal terms becomes

$$i_t = i_t^i + E_t (e_{t+1}^i - e_t^i).$$

If the domestic nominal rate is above the nominal rate in country  $i$ , then agents must expect the domestic currency to depreciate ( $e^i$  to rise). It is this loss in the value of the domestic economy's currency that ensures expected returns at home and in country  $i$  are equal. Aggregating over all  $i$ ,

$$i_t = i_t^* + E_t (e_{t+1} - e_t). \quad (9.71)$$

### 9.3.3 Domestic Firms

The analysis of domestic firms parallels the approach followed in chapter 8.

In each period, there is a fixed probability  $1 - \omega$  that the firm can adjust its price. When it can adjust, it does so to maximize the expected discounted value of profits. Each domestic firm faces the identical production function,

$$Y_t^h(h) = A_t N_t(h),$$

where  $N_t(h)$  is employment at firm  $j$  and  $A_t$  is a common domestic productivity shock. The firm also faces a constant elasticity demand curve for its output. The firm's real marginal cost is equal to

$$MC_t = \frac{W_t/P_t^h}{A_t},$$

where  $W_t/P_t^h$  is the real product wage (the same for all firms as they hire from a common labor market), and  $A_t$  is the marginal product of labor. In terms of percentage deviations around the steady state, this expression becomes

$$mc_t = w_t - p_t^h - a_t. \quad (9.72)$$

Households who supply labor care about the real wage in terms of the CPI, and

$$w_t - p_t = w_t - p_t^h - (p_t - p_t^h) = w_t - p_t^h - \gamma s_t.$$

A currency depreciation lowers the real wage from the households' perspective; to obtain the same supply of labor, the nominal wage needs to rise, increasing firms' marginal costs.

Following the derivation of chapter 8, the inflation rate for the price index of domestically produced goods is

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa mc_t, \quad (9.73)$$

where  $\kappa = (1 - \omega)(1 - \beta\omega)/\omega$ .

Because the model assumes sticky prices but flexible wages, the real wage expressed in terms of the CPI,  $w_t - p_t$ , is from (9.60) equal to  $\varphi n_t + \sigma c_t$ . In the closed economy, to first-order  $n_t = y_t - a_t$  and goods market clearing implied  $c_t = y_t$ . These relationships allowed the real wage to be expressed in terms of output as  $(\varphi + \sigma)y_t - \varphi a_t$ . In the open economy, however,  $c_t$  consists of home-produced goods and foreign-produced goods, and some of domestic output is sold abroad. Thus,  $c_t$  and  $y_t$  can differ. To obtain the relationship that will hold between  $c_t$  and  $y_t$ , the model's equilibrium conditions need to be used.

### 9.3.4 Equilibrium Conditions

Goods market clearing in the home country requires, for each  $h$ , that

$$Y_t(h) = C_{h,t}(h) + \int_0^1 C_{h,t}^i(h) di,$$

where  $C_{h,t}^i(h)$  is the foreign demand for home-produced good  $h$ . Using the demand relationships (9.53) and (9.54), together with (9.56),

$$Y_t(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} \left[ (1 - \gamma) \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} C_t + \gamma FD_t \right], \quad (9.74)$$



where, because preferences are taken to be symmetric, foreign demand  $FD_t$  is given by

$$FD_t \equiv \int_0^1 \left( \frac{P_{h,t}}{\mathcal{E}_{i,t} P_{f,t}^i} \right)^{-\alpha} \left( \frac{P_{f,t}^i}{P_t^i} \right)^{-\theta} C_t^i di. \quad (9.75)$$

Define aggregate output as

$$Y_t \equiv \left[ \int_0^1 Y_t(h)^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}}.$$

Using (9.74),

$$Y_t = \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} \left[ (1-\gamma)C_t + \gamma \int_0^1 \left( \frac{\mathcal{E}_{i,t} P_{f,t}^i}{P_{h,t}} \right)^{\alpha-\theta} Q_{i,t}^\theta C_t^i di \right].$$

But international risk sharing implied  $C_t^i = Q_{i,t}^{-\frac{1}{\sigma}} C_t$ , and  $\mathcal{E}_{i,t} P_{f,t}^i / P_{h,t}$  can be expressed as  $(\mathcal{E}_{i,t} P_{f,t}^i / P_{i,t}^i)(P_{i,t}^i / P_{h,t}) = (\mathcal{E}_{i,t} P_{f,t}^i / P_{i,t}^i) S_{i,t} = S_t^i S_{i,t}$ , where the bilateral terms of trade variable  $S_{i,t}$  was defined in (9.61), and  $S_t^i$  represents the effective terms of trade of country  $i$ . Therefore,

$$Y_t = \left( \frac{P_{h,t}}{P_t} \right)^{-\theta} \left[ (1-\gamma) + \gamma \int_0^1 (S_t^i S_{i,t})^{\alpha-\theta} Q_{i,t}^{\theta-\frac{1}{\sigma}} di \right] C_t. \quad (9.76)$$

When this goods-clearing condition is log-linearized around the steady state, one obtains

$$y_t = c_t + \gamma \alpha s_t + \frac{\gamma}{\sigma} (\sigma \theta - 1) q_t = c_t + \left( \frac{\gamma \phi}{\sigma} \right) s_t, \quad (9.77)$$

where  $\phi \equiv \sigma \alpha + (1-\gamma)(\sigma \theta - 1)$ . Using this in the Euler equation (9.59),

$$y_t = E_t y_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - \rho) - \left( \frac{\gamma \phi}{\sigma} \right) E_t (s_{t+1} - s_t).$$

The final step in this derivation is to note that for the entire global economy,  $y_t^* = c_t^*$ , and the consumption risk-sharing relationship can be used to obtain

$$s_t = \left( \frac{\sigma}{1-\gamma+\gamma\phi} \right) (y_t - y_t^*). \quad (9.78)$$

Using (9.67) to eliminate  $\pi_{t+1}$ , one then obtains

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\gamma} (i_t - E_t \pi_{h,t+1} - \rho_t), \quad (9.79)$$

where  $\sigma_\gamma \equiv \sigma / (1-\gamma+\gamma\phi)$  and  $\rho_t \equiv \rho + \sigma_\gamma \gamma (\phi - 1) E_t (y_{t+1}^* - y_t^*)$ .

The parameter  $\phi$  may be greater than, equal to, or less than 1, implying the impact of the terms of trade on domestic output is ambiguous. Suppose production in the rest of

the world increases, causing (from the home country's perspective) a fall in the prices of imported goods and a fall in the terms of trade. This causes expenditure switching, as domestic and foreign residents substitute away from the now relatively more expensive domestic goods. This channel causes a fall in aggregate demand in the home country and is why  $\phi$  is increasing in  $\theta$ , the elasticity of substitution between home and foreign goods. This is not the only channel, however, through which the rise in output in the rest of the world affects domestic demand. An income rise abroad increases the demand for domestic output and through (9.69) also increases domestic consumption. The effect is increasing in  $\gamma$ , the expenditure share of foreign consumption devoted to goods produced in the domestic country. Both these factors increase domestic aggregate demand. It is commonly assumed in calibration exercises that  $\sigma = \theta = \alpha = 1$ , in which case  $\phi = 1$ . In this special case, the effects of expected world output growth drop out of the definition of  $\rho_t$  in (9.79).

Recall from (9.73) that inflation in domestic goods prices depended on real marginal cost. Using the previous results on the relationship between consumption and output, real marginal cost for domestic firms is

$$\begin{aligned} mc_t &= (w_t - p_{h,t}) - a_t = \sigma c_t + \eta n_t + \gamma s_t - a_t \\ &= \sigma \left[ y^* + \left( \frac{1 - \gamma}{\sigma} \right) s_t \right] + \eta (y_t - a_t) + \gamma s_t - a_t \\ &= \sigma y^* + \eta y_t + s_t - (1 + \eta) a_t. \end{aligned}$$

Increases in the terms of trade or world output increase domestic real marginal cost via a wealth effect on domestic consumption that reduces labor supply and so increases real wages. From (9.78), and recalling that  $\sigma_\gamma \equiv \sigma / (1 - \gamma + \gamma\phi)$ ,

$$mc_t = (\eta + \sigma_\gamma) y_t + \gamma \sigma_\gamma (\phi - 1) y^* - (1 + \eta) a_t.$$

This expression allows the domestic price inflation equation to be written as

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa (\sigma_\gamma + \eta) (y_t - \tilde{y}_t),$$

where

$$\tilde{y}_t \equiv \left( \frac{1 + \eta}{\sigma_\gamma + \eta} \right) a_t - \gamma \sigma_\gamma \left( \frac{\phi - 1}{\sigma_\gamma + \eta} \right) y_t^*.$$

Thus, we have written domestic inflation in terms of an output gap.<sup>13</sup>

In open economies that target inflation, the target is always set in terms of CPI inflation,  $\pi_t$ . From (9.67),

$$\pi_t = \pi_{h,t} + \gamma \Delta s_t = \pi_{h,t} + \gamma \sigma_\gamma (\Delta y_t - \Delta y_t^*).$$

13. Note that in the closed economy,  $\gamma = 0$ , and so  $\sigma = \sigma_\gamma$  and  $\tilde{y}_t = [(1 + \eta) / (\sigma + \eta)] a_t$ , which was equal to flexible-price equilibrium output. See chapter 8.

### 9.3.5 Monetary Policy in the Linear Model

The equilibrium conditions for the linearized model involving output, flexible-price output, the output gap, CPI inflation, domestic produce price inflation, and the terms of trade can be collected in the form of the following six equations:<sup>14</sup>

$$y_t = \tilde{y}_t + x_t,$$

$$\tilde{y}_t = \left( \frac{1 + \eta}{\sigma_\gamma + \eta} \right) a_t - \gamma \sigma_\gamma \left( \frac{\phi - 1}{\sigma_\gamma + \eta} \right) y_t^*, \quad (9.80)$$

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma_\gamma} \right) (i_t - E_t \pi_{h,t+1} - \tilde{\rho}_t), \quad (9.81)$$

$$\pi_t = \pi_{h,t} + \gamma \Delta s_t,$$

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa (\sigma_\gamma + \eta) x_t, \quad (9.82)$$

$$s_t = \sigma_\gamma (y_t - y_t^*). \quad (9.83)$$

The equilibrium real interest rate consistent with a zero output gap is defined as

$$\tilde{\rho}_t = \rho_t = \rho + \sigma_\gamma (E_t \tilde{y}_{t+1} - \tilde{y}_t) \quad (9.84)$$

$$= \rho + \sigma_\gamma \alpha (\phi - 1) (E_t y_{t+1}^* - y_t^*) + \sigma_\gamma (E_t \tilde{y}_{t+1} - \tilde{y}_t).$$

To close the model, a specification for monetary policy must be added. In keeping with the analysis earlier in this chapter and in chapter 8, policy is represented either as an instrument rule for the nominal interest rate or as a loss function that the monetary authority attempts to minimize. In an open economy, monetary policy may also be oriented toward using the nominal exchange rate as a policy instrument. To incorporate this possibility, or the case in which the nominal interest rate is the policy instrument but the central bank reacts to exchange rate movements, the following two equations must be added to the previous six equations:

$$p_{h,t} = p_{h,t-1} + \pi_{h,t},$$

$$e_t = s_t + p_{h,t} - p_t^*.$$

Notice that the nominal exchange rate does not appear in the other equilibrium conditions. Thus, it may appear that movements in the nominal exchange rate do not affect domestic output or inflation. However, exchange rate effects on aggregate demand are present; they are summarized in the coefficient  $\sigma_\gamma$ . In the closed economy, the elasticity of aggregate

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14. Galí and Monacelli (2005) had an additional term in their definition of  $\tilde{y}_t$  to reflect the role of the average markup and a fiscal tax on firm revenue. If the fiscal tax (actually, a subsidy) is used to offset the average markup so that the flexible output corresponds to the efficient level of output, this additional term is zero, as is assumed here.

demand with respect to the interest rate is  $1/\sigma$ . In an open economy, changes in  $i_t$  affect the nominal exchange rate consistent with uncovered interest parity and, when prices are sticky, also affect the relative prices of foreign and domestic goods. A rise in  $i_t$ , holding  $E_t e_{t+1}$  constant, leads to a fall in  $e_t$ , that is, an appreciation. This causes domestic households to substitute toward the now cheaper foreign goods; this is one reason the elasticity of aggregate demand with respect to the interest rate in the open economy is  $1/\sigma_\gamma$ , not  $1/\sigma$ .

Galí and Monacelli (2005) consider the special case in which  $\theta = \sigma = \alpha = 1$ . They showed, consistent with the discussion in section 9.2.5, these parameter variables, combined with the appropriate fiscal subsidies to eliminate the steady-state distortions arising from monopolistic competition and the incentive to engage in a competitive depreciation, imply the quadratic approximation to the welfare of the representative domestic household involves the volatility of domestic price inflation and the output gap. That is, the domestic central bank should minimize a quadratic loss function given by

$$L_t = - \left( \frac{1}{2} \right) \sum_{i=0}^{\infty} \beta^{t+i} \left[ \pi_{h,t+i}^2 + \frac{\kappa(1+\eta)}{\xi} x_{t+i}^2 \right], \quad (9.85)$$

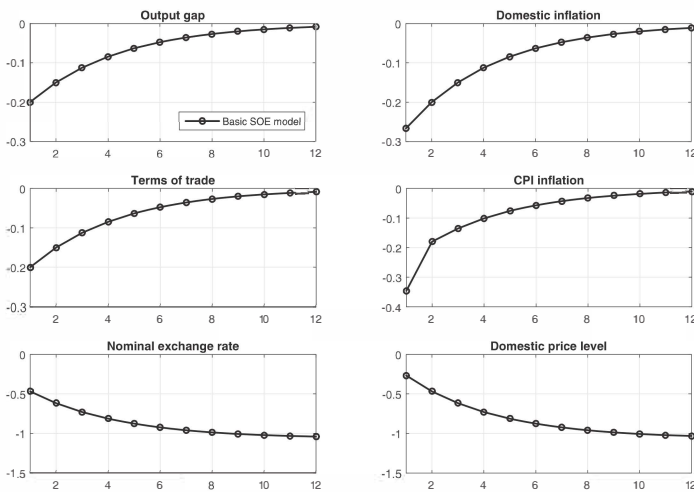
which is the same form as (9.42) obtained for the two-country model.

Suppose the monetary authority wishes to minimize the loss function  $L_t$  defined in (9.85). Then, in terms of the linearized version of the model, the only two equations that are relevant for the policymaker are (9.81) and (9.82). As seen earlier in the two-country model of Clarida, Galí, and Gertler (2002), the model's structure is exactly parallel to that of the closed-economy NK model studied in chapter 8, and the basic conclusions about monetary policy discussed there also apply to this small open economy. For example, interest rate rules have to satisfy the Taylor principle to ensure there is a locally unique stationary rational-expectations equilibrium.

To illustrate the manner in which a monetary policy shock affects the small open economy, consider a policy rule of the form

$$\dot{i}_t = \rho + \phi_{\pi_h} \pi_{h,t} + \phi_{\pi} \pi_t + \phi_e e_t + v_t, \quad (9.86)$$

where  $v_t$  is the policy shock and  $\phi_s$  is the policy response coefficient for  $s \in \{\pi_{h,t}, \pi_t, e_t\}$ . The policy shock is taken to be AR(1) and fairly persistent:  $v_t = 0.75v_{t-1} + e_{v,t}$ . The other parameters are calibrated to be consistent with Galí and Monacelli (2005). They set  $\beta = 0.99$ ,  $\omega = 0.75$ , implying that each period only 25 percent of firms adjust prices, the elasticity of demand for individual goods at  $\xi = 6$ , and  $\gamma = 0.4$ . They also set  $\theta = \alpha = \sigma = 1$ , yielding the case discussed in section 9.2.5, in which the quadratic loss function derived from an approximation to welfare involves the output gap and domestic price inflation. To ensure determinacy,  $\phi_{\pi_h} = 1.5$ , while the other policy parameters are initially set to zero:  $\phi_{\pi} = \phi_e = 0$ .


**Figure 9.3**

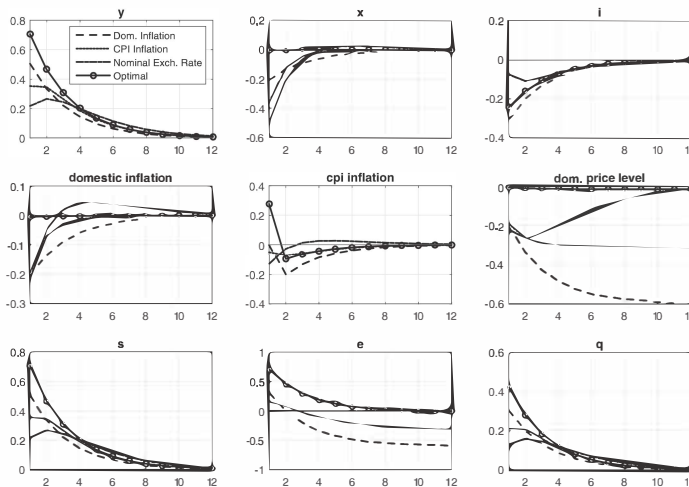
Responses to a monetary policy shock when policy is  $i_t = \rho + \phi_{\pi_h} \pi_{h,t} + v_t$  and  $v_t = 0.75v_{t-1} + e_{v,t}$ .

Figure 9.3 illustrates the effects of a contractionary interest rate shock. The upper panel of the figure shows that the output gap and domestic price inflation fall, just as they would do in a basic closed-economy NK model, as (9.81) and (9.82) are identical in form to their closed-economy counterparts. The middle panel shows that the terms of trade decline as domestic output falls (see 9.83). The associated real appreciation resulting from the contractionary monetary shock (recall  $q_t = (1 - \gamma)s_t$ ) causes foreign prices in the domestic currency to fall, decreasing the CPI, so  $\pi_t$  declines more than  $\pi_{h,t}$ . The lower panel of the figure shows the responses of the nominal exchange rate and the domestic price level. Both fall and converge to new, permanently lower levels. The lower price level is a reflection of the fall in domestic price inflation; while  $\pi_{h,t}$  returns to its steady-state value, the periods of negative inflation are not offset by positive inflation, so the price level remains below its initial level. The same is true of the nominal exchange rate. However, the terms of trade and the real exchange rate initially fall but then rise over time to return to their initial levels because they are relative prices that are not permanently affected by monetary policy.

Galí and Monacelli (2005) compared an optimal policy with simple policy rules involving either domestic price inflation  $\pi_{h,t}$  or CPI inflation  $\pi_t$ , or that implement a nominal exchange rate peg. These are all special cases of the policy rule (9.86). To represent inflation-targeting regimes, the rule used to construct figure 9.3 set  $\phi_{\pi_h} = 1.5$ ,  $\phi_{\pi} = 0$ , and  $\phi_e = 0$ . For CPI inflation targeting, set  $\phi_{\pi_h} = 0$ ,  $\phi_{\pi} = 1.5$ , and  $\phi_e = 0$ . For large values of  $\phi_{\pi_h}$ , the policy rule effectively stabilizes  $\pi_h$ , mimicking the optimal policy, while with a large value of  $\phi_e$ , the policy rule stabilizes the nominal exchange rate.

Now consider the economy's response to a domestic productivity shock under different policies. From (9.80), a positive innovation to  $a_t$  increases  $\tilde{y}_t$ . The responses of other variables depend on the specification of monetary policy. Results for four alternative policy rules are shown in figure 9.4. The optimal policy maintains a zero output gap and zero domestic inflation. Thus,  $y_t$  moves one-to-one with  $\tilde{y}_t$ , and from (9.83) the terms of trade increase. To ensure aggregate demand increases with the rise in productivity, the nominal interest rate is reduced under the optimal policy. This represents a real depreciation, and with domestic prices constant, the nominal exchange rate rises. The impulse responses of  $s$ ,  $e$ ,  $q$  are illustrated in the lower panel of figure 9.4. The depreciation results in a rise in CPI inflation as the price of foreign goods increases. Under the optimal policy, the domestic price level and the nominal exchange rate eventually return to their initial levels.

The dashed lines and the dotted lines in the figure correspond to policies that respond to domestic price inflation ( $\phi_{\pi_h} = 1.5$ ,  $\phi_{\pi} = 0$ ,  $\phi_e = 0$ ) and CPI inflation ( $\phi_{\pi_h} = 0$ ,  $\phi_{\pi} = 1.5$ ,  $\phi_e = 0$ ), respectively. The productivity increase immediately increases flexible-price output. Because flexible-price output is expected to return to its steady-state value,  $\tilde{\rho}_t$  falls (see 9.84). Neither the domestic inflation rule nor the CPI inflation rule respond directly to  $\tilde{\rho}_t$ , so neither rule reduces  $i_t$  enough to prevent a negative output gap. The nominal interest rate falls less when policy focuses on CPI inflation to limit the depreciation that pushes up  $\pi_t$ . As a consequence of this less expansionary policy, output does not rise by the full amount that  $\tilde{y}_t$  has increased, implying the output gap becomes negative. The negative



**Figure 9.4**

Responses to a domestic productivity shock under four policies: optimal policy (*solid lines with circles*), domestic price inflation targeting,  $\phi_{\pi_h} = 1.5$  (*dashed lines*), CPI inflation targeting,  $\phi_{\pi} = 1.5$  (*dotted lines*), outcomes with a nominal exchange rate peg (*dot-dashed lines*).

output gap pushes down domestic price inflation. Finally, an exchange rate peg produces the largest fall in the output gap. The uncovered interest parity condition (9.71) implies that pegging the exchange rate requires that the nominal interest rate move one-to-one with the foreign rate  $i_t^*$ . Since the latter is not affected by the domestic productivity shock,  $i_t$  remains constant, as shown in figure 9.4. Thus, an exchange rate peg produces the largest fall in the output gap and domestic price inflation.

In their comparison of these alternative policies, Galí and Monacelli (2005) concluded that the responses to domestic productivity under an exchange rate peg are similar to those obtained under the policy that reacts to CPI inflation. Both these policies generate negative output gaps to counteract the real depreciation caused by the productivity increase. A peg, unlike either policy rule reacting to inflation rates, generates a stationary domestic price level.<sup>15</sup> The optimal policy also leads to a stationary domestic price level, but it allows much larger movements in the terms of trade and the nominal exchange rate than any of the other policies. The terms of trade move the least under the exchange rate peg, but by using the quadratic approximation to welfare, Galí and Monacelli (2005) showed that the peg yields the largest welfare losses.

## 9.4 Additional Sources of Nominal Distortions

The two-country model and the small open-economy model (sections 9.2 and 9.3) involved only a single nominal distortion—sticky prices. In fact, the isomorphism between the open and closed versions of the basic new Keynesian model is a consequence of incorporating only a single nominal rigidity. As discussed in section 8.5.1 in the context of a closed economy, specifying additional nominal distortions such as sticky wages will force the monetary policy authority to make additional trade-offs. For example, a policy of stabilizing prices will allow the economy to adjust efficiently to productivity shocks when wages are flexible but not when they are sticky. In this section, three sources of nominal distortions that arise in the open-economy case are considered: imperfect pass-through, local currency pricing, and sticky prices in both tradeable and nontradeable goods-producing sectors.

### 9.4.1 Imperfect Pass-Through

In sections 9.2 and 9.3, the domestic currency price of foreign goods was assumed to equal the exchange rate times the foreign price of the good. This reflected the assumption of complete pass-through. Given the exchange rate, a change in the foreign currency price of an imported good translated one-to-one into a change in the domestic currency price of the good. Similarly, the home country prices of foreign-produced goods moved one-to-one with the nominal exchange rate. Empirical evidence suggests that imperfect pass-through

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15. See right most graph of middle panel in figure 9.4.

is more common. This could arise because changes in foreign prices are not immediately passed through to domestic currency prices or because foreign producers adjust their prices to offset partially movements in the exchange rate.

Corsetti and Presenti (2002), Monacelli (2005), and Adolfson (2007) provided examples of models that allow for incomplete pass-through. When pass-through is incomplete, the law of one price no longer holds. When the law of one price does hold, the domestic currency price of foreign goods,  $p_{f,t}$ , is equal to  $e_t + p_t^*$ , where  $e_t$  is the nominal exchange rate and  $p_t^*$  is the foreign currency price of foreign goods (all expressed as percentage deviations from their steady-state values).<sup>16</sup> The terms of trade are then equal to  $e_t + p_t^* - p_{h,t}$ . With incomplete pass-through, however,  $p_{f,t}$  and  $e_t + p_t^*$  can differ. Define the deviation from the law of one price as

$$\psi_t \equiv (e_t + p_t^*) - p_{f,t}.$$

In the models of the previous sections,  $\psi_t$  was identically equal to zero. Using (9.62), the terms of trade can be expressed as

$$s_t = p_{f,t} - p_{h,t} = (e_t + p_t^* - \psi_t) - p_{h,t}.$$

This equation allows the real exchange rate  $q_t$  to be written,<sup>17</sup> using the notation of section 9.3, as

$$q_t = e_t + p_t^* - p_t = s_t + \psi_t + p_{h,t} - p_t = (1 - \gamma)s_t + \psi_t.$$

When this expression for  $q_t$  is used in the risk-sharing condition (9.69), one obtains

$$c_t = c_t^* + \left(\frac{1 - \gamma}{\sigma}\right) s_t + \left(\frac{1}{\sigma}\right) \psi_t. \quad (9.87)$$

Following the steps that lead to (9.77) as the goods-clearing condition but noting that  $q_t = (1 - \gamma)s_t + \psi_t$  with imperfect pass-through, Monacelli (2005) showed that

$$y_t = c_t + \left(\frac{\gamma\phi}{\sigma}\right) s_t + \frac{\gamma}{\sigma} (\sigma\theta - 1) \psi_t,$$

where  $\phi \equiv \sigma\alpha + (1 - \gamma)(\sigma\theta - 1)$ . Combining these last two equations and using the global goods-clearing condition  $c_t^* = y_t^*$ ,

$$y_t - y_t^* = \left(\frac{1}{\sigma_\gamma}\right) s_t + \left(\frac{1}{\sigma_\psi}\right) \psi_t, \quad (9.88)$$

16. While prices are expressed here in terms of price indexes, the assumption of complete pass-through and the law of one price would apply at the level of individual prices:  $p_{i,t}(j) = e_{i,t} + p_{i,t}^j(j)$ , where  $p_{i,t}(j)$  is the home price of good  $j$  imported from country  $i$ ,  $p_{i,t}^j(j)$  is the price of the same good in country  $i$ , and  $e_{i,t}$  is the bilateral nominal exchange rate with country  $i$ .

17. This uses (9.63), which defines the consumer price index as  $p_t = (1 - \gamma)p_{h,t} + \gamma p_{f,t} = p_{h,t} + \gamma s_t$ .



where  $\sigma_\gamma \equiv \sigma / (1 - \gamma + \gamma\phi)$  as before and  $\sigma_\psi \equiv \sigma / [1 + \gamma(\sigma\theta - 1)]$ . These results can be used to derive an expression for the real marginal cost variable that will be the driver for domestic price inflation.<sup>18</sup>

Assume domestic firms face a Calvo process for adjusting prices, as in the model of section 9.3. Real marginal cost, expressed as a percent deviation around the steady state, is  $mc_t = w_t - p_t + \gamma s_t - a_t$ , while the real wage from the household's perspective is set equal to the marginal rate of substitution between leisure and consumption. Using the international risk-sharing condition and the aggregate production function, which takes the form  $y_t = n_t + a_t$ , one obtains

$$w_t - p_t = \sigma c_t + \eta n_t = \sigma y_t^* + (1 - \gamma) s_t + \psi_t + \eta (y_t - a_t).$$

Finally, use (9.88) to obtain

$$mc_t = (\sigma_\gamma + \eta) x_t + \left(1 - \frac{\sigma_\gamma}{\sigma_\psi}\right) \psi_t,$$

where  $x_t = y_t - \tilde{y}_t$ , and

$$\tilde{y}_t \equiv \left(\frac{1 + \eta}{\sigma_\gamma + \eta}\right) a_t - \gamma \sigma_\gamma \left(\frac{\phi - 1}{\sigma_\gamma + \eta}\right) y_t^*.$$

The output measure  $\tilde{y}_t$  is the economy's equilibrium output when domestic prices are flexible *and* the law of one price holds.

The impact of deviations from the law of one price on real marginal cost is

$$\left(1 - \frac{\sigma_\gamma}{\sigma_\psi}\right) = -\gamma \left[\frac{(1 - \sigma\alpha) + \gamma(\sigma\theta - 1)}{1 - \gamma + \gamma\phi}\right].$$

As noted previously, it is common to assume  $\sigma = \theta = \alpha = 1$ , in which case  $\phi = 1$  and  $\sigma\theta = 1$ . Under these conditions,  $\psi_t$  does not affect real marginal costs. In general, however, domestic price inflation is given by

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa (\sigma_\gamma + \eta) x_t + \kappa \left(1 - \frac{\sigma_\gamma}{\sigma_\psi}\right) \psi_t.$$

If  $1 - \sigma_\gamma/\sigma_\psi \neq 0$ ,  $\psi_t$  acts like a cost or markup shock in the inflation adjustment equation, leading to the types of policy trade-offs between stabilizing  $\pi_{h,t}$  and stabilizing  $x_t$ . However, unlike an exogenous shock,  $\psi_t$  is a function of endogenous variables whose evolution needs to be explained.

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18. Kirsanova, Leith, and Wren-Lewis (2006) and Wren-Lewis and Leith (2006) showed that stochastic shocks to the uncovered interest parity condition lead to an international risk-sharing condition of the form

$$c_t = c_t^* + \left(\frac{1 - \gamma}{\sigma}\right) s_t + \zeta_t,$$

where  $\zeta_t$  is the shock to the UIP condition, which can be compared to (9.87). Shocks to UIP are common additions to empirical open-economy DSGE models, e.g., Adolfson et al. (2007; 2008).

In the model of Monacelli (2005), pass-through is incomplete because of nominal rigidity in the price of imports, with only a fraction of importers adjusting their price each period as in a standard Calvo-type model of price adjustment. These importing firms purchase foreign goods at the price  $e_t + p_{f,t}^*$  (in domestic currency) and sell them in the domestic market at the price  $p_{f,t}$ . Thus,  $\psi_t = e_t + p_{f,t}^* - p_{f,t}$  represents the marginal cost of importers, and the rate of inflation in the average domestic currency price of foreign imports takes the form

$$\pi_{f,t} = \beta E_t \pi_{f,t+1} + \kappa^f \psi_t,$$

where  $\pi_{f,t} = p_{f,t} - p_{f,t-1}$  and the parameter  $\kappa^f$  depends on the fraction of import prices that adjust each period.

Imperfect pass-through represents a second nominal rigidity when combined with the assumption of sticky domestic producer prices. Not surprisingly, therefore, it introduces policy trade-offs, much as the addition of sticky wages did in the basic new Keynesian model of chapter 8. Both the output gap and deviations from the law of one price affect real marginal cost and, as a result, inflation. Stabilizing inflation in the face of a movement in  $\psi_t$  requires that the output gap be allowed to fluctuate; stabilizing the output gap in the face of a movement in  $\psi_t$  requires that inflation fluctuate. With two sources of nominal rigidity, sticky prices and imperfect pass-through, the central bank cannot undo the effects of both distortions with a single policy instrument.

### 9.4.2 Local Currency Pricing

A large literature has studied the implications of local currency pricing (LCP) versus producer currency pricing (PCP). Under local currency pricing, a domestic firm sets its price in terms of the currency of the local market to which it is exporting. Under producer currency pricing, the domestic firm sets prices in terms of its own domestic currency. The models of sections 9.2 and 9.3 assumed PCP. If the law of one price holds and firms set prices in terms of their own currency, the good produced by firm  $f$  sells for  $P_{h,t}(f)$  in its home country and sells at a price  $P_{h,t}(f)/\mathcal{E}_{i,t}$  in country  $i$ , where  $\mathcal{E}_{i,t}$  is the nominal exchange rate between country  $i$  and the domestic economy. An important consequence of this assumption is that the dispersion of demand among domestic firms is the same at home and in export markets. In this section, the focus is on domestic exporting firms who set prices in the local currency of their export markets. LCP and the relevant literature are discussed in the survey by Corsetti, Dedola, and Leduc (2010), and the welfare implications of LCP versus PCP are the focus of Engel (2011).

In the small open-economy model of section 9.3 with symmetric preferences, (9.74) and (9.75) can be used to express the demand faced by domestic firm  $h$  as

$$Y_t(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} \Phi_t,$$

where  $\Phi_t$  is independent of  $h$ . Because employment at firm  $j$  is  $N_t(h) = Y_t(h)/A_t$ , where  $A_t$  is the aggregate stochastic productivity level, aggregate employment is

$$\int N_t(h)dh = A_t^{-1} \int \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} dh \Phi_t = A_t^{-1} \Delta_{h,t} \Phi_t,$$

where  $\Delta_{h,t} \geq 1$  is the measure of price dispersion among domestic firms. It is this dispersion of relative prices that generates a welfare loss, and under the law of one price, the cross-sectional dispersion of prices in the domestic market is the same as in foreign markets.

Now suppose instead of setting one price in the domestic currency, domestic firms follow a strategy of setting prices in the local currency of their export markets. For simplicity, consider the case of a two-country model. Let the price of the output of domestic firm  $h$  in the domestic market continue to be denoted by  $P_{h,t}(h)$ , but now let  $P_{h,t}^*(h)$  denote the foreign currency price of the output of domestic firm  $h$  in the foreign economy. Let  $P_{h,t}^*$  be the price index of domestic-produced goods in the foreign country. The demand facing firm  $h$  consists of the demand from domestic households and the demand from foreign households. The former is

$$C_{h,t}(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} C_{h,t}.$$

The latter is

$$C_{h,t}^*(h) = \left( \frac{P_{h,t}^*(h)}{P_{h,t}^*} \right)^{-\xi} C_{h,t}^*.$$

Goods market clearing requires output equal total demand facing home firm  $h$ , or

$$Y_t(h) = \left( \frac{P_{h,t}(h)}{P_{h,t}} \right)^{-\xi} C_{h,t} + \left( \frac{P_{h,t}^*(h)}{P_{h,t}^*} \right)^{-\xi} C_{h,t}^*,$$

for all  $h$ . To produce  $Y_t(h)$ , firm  $h$  employs  $N_t(h) = A_t^{-1} Y_t(h)$  workers. Aggregating over all domestic firms, employment in the home country is

$$\int N_t(h)dh = A_t^{-1} \int Y_t(h)dh = A_t^{-1} [\Delta_{h,t} C_{h,t} + \Delta_{h,t}^* C_{h,t}^*],$$

where  $\Delta_{h,t}$  is the dispersion of prices at home and  $\Delta_{h,t}^*$  is a measure of the dispersion of the prices for these same goods in the foreign market. Under the law of one price, or when the domestic firms always set  $P_{h,t}^*(h) = P_{h,t}(h)/\mathcal{E}_t$ , then  $\Delta_{h,t} = \Delta_{h,t}^*$ . But if domestic firms have sticky prices and set prices in the home and foreign markets separately, then these two measures of price dispersion can differ. And each source of price dispersion generates a distortion in the use of domestic labor. Thus, LCP combined with sticky prices introduces an additional distortion that is absent under PCP.

### 9.4.3 Sticky Tradeable and Nontradeable Goods Prices

Up to this point, the discussion has focused on open-economy models in which all goods are tradeable. Suppose instead that each economy produces tradeable and nontradeable goods. If prices for both types of goods are sticky, additional sources of nominal distortions arise. These distortions were analyzed by Wren-Lewis and Leith (2006), who extended the basic Galí and Monacelli (2005) model to incorporate firms that produce traded and nontraded goods. Depending on the nature of the shocks that affect the economy, an efficient allocation may require an adjustment of the relative price of traded goods and nontraded goods. For instance, an increase in productivity in the nontraded sector would, if all prices were flexible, lead to a fall in the relative price of nontradeables. However, if prices are sticky in both the traded and nontraded sectors, this relative price no longer behaves as it would under flexible prices. This distortion is similar to the one that arises when both wages and prices are sticky. As discussed in section 8.5.1, with prices and wages sticky the real wage is unable to adjust in the face of productivity shocks in a manner consistent with efficiency. The single instrument of monetary policy cannot undo the effects of two nominal rigidities. A similar issue arises in a closed economy with two sectors that exhibit different degrees of price stickiness, a situation analyzed by Aoki (2002).

A second distortion arises because domestic labor is employed in producing nontradeables. Price dispersion leads to an inefficiency because too much labor is needed to produce a given consumption bundle relative to the case in which all firms charge the same price. This type of labor distortion now occurs with respect to the use of labor by traded goods-producing firms *and* nontraded goods-producing firms when prices are sticky in both sectors.

## 9.5 Currency Unions

In the models of sections 9.2 and 9.3, each country had its own monetary authority. The basic models of those sections can, however, also be used to analyze policy in a currency union in which several countries share a single currency and in which monetary policy decisions are taken by a single central bank. Benigno (2004) employed a two-country model to investigate monetary policy in a currency union, while Galí and Monacelli (2008) did so in the small open-economy setting of Galí and Monacelli (2005). The discussion here follows Benigno (2004).

Suppose there are two regions with a single monetary authority.<sup>19</sup> There is a continuum of households of measure 1. Households are indexed by  $j$ , and regions by  $i = H$  for the

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19. Benigno (2004) assumed each region has its own fiscal authority that purchases goods from its own region. For simplicity, fiscal purchases are ignored here. Galí and Monacelli (2008) focused on the optimal monetary and fiscal policies in a currency union.

home region and  $i = F$  for the foreign region. Assume the populations of  $H$  and  $F$  are  $1 - \gamma$  and  $\gamma$ , respectively. Thus,  $i = H$  if  $j \in [0, 1 - \gamma]$ ,  $i = F$  if  $j \in [1 - \gamma, 1]$ . The preferences of household  $j$  in region  $i$  are given by

$$U_t^j = E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{(C_{t+s}^j)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_{t+s}^j)^{1+\eta}}{1+\eta} \right],$$

where  $C_t^j$  is consumption and  $N_t^j$  is labor hours. Each household is a consumer of all goods and a producer of a differentiated product.

The consumption bundle of household  $j$  is defined as

$$C_t^j \equiv \frac{(C_{H,t}^j)^{1-\gamma} (C_{F,t}^j)^\gamma}{\gamma^\gamma (1-\gamma)^{(1-\gamma)}}, \quad \gamma \in [0, 1], \quad (9.89)$$

where

$$C_{H,t}^j \equiv \left[ \left( \frac{1}{1-\gamma} \right)^{\frac{1}{\xi}} \left( \int_0^{1-\gamma} c_t^j(h)^{\frac{\xi-1}{\xi}} dh \right) \right]^{\frac{\xi}{\xi-1}},$$

$$C_{F,t}^j \equiv \left[ \left( \frac{1}{\gamma} \right)^{\frac{1}{\xi}} \left( \int_{1-\gamma}^1 c_t^j(f)^{\frac{\xi-1}{\xi}} df \right) \right]^{\frac{\xi}{\xi-1}},$$

with  $\xi > 1$ . In this notation,  $c_t^j(h)$  is the consumption by household  $j$  of good  $h$  produced in  $H$ , and  $c_t^j(f)$  is the household's consumption of good  $f$  produced in  $F$ . The elasticity across goods within a region is  $\xi$ , and between  $C_H$  and  $C_F$  it is 1.

When the household takes individual prices as given, the solutions to the standard problems of picking the least-cost combinations of  $c^j(h)$  and  $c^j(f)$  to yield a given  $C_{H,t}^j$  and  $C_{F,t}^j$  and the least-cost combinations of  $C_{H,t}^j$  and  $C_{F,t}^j$  to yield a given  $C_t^j$  produce the definitions of the relevant price indexes. The price of the consumption bundle for region  $i$  is

$$P_t^i \equiv (P_{H,t}^i)^{1-\gamma} (P_{F,t}^i)^\gamma, \quad (9.90)$$

where

$$P_{H,t}^i \equiv \left[ \left( \frac{1}{1-\gamma} \right) \left( \int_0^{1-\gamma} p_t^i(h)^{1-\xi} dh \right) \right]^{\frac{1}{1-\xi}},$$

$$P_{F,t}^i \equiv \left[ \left( \frac{1}{\gamma} \right) \left( \int_{1-\gamma}^1 p_t^i(f)^{1-\xi} df \right) \right]^{\frac{1}{1-\xi}}.$$

In these definitions,  $p_t^i(h)$  is the price of the home-produced good  $h$  sold in region  $i$ , and  $p_t^i(f)$  is the price of the foreign-produced good sold in region  $i$ .

Assuming the law of one price holds, all goods sell at the same price in both  $H$  and  $F$ , so  $p^H(h) = p^F(h) = p(h)$  and  $p^H(f) = p^F(f) = p(f)$ . This implies in turn that the CPI price index is the same in each region, or  $P_t^H = P_t^F = P_t$ . The average prices of the goods produced in  $H$  and  $F$  may differ, however. Let  $P_{i,t}$  denote the price index of goods produced in region  $i$ . The terms of trade are

$$S_t = \frac{P_{F,t}}{P_{H,t}}.$$

From (9.90),  $P_t = P_{H,t} S_t^\gamma = P_{F,t} S_t^{\gamma-1}$ .

Benigno (2004) showed the demands for the individual goods are given by

$$c_t^j(h) = \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\xi} S_t^\gamma C_t^j,$$

$$c_t^j(f) = \left[ \frac{p_t(f)}{P_{F,t}} \right]^{-\xi} S_t^{\gamma-1} C_t^j.$$

The demands for the basket of home-produced and foreign-produced goods are

$$C_{H,t}^i = (1 - \gamma) \left( \frac{P_{H,t}^i}{P_t^i} \right) C_t^i, \quad (9.91)$$

$$C_{F,t}^i = \gamma \left( \frac{P_{F,t}^i}{P_t^i} \right) C_t^i. \quad (9.92)$$

The total demand facing firm  $h$  in region  $i$  is  $(1 - \gamma) c_t^H(h) + \gamma c_t^F(h)$ . Market clearing for each good requires that production  $y_t(h)$  equal demand. For goods produced in region  $H$ , this requires

$$y_t(h) = \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\xi} S_t^\gamma C_t^W,$$

where  $C_t^W \equiv (1 - \gamma) C_t^H + \gamma C_t^F = \int_0^1 C^j dj$  is total consumption in the currency union. Similarly, for firms in region  $F$ ,

$$y_t(f) = \left[ \frac{p_t(f)}{P_{F,t}} \right]^{-\xi} S_t^{\gamma-1} C_t^W.$$

Note that when combined with the goods-clearing conditions, these imply  $Y_t^H = S_t^\gamma C_t^W$  and  $Y_t^F = S_t^{\gamma-1} C_t^W$ . Consequently,

$$\frac{Y_t^H}{Y_t^F} = \left( \frac{S_t^\gamma}{S_t^{\gamma-1}} \right) = S_t.$$

The two-country model of section 9.2 assumed similar preferences over the goods from each country, as assumed in (9.89). Assuming a complete set of state-contingent securities traded across the union, the results in that earlier section imply  $C_t^H = C_t^F$ .<sup>20</sup> The policy instrument of the central bank in the monetary union is the nominal interest rate on an internationally traded nominal bond. Denoting this interest rate by  $i_t$ , an Euler equation holds for households in each region, and it takes the form

$$(C_t^i)^{-\sigma} = (1 + i_t) \beta E_t \left[ \frac{P_t}{P_{t+1}} (C_{t+1}^i)^{-\sigma} \right], \quad i = H, F.$$

The specification of the pricing decisions of individual firms follows the standard Calvo specification. The frequency of adjustment,  $\omega^i$ , may differ across regions. When a firm has the opportunity to adjust, it bases its pricing decision on current and expected future real marginal cost. Assume a constant returns to scale production technology with labor as the only input and with aggregate labor productivity an exogenous stochastic variable denoted by  $A_t^i$ . For a firm in region  $i$ , real marginal cost is given by

$$MC_t^i = \frac{W_t^i / P_{i,t}}{A_t^i},$$

where  $W_t^i$  is the nominal wage in region  $i$ . The optimality condition for household labor supply in region  $H$ , for example, is

$$\frac{\chi (N_t^H)^\eta}{(C_t^H)^{-\sigma}} = \frac{W_t^H}{P_t} = \frac{W_t^H}{P_{H,t}} \left( \frac{P_{H,t}}{P_t} \right) = \left( \frac{W_t^H}{P_{H,t}} \right) S_t^{-\gamma}.$$

Thus, real marginal cost in region  $H$  expressed as a log deviation around the steady state is

$$mc_t^H = \eta n_t^H + \sigma c_t^H + \gamma s_t - a_t^H. \quad (9.93)$$

From the definition of  $P_t$ ,  $P_{H,t} = P_t S_t^{-\gamma}$ . In addition,  $P_{H,t} Y_t^H = P_t C_t^W$ . Thus,  $S_t^{-\gamma} Y_t^H = C_t^W$ , or in log-linearized terms,  $c_t^W = y_t^H - \gamma s_t$ . These results then imply  $mc_t^H$  can be written as

$$mc_t^H = (\eta + \sigma) y_t^H - (\sigma - 1) \gamma s_t - (1 + \eta) a_t^H.$$

Inflation in each region is determined by a standard new Keynesian Phillips curve. Define  $\hat{s}_t \equiv s_t - \bar{s}_t$  as the terms of trade gap, where  $\bar{s}_t$  denotes the terms of trade under flexible prices. Then inflation for region  $H$  is

$$\pi_t^H = \beta E_t \pi_{t+1}^H + \kappa^H (\eta + \sigma) x_t^H - \kappa^H (\sigma - 1) \gamma \hat{s}_t. \quad (9.94)$$

20. Benigno (2004) showed that this same result can be obtained under less restrictive conditions on the set of available financial assets.

For region  $F$  it is

$$\pi_t^F = \beta E_t \pi_{t+1}^F + \kappa^F (\eta + \sigma) x_t^F + \kappa^F (\sigma - 1) (1 - \gamma) \hat{s}_t. \quad (9.95)$$

In both equations,  $\kappa^i = (1 - \omega^i) (1 - \beta \omega^i) / \omega^i$ . Inflation in each region depends, as usual, on an output gap measure. But, in addition, inflation rates depend on the behavior of the terms of trade. Because  $S_t = P_{F,t} / P_{H,t}$ ,

$$s_t - s_{t-1} = \pi_t^F - \pi_t^H. \quad (9.96)$$

There is a parallel between the three equations given by (9.94)–(9.96) describing the determination of the inflation rate in each region and the evolution of the terms of trade and the sticky price, sticky-wage framework of section 8.5.1. With sticky prices and wages, forward-looking Phillips curves were obtained for price inflation and wage inflation. Each involved the real wage, so the system was closed by an equation that linked the log change in the real wage to the difference between nominal wage inflation and price inflation.

In the model of the currency union, movements of the terms of trade gap  $\hat{s}_t$  act as a cost shock for each region. A rise in  $\hat{s}_t$  has two effects on region  $H$ . First, the rise in the prices of foreign goods increases the domestic CPI relative to the price of domestic output. Since workers care about the real wage in terms of the CPI, the nominal wage in  $H$  rises, increasing real marginal costs of domestic firms (see 9.93). However, the goods-clearing condition and international risk sharing imply  $S_t^{-\gamma} Y_t^H = C_t^H = C_t^W$ , or in log-linearized terms,  $c_t^H - \tilde{c}_t^H = x_t^H - \gamma \hat{s}_t$ . The fall in domestic consumption for a given level of  $y_t^H$  raises the marginal utility of consumption by  $\sigma (c_t^H - \tilde{c}_t^H)$ . With consumption more valuable, households in region  $H$  supply more labor, pushing down the real wage and marginal costs. Which effect dominates depends on the sign of  $\sigma - 1$ . For region  $F$ , the same two opposing effects are at work, but in reverse of their effects in region  $H$ . If  $\sigma < 1$ , an increase in  $\hat{s}_t$  increases marginal costs and inflation in region  $H$  and lowers them in region  $F$ .

The parallels drawn between the currency union and a closed economy with sticky prices and wages carries over to the implications for monetary policy in the currency union. Inflation-induced relative price dispersion within either region generates a distortion in the use of labor hours. With two nominal rigidities, either sticky prices and wages or different degrees of nominal price stickiness in the two regions of the union, monetary policy cannot simultaneously maintain zero inflation in both regions and zero output gaps in both regions. Stabilizing domestic goods prices in one region, when both have sticky producer prices, means the important relative price measured by the terms of trade cannot flexibly adjust in response to relative productivity shocks in either region. The single monetary policy in the union will be unable to eliminate both nominal distortions.

With multiple distortions, what policy should the central bank pursue in a currency union? Under the standard assumption that a fiscal subsidy offsets the steady-state distortions due to monopolistic competition in each region, Benigno (2004) derived the quadratic approximation to welfare, defined as the sum of the utility of all members of the union. He



found that the central bank in the currency union should minimize the present discounted value of a loss function given by

$$L_t^{\text{cu}} = a_1 (x_t^W)^2 + a_2 \hat{\delta}_t^2 + \delta (\pi_t^H)^2 + (1 - \delta) (\pi_t^F)^2, \quad (9.97)$$

where  $x_t^W \equiv (1 - \gamma)x_t^H + \gamma x_t^F$  is the unionwide output gap. According to (9.97), the central bank should stabilize the output gap of the entire union, deviations of the terms of trade from their value under flexible prices, and a weighted average of inflation variability in the two regions. The relative weight to put on the inflation rates in each region is given by

$$\delta = \frac{1 - \gamma}{(1 - \gamma) + \gamma (\kappa^H / \kappa^F)}. \quad (9.98)$$

To interpret  $\delta$ , consider the case in which the degree of price stickiness is equal in both regions:  $\omega^H = \omega^F$ . In this case,  $\kappa^H = \kappa^F$  and  $\delta = \gamma$ .<sup>21</sup> The optimal weights reflect country size. If prices are stickier in region  $H$  than in  $F$ ,  $\kappa^H < \kappa^F$  and  $\delta > \gamma$ ; more weight is placed on stabilizing  $\pi_t^H$  than is warranted based solely on country size. Increased weight is placed on stabilizing inflation in the region with the stickier prices. This result is consistent with the discussion in 8.5.1 that the weight to place on stabilizing price inflation relative to wage inflation depends on their relative rigidity. It is also in line with the findings of Aoki (2002) in the case of a two-sector closed-economy model in which price rigidity differs across the regions.

The inflation terms in (9.97) can be written as

$$\delta (\pi_t^H)^2 + (1 - \delta) (\pi_t^F)^2 = (\pi_t^W)^2 + \gamma(1 - \gamma) (\pi_t^R)^2,$$

where  $\pi_t^W = (1 - \gamma)\pi_t^H + \gamma\pi_t^F$  is unionwide inflation and  $\pi_t^R = \pi_t^H - \pi_t^F = s_t - s_{t-1}$  is relative inflation. Benigno showed that the optimal monetary policy is to set  $\pi_t^W = 0$ , since the adjustment of relative prices across the two regions cannot be controlled by monetary policy. He argued this objective is consistent with the European Central Bank policy of defining its objective in terms of a euro area price index.

## 9.6 Summary

This chapter has reviewed various open-economy new Keynesian models based on households maximizing utility and firms maximizing prices but doing so in the presence of nominal rigidities. The model due to Clarida, Galí, and Gertler (2002) extended a basic closed-economy new Keynesian model to a two-country environment. An example of a new Keynesian model for a small open economy, following Galí and Monacelli (2005), was also developed. In some cases, these models result in reduced-form equations for

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21. Recall that  $\kappa^i = (1 - \omega^i) / (1 - \omega^i \beta)$  is decreasing in  $\omega^i$ .

the output gap and inflation that are identical in form to the closed-economy equivalents. However, as in closed-economy models, policy trade-offs are affected by the addition of multiple sources of nominal rigidity. The model of imperfect pass-through provided one empirically relevant example of a nominal rigidity that is absent in the closed economy, while local currency pricing and traded and nontraded goods with sticky prices provided further examples.

Several authors have taken new Keynesian open-economy frameworks discussed in this chapter to the data. Adolfson et al. (2007; 2008) provided early examples of estimated open-economy DSGE models based on models of a small open economy that can be used for policy analysis. Lubik and Schorfheide (2005) estimated a two-country open-economy DSGE model with nominal rigidities using U.S. and euro area data. These empirical models incorporate multiple sources of nominal (and real) frictions into the basic model structures reviewed in this chapter.

## 9.7 Appendix

In section 9.2, the two-country model of Clarida, Galí, and Gertler (2002) was simulated under alternative policies. The linearized equations characterizing equilibrium in this model consist of the following:

$$x_t = y_t - y_t^f,$$

$$[\eta + \sigma - \gamma(\sigma - 1)]y_t^f + \gamma(\sigma - 1)y_t^{*f} = (1 + \eta)a_t - \mu_t,$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \bar{\rho}_t) - \gamma (E_t s_{t+1} - s_t),$$

$$\bar{\rho}_t \equiv \rho + \sigma (E_t y_{t+1}^f - y_t^f),$$

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \bar{\kappa} x_t + \mu_t,$$

$$p_{h,t} = p_{h,t-1} + \pi_{h,t},$$

$$\pi_t = \pi_{h,t} + \gamma (s_t - s_{t-1}),$$

$$s_t = y_t - y_t^*,$$

$$e_t = s_t + p_{h,t} - p_{f,t}^*,$$

$$x_t^* = y_t^* - y_t^{*f},$$

$$(1 - \gamma)(\sigma - 1)y_t^{*f} + [\eta + \sigma - (1 - \gamma)(\sigma - 1)]y_t^* = (1 + \eta)a_t^* - \mu_t^*,$$

$$x_t^* = E_t x_{t+1}^* - \frac{1}{\sigma} (i_t^* - E_t \pi_{t+1}^* - \bar{\rho}_t^*) + (1 - \gamma)(E_t s_{t+1}^* - s_t^*),$$

$$\bar{\rho}_t^* \equiv \rho + \sigma (E_t y_{t+1}^{*f} - y_t^{*f}),$$

$$\pi_{f,t}^* = \beta E_t \pi_{f,t+1}^* + \bar{\kappa}^* x_t^* + \mu_t^*,$$

$$p_{f,t}^* = p_{f,t-1}^* + \pi_{f,t}^*,$$

$$\pi_t^* = \pi_{f,t}^* + (\gamma - 1)(s_t - s_{t-1}),$$

plus the specification of monetary policy in each country. It was assumed the foreign country followed the optimal policy under discretion, implying

$$\bar{\kappa}^* \pi_{h,t}^* + \lambda x_t^* = 0.$$

For the domestic economy, five alternative policies were used: optimal discretion, optimal commitment, a simple instrument rule that responds to domestic price inflation, a rule reacting to CPI inflation, and a policy that mimics a fixed exchange rate:

$$\text{Discretion: } \bar{\kappa} \pi_{h,t} + \lambda x_t = 0,$$

$$\text{Commitment: } \bar{\kappa} \pi_{h,t} + \lambda (x_t - x_{t-1}) = 0,$$

$$\text{Domestic price inflation rule: } i_t = 1.5\pi_{h,t},$$

$$\text{CPI inflation rule: } i_t = 1.5\pi_t,$$

$$\text{Fixed exchange rate: } i_t = 1.5\pi_{h,t} + 10e_t.$$

## 9.8 Problems

1. Show that the cost minimization problem given by

$$\min_{C_{h,t}, C_{f,t}} P_{h,t} C_{h,t} + P_{f,t} C_{f,t} + \lambda_t \left( C_t - C_{h,t}^{1-\gamma} C_{f,t}^\gamma \right)$$

implies that the marginal cost of the consumption basket  $C_t$  is given by  $P_t$  as defined in (9.3).

2. In the model of section 9.2.3, the final goods-producing firm combines intermediate goods  $Y_t(h)$  to produce final output according to

$$Y_t = \left( \int_0^1 Y_t(h)^{\frac{\xi-1}{\xi}} dh \right)^{\frac{\xi}{\xi-1}}, \quad \xi > 1.$$

Show that the demand for  $Y_t(h)$  by a cost-minimizing final goods firm is given by (9.18), and the price index of the home goods is given by (9.19).

3. In the NK two-country model of section 9.2, use the households' demands for home and foreign goods, the goods-clearing condition, the definitions of the CPI in the home and foreign countries, and  $S_t P_t^* = P_t$  to show that

$$P_h Y_t = P_t C_t$$

and

$$P_{f,t}^* Y_t^* = P_t^* C_t^*.$$

Show that these two equations imply the trade balance between the two countries is zero.

4. Show that (9.26) and (9.27) together with complete international risk sharing imply uncovered interest rate parity.
5. Show that the solution to

$$E_t \hat{\pi}_{t+1} = \phi_\pi \hat{\pi}_t + \hat{v}_t$$

takes the form  $\hat{\pi}_t = H \hat{v}_t$  if  $\phi_\pi > 1$ , and show  $H$  is equal to  $-1/(\phi_\pi - \rho)$  if  $\hat{v}_t = \rho \hat{v}_{t-1} + \varepsilon_t$ ,  $0 \leq \rho < 1$ .

6. Consider the two-country model of section 9.2 and assume prices are flexible in both countries.
  - a. Assume the log money supply processes are  $m_t = m_0 + \mu t + \varepsilon_t$  and  $m_t^* = m_0^* + \mu^* t + \varepsilon_t^*$ , where  $\varepsilon_t$  and  $\varepsilon_t^*$  are white noise processes. Show that (9.34) implies  $e_t = e_0 + (\mu - \mu^*)t + d_0(\varepsilon_t - \varepsilon_t^*)$ .
  - b. Assume  $m_t = m_{t-1} + \mu + \varepsilon_t$  and  $m_t^* = m_{t-1}^* + \mu^* + \varepsilon_t^*$ . Use (9.34) to find an expression for the nominal exchange rate.
  - c. In parts (a) and (b), the unconditional expected growth rates of money are  $\mu$  and  $\mu^*$ . Explain why the exchange rate solution is different in the two cases.
7. Suppose  $m_t = m_0 + \rho_m m_{t-1} + \varepsilon_t$  and  $m_t^* = m_0^* + \rho_m^* m_{t-1}^* + \varepsilon_t^*$ . Use (9.34) to show how the behavior of the nominal exchange rate under flexible prices depends on the degree of serial correlation exhibited by the home and foreign money supplies (i.e., it depends on  $\rho_m$  and  $\rho_m^*$ ).
8. In section 9.2.4, (9.35) and (9.36) gave two conditions for the equilibrium values of domestic and foreign output under flexible prices. Take logs of these two equations and solve to obtain explicit expressions for the logs of domestic and foreign output. Explain how domestic and foreign output are affected by productivity in the home country when prices are flexible. How are your results affected if  $\sigma = 1$ ? Explain. How does foreign output affect domestic output if  $\sigma < 1$ ? Explain.
9. Suppose household utility depends on a consumption aggregate defined as

$$C_t = \left[ \alpha_H^{\frac{1}{\theta}} C_{h,t}^{\frac{\theta-1}{\theta}} + (1 - \alpha_H)^{\frac{1}{\theta}} C_{f,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where  $C_{j,t}$  is a consumption bundle of final goods produced in country  $j = h, f$ . The price indexes for home- and foreign-produced goods are  $P_{h,t}$  and  $P_{f,t}$ . Assume

households purchase  $C_{h,t}$  and  $C_{f,t}$  to minimize the cost of achieving a given  $C_t$ . Derive the demand equations for  $C_{h,t}$  and  $C_{f,t}$  and find the formula for the price index of total consumption.

10. Suppose the baskets  $C_{h,t}$  and  $C_{f,t}$  in problem 9 are defined as

$$C_{h,t} = \left[ \left( \frac{1}{1-\gamma} \right)^{\frac{1}{\xi}} \int_0^{1-\gamma} c_t(h)^{\frac{\xi-1}{\xi}} dh \right],$$

$$C_{f,t} = \left[ \left( \frac{1}{\gamma} \right)^{\frac{1}{\xi}} \int_0^{1-\gamma} c_t(f)^{\frac{\xi-1}{\xi}} df \right],$$

where  $c_t(h)$  and  $c_t(f)$  are individual home- and foreign-produced goods whose prices in the home economy are  $p_t(h)$  and  $p_t(f)$ .

- Find the demand curves for  $c_t(h)$  and  $c_t(f)$  for given  $C_{h,t}$  and  $C_{f,t}$ . Find the associated price indexes for  $C_{h,t}$  and  $C_{f,t}$ .
  - Using the results from the previous problem, express the demand for  $c_t(h)$  as a function of  $C_t$  and the two relative prices  $p_t(h)/P_{h,t}$  and  $P_{h,t}/P_t$ . What is the elasticity of demand with respect to each of the two relative prices?
11. In the model of section 9.3, show that if  $\sigma = \theta = \alpha = 1$ ,

$$Y_t = \left( \frac{P_{h,t}}{P_t} \right)^{-1} C_t = \left( \frac{P_t}{P_{h,t}} \right) C_t = \left( \frac{P_{f,t}}{P_{h,t}} \right)^{\gamma} C_t = S_t^{\gamma} C_t$$

when the definition of  $P_t$  as equal to  $P_{h,t}^{1-\gamma} P_{f,t}^{\gamma}$  is used.

12. For the Monacelli (2005) model of imperfect pass-through, show that real marginal cost, expressed as a log deviation from the steady state is given by

$$mc_t = (\sigma_{\gamma} + \varphi) x_t + \left( 1 - \frac{\sigma_{\gamma}}{\sigma_{\psi}} \right) \psi_t,$$

where  $x_t$  is the output gap,  $\psi_t$  is the deviation from the law of one price, and the parameters are defined in the text.

13. Section 9.3 demonstrated how a simple open-economy model with nominal price stickiness could be expressed in a form that paralleled the closed-economy new Keynesian model of chapter 8. Would this same conclusion hold in a model with sticky wages but flexible prices? What if both wages and prices were sticky?
14. McCallum and Nelson (2000) proposed a new Keynesian open-economy model in which imported goods are only used as inputs into the production of the domestic good and households consume only the domestically produced good. If  $e_t$  is the nominal

exchange rate, and  $s_t$  is the real exchange rate, the model can be summarized by the following equations:

$$c_t = E_t c_{t+1} - b_1 [R_t - E_t \pi_{t+1} - r_t],$$

$$\text{im}_t = y_t - \sigma s_t,$$

$$\text{ep}_t = y_t^* + \sigma^* s_t,$$

$$s_t = e_t - p_t + p_t^*,$$

$$R_t = R_t^* + E_t e_{t+1} - e_t,$$

$$y_t = (1 - \alpha)(n_t + \varepsilon_t) + \alpha \text{im}_t,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t,$$

where  $\text{im}_t$  denotes imports,  $\text{ep}_t$  denotes exports, and all variables are expressed relative to their flexible-price equivalents. Foreign variables are denoted by \*. The linearized production function is

$$y_t = (1 - \alpha)(n_t + \varepsilon_t) + \alpha \text{im}_t,$$

and the goods market equilibrium condition takes the form

$$y_t = \omega_1 c_t + \omega_2 g_t + \omega_3 \text{ep}_t.$$

Show that this open-economy model can be reduced to two equations corresponding to the IS relationship and the Phillips curve that, when combined with a specification of monetary policy, could be solved for the equilibrium output gap and inflation rate. How does the interest elasticity of the output gap depend on the openness of the economy?

15. Assume the utility function of the representative household in a small open economy is

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t) - \frac{N_t^{1+\eta}}{1+\eta} + \frac{a_m}{1-\gamma_m} \left( \frac{M_t}{P_t} \right)^{1-\gamma_m} \right\},$$

where  $C_t$  is total consumption,  $N_t$  is labor supply, and  $M_t/P_t$  is real money holdings.  $C_t$  is defined by (9.52), and utility is maximized subject to the sequence of constraints given by

$$P_t C_t + M_t + e_t \frac{1}{1+i_t^*} B_t^* + \frac{1}{1+i_t} B_t \leq W_t N_t + M_{t-1} + e_t B_{t-1}^* + B_{t-1} + \Pi_t - \tau_t,$$

where  $B$  ( $B^*$ ) equals domestic (foreign) currency denominated bonds paying a nominal interest rate of  $i$  ( $i^*$ ),  $\Pi$  equals any profit income, and  $\tau$  equals lump-sum taxes net of transfers. Let  $P_t^h$  ( $P_t^f$ ) be the average price of domestically (foreign) produced consumption goods.

- a. Derive the first-order conditions for the household's problem.
  - b. Show that the choice of domestic-produced consumption goods relative to the foreign-produced consumption good basket depends on the terms of trade.
  - c. Derive an expression for the price index  $P_t$ .
16. Suppose the consumption bundle that yields utility is defined by (9.89). If the household minimizes the cost of achieving a given level of the consumption bundle, derive the demand functions for the home-produced and foreign-produced consumption bundles and show that the marginal cost of consumption is given by the price index in (9.90).





# 10 Financial Markets and Monetary Policy

## 10.1 Introduction

Central banks in the major industrialized economies implement policy by intervening in the money market to achieve a target level for a short-term interest rate.<sup>1</sup> Section 10.2 takes up a traditional topic related to interest rate rules by investigating the connection between interest rate policies and price level determinacy. Determinacy was also discussed in chapter 8, but there the focus was on the existence of a unique rational-expectations equilibrium for the inflation rate, not the price level. Some issues concerning interest rate policies in flexible-price general equilibrium models are then discussed.

Monetary models such as the basic new Keynesian model of chapter 8 assume the central bank policy rate, once adjusted for the expected rate of inflation, is also the interest rate that affects aggregate spending. Yet there are many interest rates. These differ from the central bank's policy rate for a number of reasons. One reason relates to term to maturity; the interest rate on a government bond that matures in 5 years differs from the rate on a government security that matures in 3 months, for example. Term structure theory provides the basic model for understanding the relationship between short-term and long-term interest rates, and section 10.3 discusses the basic expectations model of the term structure. Section 10.4 reviews recent work that links the affine term structure models used in the finance literature to the types of macroeconomic models commonly used to investigate monetary policy issues. The relationship between monetary policy and the term structure is discussed in section 10.5.

The role of financial frictions in affecting the linkages between the interest rate affected directly by monetary policy and the broad range of market interest rates and credit conditions that affect investment and consumption spending is taken up in section 10.6. If credit markets are imperfect, interest rates (i.e., prices) may not be sufficient to capture the impact of monetary policy on the economy; quantities may also be important, particularly

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1. Why central banks use a short-term interest rate as a policy instrument and the operating procedures used to achieve the targeted value they set for this rate are discussed in chapter 12.

if nonprice rationing occurs. Section 10.6 examines these frictions. Their implications for macroeconomics and the impact of monetary policy are discussed in section 10.7.

## 10.2 Interest Rates and Monetary Policy

In this section, two issues are explored: first, the connection between interest rate policies and price level determinacy, and second, interest rate policies in flexible-price general equilibrium models.

### 10.2.1 Interest Rate Rules and the Price Level

Monetary policy can affect nominal rates, both in the short run and in the long run. But the Fisher relationship links the real rate, expected inflation, and the nominal rate of interest, implying targets for nominal interest rates and inflation cannot be independently chosen, and controlling the nominal interest rate has important implications for the behavior of the aggregate price level. For models such as the new Keynesian model of chapter 8 in which expectations of future output and inflation play important roles, it was shown that any interest rate rule implemented by the central bank needs to respond to endogenous variables such as inflation to ensure a locally unique stationary rational-expectations equilibrium. This section considers an older class of models that were used to understand the links between interest rate policies, control of the money supply, and determinacy of the price level.

In section 7.2.1, a simple model with one-period sticky wages was developed that could be expressed in the following form:

$$y_t = y^c + a(p_t - E_{t-1}p_t) + e_t, \quad (10.1)$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t, \quad (10.2)$$

$$m_t - p_t = y_t - c i_t + v_t, \quad (10.3)$$

$$i_t = r_t + (E_t p_{t+1} - p_t), \quad (10.4)$$

where  $y$ ,  $m$ , and  $p$  are the natural logs of output, the money stock, and the price level, and  $r$  and  $i$  are the real and nominal rates of interest. Although central banks may closely control the nominal rate  $i$ , it is the expected real rate of interest  $r$  that influences consumption and investment decisions and therefore aggregate demand.<sup>2</sup> This distinction has important implications for the feasibility of an interest targeting rule.

Suppose that the central bank conducts policy by pegging the nominal interest rate at some targeted value:

$$i_t = i^T. \quad (10.5)$$

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2. Term structure considerations are postponed until section 10.3.

Under an interest rate peg, the basic aggregate demand and supply system comprises (10.1), (10.2), and (10.4):

$$y_t = y^c + a(p_t - E_{t-1}p_t) + e_t, \quad (10.6)$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t, \quad (10.7)$$

$$i^T = r_t + (E_t p_{t+1} - p_t). \quad (10.8)$$

The money demand equation, (10.3), is no longer relevant because the central bank must allow the nominal money stock to adjust to the level of money demand at the targeted interest rate and the equilibrium level of output.

Note that the price level only appears in the form of an expectation error (i.e., as  $p_t - E_{t-1}p_t$  in the aggregate supply equation) or as an expected rate of change (i.e., as  $E_t p_{t+1} - p_t$  in the Fisher equation). This structure implies that the price level is indeterminate. That is, if the sequence  $\{p_{t+i}^*\}_{i=0}^{\infty}$  is an equilibrium, so is any sequence  $\{\hat{p}_{t+i}\}_{i=0}^{\infty}$  where  $\hat{p}$  differs from  $p^*$  by any constant  $\kappa$ :  $\hat{p}_t = p_t^* + \kappa$  for all  $t$ . Since  $\kappa$  is an arbitrary constant,  $p_t^* - E_{t-1}p_t^* = \hat{p}_t - E_{t-1}\hat{p}_t$ ; hence,  $y_t$  is the same under either price sequence. From (10.7), the equilibrium real interest rate is equal to  $(\alpha_0 - y_t + u_t) / \alpha_1$ , so it, too, is the same. With expected inflation the same under either price sequence, the only restriction on the price path is that the expected rate of inflation be such that  $i^T = (\alpha_0 - y_t + u_t) / \alpha_1 + E_t p_{t+1}^* - p_t^*$ .

The indeterminacy of the price level is perhaps even more apparent if (10.6)–(10.8) are rewritten explicitly in terms of the rate of inflation. By adding and subtracting  $ap_{t-1}$  to (10.6), the equilibrium conditions become

$$y_t = y^c + a(\pi_t - E_{t-1}\pi_t) + e_t,$$

$$y_t = \alpha_0 - \alpha_1 r_t + u_t,$$

$$i^T = r_t + E_t \pi_{t+1}.$$

These three equations can be solved for output, the real rate of interest, and the rate of inflation. Since the price level does not appear, it is formally indeterminate.<sup>3</sup> In a forward-looking model, an interest rate peg would also leave the inflation rate indeterminate (see chapter 8).

As stressed by McCallum (1986), the issue of indeterminacy differs from the problem of multiple equilibria. The latter involves situations in which multiple equilibrium price paths are consistent with a given path for the nominal supply of money. One example of such a multiplicity of equilibria was seen in the model of hyperinflation studied in chapter 4. With indeterminacy, neither the price level nor the nominal supply of money is determined by

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3. Employing McCallum's (1983) minimum state solution method, the equilibrium inflation rate is  $\pi_t = i^T + (y^c - \alpha_0) / \alpha_1 + (u_t - e_t) / a$  when  $u$  and  $e$  are serially uncorrelated and the target nominal interest rate is expected to remain constant. In this case,  $E_t \pi_{t+1} = i^T + (y^c - \alpha_0) / \alpha_1$ , so permanent changes in the target rate  $i^T$  do not affect the real interest rate:  $r_t = i^T - E_t \pi_{t+1} = -(y^c - \alpha_0) / \alpha_1$ .

the equilibrium conditions of the model. If the demand for real money balances is given by (10.3), then the price sequence  $p^*$  is associated with the sequence  $m_t^* = p_t^* + y_t - ci_t^T + v_t$ , while  $\hat{p}$  is associated with  $\hat{m}_t = \hat{p}_t + y_t - ci_t^T + v_t = m^* + \kappa$ . The price sequences  $p^*$  and  $\hat{p}$  are associated with different paths for the nominal money stock.

Intuitively, if all agents expect the price level to be 10 percent higher permanently, such an expectation is completely self-fulfilling. To peg the nominal rate of interest, the central bank simply lets the nominal money supply jump by 10 percent. This stands in contrast to the case in which the central bank controls the nominal quantity of money; a jump of 10 percent in the price level would reduce the real quantity of money, thereby disturbing the initial equilibrium. Under a rule such as (10.5), which has the policymaker pegging the nominal interest rate, the central bank lets the nominal quantity of money adjust as the price level does, leaving the real quantity unchanged.<sup>4</sup>

Price level indeterminacy is often noted as a potential problem with pure interest rate pegs; if private agents don't care about the absolute price level—and under pure interest rate control, neither does the central bank—nothing pins down the price level. Simply pegging the nominal interest rate does not provide a *nominal anchor* to pin down the price level. However, this problem will not arise if the central bank's behavior does depend on a nominal quantity such as the nominal money supply.

For example, suppose the nominal money supply (or a narrow reserve aggregate) is the actual instrument used to affect control of the interest rate, and assume it is adjusted in response to interest rate movements, as Canzoneri, Henderson, and Rogoff (1983) and McCallum (1986) proposed:

$$m_t = \mu_0 + m_{t-1} + \mu(i_t - i^T). \quad (10.9)$$

Under this policy rule, the monetary authority adjusts the nominal money supply growth rate,  $m_t - m_{t-1}$ , in response to deviations of the nominal interest rate from its target value. If  $i_t$  fluctuates randomly around the target  $i^T$ , then the average rate of money growth will be  $\mu_0$ . As  $\mu \rightarrow \infty$ , the variance of the nominal rate around the targeted value  $i^T$  will shrink to zero, but the price level can remain determinate. (See problem 1 at the end of this chapter.)

The nominal money stock is  $I(1)$  under the policy rule given by (10.9). That is,  $m_t$  is nonstationary and integrated of order 1. This property of  $m$  causes the price level to be nonstationary also.<sup>5</sup> One implication is that the error variance of price level forecasts increases with the forecast horizon.

As McCallum (1986) demonstrated, a different equilibrium describing the stochastic behavior of the nominal interest rate and the price level is obtained if the money supply

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4. See Patinkin (1965) for an early discussion of price level indeterminacy and Schmitt-Grohe and Uribe (2000) for a more recent discussion.

5. In contrast, the nominal interest rate is stationary because both the real rate of interest and the inflation rate (and therefore expected inflation) are stationary.

process takes the trend stationary form

$$m_t = \mu' + \mu_0 t + \mu(i_t - i^T), \quad (10.10)$$

even though (10.10) and (10.9) both imply that the average growth rate of money equals  $\mu_0$  (see problem 2 at the end of this chapter). With the money supply process (10.10), the equilibrium price level is trend stationary, and the forecast error variance does not increase without limit as the forecast horizon increases.

It is not surprising that (10.9) and (10.10) lead to different solutions for the price level. Under (10.9), the nominal money supply is a nontrend stationary process; random target misses have permanent effects on the future level of the money supply and therefore on the future price level. In contrast, (10.10) implies that the nominal money supply is trend stationary. Deviations of money from the deterministic growth path  $\mu' + \mu_0 t$  are temporary, so the price level is also trend stationary.

This discussion leads to two conclusions. First, monetary policy can be implemented to reduce fluctuations in the nominal interest rate without leading to price level indeterminacy. Canzoneri, Henderson, and Rogoff (1983) and McCallum (1986) showed that by adjusting the money supply aggressively in response to interest rate movements, a central bank can reduce the variance of the nominal rate around its target level while leaving the price level determinate. However, the level at which the nominal rate can be set is determined by the growth rate of the nominal money supply, since the latter equals the expected rate of inflation. The choice of  $\mu_0$  determines the feasible value of  $i^T$  (or equivalently, the choice of  $i^T$  determines  $\mu_0$ ). Targets for the nominal interest rate and rate of inflation cannot be independently determined.

Second, the underlying behavior of the nominal money supply is not uniquely determined by the assumption that the nominal rate is to be fixed at  $i^T$ ; this target can be achieved with different money supply processes. And the different processes for  $m$  lead to different behaviors of the price level. A complete description of policy, even under a nominal interest rate targeting policy, requires a specification of the underlying money supply process.

### 10.2.2 Interest Rate Policies in General Equilibrium

The analysis in the previous section employed a model that was not derived directly from the assumption of optimizing behavior on the part of the agents in the economy. Carlstrom and Fuerst (1995) developed a general equilibrium model with optimizing agents and studied interest rate policies. They employed a cash-in-advance (CIA) framework in which consumption must be financed from nominal money balances to address welfare issues associated with interest rate policies. As seen in chapter 3, a positive nominal interest rate represents a distorting tax on consumption, affecting the household's choice between cash goods (i.e., consumption) and credit goods (i.e., investment and leisure). Introducing

one-period price stickiness into their model, Carlstrom and Fuerst (1995) concluded that a constant nominal interest rate eliminates the distortion on capital accumulation, an interest rate peg Pareto-dominates a fixed money rule, and for any interest rate peg, there exists a money growth process that replicates the real equilibrium in the flexible-price version of their model. That is, an appropriate movement in the nominal money growth rate can undo the effects of the one-period price stickiness.

To illustrate the basic issues in a simple manner, consider the following five equilibrium conditions for a basic CIA economy with a positive nominal interest rate:

$$\frac{u_{c,t}}{1+i_t} = \beta E_t R_t \left( \frac{u_{c,t+1}}{1+i_{t+1}} \right),$$

$$\frac{u_{l,t}}{u_{c,t}} = \frac{\text{MPL}_t}{1+i_t},$$

$$R_t = 1 + E_t (\text{MPK}_{t+1}),$$

$$m_t = \frac{M_t}{P_t} = c_t,$$

$$1+i_{t+1} = E_t \left( \frac{R_t P_{t+1}}{P_t} \right),$$

where  $u_{c,t}$  is the marginal utility of consumption at time  $t$ ,  $\beta$  is the subjective rate of time preference,  $R_t$  is 1 plus the real rate of return,  $u_{l,t}$  is the marginal utility of leisure at time  $t$ ,  $i_t$  is the nominal interest rate,  $\text{MPL}_t$  ( $\text{MPK}_t$ ) is the marginal product of labor (capital),  $P_t$  is the price level, and  $m_t$  is the level of *real* money balances. The first of these five equations can be derived from a basic CIA model by recalling that  $u_{c,t} = (1+i_t)\lambda_t$ , where  $\lambda_t$  is the time  $t$  marginal value of wealth. (This assumes asset markets open before goods markets; see chapter 3.) Since  $\lambda_t = \beta E_t R_t \lambda_{t+1}$  (see 3.29), it follows that  $u_{c,t}/(1+i_t) = \lambda_t = \beta E_t R_t u_{c,t+1}/(1+i_{t+1})$ . The second equation equates the marginal rate of substitution between leisure and wealth to the marginal product of labor, again using the result that  $\lambda_t = u_{c,t}/(1+i_t)$ . The third equation is the definition of the real return on capital. The fourth equation is the binding CIA constraint that determines the demand for money as a function of the level of consumption. The final equation is simply the Fisher relationship linking nominal and real returns. The fourth and fifth equations of this system, as Woodford (2003a) emphasized, are traditionally interpreted as determining the price level and the nominal interest rate for an exogenous nominal money supply process. The model could be completed by adding the production function and the economywide resource constraint.

Rebelo and Xie (1999) argued that this CIA economy will replicate the behavior of a nonmonetary real economy under any nominal interest rate peg. To demonstrate the conditions under which their result holds, assume that the nominal interest rate is pegged at a

value  $\bar{i}$  for all  $t$ . Under an interest rate peg, the first two equations of the basic CIA model become

$$\frac{u_{c,t}}{1+\bar{i}} = \beta E_t R_t \left( \frac{u_{c,t}}{1+\bar{i}} \right) \Rightarrow u_{c,t} = \beta E_t R_t u_{c,t+1},$$

$$\frac{u_{l,t}}{u_{c,t}} = \frac{\text{MPL}_t}{1+\bar{i}}.$$

The Euler condition is now identical to the form obtained in a real, nonmonetary economy, an economy not facing a CIA constraint.<sup>6</sup> The level at which the nominal interest rate is pegged only appears in the labor market equilibrium condition. Thus, Rebelo and Xie concluded that if labor supply is inelastic, the equilibrium with an interest rate peg is the same as the equilibrium in the corresponding nonmonetary real economy. Any equilibrium of the purely real economy can be achieved by a CIA model with a nominal interest rate peg if labor supply is inelastic. If labor supply is elastic, however, the choice of  $\bar{i}$  does have effects on the real equilibrium.

Under an interest rate peg, the price level process must satisfy

$$E_t \left( \frac{R_t P_{t+1}}{P_t} \right) = 1 + \bar{i},$$

while the nominal money supply must satisfy

$$M_t = P_t c_t.$$

These requirements do not, however, uniquely determine the nominal money supply process. For example, suppose the utility of consumption is  $\ln c_t$ . Then  $u_{c,t} = 1/c_t$ , and the Euler condition under an interest rate peg can be written as

$$\frac{1}{c_t} = \frac{P_t}{M_t} = \beta E_t R_t \left( \frac{P_{t+1}}{M_{t+1}} \right).$$

Rearranging this equation yields

$$1 = \beta E_t R_t \left( \frac{P_{t+1}}{P_t} \frac{M_t}{M_{t+1}} \right).$$

If this equation is linearized around the steady state, one obtains

$$r_t + E_t \pi_{t+1} - E_t \mu_{t+1} = \bar{i} - E_t \mu_{t+1} = 0,$$

where  $E_t \mu_{t+1}$  is the expected growth rate of money. In this formulation, while real money balances are determined ( $m_t = c_t$ ), there are many nominal money supply processes consistent with equilibrium as long as they all generate the same expected rate of nominal money growth.

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6. If output follows an exogenous process and all output is perishable, equilibrium requires that  $c_t$  equal output; the Euler condition then determines the real rate of return.

As noted in section 10.2.1, the price level is indeterminate under such an interest rate–pegging policy. However, assuming that  $P_t$  is predetermined because of price level stickiness still allows the money demand equation and the Fisher equation to determine  $P_{t+1}$  and  $m_t$  (and so the implied nominal supply of money) without affecting the real equilibrium determined by the Euler condition. In that sense, Carlstrom and Fuerst (1995) concluded that there exists a path for the nominal money supply in the face of price stickiness that leads to the same real equilibrium under an interest rate peg as would occur with a flexible price level.

Carlstrom and Fuerst (1995) provided some simulation evidence to suggest that nominal interest rate pegs dominate constant money growth rate policies. While this suggests that a constant nominal interest rate peg is desirable within the context of their model, Carlstrom and Fuerst did not explicitly derive the optimal policy. Instead, their argument was based on quite different grounds than the traditional Poole (1970) argument for an interest rate–oriented policy.<sup>7</sup> In Poole’s analysis, stabilizing the interest rate insulated the real economy from purely financial disturbances. In contrast, Carlstrom and Fuerst appealed to standard tax-smoothing arguments to speculate, based on intertemporal tax considerations, that an interest rate peg might be optimal (see chapter 4).

The tax-smoothing argument for an interest rate peg is suggestive, but it is unlikely to be robust in the face of financial market disturbances. For example, in an analysis of optimal policy defined as money growth rate control, Ireland (1996) introduced a stochastic velocity shock by assuming the CIA constraint applies to only a time-varying fraction  $v_t$  of all consumption. In this case, the CIA constraint takes the form  $P_t v_t c_t \leq Q_t$ , where  $Q_t$  is the nominal quantity out of which cash goods must be purchased. It is straightforward to show that the Euler condition must be modified in this case to become

$$\frac{u_c(c_t)}{1 + v_t i_t} = \beta E_t R_t \left( \frac{u_c(c_{t+1})}{1 + v_{t+1} i_{t+1}} \right).$$

If  $v_t \equiv 1$ , the case considered by Carlstrom and Fuerst is obtained. If  $v_t$  is random, eliminating the intertemporal distortion requires that  $v_t i_t$  be pegged and that the nominal interest rate vary over time to offset the stochastic fluctuations in  $v_t$ . The introduction of a stochastic velocity disturbance suggests that an interest rate peg would not be optimal.

### 10.3 The Term Structure of Interest Rates

The distinction between real and nominal rates of interest is critical for understanding monetary policy issues, but another important distinction is that between short-term and long-term interest rates. Changes in the short-term interest rate that serves as the operational target for implementing monetary policy will affect aggregate spending decisions

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7. See chapter 12 for a discussion of Poole’s analysis of the choice of monetary policy operating procedures.



only if longer-term real interest rates are affected. While the use of an interest rate-oriented policy reduces the importance of money demand in the transmission of policy actions to the real economy, it raises to prominence the role played by the term structure of interest rates.

### 10.3.1 The Basic Expectations Theory

The exposition here builds on the expectations theory of the term structure. For a systematic discussion of the traditional theory of the term structure, see Cox, Ingersoll, and Ross (1985), Shiller (1990), or Campbell and Shiller (1991). Under the expectations hypothesis of the term structure, long-term nominal interest rates depend on expectations of the future path of nominal short-term interest rates. These future short-term rates will be functions of monetary policy, so expectations about future policy play an important role in determining the shape of the term structure.

Under the expectations theory of the term structure, the  $n$  period interest rate equals an average of the current short-term rate and the future short-term rates expected to hold over the  $n$  period horizon. For example, if  $i_{n,t}$  is the nominal yield to maturity at time  $t$  on an  $n$  period discount bond, while  $i_t$  is the one-period rate, the pure expectations hypothesis in the absence of uncertainty would imply<sup>8</sup> that

$$(1 + i_{n,t})^n = \prod_{i=0}^{n-1} (1 + i_{t+i}).$$

This condition ensures that the holding period yield on the  $n$ -period bond is equal to the yield from holding a sequence of one-period bonds. Taking logs of both sides and recalling that  $\ln(1 + x) \approx x$  for small  $x$  yields a common approximation:

$$i_{n,t} \approx \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}.$$

Since an  $n$ -period bond becomes an  $n - 1$  period bond after one period, these two relationships can also be written as

$$(1 + i_{n,t})^n = (1 + i_t) \prod_{i=0}^{n-2} (1 + i_{t+1+i}) = (1 + i_t) (1 + i_{n-1,t+1})^{n-1}$$

and

$$i_{n,t} \approx \left(\frac{1}{n}\right) i_t + \left(\frac{n-1}{n}\right) i_{n-1,t+1}.$$

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8. A constant risk premium could easily be incorporated. A time-varying risk premium is added to the analysis following.

These conditions will not hold exactly under conditions of uncertainty for two reasons. First, if risk-neutral investors equate expected one-period returns, then the one-period rate  $1 + i_t$  will equal  $E_t(1 + i_{n,t})^n / (1 + i_{n-1,t+1})^{n-1}$ , which, from Jensen's inequality, is not the same as  $(1 + i_{n,t})^n = (1 + i_t)E_t(1 + i_{n-1,t+1})^{n-1}$ .<sup>9</sup> Second, Jensen's inequality implies that  $\ln E_t(1 + i_{n-1,t+1})$  is not the same as  $E_t \ln(1 + i_{n-1,t+1})$ .

These two issues are ignored, however, to illustrate the basic linkages between the term structure of interest rates and monetary policy. It is sufficient to simplify further by dealing only with one- and two-period interest rates. Letting  $I_t \equiv i_{2,t}$  be the two-period rate (the long-term interest rate), the term structure equation becomes

$$(1 + I_t)^2 = (1 + i_t)(1 + E_t i_{t+1}), \quad (10.11)$$

and this is approximated as

$$I_t = \frac{1}{2} (i_t + E_t i_{t+1}). \quad (10.12)$$

The critical implication of this relationship for monetary policy is that the current term structure of interest rates will depend on current short-term rates *and* on market expectations of future short-term rates. Since the short-term rate is affected by monetary policy,  $I_t$  depends on expectations about future policy.

Equation (10.12) has a direct and testable empirical implication. Subtracting  $i_t$  from both sides, the equation can be rewritten as

$$I_t - i_t = \frac{1}{2} (E_t i_{t+1} - i_t).$$

If the current two-period rate is greater than the one-period rate (i.e.,  $I_t - i_t > 0$ ), then agents must expect the one-period rate to rise ( $E_t i_{t+1} > i_t$ ). Because we can always write  $i_{t+1} = E_t i_{t+1} + (i_{t+1} - E_t i_{t+1})$ , it follows that

$$\begin{aligned} \frac{1}{2} (i_{t+1} - i_t) &= I_t - i_t + \frac{1}{2} (i_{t+1} - E_t i_{t+1}) \\ &= a + b (I_t - i_t) + \theta_{t+1}, \end{aligned} \quad (10.13)$$

where  $a = 0$ ,  $b = 1$ , and  $\theta_{t+1} = \frac{1}{2} (i_{t+1} - E_t i_{t+1})$  is the error the private sector makes in forecasting the future short-term interest rate. Under the assumption of rational expectations,  $\theta_{t+1}$  will be uncorrelated with information available at time  $t$ . In this case, (10.13) forms a regression equation that can be estimated consistently by least squares. Unfortunately, estimates of such equations usually reject the joint hypothesis that  $a = 0$  and  $b = 1$ , generally obtaining point estimates of  $b$  significantly less than 1. Some of this empirical

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9. Suppose  $P_{n,t}$  is the time  $t$  price of an  $n$  period discount bond. Then  $P_{n,t}^{-1} = (1 + i_{n,t})^n$ . Since at time  $t + 1$  this becomes an  $n - 1$  period bond, the one-period gross return is

$$E_t P_{n-1,t+1} / P_{n,t} = E_t (1 + i_{n,t})^n / (1 + i_{n-1,t})^{n-1}.$$

evidence is summarized in Rudebusch (1995b) and McCallum (1984b). In section 10.5.1, the observed relationship between long- and short-term rates, as well as the way in which interest rates react to monetary policy, is shown to depend on the manner in which policy is conducted.

The one-period ahead forward rate is defined as

$$f_t^1 = \frac{(1 + I_t)^2}{1 + i_t} - 1.$$

If the pure expectations hypothesis of the term structure holds, (10.11) implies that  $f_t^1$  is equal to the market's expectation of the future one-period rate. Hence, forward rates derived from the term structure are often used to gain information on expectations of future interest rates (see Dahlquist and Svensson 1996; Söderlind and Svensson 1997; Rudebusch 2002b).

### 10.3.2 Expected Inflation and the Term Structure

The term structure plays an important role as an indicator of inflationary expectations. Since market interest rates are the sum of an expected real return and an expected inflation premium, the nominal interest rate on an  $n$ -period bond can be expressed as

$$i_t^n = \frac{1}{n} \sum_{i=0}^n E_t r_{t+i} + \frac{1}{n} E_t \bar{\pi}_{t+n},$$

where  $E_t r_{t+i}$  is the one-period real rate expected at time  $t$  to prevail at  $t + i$  and  $E_t \bar{\pi}_{t+n} \equiv E_t p_{t+n} - p_t$  is the expected change in log price from  $t$  to  $t + n$ . If real rates are stationary around a constant value  $\bar{r}$ , then  $\frac{1}{n} \sum_{i=0}^n E_t r_{t+i} \approx \bar{r}$  and

$$i_t^n \approx \bar{r} + \frac{1}{n} E_t \bar{\pi}_{t+n}.$$

In this case, fluctuations in the long-term rate are caused mainly by variations in expected inflation. Based on a study of interest rates on nominal and indexed government bonds in the United Kingdom, Barr and Campbell (1997) concluded that “almost 80 percent of the movement in long-term nominal rates appears to be due to changes in expected long-term inflation.” For this reason, increases in long-term nominal rates of interest are often interpreted as signaling an increase in expected inflation.

When both nominal bonds and bonds whose returns are indexed to inflation are traded, a comparison of the returns on the two assets provides information about the expected rate of inflation. However, if there are time-varying inflation risk premiums, this comparison may make it difficult to tell whether the different interest rates on the nominal bond and the real indexed bond are reflecting changes in expected inflation or changes in the risk premium. Ravenna and Seppälä (2007), using a new Keynesian model calibrated to U.S. data, found that inflation risk premiums are small and not highly volatile. Thus, indexed bonds can be used to extract information about expected inflation.

A policy-induced rise in short-term rates that is accompanied by a decline in long-term rates would be interpreted as meaning that the contractionary policy (the rise in short-term rates) is expected to lower future inflation, thereby lowering nominal long-term interest rates and future short-term rates. Conversely, a cut in the short-run policy rate that is accompanied by a rise in long-term rates would provide evidence that the central bank was following an inflationary policy. Goodfriend (1993) provided an interpretation of U.S. monetary policy in the period 1979–1992 based on the notion that long-term interest rates provide important information on market inflation expectations.

Buttiglione, Del Giovane, and Tristani (1998) examined the impact of policy rate changes on forward rates in OECD countries. Under the hypothesis that changes in monetary policy do not affect the expected real interest rate far in the future, changes in the forward rates implied by the term structure should reflect the impact of the policy change on expected future inflation. The forward interest rate on a one-period discount bond  $n$  periods in the future can be derived from the rates on  $n$  and  $n + 1$  period bonds and is equal to

$$f_t^n = \frac{(1 + i_{n+1,t})^{n+1}}{(1 + i_{n,t})^n} - 1 \approx (n + 1)i_{n+1,t} - ni_{n,t}.$$

Thus, if long-term expected real rates are constant, then for large  $n$ ,  $f_t^n \approx \bar{r} + E_t \bar{\pi}_{t+n+1} - E_t \bar{\pi}_{t+n} = \bar{r} + E_t[(p_{t+n+1} - p_{t+n})]$ , or  $f_t^n \approx \bar{r} + E_t \pi_{t+n+1}$ . The forward rate then provides a direct estimate of future expected rates of inflation.<sup>10</sup> Interestingly, Buttiglione, Del Giovane, and Tristani found that a contractionary shift in policy (a rise in the short-term policy interest rate) lowered forward rates for some countries and raised them for others. The response of forward rates was closely related to a country's average inflation rate; for low-inflation countries, a policy action that increased short-term rates was estimated to lower forward rates. This response is consistent with the hypothesis that the increases in the short-term rate represented a credible policy expected to reduce inflation. In countries with high-inflation experiences, increases in short-term rates were not associated with decreases in forward rates.

A key maintained hypothesis in the view that movements in interest rates reveal information about inflation expectations is that the Fisher hypothesis, the hypothesis that nominal interest rates incorporate a premium for expected inflation, holds. Suppose that the real rate is stationary around an average value of  $\bar{r}$ . Then, since  $i_t = r_t + \pi_{t+1}^e = r_t + \pi_{t+1} + e_{t+1}$ , where  $e_{t+1}$  is the inflation forecast error (which is stationary under rational expectations), the ex post real rate  $i_t - \pi_{t+1}$  is stationary. Thus, if the nominal interest rate and the inflation rate are nonstationary, they must be cointegrated under the Fisher hypothesis. This is the sense in which long-term movements in inflation should be reflected in the nominal

10. Söderlind and Svensson (1997) provided a survey of techniques for estimating market expectations from the term structure.

interest rate. Mishkin (1992) adopted this cointegrating interpretation of the Fisher relationship to test for the presence of a long-term relationship between inflation and nominal interest rates in the United States. If over a particular time period neither  $i$  nor  $\pi$  is integrated of order 1 but instead both are stationary, there is no real meaning to the statement that permanent shifts in the level of inflation should cause similar movements in nominal rates because such permanent shifts have not occurred. If either  $i$  or  $\pi$  is  $I(1)$ , they should both be  $I(1)$ , and they should be cointegrated. Mishkin found the evidence to be consistent with the Fisher relationship.

## 10.4 Macrofinance

A recent literature identifies the latent factors employed in finance models of the term structure with macroeconomic variables such as inflation, real economic activity, and monetary policy. The term structure is represented using linear, affine, no-arbitrage models, as in Dai and Singleton (2000). For discussions of no-arbitrage, affine models, see Piazzesi (2010) and Hamilton and Wu (2012a). The unobserved latent variables that determine bond prices in these models are linked to macroeconomic variables, either through nonstructural statistical models such as a VAR (e.g., Ang and Piazzesi 2003) or by using a new Keynesian model to represent macroeconomic and monetary policy outcomes (e.g., Rudebusch and Wu 2007; 2008). Diebold, Piazzesi, and Rudebusch (2005) provided an overview of this growing research area and discussed some of the issues that arise in linking finance model and macroeconomic models. Wu and Xia (2016) extended the basic affine model to the situation in which the short-term rate is constrained at zero and derived a shadow rate that has been used to assess the stance of monetary policy when the normal policy rate is at zero.

### 10.4.1 Affine Models of the Term Structure

Suppose there are two latent (unobserved) factors that determine bond prices.<sup>11</sup> Following Rudebusch and Wu (2007), denote these factors by  $L_t$  and  $S_t$ , and assume they follow a VAR process given by

$$F_t \equiv \begin{bmatrix} L_t \\ S_t \end{bmatrix} = \rho \begin{bmatrix} L_{t-1} \\ S_{t-1} \end{bmatrix} + \Sigma e_t = \rho F_{t-1} + \Sigma e_t, \quad (10.14)$$

where  $e_t$  is independently and identically distributed as a normal mean zero unit variance process, and  $\Sigma$  is a  $2 \times 2$  nonsingular matrix. Assume further that the short-term interest rate  $i_t$  can be written as a function of the two factors. Specifically,

$$i_t = \delta_0 + \delta_1 F_t. \quad (10.15)$$

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11. Rudebusch and Wu (2008) found that two-factor models are rich enough to fit the data adequately.

Finally, assume the prices of risk associated with each factor are linear functions of the two factors, so that if  $\Lambda_{i,t}$  is the price of risk associated with conditional volatility of factor  $i$ :

$$\Lambda_t = \begin{bmatrix} \Lambda_{L,t} \\ \Lambda_{S,t} \end{bmatrix} = \lambda_0 + \lambda_1 F_t. \quad (10.16)$$

If  $i_t$  is the return on a one-period bond, then the structure given by (10.14)–(10.16), together with the assumption that no-arbitrage opportunities exist, allows one to price longer-term bonds. In particular, if  $b_{j,t}$  is the log price of a  $j$ -period nominal bond, one can show that

$$b_{j,t} = \bar{A}_j + \bar{B}_j F_t,$$

where

$$\bar{A}_1 = -\delta_0, \bar{B}_1 = -\delta_1,$$

and for  $j = 2, \dots, J$ ,

$$\bar{A}_{j+1} - \bar{A}_j = \bar{B}_j (-\Sigma \lambda_0) + \frac{1}{2} \bar{B}_j \Sigma \Sigma' \bar{B}_j + \bar{A}_1,$$

$$\bar{B}_{j+1} = \bar{B}_j (\rho - \Sigma \lambda_1) + \bar{B}_1.$$

Empirical research aimed at estimating this type of no-arbitrage model generally finds that one factor affects yields at all maturities and so is called the level factor, while the other factor affects short- and long-term rates differently and so is called the slope factor.

The macrofinance literature has attempted to identify the level and slope factors with macroeconomic factors. For example, in new Keynesian models, the short-term interest rate is often represented in terms of a Taylor rule of the form

$$i_t = r^* + \pi_t^T + a_\pi (\pi_t - \pi_t^T) + a_x x_t,$$

where  $\pi_t^T$  is the central bank's inflation target and  $x_t$  is the output gap. In this case, changes in the inflation target should affect nominal interest rates at all maturities by altering inflation expectations. Thus, it would seem to be a prime candidate for the level factor. The slope factor might then be capturing the central bank's policy actions intended to stabilize the economy in the short run. Thus, one could model the factors explicitly in terms of the policy behavior of the central bank.<sup>12</sup>

Of course, this approach requires that the behavior of inflation, the inflation target, and the output gap also be modeled. As noted, Ang and Piazzesi (2003) represented the behavior of the macroeconomic variables using a VAR representation. They grouped variables into a set related to inflation and a set related to real activity. By then using the principal

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12. See section 10.5.

component from each group, they obtained the two factors that determine the term structure. Macroeconomic factors are found to explain movements of short- and medium-term interest rates but little of the long-term interest rate. Rudebusch and Wu (2008) employed a simplified new Keynesian model to model the behavior of macroeconomic variables, and Rudebusch and Wu (2007) argued that shifts in the pricing of risk associated with the Fed's inflation target can account for shifts in the behavior of the term structure in the United States.

#### 10.4.2 A Preferred Habitat Term Structure Model

The standard model of the term structure assumes that investors are indifferent between a  $j$ -period bond and a  $k$ -period bond if their expected instantaneous holding period returns are equal. During the 1960s, Modigliani and Sutch (1967) developed a model of the term structure of interest rates in which investors had preferences for bonds of different maturities such that an investor might prefer the  $j$ -period bond to the  $i$ -period bond even if their expected returns were equal. For example, a pension fund might prefer, or be legally required, to hold a certain fraction of its portfolio in long-term bonds. Even if the expected return on a short-term bond were greater than that on the long-term bond, the pension fund would still want to hold long-term bonds.

When investors prefer to hold bonds at some maturities relative to other maturities, they are said to have preferred habitats. If, for example, investors in the aggregate prefer short-term government bonds over long-term government bonds, then a reduction of the supply of short-term bonds will cause their price to rise (their yield to fall) relative to long-term bonds. Thus, changes in the relative supply of bonds of different maturities will lead to changes in yields. Preferred habitat models implied that balance sheet policies undertaken in recent years by, for example, the Federal Reserve, in which the central bank sells short-term government bonds from its portfolio and purchases long-term bonds, could affect long-term interest rates relative to short-term interest rates. In this example, the private sector ends up holding more short-term bonds and fewer long-term bonds, and the yield on long-term bonds should fall. Such policies were also undertaken in the 1960s in an attempt to twist the term structure, lowering long-term rates relative to short-term rates.<sup>13</sup>

While the preferred habitat model of Modigliani and Sutch (1967) was incorporated into the early large-scale macroeconomic models developed in the 1960s and early 1970s, this approach was replaced by the assumption that investors viewed bonds of different maturities as perfect substitutes, once adjusted for any differences in risk, so that, in equilibrium, they had to yield the same risk-adjusted rate of return.

Vayanos and Vila (2009) developed a modern theory of the term structure that incorporates preferred habitats among investors. There are two basic classes of investors. One

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13. Swanson (2011) evaluated the effects on interest rates of the 1960s Operation Twist and the more recent balance sheet policies. Balance sheet policies are discussed in section 11.5.

class has preferred habitats; an investor in this class of type  $\tau$  demands bonds of maturity  $\tau$ . The second class of investors are arbitrageurs who link together the markets for bonds of different maturity. They will sell bonds with low expected returns and buy bonds with high expected returns. If these investors were risk-neutral, their actions would arbitrage away any differences in expected returns. Vayanos and Vila assumed that arbitrageurs are risk-averse; this limits their willingness to arbitrage away all excess returns.

The model is in continuous time. There is a continuum of zero coupon bonds in zero net supply. Bonds are of maturity  $\tau \in (0, T]$ , and a bond of maturity  $\tau$  pays one dollar at  $t + \tau$ . Let  $P_{t,\tau}$  be the time  $t$  price of a bond with maturity  $\tau$ . Define  $R_{t,\tau}$  as the spot rate for maturity  $\tau$  at time  $t$ . Spot rates and prices are related by  $e^{\tau R_{t,\tau}} P_{t,\tau} = 1$ ; for asset value to grow from  $P_{t,\tau}$  to 1 in  $\tau$  periods, it grows at rate  $R_{t,\tau}$ . Taking logs,

$$R_{t,\tau} = -\frac{\log P_{t,\tau}}{\tau}.$$

The short-term rate is defined as

$$r_t = \lim_{\tau \rightarrow 0} R_{t,\tau}.$$

This rate is taken to be exogenous, and it follows the Ornstein-Uhlenbeck process

$$dr_t = \kappa_r (\bar{r} - r_t) dt + \sigma_r dB_{r,t}, \quad (10.17)$$

where  $(\bar{r}, \kappa_r, \sigma_r)$  are positive constants and  $B_{r,t}$  is a Brownian motion.<sup>14</sup>

The demand for bonds of maturity  $\tau$  by preferred habitat investors is assumed to be

$$y_{t,\tau} = \alpha(\tau) \tau (R_{t,\tau} - \beta_{t,\tau}), \quad (10.18)$$

where the function  $\alpha(\tau)$  is only restricted to be positive. Thus, if the return on a bond of maturity  $\tau$  increases, the demand for it increases.

Suppose the only investors were preferred habitat investors. Then since the supply of maturity  $\tau$  bonds was assumed to be zero, equilibrium requires that demand also equal zero, or

$$y_{t,\tau} = \alpha(\tau) \tau (R_{t,\tau} - \beta_{t,\tau}) = 0$$

for all  $\tau$ . It follows that

$$R_{t,\tau} = \beta_{t,\tau}.$$

If  $\beta_{t,\tau}$  is unrelated across maturities, then the model would be characterized by complete market segmentation. The return at each maturity would be independent of the returns at other maturities. For example, a rise in  $\beta_{t,s}$  for  $s \neq \tau$  would have no effect on the demand

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14. The Ornstein-Uhlenbeck process can also be considered as the continuous-time analogue of a discrete-time AR(1) process.



for maturity  $\tau$  bonds and consequently no effect on  $R_{t,\tau}$ . To further simplify, assume  $\beta_{t,\tau} = \bar{\beta}$ .<sup>15</sup>

Vayanos and Vila (2009) now introduced arbitrageurs. These agents ensure that bonds of near equal maturities trade at similar prices. The representative arbitrageur chooses a portfolio by investing  $x_{t,\tau}$  in maturity  $\tau$  bonds to maximize utility, which is a function of the mean and variance of the portfolio's return. That is, arbitrageurs choose  $x_{t,\tau}$  to

$$\max \left[ E_t (dW_t) - \frac{a}{2} \text{Var}_t (dW_t) \right],$$

subject to the evolution of wealth given by

$$dW_t = \left( W_t - \int_0^T x_{t,\tau} \right) r_t dt + \int_0^T x_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}}. \quad (10.19)$$

The coefficient  $a$  is the measure of risk aversion. In (10.19) the first term is the return  $r_t$  on the holdings of the instantaneous bond, and the second term reflects the capital gains or losses due to price changes on the bond portfolio. The presence of arbitrageurs eliminates any risk-free arbitrage opportunities. Remaining differences in returns across maturities reflect risk differences.

Because the only risk factor is the exogenous short-term rate  $r_t$ , guess that the solution for the price of a  $\tau$  maturity bond is

$$P_{t,\tau} = e^{-[A_r(\tau)r_t + C(\tau)]}, \quad (10.20)$$

where  $A_r(\tau)$  and  $C(\tau)$  are unknown coefficients that depend on maturity.

From Ito's lemma and (10.17),

$$\frac{dP_{t,\tau}}{P_{t,\tau}} = \mu_{t,\tau} dt - A_r(\tau) \sigma_\tau dB_{r,t}, \quad (10.21)$$

where the instantaneous return on the bond of maturity  $\tau$  is

$$\mu_{t,\tau} \equiv A'_r(\tau) r_t + C'(\tau) - A_r(\tau) \kappa_r (\bar{r} - r_t) + \frac{1}{2} A_r(\tau)^2 \sigma_r^2. \quad (10.22)$$

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15. Vayanos and Vila (2009) allowed for cross-maturity effects by assuming  $\beta_{t,\tau}$  for each  $\tau$  is a function of  $K$  common stochastic factors  $\beta_{k,t}$ ,  $k = 1 \dots K$ :

$$\beta_{t,\tau} = \bar{\beta} + \sum_{k=1}^K \theta_k(\tau) \beta_{k,t},$$

where  $\bar{\beta}$  is a constant,  $\beta_{k,t}$  are  $K$  demand risk factors, and  $\theta_k(\tau)$  measures how each factor affects the cross-section of maturities in the absence of arbitrageurs. For each demand factor, assume

$$d\beta_{k,t} = -\kappa_{\beta,k} \beta_{k,t} dt + \sigma_{\beta,k} dB_{\beta,k,t},$$

with the Brownian motion  $B_{\beta,k,t}$  independent of  $B_{\tau,t}$  and  $B_{\beta,k',t}$  for all  $k \neq k'$ . For simplicity, attention is restricted to the  $K = 0$  case, in which the only factor is the exogenous short rate  $r_t$ .

Using (10.21) in (10.19), the first-order conditions for the arbitrageur's portfolio choice problem become

$$\mu_{t,\tau} - r_t = A_r(\tau) \lambda_{r,t} \quad (10.23)$$

for all  $\tau$ , where

$$\lambda_{r,t} \equiv a\sigma_r^2 \int_0^T x_{t,\tau} A_r(\tau) d\tau. \quad (10.24)$$

According to (10.23), the excess return  $\mu_{t,\tau} - r_t$  is proportional to a bond's sensitivity to the short-term rate as measured by  $A_r(\tau) \lambda_{r,t}$ . The proportionality coefficient  $\lambda_{r,t}$  is the same for all bonds and is the market price of short-term interest rate risk. The absence of arbitrage opportunities requires the excess return to be the same for all  $\tau$  (i.e.,  $\lambda_{r,t}$  is independent of  $\tau$ ).  $\lambda_{r,t}$  is the expected excess return arbitrageurs require for holding an additional marginal unit of risk. Risk-averse arbitrageurs require more compensation the more sensitive the portfolio is to risk and the more risk-averse they are (i.e., the higher  $a$  is).

Recall that bonds of maturity  $\tau$  are in zero net supply. Equating demand, which now arises from preferred habitat investors and arbitrageurs, to supply, the market equilibrium condition becomes

$$y_{t,\tau} + x_{t,\tau} = 0.$$

Thus, given that attention is restricted to the special case of  $\beta_{t,\tau} = \bar{\beta}$ , equilibrium requires  $x_{t,\tau} = -y_{t,\tau} = -\alpha(\tau)\tau(R_{t,\tau} - \bar{\beta})$  from (10.18). Using this equilibrium condition in (10.24),

$$\lambda_{r,t} = -a\sigma_r^2 \int_0^T \alpha(\tau)\tau(R_{t,\tau} - \bar{\beta})A_r(\tau) d\tau.$$

From (10.20), the fact that  $R_{t,\tau} = -\log P_{t,\tau}/\tau$ , and  $\beta_{t,r} = \bar{\beta}$ , the expression for  $\lambda_{r,t}$  can be written as

$$\lambda_{r,t} = a\sigma_r^2 \int_0^T \alpha(\tau)(\bar{\beta}\tau - A_r(\tau)r_t - C(\tau))A_r(\tau) d\tau, \quad (10.25)$$

which is affine (linear) and decreasing in the short-term rate  $r_t$ . Vayanos and Vila (2009) provided details on the equilibrium functions  $A_r(\tau)$  and  $C(\tau)$ .

The key implication of their model is that combining investors with maturity preferences and investors who are risk-averse arbitrageurs, one obtains a model of term premiums in which the asset stocks that arbitrageurs must hold can affect relative rates of return. In their model, asset stocks are in zero net supply. Consider, though, a variant of their model in which the assets being priced are government bonds of varying maturity. If  $b_{t,\tau}$  is the stock of  $\tau$  maturity bonds, then market equilibrium requires

$$y_{t,\tau} + x_{t,\tau} = b_{t,\tau}.$$

In this case,

$$\lambda_{r,t} = a\sigma_r^2 \int_0^T [b_{t,\tau} - \alpha(\tau)\tau(R_{t,\tau} - \bar{\beta})] A_r(\tau) d\tau.$$

Variations in  $b_{t,\tau}$  through, for example, central bank balance sheet policies of the type considered in chapter 11 would affect asset prices. Because arbitrageurs are risk-averse, if  $b_{t,\tau}$  is increased and  $b_{t,s}$  is decreased,  $\tau \neq s$ , they need to be induced by higher returns to hold the larger stock of maturity  $\tau$  bonds, while the return on maturity  $s$  bonds can fall. In fact, Hamilton and Wu (2012b) used the Vayanos and Vila model to estimate the impact on long-term interest rates of a change in the maturity composition of government debt.

## 10.5 Policy and the Term Structure

Long-term interest rates depend on expectations of future short-term rates. These expectations are affected by the central bank's monetary policy. The dependence of interest rates and the term structure on monetary policy implies that the results of empirical studies of the term structure should depend on the policy rule followed by the central bank. Evans and Marshall (1998) examined the impact of policy shocks on the terms structure; Rudebusch (1995a), Fuhrer (1996), Balduzzi et al. (1998), and McCallum (2005) examined the connection between the Fed's interest rate-setting behavior, the dynamics of short-term interest rates, and empirical tests of the expectations model of the term structure. These connections are illustrated here with a simple example based on McCallum (2005). Then an example of an affine model linking policy to the term structure due to Smith and Taylor (2009) is developed.<sup>16</sup>

### 10.5.1 A Simple Example

Following McCallum (2005), consider a two-period model of nominal interest rates in which, as before,  $I$  is the two-period rate and  $i$  is the one-period rate:

$$I_t = \frac{1}{2} (i_t + E_t i_{t+1}) + \xi_t, \quad (10.26)$$

where  $\xi$  is a random variable that represents a time-varying term premium. Equation (10.12) implied that the pure expectations model of the term structure holds exactly, without error; the term premium  $\xi$  introduced in (10.26) allows for a stochastic deviation from the exact form of the expectations hypothesis. Variation in risk factors might account for the presence of  $\xi$ . Suppose that the term premium is serially correlated:

$$\xi_t = \rho \xi_{t-1} + \eta_t, \quad (10.27)$$

where  $\eta_t$  is a white noise process.

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16. Gallmeyer, Hollifield, and Zin (2005) provided an explicit analysis of the role of the policy rule in a no-arbitrage model of the term structure.

Let  $\varepsilon_{t+1} = i_{t+1} - E_t i_{t+1}$  be the expectational error in forecasting the future one-period rate; then (10.26) implies that

$$\frac{1}{2}(i_{t+1} - i_t) = I_t - i_t - \xi_t + \frac{1}{2}\varepsilon_{t+1}, \quad (10.28)$$

which is usually interpreted to mean that the slope coefficient in a regression of one-half the change in the short-term rate on the spread between the long-term rate and the short-term rate should equal 1. As noted, actual estimates of this slope coefficient have generally been much less than 1 and have even been negative.

The final aspect of the model is a description of the behavior of the central bank. Since many central banks use the short-term interest rate as their operational policy instrument, and since they often engage in interest rate smoothing, McCallum assumes that  $i_t = i_{t-1} + \mu(I_t - i_t) + \zeta_t$ .<sup>17</sup> However, problems of multiple equilibria may arise when policy responds to forward-looking variables such as  $I_t$  (see Bernanke and Woodford 1997 and problem 4 at the end of this chapter). To avoid this possibility, assume that policy adjusts the short-term rate according to

$$i_t = i_{t-1} - \mu\xi_t + \zeta_t, \quad (10.29)$$

where  $\zeta_t$  is a white noise process and  $|\mu| < 1$ . According to (10.29), a rise in the risk premium in the long-term rate induces a policy response that lowers the short-term rate. Exogenous changes in risk that alter the term structure might also affect consumption or investment spending, leading the central bank to lower short-term interest rates to counter the contractionary effects of a positive realization of  $\xi_t$ . Because no real explanation has been given for  $\xi$  or why policy might respond to it, it is important to keep in mind that this is only an illustrative example that suggests how policy behavior might affect the term structure.

Equations (10.26)–(10.29) form a simple model that can be used to study how policy responses to the term structure risk premium (i.e.,  $\mu$ ) affect the observed relationship between short-term and long-term interest rates. From (10.29),  $E_t i_{t+1} = i_t - \mu\rho\xi_t$ , so

$$I_t = \frac{1}{2}(i_t + E_t i_{t+1}) + \xi_t = i_t + \left(1 - \frac{\mu\rho}{2}\right)\xi_t.$$

This implies that

$$\left(1 - \frac{\mu\rho}{2}\right)^{-1}(I_t - i_t) = \xi_t.$$

Using this result, (10.28) can be written as

$$\begin{aligned} \frac{1}{2}(i_{t+1} - i_t) &= I_t - i_t - \left(1 - \frac{\mu\rho}{2}\right)^{-1}(I_t - i_t) + \frac{1}{2}\varepsilon_{t+1}, \quad \text{or} \\ \frac{1}{2}(i_{t+1} - i_t) &= -\left(\frac{\mu\rho}{2 - \mu\rho}\right)(I_t - i_t) + \frac{1}{2}\varepsilon_{t+1}, \end{aligned} \quad (10.30)$$

17. McCallum actually allowed the coefficient on  $i_{t-1}$  in (10.29) to differ from 1.

so that one would expect the regression coefficient on  $I_t - i_t$  to be  $-\mu\rho/(2 - \mu\rho)$ , not 1. In other words, the estimated slope of the term structure, even when the expectations model is correct, will depend on the serial correlation properties of the term premium ( $\rho$ ) and on the policy response to the spread between long- and short-term rates ( $\mu$ ). The problem arises even though (10.28) implies that  $\frac{1}{2}(i_{t+1} - i_t) = a + b(I_t - i_t) + x_{t+1}$ , with  $a = 0$  and  $b = 1$ , because the error term  $x_{t+1}$  is equal to  $-\xi_t + \frac{1}{2}\varepsilon_{t+1}$ ; since this is correlated with  $I_t - i_t$ , ordinary least squares is an inconsistent estimator of  $b$ .

The important lesson of McCallum's analysis is that observed term structure relationships can be affected by the way monetary policy is conducted. In chapter 8, the new Keynesian model was used to analyze interest rate policies. In a general equilibrium context, it was seen that the interest rate rule had to incorporate a response to endogenous variables to ensure a locally unique stationary equilibrium. Suppose the rule (10.29) is replaced by

$$\dot{i}_t = \rho i_{t-1} + (1 - \rho)\phi\pi_t - \mu\xi_t + \zeta,$$

with  $0 \leq \rho, 1$ , and  $\phi > 1$ . Solving for the impact of the policy rule parameters ( $\rho$ ,  $\phi$ , and  $\mu$ ) now requires the specification of a complete model to determine the equilibrium behavior of inflation.<sup>18</sup>

McCallum assumed policy responded to the slope of the term structure, which in his model reflected variations in the risk premium. Cochrane and Piazzesi (2002) found evidence that the interest rate-setting behavior of the Federal Reserve has been affected by both long-term interest rates and the slope of the yield curve. If the former reflects long-term inflation expectations and the latter helps forecast real economic activity, then this behavior would be broadly consistent with Taylor rules (see chapter 8). Gallmeyer, Hollifield, and Zin (2005) provided a more modern treatment of McCallum's results in the context of models that allow for endogenous variation in risk premiums.<sup>19</sup> They showed how the policy behavior assumed by McCallum can be reconciled with Taylor rule representations of monetary policy. Ravenna and Seppälä (2006) showed how a new Keynesian model can account for rejections of the expectations model of the term structure, and McGough, Rudebusch, and Williams (2005) considered monetary policy rules that respond to long-term interest rates.

Rather than employing an equation such as (10.29) to represent policy behavior, Rudebusch (1995b) used data from periods of funds rate targeting (1974–1979 and 1984–1992) to estimate a model of the Federal Reserve's target for the funds rate. He was then able to simulate the implied behavior of the term structure, using the expectations hypothesis to link funds rate behavior to the behavior of longer-term interest rates. He found that the manner in which the Fed has adjusted its target can account for the failure of the spread between long- and short-term rates to have much predictive content for changes in long-term rates, at least at horizons of 3 to 12 months (that is, for the failure to obtain a

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18. See problem 8 at the end of this chapter.

19. These models were discussed in section 10.4.

coefficient of 1, or even a significant coefficient, in a regression of  $\frac{1}{2}(i_{t+1} - i_t)$  on  $(I_t - i_t)$ . Thus, if the 3-month rate exceeds the funds rate, (10.13) would appear to predict a rise in the funds rate. As Rudebusch demonstrated, the Fed tends to set its target for the funds rate at a level it expects to maintain. In this case, any spread between the funds rate and other rates has no implications for future changes in the funds rate (in terms of (10.29),  $\mu \approx 0$ ). Only as new information becomes available might the target funds rate change.

Fuhrer (1996) provided further evidence on the relationship between the Fed's policy rule and the behavior of long-term interest rates. He estimated time-varying parameters of a policy reaction rule for the funds rate consistent with observed long-term rates. Agents are assumed to use the current parameter values of the policy rule to forecast future short-term rates.<sup>20</sup> Fuhrer argued that the parameters he obtained are consistent with general views on the evolution of the Fed's reaction function. Balduzzi et al. (1998) found that during the 1989–1996 period of federal funds rate targeting in the United States, the term structure was consistent with a regime in which changes in the target for the funds rate occurred infrequently but were partially predictable. In related literature, Mankiw and Miron (1986) and Mankiw, Miron, and Weil (1987) studied how the founding of the Federal Reserve affected the seasonal behavior of interest rates. See also Fische and Wohar (1990), Angelini (1994a;b), and Mankiw, Miron, and Weil (1994).

### 10.5.2 An Affine Example

Smith and Taylor (2009) combined an affine model of the term structure with a Taylor rule describing monetary policy to show how long-term interest rates depend on the coefficients in the Taylor rule. They assumed the central bank sets the short-term interest rate  $r_t$  according to a Taylor rule of the form

$$r_t = \delta_\pi \pi_t + \delta_x x_t, \quad (10.31)$$

where  $r_t$  is the short-term rate,  $\pi_t$  is inflation, and  $x_t$  is a measure of real economic activity. For simplicity, only the case with  $\delta_x = 0$  and policy responding solely to inflation is considered. The yield to maturity on an  $n$  period zero coupon bond is

$$i_t^{(n)} = -\frac{1}{n} \log \left( P_t^{(n)} \right), \quad (10.32)$$

where  $P_t^{(n)}$  is the time  $t$  price of the bond. Because an  $n + 1$ -period bond becomes an  $n$ -period bond after one period, arbitrage ensures that

$$P_t^{(n+1)} = E_t m_{t+1} P_{t+1}^{(n)}, \quad (10.33)$$

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20. As Fuhrer noted, this behavior is not fully rational because agents presumably learn that the policy rule changes over time. However, the time-varying parameters approximately follow a random walk process, so using the current values to forecast future policy does not introduce large systematic errors.

where  $m_{t+1}$  is the stochastic discount factor. Smith and Taylor assumed this is given by

$$m_{t+1} = \exp\left(-r_t - 0.5\lambda_t^2 - \lambda_t\varepsilon_{t+1}\right), \quad (10.34)$$

where  $\lambda_t$  is the risk factor and  $\varepsilon_{t+1}$  is an i.i.d. random normal variable. Further, assume

$$\lambda_t = -\gamma_0 - \gamma_1\pi_t,$$

$$\pi_t = \pi_{t-1} - \phi(r_{t-1} - \pi_{t-1}) + \sigma\varepsilon_t,$$

where  $\varepsilon_t$  is an i.i.d. standard normal.

How does the term structure depend on monetary policy? In this simple example, policy is completely described by the choice of  $\delta_\pi$ , the central bank's response to inflation. If yields to maturity are linear functions of inflation of the form

$$i_t^{(n)} = a_n + b_n\pi_t,$$

the issue is to determine how  $a_n$  and  $b_n$  depend on  $\delta_\pi$ . Because  $i_t^{(1)} = r_t$ , the policy rule (10.31) implies  $i_t^{(1)} = \delta_\pi\pi_t$ , so  $a_1 = 0$  and  $b_1 = \delta_\pi$  (recall  $\delta_x$  has been set to zero). With this result for  $n = 1$ , the relationship between yields and prices given by (10.32) and the arbitrage condition given by (10.33) can be used.

The price of a one-period bond at time  $t + 1$  is

$$P_{t+1}^{(1)} = \exp(-i_{t+1}) = \exp(-r_{t+1}).$$

It follows from (10.33) that

$$P_t^{(2)} = E_t m_{t+1} P_{t+1}^{(1)} = E_t m_{t+1} \exp(-r_{t+1}) = E_t m_{t+1} \exp(-\delta_\pi \pi_{t+1}).$$

Using the definition of the stochastic discount factor, the specification for the risk factor, and the process describing the evolution inflation, Smith and Taylor showed that the coefficient on inflation in the equilibrium expression for the yield on the  $n$ -period bond is

$$b_n = \frac{\delta_\pi \sum_{i=0}^{n-1} (1 - \phi(\delta_\pi - 1) + \sigma\gamma_1)^i}{n}.$$

This implies

$$b_{n+1} = \left(\frac{n}{n+1}\right) b_n + \frac{\delta(1 - \phi(\delta_\pi - 1) + \sigma\gamma_1)^n}{n+1}.$$

They noted that

$$\delta_\pi > 1 + \frac{\sigma\gamma_1}{\phi}$$

is required to prevent  $b_n$  from exploding as  $n$  increases. The requirement that policy respond more than one-to-one with inflation is similar to the Taylor principle (see chapter 8).

Smith and Taylor showed that an increase in the policy reaction coefficient  $\delta_\pi$  should increase the term structure coefficients. They estimated a policy rule and a term structure model for 1960–1979 and 1984–2006 and found evidence that the policy response coefficient  $\delta_\pi$  increased from the earlier period to the latter period and that the term structure coefficients, the  $b_n$  coefficients, also increased. This provides supporting evidence that the systematic behavior of monetary policy affects the term structure of interest rates.

## 10.6 Financial Frictions in Credit Markets

Money has traditionally played a special role in macroeconomics and monetary theory because of the relationship between the nominal stock of money and the aggregate price level. The importance of money for understanding the determination of the general level of prices and average inflation rates, however, does not necessarily imply that the stock of money is the key variable that links the real and financial sectors or the most appropriate indicator of the short-run influence of financial factors on the economy. However, many economists have argued that monetary policy has direct effects on aggregate spending that do not operate through traditional interest rate or exchange rate channels, and a large literature has focused on credit markets as playing a critical role in the transmission of monetary policy actions to the real economy.

The *credit view* stresses the distinct role played by financial assets and liabilities. Rather than aggregate all nonmoney financial assets into a single category called *bonds*, the credit view argues that macroeconomic models need to distinguish between different nonmonetary assets, either along the dimension of bank versus nonbank sources of funds or, more generally, internal versus external financing. The credit view also highlights heterogeneity among borrowers, stressing that some borrowers may be more vulnerable to changes in credit conditions than others. Finally, investment may be sensitive to variables such as net worth or cash flow if agency costs associated with imperfect information or costly monitoring create a wedge between the cost of internal and external funds. A rise in interest rates may have a much stronger contractionary impact on the economy if balance sheets are already weak, introducing the possibility that nonlinearities in the impact of monetary policy may be important.

The credit channel also operates when shifts in monetary policy alter either the efficiency of financial markets in matching borrowers and lenders or the extent to which borrowers face rationing in credit markets so that aggregate spending is influenced by liquidity constraints. There are several definitions of nonprice credit rationing. Jaffee and Russell (1976) defined credit rationing as existing when at the quoted interest rate the lender supplies a smaller loan than the borrower demands. Jaffee and Stiglitz (1990), however, pointed out that this practice represents standard price rationing; larger loans are normally accompanied by a higher default rate and therefore carry a higher interest rate. Instead, Jaffee and Stiglitz characterized “pure credit rationing” as occurring when, among a group of agents



(firms or individuals) who appear to be identical, some receive loans and others do not. Stiglitz and Weiss (1981) defined equilibrium credit rationing as being present whenever “either (a) among loan applicants who appear to be identical some receive a loan and others do not, and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate; or (b) there are identifiable groups of individuals in the population who, with a given supply of credit, are unable to obtain loans at any interest rate, even though with a larger supply of credit, they would” (394–395). The critical aspect of this definition is that at the market equilibrium interest rate there is an unsatisfied demand for loans that cannot be eliminated through higher interest rates. Rejected loan applicants cannot succeed in getting a loan by offering to pay a higher interest rate.

It is important to recognize that credit rationing is sufficient but not necessary for a credit channel to exist. A theme of Gertler (1988), Bernanke and Gertler (1989), and Bernanke (1993) was that agency costs in credit markets will vary countercyclically; a monetary tightening that raises interest rates and generates a real economic slowdown will cause firm balance sheets to deteriorate, raising agency costs and lowering the efficiency of credit allocation. Changes in credit conditions are not reflected solely in interest rate levels. Thus, the general issue is to understand how credit market imperfections affect the macroeconomic equilibrium and the channels through which monetary policy actions are transmitted to the real economy.

The main focus is on credit markets for firms undertaking investment projects. This approach is chosen primarily for convenience; the theoretical models may also be applied to the consumer loan market, and there is evidence that a significant fraction of households behave as if they face liquidity constraints that link consumption spending more closely to current income than would be predicted by forward-looking models of consumption.<sup>21</sup>

The role of credit effects in the transmission of monetary policy arises as a result of imperfect information between parties in credit relationships. The information that each party to a credit transaction brings to the exchange has important implications for the nature of credit contracts, the ability of credit markets to match borrowers and lenders efficiently, and the role played by the rate of interest in allocating credit among borrowers. The nature of credit markets can lead to distinct roles for different types of lenders (e.g., bank versus nonbank) and different types of borrowers (e.g., small firms versus large firms).

Critical to the presence of a distinct credit channel is the presence of imperfections in financial markets. The first task, then, is to review theories of credit market imperfections based on adverse selection, moral hazard, and monitoring costs. These theories help to explain many of the distinctive features of financial markets, from collateral to debt contracts to the possibility of credit rationing. This material provides the microfoundations for the macroeconomic analysis of credit channels in section 10.7.

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21. Empirical evidence on consumption and liquidity constraints can be found in Campbell and Mankiw (1989; 1991), who provided estimates of the fraction of liquidity-constrained households for a number of OECD countries. For more recent evidence, see Kaplan, Violante, and Weidner (2014).

### 10.6.1 Adverse Selection

Jaffee and Russell (1976) analyzed a credit market model in which there are two types of borrowers, “honest” ones who always repay and “dishonest” ones who repay only if it is in their interest to do so. Ex ante, the two types appear identical to lenders. Default is assumed to impose a cost on the defaulter, and dishonest borrowers default whenever the loan repayment amount exceeds the cost of default. By assuming a distribution of default costs across the population of borrowers, Jaffee and Russell showed that the fraction of borrowers who default is increasing in the loan amount.<sup>22</sup> In a pooling equilibrium, lenders offer the same loan contract (interest rate and amount) to all borrowers because they are unable to distinguish between the two types.<sup>23</sup> If lenders operate with constant returns to scale, if there is free entry, and if funds are available to lenders at an exogenously given opportunity cost, then the equilibrium loan rate must satisfy a zero profit condition for lenders. Since the expected return on a loan is less than or equal to the interest rate charged, the actual interest rate on loans must equal or exceed the opportunity cost of funds to the lenders.<sup>24</sup>

The effects of borrower heterogeneity and imperfect information on credit market equilibria can be illustrated following Stiglitz and Weiss (1981). The lender’s expected return on a loan is a function of the interest rate charged and the probability that the loan will be repaid, but individual borrowers differ in their probabilities of repayment. Suppose borrowers come in two types. Type *G* repays with probability  $q_g$ ; type *B* repays with probability  $q_b < q_g$ . If lenders can observe the borrower’s type, each type will be charged a different interest rate to reflect the differing repayment probabilities. If the supply of credit is perfectly elastic at the opportunity cost of  $r$ , and lenders are risk-neutral and able to lend to a large number of borrowers so that the law of large numbers holds, then all type *G* borrowers can borrow at an interest rate of  $r/q_g$ , while type *B* borrowers borrow at  $r/q_b > r/q_g$ . At these interest rates, the lender’s expected return from lending to either type of borrower is equal to the lender’s opportunity cost of  $r$ . No credit rationing occurs; riskier borrowers are simply charged higher interest rates.

Now suppose the lender cannot observe the borrower’s type. It may be the case that changes in the terms of a loan (interest rate, collateral, amount) affect the mix of borrower types the lender attracts. If increases in the loan interest rate shift the mix of borrowers, raising the fraction of type *B*’s, the expected return to the lender might actually decline

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22. See Smith (1983) for a general equilibrium version of Jaffee and Russell’s model using an overlapping-generations framework.

23. This ignores the possibility of separating equilibrium, in which the lender offers two contracts and the borrowers (truthfully) signal their type by the contract they choose.

24. If the probability of default were zero, the constant-returns-to-scale assumption with free entry would ensure that lenders charge an interest rate on loans equal to the opportunity cost of funds. If default rates are positive, then the expected return on a loan is less than the actual interest rate charged, and the loan interest rate must be greater than the opportunity cost of funds.

with higher loan rates because of adverse selection. In this case, further increases in the loan rate would lower the lender's expected profits, even if an excess demand for loans remains. The intuition is similar to that of Akerlof's (1970) market for lemons. Assume that a fraction  $g$  of all borrowers are of type  $G$ . Suppose the lender charges an interest rate of  $r_l$  such that  $gq_g r_l + (1-g)q_b r_l = r$ , or  $r_l = r/[gq_g + (1-g)q_b]$ . At this loan rate, the lender earns the required return of  $r$  if borrowers are drawn randomly from the population. But at this rate, the pool of borrowers is no longer the same as in the population at large. Since  $r/q_g < r_l < r/q_b$ , the lender is more likely to attract type  $B$  borrowers, and the lender's expected return would be less than  $r$ .

Loans are, however, characterized by more than just their interest rate. For example, suppose a loan is characterized by its interest rate  $r_l$ , the loan amount  $L$ , and the collateral the lender requires  $C$ . The probability that the loan will be repaid depends on the (risky) return yielded by the borrower's project. If the project return is  $R$ , then the lender is repaid if  $L(1+r_l) < R+C$ .

If  $L(1+r_l) > R+C$ , the borrower defaults and the lender receives  $R+C$ .

Suppose the return  $R$  is  $R' + x$  with probability  $\frac{1}{2}$  and  $R' - x$  with probability  $\frac{1}{2}$ . The expected return is  $R'$ , while the variance is  $x^2$ . An increase in  $x$  represents a mean preserving spread in the return disturbance and corresponds to an increase in the project's risk. Assume that  $R' - x < (1+r_l)L - C$  so that the borrower must default when the bad outcome occurs. If the project pays off  $R' + x$ , the borrower receives  $R' + x - (1+r_l)L$ ; if the bad outcome occurs, the borrower receives  $-C$ , that is, any collateral is lost. The expected profit to the borrower is

$$E\pi^B = \frac{1}{2} [R' + x - (1+r_l)L] - \frac{1}{2}C.$$

Define

$$x^*(r_l, L, C) \equiv (1+r_l)L + C - R'. \quad (10.35)$$

Expected profits for the borrower are positive for all  $x > x^*$ . This critical cutoff value of  $x$  is increasing in  $r_l$ . Recall that increases in  $x$  imply an increase in the project's risk, as measured by the variance of returns. An increase in the loan rate  $r_l$  increases  $x^*$ , and this implies that some borrowers with less risky projects will find it unprofitable to borrow if the loan rate rises, while borrowers with riskier projects will still find it worthwhile to borrow. Because borrowers can lose no more than their collateral in the bad state, expected profits are a convex function of the project's return and therefore increase with an increase in risk (for a constant mean return).

While the expected return to the firm is increasing in risk, as measured by  $x$ , the lender's return is decreasing in  $x$ . To see this point, note that the lender's expected profit is

$$E\pi^L = \frac{1}{2} [(1+r_l)L] + \frac{1}{2} [C + R' - x] - (1+r)L,$$

where  $r$  is the opportunity cost of funds to the lender. The lender's expected profit decreases with  $x$ . Because the lender receives a fixed amount in the good state, the lender's expected return is a concave function of the project's return and therefore decreases with an increase in risk.

Now suppose there are two groups of borrowers, those with  $x = x_g$  and those with  $x = x_b$ , with  $x_g < x_b$ . Type  $x_g$  borrowers have lower-risk projects. From (10.35), if the loan rate  $r_l$  is low enough such that  $x_b > x_g \geq x^*(r_l, L, C)$ , then both types will find it profitable to borrow. If each type is equally likely, the lender's expected return is

$$\begin{aligned} E\pi^L &= \frac{1}{4} [(1+r_l)L + C + R' - x_g] + \frac{1}{4} [(1+r_l)L + C + R' - x_b] - (1+r)L \\ &= \frac{1}{2} [(1+r_l)L + C + R'] - \frac{1}{4} (x_g + x_b) - (1+r)L, \quad x^*(r_l, L, C) \leq x_g, \end{aligned}$$

which is increasing in  $r_l$ . But as soon as  $r_l$  increases to the point where  $x^*(r_l, L, C) = x_g$ , any further increase causes all  $x_g$  types to stop borrowing. Only  $x_b$  types will still find it profitable to borrow, and the lender's expected profit falls to

$$E\pi^L = \frac{1}{2} [(1+r_l)L + C + R'] - \frac{1}{2} x_b - (1+r)L, \quad x_g \leq x^*(r_l, L, C) \leq x_b.$$

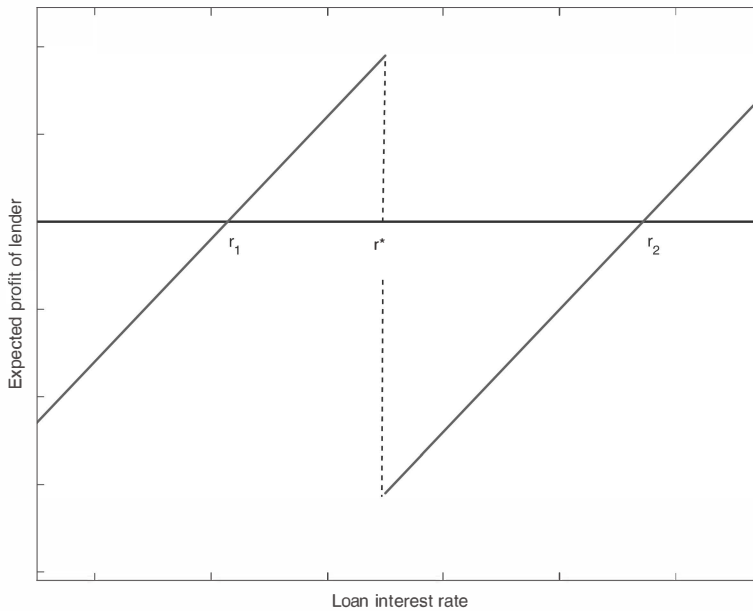
As a result, the lender's expected profit as a function of the loan rate is increasing for  $x^*(r_l, L, C) \leq x_g$  and then falls discretely at  $1+r_l = [x_g - C + R']/L$  as all low-risk types exit the market. This is illustrated in figure 10.1, where  $r^*$  denotes the loan rate that tips the composition of the pool of borrowers. For loan rates between  $r_1$  and  $r^*$ , both types borrow and the lender's expected profit is positive. Expected profits are again positive for loan rates above  $r_2$ , but in this region only  $x_b$  types borrow.

The existence of a local maximum in the lender's profit function at  $r^*$  introduces the possibility that credit rationing will occur in equilibrium. Suppose at  $r^*$  there remains an excess demand for loans. A type  $x_g$  would not be willing to borrow at a rate above  $r^*$ , but a type  $x_b$  would. If the lender responds to the excess demand by raising the loan rate, expected profits fall. Equilibrium may involve a loan rate of  $r^*$ , with some potential borrowers being rationed.<sup>25</sup> Thus, adverse selection provides one rationale for a lender's profit function that is not monotonic in the loan rate. Equilibrium credit rationing may exist because lenders find it unprofitable to raise the interest rate on loans even in the face of an excess demand for loans.

### 10.6.2 Moral Hazard

Moral hazard can arise in credit markets when the borrower's behavior is influenced by the terms of the loan contract. In the model of the previous section, the borrower decided

25. As figure 10.1 suggests, if the demand for loans is strong enough, the lender may be able to raise the loan rate sufficiently so that expected profits do rise.



**Figure 10.1**  
Expected loan profit with adverse selection.

whether to borrow, but the project’s return was exogenous. Borrowers differed in terms of the underlying riskiness of their projects, and adverse selection occurred as loan rate changes affected the pool of borrowers. Suppose instead that each borrower can choose between several projects of differing risk. If the lender cannot monitor this choice, a moral hazard problem arises. The lender’s expected return may not be monotonic in the interest rate charged on the loans. Higher loan rates lead the borrower to invest in riskier projects, lowering the expected return to the lender.

To illustrate this situation, again following Stiglitz and Weiss (1981), suppose the borrower can invest either in project A, which pays off  $R^a$  in the good state and 0 in the bad state, or in project B, which pays off  $R^b > R^a$  in the good state and 0 in the bad state. Suppose the probability of success for project A is  $p^a$  and  $p^b$  for project B, with  $p^a > p^b$ . Project B is the riskier project. Further, assume the expected payoff from A is higher:  $p^a R^a > p^b R^b$ . By investing in A, the borrower’s expected return is

$$E\pi^A = p^a [R^a - (1 + r_l)L] - (1 - p^a)C,$$

where the borrower loses collateral  $C$  if the project fails. The expected return from project B is

$$E\pi^B = p^b [R^b - (1 + r_l)L] - (1 - p^b)C.$$

The expected returns on the two projects depend on the interest rate on the loan  $r_l$ . It is straightforward to show that

$$E\pi^A > E\pi^B$$

if and only if

$$\frac{p^a R^a - p^b R^b}{p^a - p^b} > (1 + r_l)L - C.$$

The left side of this condition is independent of the loan rate, but the right side is increasing in  $r_l$ . Define  $r_l^*$  as the loan rate at which the expected returns to the borrower from the two projects are equal. This occurs when

$$(1 + r_l^*)L - C = \frac{p^a R^a - p^b R^b}{p^a - p^b}.$$

For loan rates less than  $r_l^*$ , the borrower will prefer to invest in project  $A$ ; for loan rates above  $r_l^*$ , the riskier project  $B$  is preferred. The expected payment to the lender, therefore, will be  $p^a(1 + r_l)L + (1 - p^a)C$  if  $r_l < r_l^*$ , and  $p^b(1 + r_l)L + (1 - p^b)C$  for  $r_l > r_l^*$ . Since

$$p^a(1 + r_l^*)L + (1 - p^a)C > p^b(1 + r_l^*)L + (1 - p^b)C, \quad (10.36)$$

the lender's profits fall as the loan rate rises above  $r_l^*$ ; the lender's profits are not monotonic in the loan rate.<sup>26</sup> Just as in the example of the previous section, this leads to the possibility that credit rationing may characterize the loan market's equilibrium.

### 10.6.3 Monitoring Costs

The previous analysis illustrated how debt contracts in the presence of adverse selection or moral hazard could lead to credit rationing as an equilibrium phenomenon. One limitation of the discussion, however, was the treatment of the nature of the loan contract—repayment equal to a fixed interest rate times the loan amount in some states of nature, zero or a predetermined collateral amount in others—as exogenous. Williamson (1986; 1987a; 1987b) illustrated how debt contracts and credit rationing can arise even in the absence of adverse selection or moral hazard problems if lenders must incur costs to monitor borrowers.<sup>27</sup> The intuition behind his result is straightforward. Suppose the lender can observe

26. To see this, note that using the definition of  $r_l^*$  implies that the left side of (10.36) is equal to  $p^a [(1 + r_l^*)L - C] = p^a \left( \frac{p^a R^a - p^b R^b}{p^a - p^b} \right)$ , and the right side is equal to  $p^b [(1 + r_l^*)L - C] = p^b \left( \frac{p^a R^a - p^b R^b}{p^a - p^b} \right)$ .

The direction of the inequality follows, since  $p^a > p^b$ .

27. Townsend (1979) provided the first analysis of optimal contracts when it is costly to verify the state.

the borrower's project outcome only at some positive cost. Any repayment schedule that ties the borrower's payment to the project outcome would require that the monitoring cost be incurred; otherwise, the borrower always has an incentive to underreport the success of the project. Expected monitoring costs can be reduced if the borrower is monitored only in some states of nature. If the borrower reports a low project outcome and defaults on the loan, the lender incurs the monitoring cost to verify the truth of the report. If the borrower reports a good project outcome and repays the loan, the lender does not need to incur the monitoring cost.

Following Williamson (1987b), assume there are two types of agents, borrowers and lenders. Lenders are risk-neutral and have access to funds at an opportunity cost of  $r$ . Each lender takes  $r$  as given and offers contracts to borrowers that yield, to the lender, an expected return of  $r$ . Assume there are two periods. In period 1, lenders offer contracts to borrowers who have access to a risky investment project that yields a payoff in period 2 of  $x \in [0, \bar{x}]$ . The return  $x$  is a random variable, drawn from a distribution known to both borrowers and lenders. The actual realization is observed costlessly by the borrower; the lender can observe it by first paying a cost of  $c$ . This assumption captures the idea that borrowers are likely to have better information about their own projects than do lenders. Lenders can obtain this information by monitoring the project, but such monitoring is costly.

In period 2, after observing  $x$ , the borrower reports the project outcome to the lender. Let this report be  $x^s$ . While  $x^s$  must be in  $[0, \bar{x}]$ , it need not equal the true  $x$ , since the borrower will have an incentive to misreport if doing so is in the borrower's own interest. By choice of normalization, projects require an initial resource investment of 1 unit. Although borrowers have access to an investment project, assume they have no resources of their own, so to invest they must obtain resources from lenders.

Suppose that monitoring occurs whenever  $x^s \in S \subset [0, \bar{x}]$ . Otherwise, the lender does not monitor. Denote by  $R(x)$  the payment from the borrower to the lender if  $x^s \in S$  and monitoring takes place. Because the lender monitors and therefore observes  $x$ , the repayment can be made a function of the actual  $x$ . The return to the lender net of monitoring costs is  $R(x) - c$ . If the reported value  $x^s \notin S$ , then no monitoring occurs and the borrower pays  $K(x^s)$  to the lender. This payment can only depend on the signal, not the true realization of  $x$ , since the lender cannot verify the latter. In this case, the return to the lender is simply  $K(x^s)$ . Whatever the actual value of  $x^s \notin S$ , the borrower will report the value that results in the smallest payment to the lender; hence, if monitoring does not occur, the payment to the lender must be equal to a constant,  $\bar{K}$ .<sup>28</sup> Since all loans are for 1 unit,  $\bar{K} - 1$  is the interest rate on the loan when  $x^s \notin S$ .

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28. That is, suppose  $x_1$  and  $x_2$  are project return realizations such that the borrower would report  $x_1^s$  and  $x_2^s \notin S$ . If reporting  $x_1^s$  results in a larger payment to the lender, the borrower would always report  $x_2^s$ .

If the reported signal is in  $S$ , then monitoring occurs so that the lender can learn the true value of  $x$ . The borrower will report a  $x^s$  in  $S$  only if it is in the borrower's best interest, that is, reporting  $x^s \in S$  must be incentive compatible. For this to be the case, the net return to the borrower when  $x^s \in S$ , equal to  $x - R(x)$ , must exceed the return from reporting a signal not in  $S$ ,  $x - \bar{K}$ . That is, incentive compatibility requires that

$$x - R(x) > x - \bar{K}, \quad \text{or } \bar{K} > R(x), \quad \text{for all } x^s \in S.$$

The borrower will report a signal that leads to monitoring only if  $R(x) < \bar{K}$  and will report a signal not in  $S$  (so that no monitoring occurs) if  $R(x) \geq \bar{K}$ .

The optimal contract is a payment schedule  $R(x)$  and a value  $\bar{K}$  that maximizes the borrower's expected return, subject to the constraint that the lender's expected return be at least equal to the lender's opportunity cost  $r$ . Letting  $\Pr[x < y]$  denote the probability that  $x$  is less than  $y$ , the expected return to the borrower can be written as the expected return conditional on monitoring occurring,  $E[x - R(x)|R(x) < \bar{K}]$ , times the probability that  $R(x) < \bar{K}$ , plus the expected return conditional on no monitoring occurring, times the probability that  $R(x) \geq \bar{K}$ :

$$E[R^b] \equiv E[x - R(x)|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + E[x - \bar{K}|R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}]. \quad (10.37)$$

The optimal loan contract maximizes this expected return subject to the constraint that the lender's expected return be at least  $r$ :

$$E[R(x) - c|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \bar{K} \Pr[R(x) \geq \bar{K}] \geq r. \quad (10.38)$$

The solution to this problem, and therefore the optimal loan contract, has  $R(x) = x$ . In other words, if the borrower reports a signal that leads the lender to monitor, then the lender takes the entire actual project return. This result corresponds to a loan default in which the lender takes over the project, incurs the monitoring cost  $c$  (which in this case one can think of as a liquidation cost), and ends up with  $x - c$ . If the project earns a sufficient return, that is  $R(x) = x \geq \bar{K}$ , then the borrower pays the lender the fixed amount  $\bar{K}$ . Since  $\bar{K}$  is independent of the realization of  $x$ , no monitoring is necessary. The presence of monitoring costs and imperfect information leads to the endogenous determination of the optimal loan contract.

The proof that  $R(x) = x$  whenever monitoring takes place is straightforward. In equilibrium, the constraint given by (10.38) will be satisfied with equality. Otherwise, the payment to the lender could be reduced in some states, which would increase the expected return to the borrower. Hence,

$$E[R(x) - c|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \bar{K} \Pr[R(x) \geq \bar{K}] = r.$$



Any contract that called for  $R(x) < x$  for some realizations of  $x$  could be replaced by another contract that increases repayment slightly when monitoring occurs but lowers  $\bar{K}$  to decrease the range of  $x$  for which monitoring actually takes place. This can be done such that the lender's expected profit is unchanged.<sup>29</sup> Using the constraint for the lender's expected return, the expected return to the borrower can be written as

$$\begin{aligned}
 E[R^b] &= E[x - R(x)|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + \{E[x|R(x) \geq \bar{K}] - \bar{K}\} \Pr[R(x) \geq \bar{K}] \\
 &= E[x - R(x)|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + E[x|R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}] \\
 &\quad - \{r - E[R(x) - c|R(x) < \bar{K}] \Pr[R(x) < \bar{K}]\} \\
 &= E[x - c|R(x) < \bar{K}] \Pr[R(x) < \bar{K}] + E[x|R(x) \geq \bar{K}] \Pr[R(x) \geq \bar{K}] - r \\
 &= E[x] - c \Pr[R(x) < \bar{K}] - r,
 \end{aligned} \tag{10.39}$$

where  $\Pr[R(x) < \bar{K}]$  is the probability that monitoring occurs. Equation (10.39) shows that the expected return to the borrower is decreasing in  $\bar{K}$ . Any contract that lowers  $\bar{K}$  and reduces the probability of monitoring while leaving the lender with an expected return of  $r$  will be strictly preferred by the borrower. Such a contract can be constructed if  $R(x) < x$ .<sup>30</sup>

To make the example more specific, suppose  $x$  is uniformly distributed on  $[0, \bar{x}]$ . The expected return to the lender is equal to

$$\int_0^{\bar{K}} (x - c) \frac{1}{\bar{x}} dx + \int_{\bar{K}}^{\bar{x}} \bar{K} \frac{1}{\bar{x}} dx.$$

The first term is the expected return to the lender if the borrower defaults, an outcome that occurs whenever  $x < \bar{K}$ ; the probability of this outcome is  $\bar{K}/\bar{x}$ . The second term is the fixed payment received by the lender whenever  $x \geq \bar{K}$ , an outcome that occurs with probability  $[\bar{x} - \bar{K}]/\bar{x}$ . Evaluating the expected return and equating it to  $r$  yields the following condition to determine  $\bar{K}$ :

$$\left[ \frac{1}{2} \left( \frac{\bar{K}^2}{\bar{x}} \right) - c \left( \frac{\bar{K}}{\bar{x}} \right) \right] + \bar{K} \left[ 1 - \left( \frac{\bar{K}}{\bar{x}} \right) \right] = r.$$

29.  $R(x) > x$  is ruled out by the assumption that the borrower has no other resources. If  $R(x) < x$  for some  $x$  for which monitoring occurs, then the new contract, which increases  $R(x)$  in those states, increases  $R(x) - c$  when monitoring does occur. For a given  $\bar{K}$ , this increases  $E[R(x) - c|R(x) < \bar{K}]$ , making the lender's expected profit greater than  $r$ . Since  $\bar{K}$  is then lowered, monitoring occurs in fewer states, thereby reducing the lender's expected profit so that it again equals  $r$ .

30. One implication of (10.39) is that the borrower bears the cost of monitoring; the expected return to the borrower is equal to the total expected project return net of the opportunity cost of funds ( $r$ ) and expected monitoring costs ( $c \Pr[R(x) < \bar{K}]$ ).

If  $(\bar{x} - c)^2 > 2\bar{x}r$ , this quadratic has two real solutions, one less than  $\bar{x} - c$  and one greater than  $\bar{x} - c$ .<sup>31</sup> However, the effect of  $\bar{K}$  on the lender's expected return is

$$\frac{\bar{K}}{\bar{x}} - \frac{c}{\bar{x}} + \left(1 - \frac{2\bar{K}}{\bar{x}}\right) = 1 - \frac{c + \bar{K}}{\bar{x}},$$

which becomes negative for  $\bar{K} > \bar{x} - c$ . This means that when the loan repayment amount is large, further increases in the contracted repayment would actually lower the lender's expected return; loan contracts with less monitoring (a lower  $\bar{K}$ ) would be preferred by both borrower and lender;  $\bar{K} > \bar{x} - c$  cannot be an equilibrium.

When the lender's expected profits are no longer monotonic in the loan interest rate but can actually decrease at higher interest rates, the possibility exists of an equilibrium in which some borrowers face credit rationing. In a nonrationing equilibrium, all borrowers receive loans.<sup>32</sup> The expected rate of return  $r$  is determined by the condition that loan demand equal loan supply, and the gross interest rate on loans,  $\bar{K}$ , is less than  $\bar{x} - c$ . In a credit-rationing equilibrium,  $\bar{K} = \bar{x} - c$ , and not all potential borrowers receive loans. Even though there are unsatisfied potential borrowers, the interest rate on loans will not rise because the lenders' expected profits are decreasing in the loan rate when  $\bar{K} > \bar{x} - c$ . Even though all potential borrowers were assumed to be identical ex ante, some receive loans while others do not. The ones that do not get loans would be willing to borrow at an interest rate above the market rate, yet no lenders are willing to lend.

Williamson's model illustrates that neither adverse selection nor moral hazard is necessary for rationing to characterize credit markets. The presence of monitoring costs can account for both the general form of loan contracts in which monitoring occurs only when the borrower defaults—in which case the lender takes over the entire project's return—and for rationing to arise in some equilibria.

#### 10.6.4 Agency Costs

Adverse selection, moral hazard, and monitoring costs are all important factors in any relationship in which a principal delegates decision-making authority to an agent. In credit markets, the lender delegates to a borrower control over resources. The inability to monitor the borrower's actions or to share the borrower's information gives rise to agency costs. Bernanke and Gertler (1989) and Gertler (1988) emphasized the role of agency costs that

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31. These are given by

$$\bar{x} - c \pm \sqrt{(\bar{x} - c)^2 - 2\bar{x}r}.$$

32. A complete specification of the model requires assumptions on the number of (potential) borrowers and lenders that ensure an upward-sloping supply curve of funds. See Williamson (1987b) for details on one such specification.

made external funding sources more expensive for firms than internal sources. As a consequence, a firm's balance sheet plays a role in affecting the cost of finance. In recessions, internal sources of funds decline, forcing firms to turn to external sources. But the deterioration of the firm's balance sheet worsens the agency problems and increases the cost of external funds, thereby further contracting investment spending and contributing to the recession. Thus, credit conditions can play a role in amplifying the impact of other shocks to the economy and affecting their propagation throughout the economy and through time.

In the model of Bernanke and Gertler (1989), firms are assumed to be able to observe the outcome of their own investment projects costlessly; others must incur a monitoring cost to observe project outcomes. Firms and lenders are assumed to be risk-neutral. Firms are indexed by efficiency type  $\omega$ , distributed uniformly on  $[0, 1]$ . More efficient types (ones with low  $\omega$ ) need to invest fewer inputs in a given project. Projects themselves require inputs of  $x(\omega)$ , yielding gross payoff  $\kappa_1$  with probability  $\pi_1$  and  $\kappa_2 > \kappa_1$  with probability  $\pi_2 = 1 - \pi_1$ . The function  $x(\cdot)$  is increasing in  $\omega$ . The expected project return,  $\pi_1\kappa_1 + \pi_2\kappa_2$ , is denoted  $\kappa$ . The realized outcome of a particular project can be observed costlessly by the firm undertaking the project and at cost  $c$  by others. Firms are assumed to have internal sources of financing equal to  $S$ ;  $S$  is assumed to be less than  $x(0)$ , so that even the most efficient firm must borrow to undertake a project. Finally, let  $r$  denote the opportunity cost of funds to lenders; firms that do not undertake a project also receive this rate on their funds.<sup>33</sup>

If lenders could observe project outcomes costlessly, equilibrium would involve lenders financing all projects whose expected payoff exceeds their opportunity cost of  $rx$ . Thus, all firms whose  $\omega$  is less than a critical value  $\omega^*$  defined by

$$\kappa - rx(\omega^*) = 0$$

would receive loans. Firms with  $\omega < \omega^*$  borrow  $B \equiv x(\omega) - S$ .

With imperfect information, the firm clearly has an incentive to always announce that the bad outcome,  $\kappa_1$ , occurred. It will never pay for the lender to incur the monitoring cost if the firm announces  $\kappa_2$ . Let  $p$  be the probability that the firm is audited (i.e., the lender pays the monitoring cost to observe the true outcome) when the firm announces  $\kappa_1$ . Let  $P_1^a$  be the payment to the firm when  $\kappa_1$  is announced and auditing takes place,  $P_1$  the payment when  $\kappa_1$  is announced and no auditing occurs, and  $P_2$  the payment if  $\kappa_2$  is announced. The optimal lending contract must maximize the expected payoff to the firm, subject to several constraints. First, the lender's expected return must be at least as great as the lender's opportunity cost  $rB$ . Second, the firm must have no incentive to report the bad state when in fact the good state occurred. Third, even in the bad state, limited liability requires that  $P_1^a$  and  $P_1$  be non-negative. The optimal contract is characterized by the values

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33. Bernanke and Gertler developed a general equilibrium model; a partial equilibrium version is discussed to focus on the role played by credit market imperfections in investment decisions.

of  $\{p, P_1^a, P_1, P_2\}$  that solve

$$\max \pi_1 [pP_1^a + (1-p)P_1] + \pi_2 P_2,$$

subject to

$$\pi_1 [\kappa_1 - p(P_1^a + c) - (1-p)P_1] + \pi_2 [\kappa_2 - P_2] \geq rB, \quad (10.40)$$

$$P_2 \geq (1-p)(\kappa_2 - \kappa_1 + P_1), \quad (10.41)$$

$$P_1^a \geq 0, \quad (10.42)$$

$$P_1 \geq 0, \quad (10.43)$$

and  $0 \leq p \leq 1$ .

Only the constraint given by (10.41) may require comment. The left side is the firm's income in the good state. The right side gives the firm's income if the good state occurs but the firm reports the bad state. After reporting the bad state, the firm is audited with probability  $p$ . So with probability  $1-p$  the firm is not audited, turns over  $\kappa_1 - P_1$  to the lender, and keeps  $P_1$ . But the firm now also gets to keep the amount  $\kappa_2 - \kappa_1$  because, by assumption, the good state had actually occurred. If (10.41) is satisfied, the firm has no incentive to conceal the truth in announcing the project outcome.

Assuming an interior solution, the first-order necessary conditions for this problem are

$$\pi_1 [(P_1^a - P_1) + \mu_1 (P_1 - P_1^a - c)] + \mu_2 (\kappa_2 - \kappa_1 + P_1) = 0, \quad (10.44)$$

$$\pi_1 p(1 - \mu_1) + \mu_3 = 0, \quad (10.45)$$

$$\pi_1 (1-p)(1 - \mu_1) - \mu_2 (1-p) + \mu_4 = 0, \quad (10.46)$$

$$\pi_2 (1 - \mu_1) + \mu_2 = 0, \quad (10.47)$$

where  $\mu_i$  is the (non-negative) Lagrangian multiplier associated with the constraints (10.40)–(10.43).

Since  $\mu_3 \geq 0$ , (10.45) implies that  $\mu_1 \geq 1$ . This means the constraint on the lender's return (10.40) holds with equality. With  $\pi_1 [\kappa_1 - p(P_1^a + c) - (1-p)P_1] + \pi_2 [\kappa_2 - P_2] - r(x - S) = 0$ , this can be added to the objective function, yielding an equivalent problem that the optimal contract solves given by  $\max [\pi_1 (\kappa_1 - pc) + \pi_2 \kappa_2]$ , subject to (10.41) and the non-negative constraints on  $P_1^a$  and  $P_1$ . However,  $\pi_1 (\kappa_1 - pc) + \pi_2 \kappa_2 = \kappa - \pi_1 pc$ , and with  $\kappa$  an exogenous parameter, this new problem is equivalent to minimizing expected auditing costs  $\pi_1 pc$ .

If the return to the lender,  $rB$ , is less than the project return even in the bad state  $\kappa_1$ , then no auditing is ever necessary and  $p = 0$ . Agency costs are therefore zero whenever  $\kappa_1 \geq rB$ . Recall that the amount borrowed,  $B$ , was equal to  $x(\omega) - S$ , where  $S$  represented the firm's internal funds invested in the project, so the no-agency-cost condition

can be written

$$S \geq x(\omega) - \frac{\kappa_1}{r} \equiv S^*(\omega).$$

Any type  $\omega$  with internal funds greater than or equal to  $S^*(\omega)$  can always repay the lender, so no auditing on the project is required. When  $S < S^*(\omega)$ , a situation Bernanke and Gertler labeled as one of *incomplete collateralization*, constraints (10.40)–(10.43) all hold with equality. Since auditing is costly, the optimal auditing probability is just high enough to ensure that the firm truthfully reports the good state when it occurs. From the incentive constraint (10.41),  $P_2 = (1 - p)(\kappa_2 - \kappa_1)$ , since  $P_1 = P_1^a = 0$  (the firm keeps nothing in the bad state). Substituting this into the lender's required-return condition (10.40),

$$p = \frac{r[x(\omega) - S] - \kappa_1}{\pi_2(\kappa_2 - \kappa_1) - \pi_1 c}.$$

The auditing probability is decreasing in the return in the good state ( $\kappa_2$ ) and the firm's own contribution  $S$ . If the firm invests little in the project and borrows more, then the firm receives less of the project's return in the good state, increasing its incentive to falsely claim that the bad state occurred. To remove this incentive, the probability of auditing must rise.

Bernanke and Gertler characterized the expected costs of project auditing,  $\pi_1 p c$ , as the *agency costs due to asymmetric information*. As they show, some firms with intermediate values of  $\omega$  (i.e., neither the most nor the least efficient) will find that the investment project is not worth undertaking if they have only low levels of internal funds to invest. The probability of auditing that lenders would require makes agency costs too high to justify the investment. If the firm had a higher level of internal funds, it would undertake the project. Even though the opportunity costs of funds  $r$  and the project inputs  $x$  and returns ( $\kappa_1$  and  $\kappa_2$ ) have not changed, variations in  $S$  can alter the number of projects undertaken. This illustrates how investment levels may depend on the firm's internal sources of financing. Agency costs drive a wedge between the costs of internal and external funds, so investment decisions will depend on variables such as cash flow that would not play a role if information were perfect. Since a recession will worsen firms' balance sheets, reducing the availability of internal funds, the resulting rise in agency costs and the reduction in investment may amplify the initial cause of a recession.

### 10.6.5 Intermediary-to-Intermediary Credit Flows

The previous sections have focused on frictions involving a financial intermediary having access to funds and then using these funds to make loans to firms. For example, the lenders in the model of Bernanke and Gertler (1989) have a fixed opportunity cost of funds. They are risk-neutral and will lend to firms as long as the expected return on loans exceeds their cost of funds. Agency costs account for the spread between the opportunity cost of funds to lenders and the costs to the firm of external funds used to finance projects.

However, much of the borrowing and lending in a modern economy involves not firms financing capital purchases but financial intermediaries borrowing and lending to one another. Credit frictions within the financial sector, rather than frictions affecting credit extended to the nonfinancial sector, seemed to be at the heart of the 2008–2009 global financial crisis. Gertler and Kiyotaki (2010) developed a model that focuses directly on frictions that may limit the ability of financial firms to access credit. While their model is general equilibrium in nature, with a real side in which households make optimal consumption decisions and firms engage in investment decisions, the focus here is on the frictions they highlight in the financial sector that affect the allocation of credit.<sup>34</sup>

In the Gertler and Kiyotaki model, households deposit funds with financial intermediaries (banks), and these banks then lend to nonfinancial firms. An agency problem, however, will limit the ability of banks to obtain funds from depositors. The economy consists of a large number of segmented markets, or islands. On each island, there are firms and a bank. Banks, regardless of which island they are located on, raise funds (deposits) from households in an economywide retail deposit market. Households do not lend directly to firms. Each bank can make loans to firms, but only to firms on the same island. Thus, markets are segmented to two ways. First, banks are assumed to have special skills in evaluating firms that make it more efficient for households not to lend directly to nonfinancial firms. Second, banks have localized knowledge about firms on their own island that makes it efficient not to lend to any firm not on the same island.

After the retail deposit market closes, islands receive random productivity shocks, so there may be many firms on some islands that have good investment projects, while on other islands (those getting negative productivity shocks, for example) there may be few firms with good projects. Suppose the firms on island  $i$  have lots of high-return projects for which they seek funding, while firms on island  $j$  have few projects worth financing. Suppose further that the bank on island  $i$  does not have sufficient deposits to finance all the firms with good projects on its island, while the bank on island  $j$  has deposits but no good lending opportunities. In this environment, there is a role for an interbank market in which bank  $j$  lends funds to bank  $i$ , allowing bank  $j$  to indirectly lend to firms on island  $i$  with the higher expected returns. If this interbank market were not subject to any frictions, funds would flow from island  $j$ , lowering asset prices on this island, to island  $i$ , bidding up asset values on this island. In an efficient equilibrium, this process would ensure prices adjust until expected returns are equalized across islands.

Assume the balance sheet of an individual bank  $h$  is

$$a_t^h = d_t^h + b_t^h + n_t^h, \quad (10.48)$$

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34. Christiano and Zha (2010) provided a simplified two-period exposition of the Gertler-Kiyotaki model. Brunnermeier and Sannikov (2016) discussed the importance of intermediaries in monetary economics.

where  $a_t^h$  are assets (loans to firms),  $d_t^h$  deposit liabilities,  $b_t^h$  borrowing from the interbank market (negative if the bank is lending in the interbank market), and  $n_t^h$  the bank's equity. Let  $R_{d,t}$  be the gross interest rate paid on deposits,  $R_{b,t}$  the gross interest rate on interbank borrowing, and  $R_{a,t}$  the gross return on lending to nonfinancial firms.<sup>35</sup> The bank's equity at the end of the period is

$$n_{t+1}^h = R_{a,t}a_t^h - R_{b,t}b_t^h - R_{d,t}d_t^h.$$

Ignoring any uncertainty or frictions in the loan, deposit, and interbank markets, suppose the bank maximized  $n_{t+1}^h$  subject to the balance sheet constraint (10.48), taking  $n_t^h$  as given. Using (10.48),

$$n_{t+1}^h = (R_{a,t} - R_{d,t})d_t^h + (R_{a,t} - R_{b,t})b_t^h + R_{a,t}n_t^h.$$

Equilibrium in the interbank market requires that net borrowing be zero, that is, the sum of  $b_t^h$  over all banks must equal zero. In a frictionless interbank market, this requires  $R_{a,t} = R_{b,t}$ . If this equality did not hold, either all banks would wish to borrow to finance additional lending to firms (if  $R_{a,t} > R_{b,t}$ ) or none would (if  $R_{a,t} < R_{b,t}$ ). In the market for deposits, if  $R_{a,t} > R_{d,t}$ , all banks will wish to attract more deposits from households, pushing up the interest paid on deposits until  $R_{a,t} = R_{d,t}$ . If  $R_{a,t} < R_{d,t}$ , banks would reduce their demand for funds from households and  $R_{d,t}$  would fall. Thus, in this competitive, frictionless benchmark,  $R_{a,t} = R_{b,t} = R_{d,t}$ ; all interest rate spreads are zero. The interbank market overcomes the market segmentation. If loan interest rates are higher on island  $i$  than on  $j$ , the bank on island  $j$  could reduce its own lending to firms and instead lend to bank  $i$  via the interbank market. This increases the supply of funds on island  $i$ , leading to a fall in the loan interest rate on island  $i$  and a rise in loan rates on  $j$ . Arbitrage eliminates the differences across islands.

Gertler and Kiyotaki next introduced a moral hazard friction that may limit the ability of banks to raise funds from depositors as well as a bank's ability to raise funds in the interbank market to achieve an efficient allocation of credit. Suppose that the bank owner can, after receiving deposits and borrowing on the interbank market, divert a fraction  $\theta$  of its assets for the banker's own use. The banker would have an incentive to divert funds if the amount received from not diverting funds,  $n_{t+1}^h$ , ever fell below what could be obtained by diverting funds. Neither depositors nor other banks would lend if  $n_{t+1}^h < \theta a_t^h$ , as then the bank has an incentive to divert funds. This would limit the bank's ability to borrow.

Gertler and Kiyotaki actually assumed that banks may find it more difficult to divert assets funded by loans from other banks than assets purchased using funds raised from household deposits. Assume, therefore, that the bank can divert  $\theta(a_t^h - \omega b_t^h)$ , with

35. The setup here is a simplification of Gertler and Kiyotaki (2010). They assumed banks hold equity shares in the nonfinancial firms rather than make simple loans as assumed here.

$0 \leq \omega \leq 1$ . If  $\omega = 1$ , the bank cannot divert assets purchased with funds borrowed from other banks. If  $\omega = 0$ , assets purchased using interbank funds or depositors' funds are equally subject to being diverted. To ensure the bank does not divert funds, the incentive constraint

$$n_{t+1}^h \geq \theta (a_t^h - \omega b_t^h) \quad (10.49)$$

must hold.

The bank's decision problem then involves maximizing  $n_{t+1}^h = R_{a,t}a_t^h - R_{b,t}b_t^h - R_{d,t}d_t^h$  subject to the balance sheet constraint (10.48) and the incentive constraint (10.49). Using the balance sheet constraint to eliminate  $a_t^h$  and letting  $\lambda_t$  be the Lagrangian multiplier on the incentive constraint, the bank's problem can be written as

$$\max_{d_t^h, b_t^h} (1 + \lambda_t) \left[ (R_{a,t} - R_{b,t}) b_t^h + (R_{a,t} - R_{d,t}) d_t^h + R_{a,t} n_t^h \right] - \lambda_t \left[ (1 - \omega) b_t^h + d_t^h + n_t^h \right].$$

The first-order conditions for  $b_t^h$  and  $d_t^h$  take the form

$$(1 + \lambda_t) (R_{a,t} - R_{b,t}) - \lambda_t \theta (1 - \omega) = 0,$$

$$(1 + \lambda_t) (R_{a,t} - R_{d,t}) - \lambda_t \theta = 0.$$

Rearranging these two conditions yields expressions for the interest rate spreads:

$$R_{a,t} - R_{b,t} = \left( \frac{\lambda_t}{1 + \lambda_t} \right) \theta (1 - \omega) \geq 0, \quad (10.50)$$

$$R_{a,t} - R_{d,t} = \left( \frac{\lambda_t}{1 + \lambda_t} \right) \theta \geq 0. \quad (10.51)$$

In addition, these two equations imply

$$R_{b,t} - R_{d,t} = \left( \frac{\lambda_t}{1 + \lambda_t} \right) \theta \omega \geq 0.$$

To interpret these results, suppose  $\omega = 1$ ; the bank can never divert assets purchased with funds borrowed from other banks. In this case, (10.50) shows that  $R_{a,t} = R_{b,t}$ , as in the frictionless environment previously considered. If the return on assets exceeds the cost of borrowing in the interbank market, banks with opportunities to buy assets will increase their borrowing, and this process will continue until the price of assets and the cost of borrowing adjust until  $R_{a,t} = R_{b,t}$ . Similarly, if  $\theta = 0$ , depositors do not need to worry about the bank diverting funds and so will be willing to lend (hold deposits). Arbitrage will again ensure  $R_{a,t} = R_{d,t}$ . Spreads will also be zero even if  $\theta > 0$  and  $\omega < 1$  if  $\lambda_t = 0$ . That is, if the incentive constraint does not bind, the bank will be able to raise funds from depositors or other banks and arbitrage will ensure the spreads equal zero. Finally, if  $\omega > 0$ , then  $R_{b,t} > R_{d,t}$ . This is a situation in which it is easier to divert funds raised from household depositors than from other banks. This leads to a tighter limit on the ability of



the bank to raise funds in the retail deposit market. If  $R_{d,t} = R_{b,t}$ , other banks would still be willing to lend, as they know the bank is less able to divert the funds they borrow in the interbank market. The increased demand for funds in the interbank market pushes up  $R_{b,t}$  in equilibrium, so that in equilibrium  $R_{d,t} < R_{b,t} < R_{a,t}$ .

Now consider the special case in which  $\omega = 0$ , so that the moral hazard friction is present equally with respect to deposits and interbank borrowing. Then,  $R_{a,t} \geq R_{b,t} = R_{d,t}$ . If the incentive constraint is binding, (10.49) can be written as

$$n_{t+1}^h = R_{a,t}a_t^h - R_{d,t}(b_t^h - d_t^h) = \theta(b_t^h + d_t^h + n_t^h).$$

Define  $B_t^h \equiv b_t^h + d_t^h$  as the bank's total liabilities. Using (10.48) and (10.50) with  $\omega = 0$ , the binding incentive constraint becomes

$$\phi_t n_t^h = a_t^h, \tag{10.52}$$

where

$$\phi_t \equiv \frac{R_{d,t}}{[\theta - (R_{a,t} - R_{d,t})]} = \frac{(1 + \lambda_t) R_{d,t}}{\theta},$$

and (10.51) has been used to eliminate  $R_{a,t} - R_{d,t}$ . Equation (10.52) defines the bank's maximum leverage ratio  $a_t^h/n_t^h$ . The incentive constraint binds when the bank's assets are a multiple  $\phi_t$  of its equity. The constraint is tighter ( $\phi_t$  is lower), the greater the extent to which the bank can divert assets (the higher  $\theta$  is). The higher is  $R_{a,t} - R_{d,t}$ , the more profitable the bank is, as it can borrow at  $R_{d,t}$  and lend at  $R_{a,t}$ , so there is less incentive to divert funds and the bank can increase its leverage. Moral hazard generates an endogenous limit to the ability of the bank to become highly leveraged.

Gertler and Kiyotaki used their model to investigate the effects of various central bank policies that include, for example, lending directly to banks that are unable to borrow more in the interbank market. The policy implications of their approach are discussed in section 11.5.4.

## 10.7 Macroeconomic Implications

The presence of credit market imperfections can play a role in determining how the economy responds to economic disturbances and how these disturbances are propagated throughout the economy and over time. Various partial equilibrium models have provided insights into how imperfect information and costly state verification affect the nature of credit market equilibria. The next step is to embed these partial equilibrium models of the credit market within a general equilibrium macroeconomic model so that the qualitative and quantitative importance of credit channels can be assessed. As Bernanke, Gertler, and Gilchrist (1996) discussed, there are difficulties in taking this step. For one, distributional

issues are critical. Private sector borrowing and lending do not occur in a representative agent world, so agents must differ in ways that give rise to borrowers and lenders. And both the source of credit and the characteristics of the borrower matter, so not all borrowers and not all lenders are alike. Changes in the distribution of wealth or the distribution of cash flow can affect the ability of agents to obtain credit.

### 10.7.1 General Equilibrium Models

The microeconomic literature on imperfect information provides insights into the structure of credit markets. Embedding these insights in a macroeconomic framework to determine how credit markets affect the nature of the equilibrium and the manner in which the economy responds to macroeconomic disturbances is much more difficult. In representative agent models, no lending actually takes place. And with all agents identical, the distinctive features of credit markets that have been emphasized in the literature on credit channels are absent. Incorporating heterogeneity among agents in a tractable general equilibrium model is difficult, particularly when the nature of debt and financial contracts in the model economy should be derived from the characteristics of the basic technology and informational assumptions of the model environment.

Two early examples of general equilibrium models designed to highlight the role of credit factors were developed by Williamson (1987a) and Bernanke and Gertler (1989). In these models, credit markets play an important role in determining how the economy responds to a real productivity shock. Williamson embedded his model of financial intermediation with costly monitoring (see section 10.6.3) in a dynamic general equilibrium model. In response to shocks to the riskiness of investment, credit rationing increases, loans from intermediaries fall, and investment declines. The decline in investment reduces future output and contributes to the propagation of the initial shock. Bernanke and Gertler (1989) incorporated the model of costly state verification (see section 10.6.4) into a general equilibrium framework in which shocks to productivity drive the business cycle dynamics. A positive productivity shock increases the income of the owners of the production technology; this rise in their net worth lowers agency costs associated with external financing of investment projects, allowing for increased investment. This propagates the shock through time.

Kiyotaki and Moore (1997) developed a model that illustrates the role of net worth and credit constraints on equilibrium output. In their model economy, there are two types of agents. One group, called *farmers*, can combine their own labor with land to produce output. They can borrow to purchase additional land but face credit constraints in so doing. These constraints arise because farmers' labor input is assumed to be critical to production—once farmers start producing, no one else can replace them—and farmers are assumed to be unable to precommit to work. Thus, if any creditor attempts to extract too much from a farmer, the farmer can simply walk away from the land, leaving the creditor with only the value of the land; all current production is lost. The inability to precommit

to work plays a role similar to the assumption of costly state verification; in this case, the creditor is unable to monitor farmers to ensure that they continue to work. As a result, the farmers' ability to borrow is limited by the collateral value of their land.

Letting  $k_t$  denote the quantity of land cultivated by farmers, output by farmers is produced according to a linear technology:

$$y_{t+1}^f = (a + c)k_t,$$

where  $ck_t$  is nonmarketable output ("bruised fruit" in the farmer analogy) that can be consumed by the farmer.

The creditors in Kiyotaki and Moore's model are called *gatherers*. They, too, can use land to produce output, employing a technology characterized by decreasing returns to scale. The output of gatherers is

$$y_{t+1}^g = G(\bar{k} - k_t), \quad G' \geq 0, G'' \leq 0,$$

where  $\bar{k}$  is the total fixed stock of land, so  $\bar{k} - k_t$  is the land cultivated by gatherers.

Utility of both farmers and gatherers is assumed to be linear in consumption, although gatherers are assumed to discount the future more. Because of the linear utility, and the assumption that labor generates no disutility, the socially efficient allocation of the fixed stock of land between the two types of agents would ensure that the marginal product of land is equalized between the two production technologies, or

$$G'(\bar{k} - k^*) = a + c, \tag{10.53}$$

where  $k^*$  is the efficient amount of land allocated to farmers.

Consider the market equilibrium. Taking the gatherers first, given that they are not credit-constrained and have linear utility, the real rate of interest is simply equal to the inverse of their subjective rate of time preference:  $R = 1/\beta$ .<sup>36</sup> Again exploiting the unconstrained nature of the gatherers' decision, the value of a unit of land,  $q_t$ , must satisfy

$$q_t = \beta [G'(\bar{k} - k_t) + q_{t+1}].$$

The present value of a unit of land is just equal to the discounted marginal return  $G'$  plus its resale value at time  $t + 1$ . Since  $\beta = R^{-1}$ , this condition can be rewritten as

$$\frac{1}{R}G'(\bar{k} - k_t) = q_t - \frac{q_{t+1}}{R} \equiv u_t. \tag{10.54}$$

The variable  $u_t$  will play an important role in the farmers' decision problem. To interpret it,  $q_{t+1}/R$  is the present value of land in period  $t + 1$ . This represents the collateralized value of a unit of land; a creditor who lends  $q_{t+1}/R$  or less against a piece of land is sure of being

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36. The standard Euler condition for optimal consumption requires that  $u_c(t) = \beta R u_c(t + 1)$ , where  $u_c(s)$  is the marginal utility of consumption at date  $s$ . With linear utility,  $u_c(t) = u_c(t + 1) = h$  for some constant  $h$ . Hence,  $h = \beta R h$ , or  $R = 1/\beta$ .

repaid. The price of a unit of land at time  $t$  is  $q_t$ , so  $u_t$  is the difference between the cost of the land and the amount that can be borrowed against the land. It thus represents the down payment a farmer will need to make in order to purchase more land.

Kiyotaki and Moore construct the basic parameters of their model to ensure that farmers will wish to consume only their nonmarketable output ( $ck_{t-1}$ ). Farmers then use the proceeds of their marketable output plus new loans minus repayment of old loans (inclusive of interest) to purchase more land. However, the maximum a farmer can borrow will be the collateralized value of the land, equal to  $q_{t+1}k_t/R$ . Hence, if  $b_t$  is the farmer's debt,

$$b_t \leq \frac{q_{t+1}k_t}{R}. \quad (10.55)$$

This can be shown to be a binding constraint in equilibrium, and the change in the farmer's land holdings will be

$$q_t(k_t - k_{t-1}) = ak_{t-1} + \frac{q_{t+1}k_t}{R} - Rb_{t-1},$$

where  $b_{t-1}$  is debt incurred in the previous period. Rearranging,

$$k_t = \frac{(a + q_t)k_{t-1} - Rb_{t-1}}{u_t}. \quad (10.56)$$

The numerator of this expression represents the farmer's net worth—current output plus land holdings minus existing debt. With  $u_t$  equal to the required down payment per unit of land, farmers invest their entire net worth in purchasing new land.

To verify that the borrowing constraint is binding, it is necessary to show that the farmer always finds it optimal to use all marketable output to purchase additional land (after repaying outstanding loans). Suppose instead that the farmer consumes a unit of output over and above  $ck_{t-1}$ . This yields marginal utility  $u_c$  (a constant by the assumption of linear utility), but by reducing the farmer's land in period  $t$  by  $1/u_t$ , this additional consumption costs

$$u_c \left[ \beta_f \frac{c}{u_t} + \beta_f^2 \left( \frac{a}{u_t} \left( \frac{c}{u_{t+1}} + \beta_f \left( \frac{a}{u_{t+1}} \left( \frac{c}{u_{t+2}} + \dots \right) \dots \right) \dots \right) \right) \right],$$

since the  $1/u_t$  units of land purchased at time  $t$  would have yielded additional consumption  $c/u_t$  plus marketable output  $a/u_t$  that could have been used to purchase more land that would have yielded  $c/u_{t+1}$  in consumption, and so on. Each of these future consumption additions must be discounted back to time  $t$  using the farmer's discount rate  $\beta_f$ . It will be demonstrated subsequently that the steady-state value of  $u$  will be  $a$ . Making this substitution, the farmer will always prefer to use marketable output to purchase land if

$$1 < \left[ \beta_f \frac{c}{a} + \beta_f^2 \left( \frac{a}{a} \left( \frac{c}{a} + \beta_f \left( \frac{a}{a} \left( \frac{c}{a} + \dots \right) \dots \right) \dots \right) \right) \right] = \frac{\beta_f c}{1 - \beta_f a},$$

or

$$\frac{a + c}{a} > \frac{1}{\beta_f} > R. \quad (10.57)$$

Kiyotaki and Moore assumed that  $c$  is large enough to ensure that this condition holds. This means farmers would always like to postpone consumption and will borrow as much as possible to purchase land. Hence, the borrowing constraint will bind.

Equation (10.56) can be written as  $u_t k_t = (a + q_t)k_{t-1} - Rb_{t-1}$ . But  $Rb_{t-1} = q_t k_{t-1}$  from (10.55), so  $u_t k_t = ak_{t-1}$ . Now using (10.54) to eliminate  $u_t$ , the capital stock held by farmers satisfies the following difference equation:

$$\frac{1}{R}G'(\bar{k} - k_t)k_t = ak_{t-1}. \quad (10.58)$$

Assuming standard restrictions on the gatherers' production function, (10.58) defines a convergent path for the land held by farmers.<sup>37</sup> The steady-state value of  $k$  is then given as the solution  $k^{ss}$  to

$$\frac{1}{R}G'(\bar{k} - k^{ss}) = a. \quad (10.59)$$

Multiplying through by  $R$ ,  $G'(\bar{k} - k^{ss}) = Ra$ . From (10.54) this implies

$$u^{ss} = a.$$

Equation (10.59) can be compared with (10.53), which gives the condition for an efficient allocation of land between farmers and gatherers. The efficient allocation of land to farmers,  $k^*$ , was such that  $G'(\bar{k} - k^*) = a + c > Ra = G'(\bar{k} - k^{ss})$ , where the inequality sign is implied by (10.57). Since the marginal product of gatherers' output is positive but declines with the amount of land held by gatherers, it follows that  $k^{ss} < k^*$ . The market equilibrium is characterized by too little land in the hands of farmers. As a consequence, aggregate output is too low.

Using the definition of  $u$ , the steady-state price of land is equal to  $q^{ss} = Ra/(R - 1)$ , and steady-state debt is equal to  $b^{ss} = q^{ss}k^{ss}/R = ak^{ss}/(R - 1)$ . The farmer's debt repayments each period are then equal to  $Rb^{ss} = [R/(R - 1)]ak^{ss} > ak^{ss}$ .

Kiyotaki and Moore extended this basic model to allow for reproducible capital and were able to study the dynamics of the more general model. The simple version, though, allows the key channels through which credit affects the economy's equilibrium to be highlighted. First, output is inefficiently low because of borrowing restrictions; even though farmers have access to a technology that, at the steady state, is more productive than that of gatherers, they cannot obtain the credit necessary to purchase additional land. Second, the ability of farmers to obtain credit is limited by their net worth. Equation (10.56) shows how the borrowing constraint makes land holdings at time  $t$  dependent on net worth (marketable output plus the value of existing land holdings minus debt). Third, land purchases by farmers will depend on asset prices. A fall in the value of land that is expected to persist (so  $q_t$

37. As long as  $G'(\bar{k} - k)$  is monotonically increasing in  $k$ ,  $G'(\bar{k}) < a$ , and  $G'(0) > a$ , there will be a single stable equilibrium.

and  $q_{t+1}$  both fall) reduces the farmers' net worth and demand for land. This follows from (10.56), which can be written as  $k_t = (q_t k_{t-1}/u_t) + (ak_{t-1} - Rb_{t-1})/u_t$ . A proportional fall in  $q_t$  and  $q_{t+1}$  leaves the first term,  $q_t k_{t-1}/u_t$ , unchanged. The second term increases in absolute value, but at the steady state,  $Rb > ak$ , so this term is negative. Thus, farmers' net worth declines with a fall in land prices.

These mechanisms capture the financial accelerator effects, as can be seen by considering the effects of an unexpected but transitory productivity shock. Suppose the output of both farmers and gatherers increases unexpectedly at time  $t$ . If the economy was initially at the steady state, then if  $\Delta$  is the productivity increase for farmers, (10.56) implies

$$u(k_t)k_t = (a + \Delta a + q_t - q^{ss})k^{ss}, \quad (10.60)$$

since  $q^{ss}k^{ss} = Rb^{ss}$  from the borrowing constraint, and the required down payment  $u$  is written as a function of  $k$ .<sup>38</sup> Two factors are at work in determining the impact of the productivity shock on the farmers' demand for land. First, because marketable output rises by  $\Delta ak^{ss}$ , this directly increases farmers' demand for land. Second, the term  $(q_t - q^{ss})k^{ss}$  represents a capital gain on existing holdings of land. Both factors act to increase farmers' net worth and their demand for land.

One way to highlight the dynamics is to examine a linear approximation to (10.60) around the steady state. Letting  $e$  denote the elasticity of the user cost of land  $u(k)$  with respect to  $k$ , the left side of (10.60) can be approximated by

$$ak^{ss}[1 + (1 + e)\hat{k}].$$

Using the fact that  $u(k^{ss}) = a$  and letting  $\hat{x}$  denote the percentage deviation of a variable  $x$  around the steady state,<sup>39</sup> the right side is approximated by

$$(a + \Delta a + q^{ss}\hat{q}_t)k^{ss}.$$

Equating these two and using the steady-state result that  $q^{ss} = Ra/(R - 1)$  yields

$$(1 + e)\hat{k} = \Delta + \frac{R}{R - 1}\hat{q}_t. \quad (10.61)$$

The capital gain effect on farmers' land purchases is, as Kiyotaki and Moore emphasize, scaled up by  $R/(R - 1) > 1$  because farmers are able to leverage their net worth. This factor can be quite large; if  $R = 1.05$ , the coefficient on  $\hat{q}_t$  is 21.

The asset price effects of the temporary productivity shock reinforce the original disturbance. These effects also generate a channel for persistence. When more land is purchased in period  $t$ , the initial rise in aggregate output persists.<sup>40</sup>

38. Recall that  $u_t = G'(\bar{k} - k_t)/R$  (see (10.54)).

39. The elasticity  $e$  is equal to  $[u'(k^{ss})k^{ss}]/u(k^{ss}) = u'(k^{ss})k^{ss}/a$ , where  $u'$  denotes the derivative of  $u$  with respect to  $k$ . Since  $u$  is increasing in  $\bar{k} - k$ ,  $u' < 0$ .

40. Recall that at the margin, farmers are more productive than gatherers; a shift of land from gatherers to farmers raises total output.

### 10.7.2 Agency Costs and General Equilibrium

Carlstrom and Fuerst (1997) embedded a model of agency costs based on Bernanke and Gertler (1989) in a general equilibrium framework that can then be used to investigate the model's qualitative and quantitative implications. In particular, they studied the way agency costs arising from costly state verification affect the impact that shocks to net worth have on the economy.<sup>41</sup>

In their model, entrepreneurs borrow external funds in an intraperiod loan market to invest in a project that is subject to idiosyncratic productivity shocks. Suppose entrepreneur  $j$  has a net worth of  $n_j$  and borrows  $i_j - n_j$ . The project return is  $\omega_j i_j$ , where  $\omega_j$  is the idiosyncratic productivity shock. Entrepreneurs have private information about this shock, whereas lenders can observe it only by incurring a cost. If the interest rate on the loan to entrepreneur  $j$  is  $r_j^k$ , then the borrower defaults if

$$\omega_j < \frac{(1 + r_j^k)(i_j - n_j)}{i_j} \equiv \bar{\omega}_j.$$

If the realization of  $\omega_j$  is less than  $\bar{\omega}_j$ , the entrepreneur's resources,  $\omega_j i_j$ , are less than the amount needed to repay the loan,  $(1 + r_j^k)(i_j - n_j)$ . If default occurs, the lender monitors the project at a cost  $\mu i_j$ .

Carlstrom and Fuerst derived the optimal loan contract between entrepreneurs and lenders and showed that it is characterized by  $i_j$  and  $\bar{\omega}_j$ . Given these two parameters, the loan interest rate is

$$1 + r_j^k = \frac{\bar{\omega}_j i_j}{i_j - n_j}.$$

Suppose the distribution function of  $\omega_j$  is  $\Phi(\omega_j)$ . The probability of default is  $\Phi(\bar{\omega}_j)$ . Let  $q$  denote the end-of-period price of capital. An entrepreneur not defaulting receives  $q\omega_j i_j - (1 + r_j^k)(i_j - n_j)$ . Entrepreneurs who default receive nothing. If  $f(\bar{\omega})$  is defined as the fraction of expected net capital output received by the entrepreneur, then

$$\begin{aligned} q i_j f(\bar{\omega}_j) &\equiv q \left\{ \int_{\bar{\omega}}^{\infty} \omega_i \Phi(d\omega) - [1 - \Phi(\bar{\omega}_j)] (1 + r_j^k)(i_j - n_j) \right\} \\ &= q i_j \left\{ \int_{\bar{\omega}}^{\infty} \omega \Phi(d\omega) - [1 - \Phi(\bar{\omega}_j)] \bar{\omega}_j \right\}. \end{aligned} \quad (10.62)$$

The expected income of the lender is

$$q \left\{ \int_0^{\bar{\omega}} \omega_i \Phi(d\omega) - \mu i_j \Phi(\bar{\omega}_j) + [1 - \Phi(\bar{\omega}_j)] (1 + r_j^k)(i_j - n_j) \right\}.$$

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41. See also Kocherlakota (2000).

If  $g(\bar{\omega}_j)$  is defined as the fraction of expected net capital output received by the lender, then

$$q_{ij}g(\bar{\omega}_j) \equiv q_{ij} \left\{ \int_0^{\bar{\omega}} \omega \Phi(d\omega) - \mu \Phi(\bar{\omega}_j) + [1 - \Phi(\bar{\omega}_j)] \bar{\omega}_j \right\}. \quad (10.63)$$

By adding together (10.62) and (10.63), one finds that

$$f(\bar{\omega}_j) + g(\bar{\omega}_j) = 1 - \mu \Phi(\bar{\omega}_j) < 1. \quad (10.64)$$

Hence, the total expected income to the entrepreneur and the lender is less than the total expected project return (the fractions sum to less than 1) because of the expected monitoring costs.

The optimal lending contract maximizes  $qif(\bar{\omega})$  subject to

$$qig(\bar{\omega}) \geq i - n, \quad (10.65)$$

$$qif(\bar{\omega}) \geq n,$$

where, for convenience, the  $j$  notation has been dropped. The first constraint reflects the assumption these are intraperiod loans, so the lender just needs to be indifferent between lending and retaining funds. The second constraint must hold if the entrepreneur is to participate; it ensures that the expected payout to the entrepreneur is greater than the net worth the entrepreneur invests in the project. Carlstrom and Fuerst showed that this second constraint always holds, so it is ignored in the following. Using (10.64), the optimal loan contract solves

$$\max_{i, \bar{\omega}} \{qif(\bar{\omega}) + \lambda [qi(1 - \mu \Phi - f(\bar{\omega})) - i + n]\}.$$

The first-order conditions for  $i$  and  $\bar{\omega}$  are

$$qf(\bar{\omega}) + \lambda [q(1 - \mu \Phi - f(\bar{\omega})) - 1] = 0, \quad (10.66)$$

$$qif'(\bar{\omega}) - \lambda qi(\mu \phi + f'(\bar{\omega})) = 0,$$

where  $\phi = \Phi'$  is the density function for  $\omega$ . Solving this second equation for  $\lambda$ ,

$$\lambda \left[ 1 + \frac{\mu \phi(\bar{\omega})}{f'(\bar{\omega})} \right] = 1. \quad (10.67)$$

Now multiplying both sides of (10.66) by  $[1 + \mu \phi(\bar{\omega})/f'(\bar{\omega})]$  and using (10.67) yields, after some rearrangement,

$$q \left[ 1 - \mu \Phi + \mu \phi(\bar{\omega}) \frac{f(\bar{\omega})}{f'(\bar{\omega})} \right] = 1. \quad (10.68)$$

Finally, from the constraint (10.65),

$$qig(\bar{\omega}) = i - n. \quad (10.69)$$

Equation (10.68) determines  $\bar{\omega}$  as a function of the price of capital  $q$ , the distribution of the shocks, and the cost of monitoring. All three of these factors are the same for



all entrepreneurs, so all borrowers face the same  $\bar{\omega}$ , justifying the dropping of the  $j$  subscript. Writing  $\bar{\omega} = \bar{\omega}(q)$ , investment  $i$  can be expressed using (10.69) as a function of  $q$  and  $n$ :

$$i(q, n) = \left[ \frac{1}{1 - qg(\bar{\omega}(q))} \right] n. \quad (10.70)$$

Expected capital output is

$$I^s(q, n) = i(q, n) [1 - \mu \Phi(\bar{\omega})]. \quad (10.71)$$

The optimal contract has been derived while taking the price of capital,  $q$ , as given. In a general equilibrium analysis, this price must also be determined. To complete the model specification, assume that firms produce output using a standard neoclassical production function employing labor and capital:

$$Y_t = \theta_t F(K_t, H_t),$$

where  $\theta_t$  is an aggregate productivity shock. Factor markets are competitive. Households supply labor and rent capital to firms. If households wish to accumulate more capital, they can purchase investment goods at the price  $q_t$  from a mutual fund that lends to entrepreneurs. These entrepreneurs then create capital goods using the project technology just described and end the period by making their consumption decision.<sup>42</sup> This last choice then determines the net worth entrepreneurs carry into the following period.

If net worth is constant, Carlstrom and Fuerst showed their general equilibrium model can be mapped into a standard real business cycle model with capital adjustment costs. They argued that agency costs therefore provide a means of endogenizing adjustment costs. Because net worth is not constant in their model, however, variations in entrepreneur net worth can serve to propagate shocks over time. For example, a positive productivity shock increases the demand for capital, and this pushes up the price of capital. By increasing entrepreneurs' net worth, the rise in the price of capital increases the production of capital (see 10.71). By boosting the return on internal funds, the rise in the price of capital also induces entrepreneurs to reduce their own consumption to build up additional net worth. The endogenous response of net worth causes investment to display a hump-shaped response to an aggregate productivity shock. This type of response is more consistent with empirical evidence than is the response predicted by a standard real business cycle model in which the maximum impact of a productivity shock on investment occurs in the initial period.

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42. Carlstrom and Fuerst assumed that entrepreneurs discount the future more heavily than households and that their utility is linear. The Euler condition for entrepreneurs is

$$q_t = \beta \gamma E_t [q_{t+1} (1 - \delta) + F_K(t+1)] \left[ \frac{q_{t+1} f'(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right], \quad 0 < \gamma < 1,$$

where the first term on the right side is the return to capital and the second term is the additional return on internal funds.

### 10.7.3 Agency Costs and Sticky Prices

In chapter 8, nominal rigidities played an important role in transmitting monetary policy disturbances to the real economy. Bernanke, Gertler, and Gilchrist (1999) combined nominal rigidities with an agency cost model to explore the interactions between credit market factors and price stickiness. They developed a tractable model with the complications introduced by both credit factors and sticky prices by employing a model with three types of agents: households, entrepreneurs, and retailers. Entrepreneurs borrow to purchase capital. Costly state verification in the Bernanke, Gertler, and Gilchrist model implies that investment will depend positively on entrepreneurs' net worth, just as it did in the Carlstrom and Fuerst (1997) model (see (10.70)). Entrepreneurs use capital and labor to produce wholesale goods. These wholesale goods are sold in a competitive goods market to retailers. Retailers use wholesale goods to produce differentiated consumer goods that are sold to households. Wholesale prices are flexible, but retail prices are sticky. This model exhibits a financial accelerator (Bernanke, Gertler, and Gilchrist 1996); movements in asset prices affect net worth and amplify the impact of an initial shock to the economy.

Sticky price adjustment in the retail sector is modeled following Calvo (see chapter 7) so that each period there is a fixed probability that the individual retail firm can adjust its price. When a firm does adjust, it sets its price optimally. As a result, the rate of inflation of retail prices is a function of expected future inflation and given by real marginal cost in the retail sector. Since retail firms simply purchase wholesale goods at the competitive wholesale price  $P_t^w$  and resell these goods to households, real marginal cost for retailers is just the ratio of wholesale to retail prices.

Bernanke, Gertler, and Gilchrist (1999) calibrated a log-linearized version of their model to study the role the financial accelerator plays in propagating the impact of a monetary policy shock. They found that it increases the real impact of a policy shock. A positive nominal interest rate shock reduces the demand for capital, and this lowers the price of capital. The decline in the value of capital lowers entrepreneurs' net worth. As a consequence, the finance premium demanded by lenders rises, and this further reduces investment demand. Thus, a multiplier effect amplifies the initial impact of the interest rate rise. The contraction in the wholesale sector lowers wholesale prices relative to sticky retail prices. The retail price markup increases, reducing retail price inflation. More recently, however, Carlstrom, Fuerst, and Paustian (2016) derived the optimal lending contract in the Bernanke-Gertler-Gilchrist model and showed that it involved indexation to the aggregate return to capital, consumption, and the return to internal funds. They then showed that under this optimal contract, the financial accelerator in the Bernanke-Gertler-Gilchrist model is virtually eliminated.

The financial crisis of 2007–2009 has led to a rapidly growing literature that incorporates financial frictions, often based on the Bernanke, Gertler, and Gilchrist (1999) approach, in models with nominal rigidities designed to address monetary policy issues. For example, in a model without capital, Demirel (2009) assumed firms must borrow to finance inputs into

the production process. Christiano, Motto, and Rostagno (2010) embedded the Bernanke-Gertler-Gilchrist model of agency costs in a DSGE model with sticky wages and prices, which they then fit to U.S. and euro area data. De Fiore and Tristani (2013) developed a model with sticky prices and costly state verification that leads to agency costs, as firms must borrow to finance their wage payments. Cúrdia and Woodford (2010) allowed for interest rates paid by borrowers and received by savers to differ. They found that the optimal Taylor rule calls for responding to credit spreads. In the context of open-economy models, Gertler, Gilchrist, and Natalucci (2007) embedded the financial accelerator into a model of a small open economy to study the role of exchange rate regimes. They found that financial frictions play a significant role in accounting for output declines in the face of an exogenous rise in the country's risk premium. Monacelli (2008) added financial frictions to a small open economy model by incorporating the presence of collateral constraints on borrowing by the household sector.

## 10.8 Summary

This chapter has examined a number of issues related to financial markets and monetary policy, including the role of the term structure of interest rates and frictions in credit markets. The economics of imperfect information provides numerous insights into the structure of credit markets. Adverse selection and moral hazard account for many of the distinctive features of credit contracts when monitoring is costly. Credit market imperfections commonly lead to situations in which the lender's expected profits are not monotonic in the interest rate charged on a loan; expected profits initially rise with the loan rate but can then reach a maximum before declining. This produces the possibility that equilibrium may be characterized by credit rationing; excess demand fails to induce lenders to raise the loan rate, as doing so lowers their expected profits. Perhaps more important, balance sheets matter. Variations in borrowers' net worth affect their ability to gain credit. A recession that lowers cash flows or a decline in asset prices that lowers net worth will reduce credit availability and increase the wedge between the costs of external and internal finance. The resulting impact on aggregate demand can generate a financial accelerator effect.

## 10.9 Problems

1. For the model of (10.1)–(10.4) and the policy rule (10.9), find the rational-expectations equilibrium expression for the price level as a function of  $m_{t-1}$  and the model shocks. Verify that  $i_t$  fluctuates randomly around the target  $i^T$ , and that as  $\mu \rightarrow \infty$ , the variance of the nominal rate around the targeted value  $i^T$  will shrink to zero, but the price level can remain determinate. (*Hint*: The model can be solved using the method of undetermined coefficients. See Attfield, Demery, and Duck (1991), Wickens (2008), DeJong and Dave (2011), or Miao (2014).)

2. Redo problem 1 using the policy rule (10.10) instead of (10.9).
3. Suppose the money supply process in section 10.2.1 is replaced with

$$m_t = m_{t-1} + \xi_t - \gamma \xi_{t-1}.$$

Does  $i_t$  depend on  $\gamma$ ? Does  $I_t$ ? Explain.

4. McCallum (2005). Suppose the central bank adjusts the short-term rate  $i_t$  in response to the slope of the term structure:  $i_t = i_{t-1} + \lambda(I_t - i_t) + \zeta_t$ , where  $\zeta$  is a white noise process and  $|\lambda| < 1$ , and  $I_t$  is the two-period rate.
  - a. If the long-term rate is given by (10.26) and  $\xi_t = \rho \xi_{t-1} + u_t$ , show that the short-term rate must satisfy  $(1 + \lambda)i_t = i_{t-1} + \frac{\lambda}{2}(i_t + E_t i_{t+1}) + \lambda \xi_t + \zeta_t$ .
  - b. Now suppose the solution for  $i_t$  is of the form

$$i_t = \phi_0 + \phi_1 i_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t.$$

Assuming that private agents can observe the contemporaneous values of the two shocks  $\xi_t$  and  $\zeta_t$ , show that

$$i_t = i_{t-1} + \frac{\lambda}{1 - \frac{\lambda\rho}{2}} \xi_t + \zeta_t$$

and

$$i_t = \frac{2}{\lambda} i_{t-1} + \frac{2}{1 - \rho} \xi_t + \frac{2}{\lambda} \zeta_t$$

are both consistent with equilibrium but that the second of these solutions implies explosive behavior of the short-term rate.

5. Assume the central bank's policy rule is

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [r^n + \pi^T + \phi(\pi_t - \pi^T)] + v_t,$$

with  $\phi > 1$ ,  $v_t$  a white noise disturbance,  $r^n$  a constant, and  $\pi^T$  equal to the central bank's inflation target. The rest of the model consists of

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right) (i_t - E_t x_{t+1} - r^n),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

where  $x_t$  is the output gap and  $\pi$  is inflation. Assume  $u_t$  is white noise. Under the expectations hypothesis of the term structure, the two-period rate is given by (10.11).

- a. Obtain an expression for the equilibrium behavior of the two-period rate (i.e., express it as a function of the state variables  $v_t$ ,  $u_t$ , and  $i_{t-1}$  and the parameters of the model). How does it depend on  $\rho_i$ ? (*Hint*: Solve using the method of undetermined coefficients. Because the shocks are serially uncorrelated, guess that  $x_t = a_1 i_{t-1} + a_2 v_t + a_3 u_t$  and  $\pi_t = b_0 \pi^T + b_1 i_{t-1} + b_2 v_t + b_3 u_t$ .)

- b. How does the central bank's inflation target affect the level of the one-period and the two-period interest rates?
6. Consider the model due to Smith and Taylor (2009) that was discussed in section 10.5.2. There, it was shown that

$$P_t^{(2)} = E_t m_{t+1} P_{t+1}^{(1)} = E_t m_{t+1} \exp(-r_{t+1}) = E_t m_{t+1} \exp(-\delta_\pi \pi_{t+1}).$$

Using (10.34) and the specification of  $\lambda_t$  and  $\pi_t$  given in the text, verify that

$$i_t^{(2)} = a_2 + b_2 \pi_t,$$

where

$$a_2 = \delta \left( 0.5 \delta \sigma^2 - \sigma \gamma_0 \right),$$

$$b_n = -\delta_\pi (2 - \phi(\delta_\pi - 1) + \sigma \gamma_1).$$

Is  $b_n$  increasing or decreasing in the policy coefficient  $\delta_\pi$ ? *Hint:* Because  $\varepsilon_{t+1}$  is a standard normal,

$$E_t \exp[a \varepsilon_{t+1}] = \exp\left(\frac{1}{2} a^2\right).$$

7. Consider a firm that can invest in one of two projects. Project  $i = 1, 2$  yields a gross rate of return of  $R - x_i$  with probability  $1/2$  and  $R + x_i$  with probability  $1/2$ . Assume  $x_2 > x_1$ , so project 2 is the riskier project. The firm borrows  $L$  to undertake the project and has collateral  $C$ . The lender's opportunity cost of funds (the rate of return it can earn if it does not lend to the firm) is  $r$ . Both the firm and the lender are risk-neutral. Assume the firm defaults when  $R - x_i$  occurs.
- a. Suppose the lender can, without cost, monitor which project the firm chooses. What interest rate will the lender charge the firm if the firm picks project 1? What interest rate will it charge if the firm picks project 2? Assume that the firm defaults on the loan if the bad state (when the return is  $R - x_i$ ) occurs, in which case the lender gets  $R - x_i + C$ . (*Hint:* For either project, the expected rate of return to the lender must equal  $r$ .)
- b. Now suppose the firm chooses which project to undertake after it receives the loan, and the lender cannot observe which project is undertaken. What interest rate will the bank charge on loans? Can low-risk projects get funding? Explain.
8. Suppose the model consists of standard IS and inflation equations of the form

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right) (i_t - E_t \pi_{t+1}),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

and the policy rule

$$i_t = \rho i_{t-1} + (1 - \rho) \phi \pi_t + v_t.$$

Assume the policy shock is white noise. Assume further that  $\phi = 1.5 > 1$  so that the Taylor principle is satisfied. For the other parameters, assume  $\sigma = 1$ ,  $\beta = 0.99$ , and  $\kappa = 0.15$ . Solve the model numerically for  $\rho \in [0.4 \ 0.8 \ 1.0]$ , and show how the response of the  $n$ -period interest rate depends on  $\rho$ .

9. In the Gertler and Kiyotaki (2010) model (see section 10.6.5), consider the special case in which  $\omega = 1$  so that the moral hazard friction is absent from interbank lending. Then,  $R_{a,t} = R_{b,t} \geq R_{d,t}$ . If the incentive constraint is binding, the bank's maximum leverage ratio,  $\phi_t$ , was defined by (10.52). Carefully explain why  $\phi_t$  is increasing in  $R_{d,t}$  and decreasing in  $\theta$  for given  $\lambda_t$ .

# 11

## The Effective Lower Bound and Balance Sheet Policies

### 11.1 Introduction

Between December 2008 and December 2015, the target rate for the standard instrument of U.S. monetary policy, the federal funds rate, was fixed at 0.0–0.25 percent, a level viewed by the Federal Reserve as its effective lower bound (Bernanke and Reinhart 2004). This lower bound was ignored in earlier chapters. This chapter begins in section 11.2 with a discussion of why standard models imply the nominal interest is bounded below by zero. Then, in section 11.3, it is shown how two core components of many monetary policy models—the Fisher equation linking real and nominal returns and an instrument rule for monetary policy—imply a seemingly sensible policy rule may lead the economy into a liquidity trap in which the nominal interest rate is zero even in the absence of any exogenous shocks.

The focus turns in section 10.4 to situations in which the economy experiences a large contractionary shock to aggregate demand. The appropriate policy response is a cut in interest rates, but if the shock is big enough, the central bank may eventually push the nominal rate to its lower bound. If the policy rate cannot be reduced further, can the central bank still effectively stabilize the economy? This question is investigated under the assumption that an interest rate remains the only instrument of the central bank. Promises about the future path of interest rates, that is, forward guidance, play a critical role in allowing monetary policy to still affect the economy.

Section 11.5 turns to more unconventional policies, which are often referred to as quantitative-easing (QE), credit-easing, or balance sheet policies. These policies involve changing the size or composition of the central bank's balance sheet. Several models that investigate channels through which balance sheet policies may affect interest rates and aggregate spending are reviewed.

Recent experiences make it clear that zero is not the true lower bound on nominal interest rates. By early 2016, the European Central Bank and the central banks of Denmark, Japan, Sweden, and Switzerland had all set negative policy rates, and rates on 10-year Swiss government bonds had fallen to negative levels. It is better, therefore, to speak of

an *effective lower bound* (ELB) for nominal rates rather than a zero lower bound (ZLB). However, because the exact value of the ELB is uncertain, and because negative rates are a very recent phenomenon, the models reviewed in this chapter generally take the value of the ELB to be zero.

## 11.2 The Effective Lower Bound

Conventional monetary models assume that the nominal rate of interest must be non-negative. To understand why, consider the typical budget constraint of a representative household. Because the key point can be made ignoring capital and labor supply, assume the representative agent simply receives an endowment  $y_t$  each period and chooses consumption, money, and bonds subject to a budget constraint of the form

$$y_t + \frac{(1 + i_{t-1})b_{t-1} + m_{t-1}}{1 + \pi_t} + \tau_t = c_t + b_t + m_t,$$

where  $b_{t-1}$  and  $m_{t-1}$  are the real values of bond and money holdings carried over from the previous period,  $i_{t-1}$  is the nominal interest yield on bonds,  $\pi_t$  is the inflation rate,  $\tau_t$  equals any lump-sum transfers, and  $c_t$  is consumption. Adding  $i_{t-1}m_{t-1}/(1 + \pi_t)$  to both sides and letting  $d_t = b_t + m_t$  equal the real value of bond and money holdings, the budget constraint can be written as

$$y_t + (1 + r_t)d_{t-1} + \tau_t = c_t + \left( \frac{i_{t-1}}{1 + \pi_t} \right) m_{t-1} + d_t,$$

where  $1 + r_t \equiv (1 + i_{t-1}) / (1 + \pi_t)$ . When  $i_{t-1} \geq 0$ , the term  $i_{t-1}m_{t-1}/(1 + \pi_t)$  represents the cost of holding money expressed in terms of forgone interest. Recursively solving this equation forward yields

$$d_{t-1} + \sum_{i=0}^{\infty} \prod_{j=0}^i \left( \frac{1}{1 + r_{t+j}} \right) (y_{t+i} + \tau_{t+i}) \geq \sum_{i=0}^{\infty} \prod_{j=0}^i \left( \frac{1}{1 + r_{t+j}} \right) \left[ c_{t+i} + \left( \frac{i_{t+i-1}}{1 + \pi_{t+i}} \right) m_{t+i-1} \right]. \quad (11.1)$$

This intertemporal budget constraint requires that the household's current assets plus the present discounted value of current and future income plus transfers be greater than or equal to the present discounted value of current and future consumption plus the cost of holding money.

If  $i_{t+i-1} < 0$ , however, the agent does not face a bounded decision problem. With the present discounted value of resources fixed, the agent can increase both consumption and money holdings without limit without violating the budget constraint if the nominal rate is negative. As long as the marginal utility of consumption is positive, the agent can increase utility by increasing both consumption and holdings of money. Taking interest rates as



given, the individual will have an unbounded demand for money, and the problem of maximizing utility does not have a bounded solution.

At negative nominal interest rates, the demand for money should be infinite. Rather than incurring a cost in holding money—the forgone positive interest one could have earned by holding a bond—negative rates mean one is paid to hold money. Nothing in standard models would limit the demand for money.

While theory would seem to rule out a negative nominal interest rate in equilibrium, negative interest rates are observed. The Swiss National Bank policy range for the 3-month Libor rate was  $-1.25$  percent to  $-0.25$  percent in 2015. And not just short-term rates have been negative. The interest rate on 10-year Swiss government debt fell to  $-0.38$  percent in December 2015. Negative nominal interest rates, even on long-term risk-free debt, are clearly possible.<sup>1</sup> The pure expectations model of the term structure (see chapter 10) implies the long-term nominal rate on a riskless government bond should equal the average of the riskless short-term rates over the term of the bond. Thus, for a long-term rate such as the Swiss 10-year bond rate to be negative, the expectations hypothesis implies that short-term rates are expected to be negative on average over the next ten years.

Issues that would arise with a negative interest rate can be illustrated using the money-in-the-utility function (MIU) model of chapter 2. In that model, the first-order conditions for bond and money holdings took the form

$$U_c(c_t, m_t) = \beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}), \quad (11.2)$$

$$U_c(c_t, m_t) \geq U_m(c_t, m_t) + \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) U_c(c_{t+1}, m_{t+1}), \quad (11.3)$$

where  $U_c$  is the marginal utility of consumption and  $U_m$  is the marginal utility of money. In chapter 2, (11.3) was assumed to hold with equality, with agents holding positive money balances. The inequality sign appears if private agents cannot issue money, implying  $m_t \geq 0$ . If the inequality (11.3) were strict, the agent would hold zero money balances.<sup>2</sup>

The two conditions (11.2) and (11.3) can be combined to yield

$$\left( \frac{i_t}{1 + i_t} \right) U_c(c_t, m_t) \geq U_m(c_t, m_t). \quad (11.4)$$

Assume the marginal utility of consumption is positive and of money is non-negative. If  $i_t = 0$ , the condition requires  $U_m = 0$ . If there exists  $\bar{m} > 0$  such that  $U_m(c, m) = 0$  for

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1. As an example of the effects if interest rates on bank deposits become negative, the *Financial Times* reported that the Swiss canton of Zug eliminated discounts for early payment of taxes as with negative interest rates it was costly for the canton to have excess cash in its accounts (“Swiss canton tells taxpayers to delay settling bills,” (Jan. 11, 2016)).

2. Assuming private agents can issue bonds or there exists a positive supply of government bonds, the first-order condition for  $b_t$  holds with equality.

all  $m \geq \bar{m}$ , the condition would be satisfied when  $i_t = 0$  for any  $m \geq \bar{m}$ .<sup>3</sup> However, this condition can never be satisfied if  $i_t < 0$ .

One way to reconcile negative nominal rates with basic models of money demand is to assume it is costly to hold large sums of cash. One needs to safeguard them, and doing so entails real resource costs. Assume the cost of storing money is given by a function  $\tau(m)$  that is nondecreasing and convex in  $m$ ;  $\tau' \geq 0$  and  $\tau'' > 0$ . This cost function might make  $\tau(m)$  close to zero until money holdings reach a sufficiently large level, at which point storage costs begin to increase significantly. Now, instead of (11.3), the first-order condition would, assuming money is held, take the form

$$\left[ \left( \frac{i_t}{1+i_t} \right) + \tau'(m_t) \right] U_c(c_t, m_t) = U_m(c_t, m_t).$$

The left side is the marginal utility costs of holding money; these costs are the value of the forgone real income when the nominal rate is positive and the marginal storage costs. The right side is the marginal utility of additional money holdings. If the nominal interest rate is negative, money demand is well defined even if  $m > \bar{m}$ , so that  $U_m = 0$ . In this case, the condition becomes

$$\tau'(m_t) = - \left( \frac{i_t}{1+i_t} \right) \geq 0.$$

With  $\tau' > 0$  and increasing in  $m$ , there can be a well-defined finite solution for the agent's demand for money. Thus, while negative nominal interest rates can arise as equilibrium phenomena, they can only do so if standard models of money are modified by, for example, assuming storage costs.<sup>4</sup>

### 11.3 Liquidity Traps

A situation in which the nominal interest rate equals its effective lower bound is often referred to as a liquidity trap because (11.4) is satisfied for any  $m > \bar{m}$ . In a model like the one discussed in section 11.2, bonds and money are distinguished by two characteristics: money yields a nonpecuniary service (utility) that bonds do not, and bonds pay a nominal

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3. If  $\lim_{m \rightarrow \infty} U_m(c, m) > 0$  for all  $c$ , no finite level of money holdings satisfies the equilibrium condition when  $i_t = 0$ , which is why the presence of a saturation level of money balances  $\bar{m}$  is often assumed (e.g., M. Friedman 1969 assumed this in his analysis of the optimum quantity of money). See chapter 4.

4. Some have argued that the effective bound may in fact be positive. For example, Bernanke and Reinhart (2004) argued that institutions (such as money market funds in the United States) whose liabilities typically paid rates below the policy interest rate would lose funds if the policy rate were pushed to zero. Borrowers who rely on these institutions for funding would need to seek alternatives. These "short-term dislocations" would represent costs of a zero interest rate.

return that money does not. If  $i = 0$  and  $U_m = 0$ , nothing distinguishes money and bonds; they are perfect substitutes. Open-market operations that swap  $M$  for  $B$  in the hands of the public without altering  $M + B$  have no effect on the economy's equilibrium.

Benhabib, Schmitt-Grohé, and Uribe (2001a; 2001b; 2002) argued that simple and seemingly reasonable monetary policy rules, such as a rule that satisfies the Taylor principle (see chapter 8), may actually lead to macroeconomic instability by forcing the economy into a liquidity trap.

To understand their argument, recall that in a standard MIU model, one could have an accelerating hyperinflation even if the nominal quantity of money were kept fixed. An explosive deflation, however, would cause real money balances to grow to infinity, and the transversality condition for the representative agent's optimization problem would eventually be violated (see section 2.2.2). However, the nominal interest rate would be falling along such a deflationary path. Eventually, it would reach zero (or the ELB), halting the rate of deflation from exploding.

To illustrate this possibility, consider a perfect-foresight equilibrium and write the Fisher equation as

$$i_t = r + \pi_{t+1}.$$

The real interest rate  $r$  is taken to be a constant for simplicity. Now suppose the central bank follows an interest rate rule of the form

$$i_t = r + \pi^* + \delta (\pi_t - \pi^*), \quad (11.5)$$

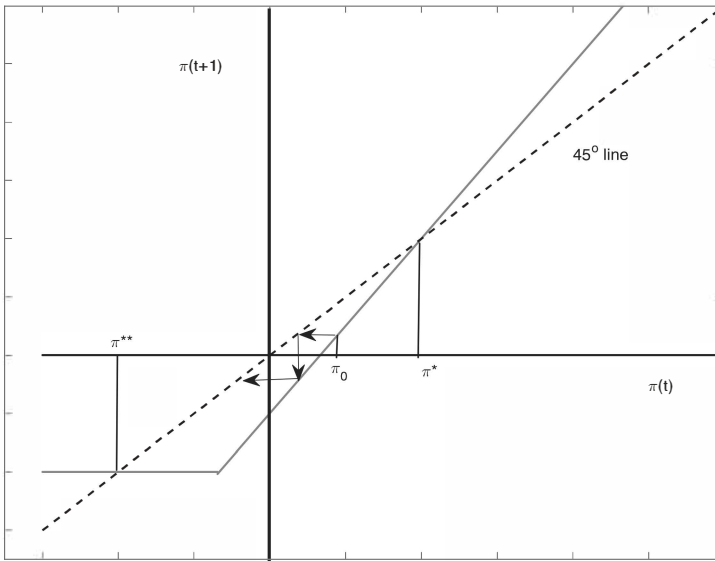
where  $\pi^*$  is the central bank's target inflation rate and  $\delta > 1$  to ensure the Taylor principle is satisfied.

If these two equations are combined, the equilibrium process for the inflation rate becomes

$$\pi_{t+1} = \pi^* + \delta (\pi_t - \pi^*),$$

which is unstable for  $\delta > 1$ , that is, for policy rules following the Taylor principle. The dynamics of the model are illustrated in figure 11.1. A stationary equilibrium exists with inflation equal to  $\pi^*$ . However, for inflation rates that start out below the target rate  $\pi^*$ ,  $\pi$  declines. As inflation declines, so does the nominal interest rate. Let  $i_L$  denote the lower bound on the nominal interest rate. If the rate of deflation is bounded below by  $i_L - r$ , the economy converges to a liquidity trap. The resulting equilibrium at  $\pi^{**}$  in the figure is stable.

Standard stability arguments in the presence of forward-looking jump variables rely on notions of saddle-path stability in which the inflation rate would jump to put the economy on a stable path converging to the unique stationary steady state. In the present context, this would involve current inflation jumping immediately to  $\pi^*$ . That is, the only



**Figure 11.1**

The dynamics of a liquidity trap.

perfect-foresight stationary equilibrium in a neighborhood of  $\pi^*$  is that associated with inflation equal to the target rate  $\pi^*$ . In contrast, in a neighborhood around the deflationary equilibrium  $\pi^{**}$ , there are many equilibrium paths consistent with a perfect-foresight equilibrium. If inflation starts out just to the left of  $\pi^*$ , the central bank cuts the nominal rate in an attempt to lower the real rate and stimulate the economy. But instead this policy reaction simply generates expectations of lower inflation, causing actual inflation to decline further. Expressed in terms of the quantity of money, the lower nominal rate increases the demand for real money balances, forcing a fall in the price level and pushing the economy into a deflationary equilibrium.

Suggestions for escaping a liquidity trap have involved both fiscal and monetary policies. Suppose fiscal policy is non-Ricardian.<sup>5</sup> The government could promise to run huge deficits whenever the inflation rate falls to an undesirably low level (Benhabib, Schmitt-Grohé, and Uribe 2002). According to the fiscal theory of the price level, this action, by increasing the government's total stock of nominal debt, would increase the equilibrium price level. This policy would rule out the low-inflation equilibrium by producing expectations of higher inflation whenever inflation becomes too low.

5. Under a non-Ricardian fiscal policy, the government's intertemporal budget constraint holds only at the equilibrium price level. See chapter 4.

Ireland (2005) departed from the standard representative agent framework to show that a traditional *real balance effect* can eliminate liquidity traps.<sup>6</sup> In his model, there are two overlapping generations. In the liquidity trap, nominal interest rates are zero, and the demand for real money balances is indeterminate. As a consequence, variations in the nominal stock of money may not affect the price level. However in a steady state with a zero nominal interest rate, prices are falling, so the nominal stock of money must also decline to keep real balances constant. This requires taxing the young to reduce the money supply. The future taxes necessary to reduce  $M$  will be paid, in part, by future generations, so the present discounted value of these taxes to the current generation is less than the value of their money holdings. In this environment, money is wealth, and aggregate demand depends on the real stock of money. This uniquely determines the level of real balances in equilibrium. But if  $M/P$  is uniquely determined, then varying  $M$  must always affect  $P$ , even in the liquidity trap.

If the central bank can conduct open-market operations in an asset that is an imperfect substitute for money, monetary policy can still affect inflation, even in a liquidity trap. McCallum (2000) and Svensson (2001), for example, argued that a central bank can generate inflation by depreciating its currency. By increasing the equilibrium price level, and thereby causing private agents to expect a positive rate of inflation, such policies can prevent nominal interest rates from falling to zero. Models in which central bank balance sheet policies have real effects are examined in section 11.5. However, before considering such models, conventional interest rate policies at the ELB are discussed. The focus is on situations in which the economy's being at the ELB is not due to the dynamics illustrated in figure 11.1. Instead, a negative shock to the equilibrium real interest rate ( $r$  in (11.5)) pushes the nominal rate to zero.

#### 11.4 Conventional Policies at the ELB

In the model analyses in chapters 2–4, a zero nominal interest rate was generally optimal; it eliminated the wedge between the private and social opportunity costs of money.<sup>7</sup> In economies characterized by nominal rigidities, the effective lower bound on the nominal interest rate can create serious problems. Because the real interest rate is  $i_t - E_t\pi_{t+1}$ , if the nominal interest rate set by the central bank has been reduced to its lower bound, yet this is still too high to be consistent with a zero output gap, a negative output gap and a decline in inflation will result. If inflation becomes negative, expectations of further deflation raise the real interest rate, worsening the decline in economic activity. An important policy issue, therefore, is to understand the options for a central bank when the nominal rate reaches its lower bound.

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6. See also McCallum (2000).

7. This is the M. Friedman (1969) rule.

In this section, the focus is on the potential of conventional policies, defined as interest rate policies, to stabilize the economy even though the current policy rate cannot be cut further.

### 11.4.1 Equilibria at the ELB

In the new Keynesian (NK) model of chapter 8, the optimal policy problem was studied in a linear-quadratic framework in which the central bank's objective was to minimize a quadratic loss function subject to linear constraints that represent the behavior of the private sector. Ignoring any lower bound constraint on the nominal interest rate, it was argued that the expectational IS relationship linking the real interest rate to aggregate demand could be ignored; only the inflation adjustment relationship represented a real constraint on the policymaker.<sup>8</sup> Any shocks appearing in the IS relationship could be prevented from affecting either the output gap or inflation by appropriately adjusting the nominal rate of interest.

For example, consider a basic NK model given by

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t^n), \quad (11.6)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (11.7)$$

$$i_t = r_t^n + \phi \pi_t, \quad (11.8)$$

where  $x_t$  is the output gap,  $\pi_t$  is inflation,  $i_t$  is the nominal interest rate, and  $r_t^n$  and  $e_t$  are exogenous stochastic shocks. Assume  $\phi > 1$  to satisfy the Taylor principle, ensuring there is a locally unique stationary equilibrium. Substituting the policy rule (11.8) into (11.6) yields

$$x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (\phi \pi_t - E_t \pi_{t+1}),$$

which together with (11.7) can be solved for the output gap and inflation independent of  $r_t^n$ , the shock to aggregate demand.

If the policymaker faces limits on its ability to reduce the nominal interest rate, then the IS relationship again becomes relevant for the determination of the output gap and inflation. Consider a negative realization of  $r_t^n$  such that for the value of  $i_t$  implied by the policy rule, the constraint  $i_t \geq i_L$  binds, and  $i_t = i_L$ . For convenience, assume  $i_L = 0$ , consistent with the standard assumption that the ELB equals zero. Equilibrium is given by the solution to

$$x_t = E_t x_{t+1} + \left( \frac{1}{\sigma} \right) (E_t \pi_{t+1} + r_t^n) \quad (11.9)$$

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8. See section 8.5 and chapter 8, problem 16.

and (11.7). The negative shock directly reduces  $x_t$  because it is no longer offset by monetary policy. The fall in the output gap reduces inflation. If this situation is persistent, expected future output and inflation may fall. The fall in  $E_t x_{t+1}$  directly amplifies the fall in  $x_t$ . If expected future inflation becomes negative, that is, if agents expected deflation, then from (11.9) there is a further negative effect on output. As expected inflation falls, the real interest rate is increased, reducing the current output gap further. Thus, a large negative realization of  $r_t^n$  can have severe consequences for output and inflation.

If the current nominal interest rate is the central bank's only policy instrument, then once it is at its effective lower bound, standard monetary policy might appear to be powerless. However, solving (11.6) forward under the assumption that the output gap converges to its steady-state equilibrium (so  $x_{t+T} \rightarrow 0$  as  $T \rightarrow \infty$ ),

$$x_t = - \left( \frac{1}{\sigma} \right) E_t \sum_{i=0}^{\infty} (i_{t+i} - \pi_{t+1+i} - r_{t+i}^n). \quad (11.10)$$

Current aggregate demand depends not just on the current nominal interest rate but on the entire future path of the nominal rate. For example, if the current nominal rate is at zero and expected to remain at zero until  $t+k$ , then

$$x_t = \left( \frac{1}{\sigma} \right) \sum_{j=0}^{k-1} E_t \pi_{t+j+1} - \left( \frac{1}{\sigma} \right) \sum_{j=k}^{\infty} E_t (i_{t+j} - \pi_{t+j+1}) + \left( \frac{1}{\sigma} \right) \sum_{j=0}^{\infty} E_t r_{t+j}^n.$$

Monetary policy can still affect current output if the central bank is able to influence expectations about future inflation while the economy is at the ELB or expectations about the path of the real interest rate once the ELB no longer binds. Thus, a key factor determining equilibrium at the ELB is the ability of the central bank to credibly commit to future policies.<sup>9</sup>

Eggertsson and Woodford (2003); Nakov (2008); and Adam and Billi (2006; 2007) analyzed the consequences of the effective lower bound under optimal discretionary policies, when the central bank cannot directly affect expectations, and under optimal commitment policies, when the central bank can make credible promises about the future paths of interest rates, inflation, and the output gap. In a discretionary policy regime, the central bank's only instrument is the current nominal interest rate. If this is at the ELB, there is nothing else the central bank can do, as any statements about future policy are not credible. If the central bank can credibly commit to future actions, the central bank has many instruments still available to it (e.g., the entire future path of the nominal rate). Not surprisingly, the

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9. In most of the monetary policy literature, the equilibrium real interest rate  $r_t^n$  is treated as exogenous. Correia et al. (2013) showed how fiscal policy can be used to affect  $r_t^n$  at the ELB. Devereux and Yetman (2014) explored the role of capital controls at the ELB in a two-country open-economy model subject to a global liquidity trap.

ELB can be very costly under discretion, but it imposes little welfare cost under optimal commitment.<sup>10</sup>

### 11.4.2 Analytics at the ELB

To focus on the consequences of the ELB for  $x_t$  and  $\pi_t$ , it is helpful to ignore shocks to the inflation adjustment equation. These shocks played a critical role in the analysis of optimal policy in chapter 8, as they presented the policymaker with a trade-off between stabilizing inflation and stabilizing the output gap when  $i_t > 0$ . For the analysis of policy in an ELB situation, however, inflation shocks are of less importance, and the primary focus is on the effect of a large negative realization of  $r_t^n$  that pushes  $i_t$  to its lower bound and prevents the policymaker from maintaining  $i_t - r_t^n$  equal to zero.

Assuming  $e_t \equiv 0$  in (11.7) for all  $t$ , the inflation adjustment equation takes the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \quad (11.11)$$

Treating the effective lower bound  $i_L$  as zero, equilibrium satisfies (11.9) and (11.11). The solution for  $x_t$  as a function of the shock  $r_t^n$  and expectations is given directly by (11.9), while substituting for  $x_t$  in (11.11) yields

$$\pi_t = \left( \beta + \frac{\kappa}{\sigma} \right) E_t \pi_{t+1} + \kappa E_t x_{t+1} + \frac{\kappa}{\sigma} r_t^n.$$

The equilibrium values of  $x_t$  and  $\pi_t$  at the ELB depend on agents' expectations about the future output gap and inflation rate and therefore on the expected evolution of the demand shock  $r_t^n$  and future monetary policy. Importantly, for a given (negative)  $r_t^n$ , equilibrium inflation and the output gap at the ELB are increasing in expected future inflation and the output gap.

The ELB equilibrium has been studied under a variety of simplifying assumptions about the demand shock process and about monetary policy. These simplifications allow rational expectations of future output and inflation to be determined. For example, suppose at time  $t$  the economy is at the ELB, it exits the ELB at  $t + T$ , and once it exits, it never returns to the ELB. In this case, the analysis of chapter 8 implies that in the absence of both the ELB constraint (both at  $t + T$  and in the future) and any inflation shocks, then  $x_{t+T+j} = \pi_{t+T+j} = 0$  for all  $j \geq 0$  is a feasible equilibrium. It is in fact the unique stationary equilibrium of (11.6), (11.8), and (11.11) and, as shown in chapter 8, it is also the equilibrium under optimal discretion with a standard quadratic loss function in inflation and the output gap.

Optimal policy under commitment displays inertia (see chapter 8); the past still matters. Thus, under commitment, optimal policy when the ELB no longer binds may still reflect

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10. Bodenstein, Hebden, and Nunes (2012) investigated the role of imperfect credibility at the ELB. A central bank that is less credible may need to make more extreme promises about future interest rates.



promises the central bank made when the economy was constrained by the ELB, and  $x_{t+T}$  and  $\pi_{t+T}$  may differ from zero. Because the solution is more complex under commitment, the analysis initially proceeds under the assumption that once it becomes feasible to raise the nominal interest rate, the ELB constraint never again binds, and both the output gap and inflation are equal to zero, consistent with the unique stationary equilibrium of (11.6), (11.8), and (11.11).<sup>11</sup>

Equilibrium when the ELB constraint binds depends on expectations about the output gap and inflation when the constraint no longer binds. It also depends on expectations about how long the constraint will bind. Following Eggertsson and Woodford (2003), one approach has been to assume a two-state process for  $r_t^n$  that takes a simple form. Suppose  $r_t^n = r^{\text{ELB}} < 0$  such that  $i_t = 0$ . With probability  $q$ ,  $r_{t+1}^n = r^{\text{ELB}}$  and the economy remains at the ELB; with probability  $1 - q$ ,  $r_{t+1}^n = r^n > 0$ . Once  $r_{t+j}^n = r^n$ , it remains at  $r^n$  for all  $j \geq 0$ , and once  $r_{t+j}^n = r^n$ , the ELB no longer binds, a zero output gap and inflation rate are feasible, and this is the equilibrium under either the policy rule (11.8) or under optimal discretion. The exogenous parameter  $q$  determines the expected length of the ELB period.

Consider the equilibrium output gap and inflation, denoted by  $x^{\text{ELB}}$  and  $\pi^{\text{ELB}}$ , while the ELB binds. Given the process for  $r^n$ ,  $E_t x_{t+1} = qx^{\text{ELB}} + (1 - q) \times 0 = qx^{\text{ELB}}$ , and  $E_t \pi_{t+1} = q\pi^{\text{ELB}} + (1 - q) \times 0 = q\pi^{\text{ELB}}$ . Equations (11.9) and (11.11) become

$$x^{\text{ELB}} = qx^{\text{ELB}} + \left(\frac{1}{\sigma}\right) (q\pi^{\text{ELB}} + r^{\text{ELB}}),$$

$$\pi^{\text{ELB}} = \beta q \pi^{\text{ELB}} + \kappa x^{\text{ELB}},$$

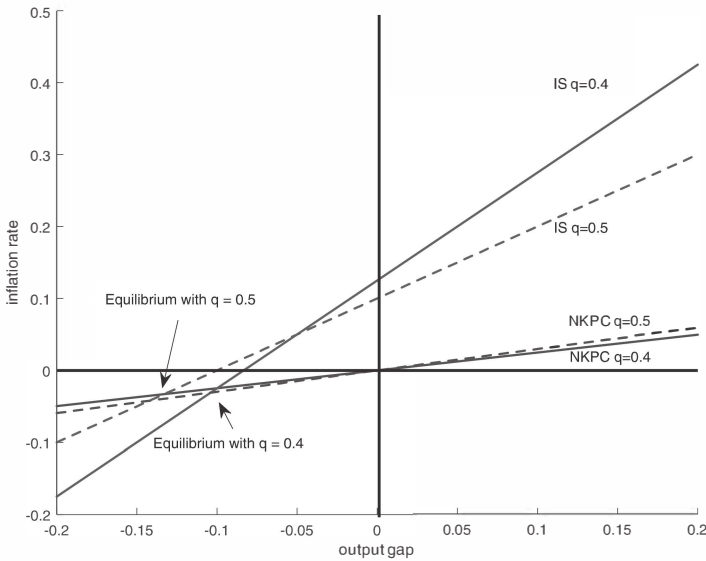
which can be solved jointly for  $x^{\text{ELB}}$  and  $\pi^{\text{ELB}}$ . Graphically, these two relationships are illustrated in figure 11.2 for  $\sigma = 1$ ,  $\beta = 0.99$ ,  $\kappa = 0.15$ , and  $r^{\text{ELB}} = -0.05$ . The solid lines represent  $q = 0.4$ ; the IS relationship derived from (11.9) and the new Keynesian Phillips curve (NKPC) derived from (11.11) intersect where both the output gap and inflation are negative. The dashed lines show the effects of a rise in  $q$ , the probability that the economy remains at the ELB, from 0.4 to 0.5. This more pessimistic assessment of the economy's prospects results in a fall in both inflation and the output gap. However, given the small value of  $\kappa$ , the NKPC is quite flat, and the rise in  $q$  affects primarily the output gap.

Solving for the equilibrium values of  $x^{\text{ELB}}$  and  $\pi^{\text{ELB}}$  yields

$$x^{\text{ELB}} = \left[ \frac{1 - \beta q}{\sigma(1 - q)(1 - \beta q) - q\kappa} \right] r^{\text{ELB}}, \quad (11.12)$$

$$\pi^{\text{ELB}} = \left[ \frac{\kappa}{\sigma(1 - q)(1 - \beta q) - q\kappa} \right] r^{\text{ELB}}. \quad (11.13)$$

11. As Nakov (2008) showed, optimal discretionary policy in a stochastic equilibrium actually suffers from a deflationary bias, and the output gap is positive for  $r_t^n > \bar{r}$  for some  $\bar{r} > 0$ .



**Figure 11.2**

Equilibrium at the ELB when the probability of remaining at the ELB is  $q$ .

Based on the calibration used for figure 11.2, the output gap falls as  $q$  increases. With a higher probability of remaining at the ELB and the output gap remaining negative, the expected future output gap falls. This fall in  $E_t x_{t+1}$  reduces current demand through two channels. First, from (11.9), current spending depends on expected future spending. In addition, through (11.11), a lower expected future output gap lowers expected future inflation. With the nominal rate at zero, the fall in expected future inflation raises the current real interest rate, further reducing the output gap. With inflation equal to  $[\kappa / (1 - \beta q)] x^{\text{ELB}}$ ,  $\pi^{\text{ELB}}$  moves in proportion to  $x^{\text{ELB}}$ .<sup>12</sup>

Several authors have studied the role of fiscal policy when the nominal rate is at zero. In normal times, monetary policy aimed at stabilizing the output gap would offset any effects of a fiscal expansion on the output gap by raising the nominal rate sufficient to boost the real rate of interest, negating the fiscal effects. Fiscal policy would still potentially affect the

12. The equilibrium illustrated in figure 11.2 assumes that the term  $\sigma(1-q)(1-\beta q) - q\kappa$  in the denominator of the expressions in (11.12) and (11.13) is positive. Braun, Körber, and Waki (2012) considered the situation in which this term is negative; this could occur if  $\kappa$  were large or  $q$  were very large (using the present notation). They characterized this as a type 2 equilibrium, which occurs when the NKPC is steeper than the IS relationship. However, they showed a type 2 equilibrium requires that  $r_t^n$  be positive. In this case, there always exists another equilibrium with  $x = \pi = 0$  in which the nominal rate is  $i_t = r_t^n > 0$  and the ELB is nonbinding. A number of papers explored the existence of multiple equilibria at the ELB, including Braun, Körber, and Waki (2012), Aruoba and Schorfheide (2013), and Gavin et al. (2015). See also Mertens and Ravn (2014).

economy's flexible price output level, but it would not affect the output gap. This crowding-out effect of higher interest rates would be absent if the nominal rate were fixed at zero, implying a fiscal expansion could raise the output gap.<sup>13</sup>

As an example of a seemingly counterintuitive result at the ELB, Eggertsson (2011) argued that a negative supply shock could be expansionary. Such a shock would raise inflation and, if persistent, also raise expected future inflation. With the nominal rate at zero, the rise in expected inflation lowers the real rate of interest and is expansionary. However, Wieland (2014) did not find evidence that negative supply shocks are expansionary.

Consider one final exercise with this simple analytical framework—raising the central bank's inflation target. The basic linearized new NKPC assumes steady-state inflation is equal to zero. Suppose instead it equals  $\pi^T$  and that firms who do not adjust prices optimally instead index prices to the central bank's target inflation rate, a common assumption in empirical dynamic stochastic general equilibrium (DSGE) models. The inflation equation with indexation is

$$\pi_t - \pi^T = \beta E_t (\pi_{t+1} - \pi^T) + \kappa x_t.$$

Upon exiting the ELB, assume  $x = 0$  and  $\pi = \pi^T$ . The equilibrium output gap and inflation rate at the ELB jointly solve

$$x^{\text{ELB}} = qx^{\text{ELB}} + \left(\frac{1}{\sigma}\right) \left[ q\pi^{\text{ELB}} + (1-q)\pi^T + r^{\text{ELB}} \right],$$

$$\pi^{\text{ELB}} - \pi^T = \beta q (\pi^{\text{ELB}} - \pi^T) + \kappa x^{\text{ELB}}.$$

Solving,

$$x^{\text{ELB}} = \frac{(1 - \beta q)(r^{\text{ELB}} + \pi^T)}{\sigma(1 - q)(1 - \beta q) - q\kappa}.$$

If the denominator in this expression is positive, a (credible) increase in the central bank's inflation target at the ELB is expansionary. By promising higher inflation when  $i_t = 0$ , the central bank lowers the real interest rate and stimulates current aggregate demand.<sup>14</sup>

These analytical results were derived based on simplifying assumptions about the demand shock  $r_t^d$ . In particular, it was assumed the economy never returned to the ELB once it exited. Nakov (2008) and Adam and Billi (2007) considered the stochastic equilibrium with more general specifications of the nature of the exogenous shocks when monetary policy is characterized by discretion. Using a standard quadratic loss function in the

13. See, for example, Christiano, Eichenbaum, and Rebelo (2011). If an expansionary fiscal policy succeeds in expanding the economy, interest rates would be increased sooner than otherwise, so a fiscal expansion would induce a future rise in rates and a delayed crowding-out effect.

14. For discussions of raising average inflation, see Williams (2009), Coibion, Gorodnichenko, and Wieland (2012), and Chung et al. (2012).

output gap and inflation to evaluate outcomes, they showed that the costs of the ELB constraint under discretion are large. Because a policymaker operating under discretion cannot influence expectations, its only instrument is the current nominal rate. If this is stuck at zero, there are no longer any policy tools available to the central bank. The economy's average level of output is also affected. In the face of expansionary shocks, the central bank can always raise its policy rate to keep the output gap at zero. For large contractionary shocks, policy may be constrained by the ELB, and so the output gap may be negative. Thus, the unconditional expectation of the output gap is no longer zero but is negative. On average, actual output will be less than the output achieved when prices are flexible.

### 11.4.3 Commitment and Forward Guidance

If the policymaker can make credible commitments about future policy actions, the consequences of the ELB constraint are quite different than when commitment is not possible. Work by Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006), and Nakov (2008) showed that making credible commitments about future policy can greatly mitigate the adverse effects of the ELB constraint. As shown by (11.10), current output depends on the current and future expected path of the real interest rate relative to  $r^n$ . Hence, even if the current nominal rate is at zero, monetary policy can still be effective if it can affect the expectation of the future path of  $i_{t+i}$  relative to  $r^n_{t+i}$ . Even a long period during which the nominal interest rate is anticipated to be at zero, for example, need not diminish significantly the central bank's ability to affect current spending if it is able to affect expectations of the future path of the nominal interest rate.

In this section, the fully optimal commitment policy is analyzed in a perfect-foresight equilibrium in which the ELB is currently binding but no longer binds from  $t + T$  on. After analyzing optimal commitment, the role of forward guidance about future policy is considered. Forward guidance may differ from optimal commitment in that it could, for example, be a promise to return to a Taylor rule such as (11.8) once the ELB period has ended. Or it could be a promise to keep interest rates at the ELB past the point at which the ELB no longer binds. Optimal policy under discretion would raise the nominal interest rate at  $t + T$  so that  $i_{t+T} - r^n_{t+T} = 0$ . This ensures the output gap and inflation equal zero as soon as the constraint no longer binds. In contrast, forward guidance might be a promise that inflation will exceed zero at  $\pi_{t+T}$ .

#### Optimal Commitment

The analysis of optimal commitment policy at the ELB follows Nakov (2008). Inflation shocks are ignored, so that inflation is given by (11.11). Also, to simplify the analysis, attention is restricted to perfect-foresight equilibria.<sup>15</sup>

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15. Werning (2011) provided results on optimal policies at the ELB in a continuous-time NK model. The analysis here is conducted in discrete time, consistent with the general approach employed in chapter 8.

Assume the central bank's objective is to minimize

$$\frac{1}{2} \sum_{j=0}^{\infty} \beta^j \left( \pi_{t+j}^2 + \lambda x_{t+j}^2 \right),$$

subject to (11.6), (11.11), and  $i_{t+j} \geq 0$  for all  $j$ . The policy problem under commitment can be written as

$$\min_{x_{t+j}, \pi_{t+j}, i_{t+j}} \sum_{j=0}^{\infty} \beta^j \left\{ \begin{array}{l} \frac{1}{2} \left( \pi_{t+j}^2 + \lambda x_{t+j}^2 \right) + \mu_{\pi, t+j} \left( \pi_{t+j} - \beta \pi_{t+1+j} - \kappa x_{t+j} \right) \\ + \mu_{x, t+j} \left[ x_{t+j} - x_{t+1+j} + \left( \frac{1}{\sigma} \right) \left( i_{t+j} - \pi_{t+1+j} - r_{t+j}^n \right) \right] \\ + \mu_{i, t+j} i_{t+j} \end{array} \right\},$$

where  $\mu_{\pi}$ ,  $\mu_x$ , and  $\mu_i$  are Lagrangian multipliers on the three constraints. When this problem was analyzed in chapter 8 and the ELB was ignored, it was shown that (11.6) did not impose a constraint on the policymaker, so that  $\mu_{x, t+j}$  was equal to zero for all  $j$ . When the ELB constraint is accounted for, the first-order conditions for this problem are, for  $j = 0$ ,

$$\pi_t + \mu_{\pi, t} = 0,$$

$$\lambda x_t - \kappa \mu_{\pi, t} + \mu_{x, t} = 0,$$

$$\left( \frac{1}{\sigma} \right) \mu_{x, t} + \mu_{i, t} = 0,$$

and for  $j > 0$ ,

$$\pi_{t+j} + \mu_{\pi, t+j} - \mu_{\pi, t-1+j} - \left( \frac{1}{\beta \sigma} \right) \mu_{x, t-1+j} = 0, \quad (11.14)$$

$$\lambda x_{t+j} - \kappa \mu_{\pi, t+j} + \mu_{x, t+j} - \left( \frac{1}{\beta} \right) \mu_{x, t-1+j} = 0, \quad (11.15)$$

$$\left( \frac{1}{\sigma} \right) \mu_{x, t+j} + \mu_{i, t+j} = 0, \quad (11.16)$$

together with (11.6), (11.11), and

$$\mu_{i, t+j} i_{t+j} = 0 \quad (11.17)$$

for all  $j \geq 0$ . From (11.16) this last condition can also be expressed as  $\mu_{x, t+j} i_{t+j} = 0$ .

If the ELB never binds, (11.14) and (11.15) imply

$$\kappa \pi_{t+j} = -\lambda (x_{t+j} - x_{t+j-1})$$

for  $j > 0$ .<sup>16</sup> From (11.6),

$$i_{t+j} = \pi_{t+j+1} + r_{t+j}^n + \sigma (x_{t+j+1} - x_{t+j}).$$

---

16. This was the targeting criterion obtained under commitment (see chapter 8).

These two relationships imply that if  $\{\pi_{t+j}^*, x_{t+j}^*, i_{t+j}^*\}$  is optimal, then

$$i_{t+j}^* = I\left(\pi_{t+1+j}^*, r_{t+j}^n\right) \equiv r_{t+j}^n + \left[1 - \left(\frac{\sigma\kappa}{\lambda}\right)\right] \pi_{t+j+1}^*.$$

Werning (2011) showed that for all  $t+j, j \geq 0$ , under the optimal commitment policy either  $i_{t+j}^* = I\left(\pi_{t+1+j}^*, r_{t+j}^n\right)$  or  $i_{t+j}^* = 0$ .

Suppose the ELB is binding at time  $t$  but is nonbinding from  $t+T$  on. Then  $\mu_{x,t+j} > 0$  for  $j = 0, \dots, T-1$ , but  $\mu_{i,t+T+j} = \mu_{x,t+T+j} = 0$  for all  $j \geq 0$ . Subtracting the first-order condition (11.15) at  $t+T+1$  from its value at  $t+T$  and using the fact that  $\mu_{i,t+T+j} = \mu_{x,t+T+j} = 0$  for  $j \geq 0$  yields

$$\lambda(x_{t+T+1} - x_{t+T}) - \kappa(\mu_{\pi,t+T+1} - \mu_{\pi,t+T}) + \left(\frac{1}{\beta}\right)\mu_{x,t+T-1} = 0.$$

But from (11.14),  $\pi_{t+T+1} = -(\mu_{\pi,t+T+1} - \mu_{\pi,t+T})$ , so

$$\lambda(x_{t+T+1} - x_{t+T}) + \kappa\pi_{t+T+1} + \left(\frac{1}{\beta}\right)\mu_{x,t+T-1} = 0. \quad (11.18)$$

From (11.18),

$$x_{t+T+1} - x_{t+T} = -\left[\frac{\kappa}{\lambda}\pi_{t+T+1} + \left(\frac{1}{\beta\lambda}\right)\mu_{x,t+T-1}\right].$$

However, (11.6) implies  $x_{t+T+1} - x_{t+T} = \sigma^{-1}(i_{t+T} - \pi_{t+T+1} - r_{t+T}^n)$ . Combining these results and solving for  $i_{t+T}$  yields

$$\begin{aligned} i_{t+T} &= r_{t+T}^n + \left[1 - \left(\frac{\sigma\kappa}{\lambda}\right)\right] \pi_{t+T+1} - \left(\frac{\sigma}{\beta\lambda}\right) \mu_{x,t+T-1} \\ &= I(\pi_{t+T+1}^*, r_{t+T}^n) - \left(\frac{\sigma}{\beta\lambda}\right) \mu_{x,t+T-1} < I(\pi_{t+T+1}^*, r_{t+T}^n). \end{aligned}$$

Thus,  $i_{t+T} \neq I(\pi_{t+T+1}^*, r_{t+T}^n)$  and Werning's result implies  $i_{t+T} = 0$ , even though the ELB no longer binds. The optimal commitment policy keeps the nominal rate at zero after the non-negativity constraint on the nominal interest rate no longer restricts monetary policy. Keeping the policy rate at zero longer results in a policy that is more expansionary than would have been optimal at  $t+T$  if the ELB had not been binding at  $t+T-1$ . Keeping the nominal rate lower than occurs under discretion at  $t+T$  implies the output gap and inflation will be higher. But higher values of  $x_{t+T}$  and  $\pi_{t+T}$  increase  $x_{t+T-j}$  and  $\pi_{t+T-j}$  for  $j \geq 1$ , limiting the negative effects of the ELB during the period when it is binding.

### Forward Guidance

At the ELB, the optimal commitment policy involves a promise not to lift the policy rate above zero the moment the non-negativity constraint on  $i$  is relaxed. In a full-information

environment, everyone understands how the policymaker behaves in a commitment equilibrium. In practice, however, the central bank may need to communicate to the public its contingent plans for the future path of the policy rate. Forward guidance refers to policy statements designed to provide information about future policy.

Jung, Teranishi, and Watanabe (2005); Levin et al. (2010); DelNegro, Giannoni, and Patterson (2012); Carlstrom, Fuerst, and Paustian (2012b); Kiley (2016); and McKay, Nakamura, and Steinsson (2016) all emphasized that the basic new Keynesian model implies credible forward guidance has a very powerful effect on equilibrium at the ELB. To understand why, consider a perfect-foresight equilibrium in which  $r_t^n = r^{\text{ELB}} < 0$  and the nominal rate is at zero from  $t$  to  $t + T - 1$ . From  $t + T$  on, the ELB never binds and the central bank ensures  $x_{t+T} = \pi_{t+T} = 0$  for  $s \geq T$ . As noted earlier, this is what a central bank would do under an optimal discretionary policy.

The equilibrium can be constructed by working backward from time  $t + T$ . Write the two equilibrium conditions (11.9) and (11.11) that hold from  $t$  to  $t + T - 1$  as

$$\begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_{t+j-1} \\ \pi_{t+j-1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x_{t+j} \\ \pi_{t+j} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ 0 \end{bmatrix} r^{\text{ELB}}$$

for  $j = 1$  to  $T$ . This is a linear difference equation with terminal conditions  $x_{t+T} = \pi_{t+T} = 0$ . Premultiplying both sides by the inverse of the matrix on the left yields

$$\begin{bmatrix} x_{t+j-1} \\ \pi_{t+j-1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix} \begin{bmatrix} x_{t+j} \\ \pi_{t+j} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r^{\text{ELB}}. \quad (11.19)$$

Defining

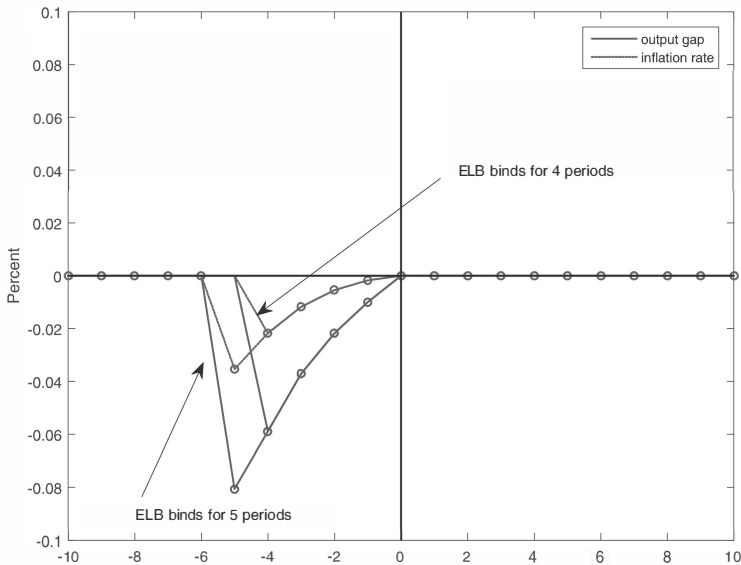
$$Q \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix},$$

one obtains through recursive substitution

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \left( I + Q + Q^2 + \dots + Q^{T-1} \right) \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r^{\text{ELB}},$$

where  $I$  is the  $2 \times 2$  identity matrix. However,  $Q$  has one eigenvalue outside the unit circle, so (11.19) has an explosive root.<sup>17</sup> Thus, the longer the economy is expected to be at the ELB (i.e., the larger is  $T$ ), the more negative are  $x_t$  and  $\pi_t$ . This is illustrated in figure 11.3, which shows the equilibrium paths for the output gap and inflation (at an annual rate) when policy ensures both equal zero once the constraint is no longer binding (at period 0 in the figure). The explosive behavior is apparent in the much larger fall in the output gap and inflation when, in this perfect-foresight equilibrium, the constraint binds for five periods rather than four periods.

17. Because the nominal interest rate is fixed at zero, the Taylor principle is violated.



**Figure 11.3**

Equilibrium output gap and inflation when the ELB binds for four periods (*solid lines*); for five periods (*circles*).

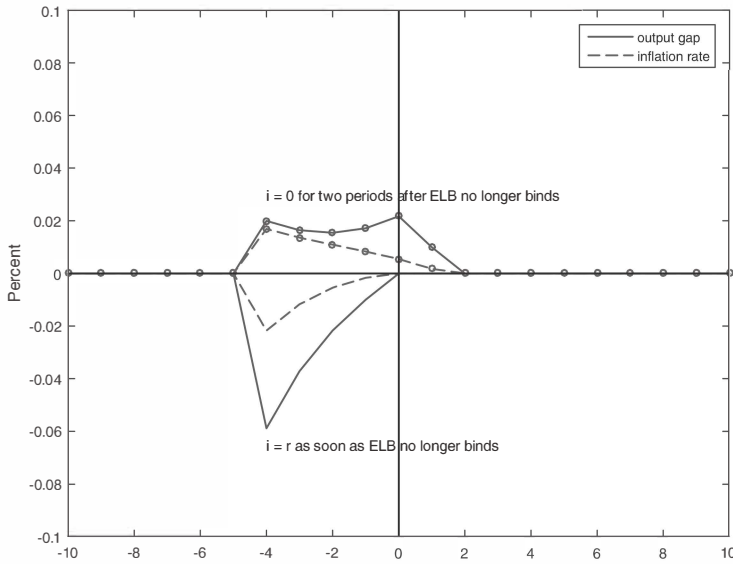
Now consider forward guidance in which the central bank announces that it will keep the nominal interest rate at zero for two periods past the time at which the ELB no longer binds. The results are shown in figure 11.4 for the case in which the ELB constraint lasts for five periods.<sup>18</sup> Not only does this promise to keep the nominal rate at zero at  $t + T$  and  $t + T + 1$  mitigate the negative consequences of the ELB, it actually creates an economic boom and positive inflation while the economy is at the ELB. The credible promise to keep the policy rate at zero ensures the equilibrium output gap and inflation never fall below zero; both are positive throughout the period during which the ELB constraint is binding.

Why is forward guidance so powerful? The linearized Euler condition (11.6) can be solved forward to obtain (11.10), showing how current aggregate demand depends on the entire future expected path of the gap between the real interest rate and the equilibrium rate consistent with a zero output gap. McKay, Nakamura, and Steinsson (2016) illustrated how (11.10) implies credible promises about future interest rates are extremely powerful. Let  $r_{t+i}^{\text{gap}} \equiv E_t(i_{t+i} - \pi_{t+1+i} - r_{t+i}^n)$ . Equation (11.10) can now be written as

$$x_t = - \left( \frac{1}{\sigma} \right) \sum_{i=0}^{\infty} r_{t+i}^{\text{gap}}.$$

18. That is, the economy is at the ELB from  $t$  to  $t + 4$ . At  $t + 5$ , the constraint no longer binds, but the nominal rate is kept at zero for  $t + 5$  and  $t + 6$ .





**Figure 11.4**

Forward guidance: nominal rate held at zero for two periods after ELB no longer binds

Because  $r_{t+i}^{\text{gap}} = 0$  is consistent with an expected future output gap of zero,  $r_{t+i}^{\text{gap}} < 0$  means policy is expected to set the nominal rate too low at  $t + i$  relative to the zero output gap value. Suppose at time  $t$  the central bank announces that  $r_{t+i}^{\text{gap}} = 0$  for all  $i < k$ ,  $r_{t+k}^{\text{gap}} = -1$ , and  $r_{t+i}^{\text{gap}} = 0$  for all  $i > k$ . That is, the interest rate gap is expected to be zero for all future periods except at  $t + k$ , when it equals  $-1$ . At  $t$ ,  $x_t$  immediately jumps up by  $1/\sigma$ . It remains there until  $t + k + 1$ , when it jumps back to zero. The cumulative rise in the output gap from  $t$  to  $t + k$  is  $k/\sigma$ , which is increasing in  $k$ . The further into the future is the promise to have a negative  $r^{\text{gap}}$ , the larger the cumulative effect on the output gap.

What is the effect on inflation at time  $t$ ? Solving (11.11) forward,

$$\pi_t = \kappa E_t \sum_{i=0}^{\infty} \beta^i x_{t+i} = \left(\frac{\kappa}{\sigma}\right) \left(\frac{1 - \beta^{k+1}}{1 - \beta}\right),$$

which is increasing in  $k$ . If  $\sigma = 1$  and  $\beta = 0.99$ , a promise to lower interest rates for one period at  $t + 1$  causes inflation to immediately jump by  $(\kappa/\sigma)(1 - \beta^2)/(1 - \beta) = 1.99(\kappa/\sigma)$ ; a promise to lower interest rates by the same amount for one period at  $t + 4$  results in inflation jumping by  $4.91(\kappa/\sigma)$ . The further out into the future is the promised interest rate cut, the larger the impact on inflation today.

The implication that the promise of an interest rate cut two years from now has a stronger effect today than a promise to cut interest rates one year from now is a reflection of the

presence of an explosive root in the model dynamics when the nominal interest rate is fixed. McKay, Nakamura, and Steinsson (2016) argued that this property arises because the coefficient on  $E_t x_{t+1}$  in the Euler equation is equal to 1. They proposed a model of precautionary savings in which this coefficient, in the linearized version of their model, is less than 1. Because future output expectations are then discounted, events further out into the future have a smaller impact on today's equilibrium. Kiley (2016) argued that a model based on sticky information also produces more reasonable effects of forward guidance than the sticky-price model does.<sup>19</sup>

Promises to maintain the nominal rate at zero in the future are not the only form of forward guidance. Cochrane (2013) focused on the central bank's choice of the inflation rate when the ELB period ends, showing that assumptions about the behavior of the economy after exiting from the ELB are crucial in determining the equilibrium while the ELB binds. As Cochrane emphasized, new Keynesian models suffer from multiple equilibria, and the situation is no different at the ELB. He argued that the standard treatment focuses on only one of many possible equilibria, and a complete specification of monetary policy must also decide which equilibrium the central bank selects. To illustrate this point, consider again a negative  $r_t^n$  shock that pushes the economy to the ELB. Assume the shock lasts from  $t = 1$  until  $t = T$  and that  $i_t = 0$  for  $t < T$  and  $i_t = r_t^n$  for  $t \geq T$ . One equilibrium is  $x_t = \pi_t = 0$  for  $t \geq T$ . This is the standard case, and the path of the output gap and inflation at  $t = T$  is used as a terminal condition in solving for  $x_t$  and  $\pi_t$  when  $t < T$ . But a policy that sets  $i_t = r_t^n$  for  $t \geq T$  violates the Taylor principle and is consistent with multiple equilibria, not just the zero output gap and inflation equilibrium.

To construct another equilibrium, suppose the central bank promises equilibrium paths  $x_t^*$  and  $\pi_t^*$  for  $t \geq T$ . Now consider the equilibrium for  $t < T$  when the constraint binds and for  $t \geq T$  when it doesn't. Rather than assume a policy rule such as (11.8) when the constraint does not bind, assume instead that the central bank implements policy according to the rule  $i_t = r_t^n + \delta (\pi_t - \pi_t^*)$ , with  $\delta > 1$ . The system for  $t \geq T$  can be written as

$$\begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\delta}{\sigma} \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} - \begin{bmatrix} \frac{\delta}{\sigma} \\ 0 \end{bmatrix} \pi_t^*. \quad (11.20)$$

But the promised equilibrium with  $x_t^*$  and  $\pi_t^*$  must also be consistent with the model's equilibrium conditions, so

$$\begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x_{t+1}^* \\ \pi_{t+1}^* \end{bmatrix} = \begin{bmatrix} 1 & \frac{\delta}{\sigma} \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t^* \\ \pi_t^* \end{bmatrix} - \begin{bmatrix} \frac{\delta}{\sigma} \\ 0 \end{bmatrix} \pi_t^*. \quad (11.21)$$

---

19. Carlstrom, Fuerst, and Paustian (2012a) considered a number of variants of the basic NK model, including the Smets and Wouters (2007) model that has been estimated on U.S. data, and found that all imply inflation responses to short periods of an interest rate peg that they viewed as unreasonably large.

Subtracting (11.21) from (11.20) yields

$$\begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \bar{x}_{t+1} \\ \bar{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\delta}{\sigma} \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{\pi}_t \end{bmatrix}, \quad (11.22)$$

where  $\bar{x}_t \equiv x_t - x_t^*$  and  $\bar{\pi}_t \equiv \pi_t - \pi_t^*$  are the deviations of the output gap and inflation from the  $(x_t^*, \pi_t^*)$  equilibrium. If the Taylor principle is satisfied, so that  $\delta > 1$ , then (11.22) has a locally unique stationary equilibrium, given by  $\bar{x}_t = \bar{\pi}_t = 0$ . Therefore,  $x_t = x_t^*$  and  $\pi_t = \pi_t^*$  is an equilibrium in which  $i_t = 0$  for  $t < T$  and  $i_t = r_t^n$  for  $t \geq T$  for any choice of the  $x_t^*$  and  $\pi_t^*$  paths consistent with (11.21). The standard analysis of the ELB focuses on one specific possible equilibrium, the one in which  $x_t^* = \pi_t^* = 0$ .

To construct a possible  $x_t^*, \pi_t^*$  equilibrium, write (11.21) as

$$\begin{aligned} Z_{t+1}^* &\equiv \begin{bmatrix} x_{t+1}^* \\ \pi_{t+1}^* \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t^* \\ \pi_t^* \end{bmatrix} \\ &= M \begin{bmatrix} x_t^* \\ \pi_t^* \end{bmatrix} = MZ_t^*, \end{aligned} \quad (11.23)$$

where

$$M \equiv \begin{bmatrix} \frac{\kappa}{\sigma\beta} + 1 & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

$M$  has one eigenvalue inside and one outside the unit circle. Let  $\Lambda$  be a  $2 \times 2$  diagonal matrix whose diagonal elements are the eigenvalues of  $M$ , and order  $\Lambda$  so the first element is the largest eigenvalue. Denote the elements of  $\Lambda$  by  $\Lambda_1 \geq 1$  and  $\Lambda_2 < 1$ . If  $V$  is the matrix of the eigenvectors of  $M$ , then  $M = V\Lambda V^{-1}$ . Now define  $z_t = [z_{1,t} \ z_{2,t}]' \equiv V^{-1}Z_t^*$ . Premultiplying (11.23) by  $V^{-1}$  yields

$$z_{t+1} \equiv V^{-1}Z_{t+1}^* = V^{-1}MZ_t^* = \Lambda V^{-1}Z_t^* = \Lambda z_t.$$

Because  $\Lambda$  is a diagonal matrix, this decomposition yields two equations:

$$z_{1,t+1} = \Lambda_1 z_{1,t},$$

$$z_{2,t+1} = \Lambda_2 z_{2,t}.$$

With  $\Lambda_1 \geq 1$ ,  $z_{1,t} = 0$  is necessary to rule out explosive solutions. With  $z_{1,t}$  equal to the first element of  $V^{-1}Z_t^*$  (and  $Z_t^* = [x_t^* \ \pi_t^*]$ ),  $z_{1,t} = 0$  implies

$$V_{1,1}^{-1}x_t^* + V_{1,2}^{-1}\pi_t^* = 0, \quad (11.24)$$

where  $V_{i,j}^{-1}$  is the  $i,j$ th element of  $V^{-1}$ . Given the choice of  $\pi_T^*$ , the inflation rate when the ELB period is exited, (11.24) can be solved for the output gap  $x_T^*$  consistent with a nonexplosive equilibrium:  $x_T^* = -\left(V_{1,2}^{-1}/V_{1,1}^{-1}\right)\pi_T^*$ .

From the second row of  $z_t = V^{-1}Z_t^*$ ,

$$z_{2,T} = V_{2,1}^{-1}x_T^* + V_{2,2}^{-1}\pi_T^*.$$

It follows that once  $\pi_T^*$  and  $x_T^*$  are determined,  $z_{2,T}$  is also uniquely determined. Given  $z_{2,T}$ ,  $z_{2,T+1} = \Lambda_2 z_{2,T}$ , the equilibrium values for  $x_{T+1}^*$  and  $\pi_{T+1}^*$  are then obtained by jointly solving

$$z_{1,T+1} = 0 = V_{1,1}^{-1}x_{T+1}^* + V_{1,2}^{-1}\pi_{T+1}^*,$$

$$z_{2,T+1} = \Lambda_2 z_{2,T} = V_{2,1}^{-1}x_{T+1}^* + V_{2,2}^{-1}\pi_{T+1}^*.$$

This process can be continued to solve for  $\pi_{T+2}^*$  and  $x_{T+2}^*$ , and so on.

With the equilibrium paths for  $x_t^*$  and  $\pi_t^*$  determined for  $t \geq T$ , one can now solve backward from  $t = T$  to obtain the equilibrium for  $t < T$  when the ELB binds. This is done using (11.9) and (11.11). These two equations imply

$$\begin{bmatrix} x_{T-1}^* \\ \pi_{T-1}^* \end{bmatrix} = Q \begin{bmatrix} x_T^* \\ \pi_T^* \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r^{\text{ELB}};$$

$Q$  was defined in an equation following (11.19).

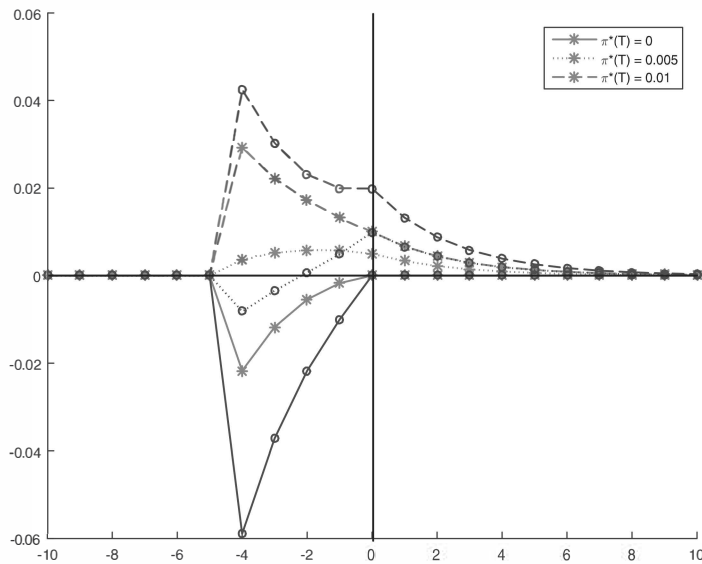
Figure 11.5 illustrates three possible equilibria. The solid lines show the output gap and inflation when inflation equals zero as soon as the ELB constraint no longer binds. This is the standard equilibrium based on the assumption the equilibrium is  $x_T = \pi_T = 0$  once the ELB is exited. The dotted lines show the output gap and inflation when the central bank commits to setting  $\pi_T^* = 0.005$  (2% at an annual rate), and the dashed lines are for the case in which  $\pi_T^* = 0.01$ . Promising even a little inflation when the ELB ends can lead to a strong expansion while the constraint binds.

It is important to emphasize that each of the equilibria shown in figure 11.5 involves exactly the same path for the nominal interest rate. The nominal rate is zero until time  $T$ , at which point it equals  $r_t^n$  for all  $t \geq T$ . A unique equilibrium, the standard one with  $x_t = \pi_t = 0$ , is selected by assuming the central bank follows a policy rule of the form  $i_t = r_t^n + \delta\pi_t$ , with  $\delta > 1$ . Cochrane (2013) argued that this policy rule is a special case of a more general rule of the form

$$i_t = r_t^n + \delta(\pi_t - \pi_t^*).$$

With  $\delta > 1$ , this policy rule ensures  $\pi_t = \pi_t^*$  is the unique equilibrium for inflation. The choice of  $\pi_t^*$  is an equilibrium selection device. The standard treatment selects the zero inflation equilibrium by setting  $\pi_t^* = 0$ , but this is not the only option. Because  $\pi_t = \pi_t^*$  in equilibrium, the path for the nominal interest rate is  $r_t^n$  and is the same regardless of  $\pi_t^*$ .<sup>20</sup>

20. It is important to note that the new Keynesian Phillips curve (11.11) was obtained by linearizing around a zero steady-state rate of inflation. Hence, the path for  $\pi_t^*$  must converge to zero, as it does in the cases illustrated in figure 11.5.



**Figure 11.5**

Multiple equilibria with same interest rate path. Each equilibrium is indexed by the inflation rate when the ELB is exited. Output gap (*circles*); inflation rate (*stars*).

#### 11.4.4 Summary on the ELB

Under discretionary monetary policy, or when the central bank follows a simple Taylor rule, the current nominal interest rate is the only instrument of monetary policy. A negative demand shock may push the nominal rate to its ELB, leading to a negative output gap and inflation. In such a situation, deflation can be a particular concern, as expectations of deflation raise the real interest rate and lead to a further drop in the output gap and inflation. The costs of the ELB constraint can be large. The situation is different when the central bank can credibly commit to future policies. In this case, even if the current policy rate is at zero, the central bank can continue to affect current output and inflation through the promises it makes about the future path of interest rates. It was shown that the optimal commitment policy will keep the nominal rate at zero past the point at which the ELB constraint is no longer binding.

Promises about the future path of the policy interest rate—forward guidance about future policy—is a powerful tool in the new Keynesian model. In fact, the structure of the basic model implies that the further into the future credible promises are made, the more powerful is their impact on the economy today. When  $i_t = 0$ , the Taylor principle is violated, implying there are multiple equilibria. Thus, an interest rate path in which  $i_t = 0$  as long as the ELB constraint is binding, and  $i_t = r_t^n$  whenever it is not binding, does not pin down a

unique equilibrium. A promise of positive inflation when the ELB period ends, rather than zero inflation as normally assumed in analyses of the ELB, can convert a major recession into a major expansion in the NK model. This may say more about the structure of new Keynesian models than about the ability of even credible central banks to overcome the constraint of the ELB.

### 11.5 Balance Sheet Policies

The extended periods of very low nominal interest rates experienced by Japan, the United States, the United Kingdom, and the euro zone have focused attention on the ways a central bank can use its balance sheet to affect the economy. Balance sheet policies involve altering the total size of the central bank's balance sheet or altering the composition of the assets the central bank holds.<sup>21</sup>

Table 11.1 presents a simplified picture of the Federal Reserve's balance sheet. The Fed's liabilities consist of currency outstanding, reserves, other liabilities (such as deposits of the federal government with the Fed), and capital. Assets consist of Treasury securities, loans to financial institutions, and other assets (such as mortgage-backed securities and agency debt). Traditional monetary policies such as open-market operations are designed to alter the size of the central bank's balance sheet. To expand the supply of base money (currency plus reserves), the central bank purchases securities from the private sector. This action increases the total liabilities (reserves) and asset holdings of the central bank, expanding the overall size of its balance sheet. Since 2008 the Fed has undertaken policies that have significantly expanded the size of its balance sheet (from approximately \$850 billion in 2007 to over \$4.5 trillion by 2016). The Fed has also altered the composition of its balance sheet by, for example, selling from its holdings of short-term Treasuries to purchase long-term Treasuries. The growth of the Fed's balance sheet and its changing composition were shown in figure 1.10.

**Table 11.1**

Assets	Liabilities and Net Worth
Short-term Treasury securities	Currency
Long-term Treasury securities	Reserves
Loans	Other
Other assets	Capital

21. See Borio and Disyatat (2010) for a general discussion of monetary policy tools that incorporate balance sheet policies. Miles and Schanz (2014) analyzed balance sheet policies in an overlapping-generations framework.

Balance sheet policies were expected to lower longer-term interest rates which would, in turn, stimulate aggregate demand and lead to increased output and employment.<sup>22</sup> However, in an important paper, Wallace (1981) demonstrated conditions under which the size and composition of the central bank's balance sheet is irrelevant for asset prices and the economy's equilibrium. To understand this result, consider an environment in which all agents can freely buy and sell all assets, which are valued only because of their pecuniary payouts in future states. Suppose in future state  $s \in S$  asset  $j$  pays  $x_j(s)$ . Further, assume  $m(s)$  is the stochastic discount factor for payments received in state  $s$ ; that is,  $m(s)$  is the value agents place today on receiving one unit of consumption in future state  $s$ . In this notation,  $x_j(s)$  equals the real value (in units of consumption) asset  $j$  yields in state  $s$ , and  $m(s)x_j(s)$  is the current value of that payout. If  $\pi(s)$  denotes the probability of state  $s$ , then standard asset pricing theory implies the current price of asset  $j$  should equal the expected value of  $m(s)x_j(s)$ , where the expectations are across all future states, or

$$p_j = \sum_{s=1}^S \pi(s)m(s)x_j(s).$$

If utility is an increasing concave function of consumption  $U[c(s)]$ , then  $m(s) = \beta U_c[c(s)]/U_c(c)$ , where  $U_c$  denotes the marginal utility of consumption and  $c$  is current consumption. Hence, for all  $j$ ,

$$p_j = \sum_{s=1}^S \pi(s) \frac{\beta U_c[c(s)]}{U_c(c_t)} x_j(s) = \beta E_t \left( \frac{U_c[c(s)]}{U_c(c_t)} \right) x_j(s). \quad (11.25)$$

The key implication of (11.25) is that asset quantities do not appear in the pricing formula. Wallace showed that in this case, asset prices are independent of the central bank's balance sheet; open-market operations and the form they take (short-term government debt, long-term government debt, private assets) are irrelevant. The operations might alter the private sector's holdings of various assets, but they would not affect asset prices. Wallace's results depend on a careful specification of monetary policy and fiscal policy. If central bank balance sheet policies alter the present discounted value of future government interest payments by altering the outstanding stocks of interest-bearing and non-interest-bearing debt, for example, then the expected future path of taxes will be affected, and this can have real effects. But these are the consequences of the change in fiscal policy and are not purely monetary effects.

The conclusion that asset quantities are irrelevant stands in contrast to a view of the monetary policy transmission process associated with James Tobin (e.g., see Tobin 1969) that was prominent in the 1960s. Tobin argued that financial assets were imperfect substitutes. If so, altering the relative quantities of assets should force relative asset prices to

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22. Evidence on the effects of balance sheet policies was reviewed in chapter 1.

adjust to reestablish equilibrium in financial markets. For example, if short-term and long-term government debt are imperfect substitutes, then a policy of issuing more short-term debt and using the proceeds to retire long-term government debt should lead to a fall in the price of the short-term debt as its supply increases, while the price of long-term debt should rise as its supply contracts. The impact of open-market operations in either short-term or long-term government debt would then depend on the degree to which different assets were imperfect substitutes.<sup>23</sup> Goodfriend (2000) considered the effects of open-market operations in long-term bonds as a policy option at the ELB, and Andrés, López-Salido, and Nelson (2004) provided an example of a modern DSGE model that incorporated some of Tobin's insights (see section 11.5.2).

During the 30 years after Wallace (1981) wrote, monetary policy models followed the modern finance literature in assuming asset prices satisfied conditions such as (11.25) and that asset quantities were irrelevant. Given the use of balance sheet policies by the Fed and other major central banks, the recent literature has moved away from the environment specified by Wallace (1981) to investigate the role balance sheet policies might play. Heterogeneous agents, segmented financial markets, and borrowing constraints play important roles in these models.

### 11.5.1 Asset Pricing Wedges

A useful starting place for understanding the various models of balance sheet policies is the MIU model. The representative agent's objective is to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i U(c_{t+i}, m_{t+i}),$$

where  $c_t$  is consumption,  $m_t$  equals real money holdings, and the maximization is subject to a sequence of budget constraints. These constraints depend on the assets available to the agent. Assume these include money, one-period bonds, two-period bonds, and a real asset that yields an exogenous stochastic dividend. Assume both bonds are issued by the government in positive net supply. The budget constraint, in nominal terms, takes the form

$$\begin{aligned} P_t y_t + P_t (q_t + d_t) s_{t-1} + (1 + i_{1,t-1}) B_{1,t-1} + B_{2,t-1} + M_{t-1} \\ = P_t c_t + P_t q_t s_t + B_{1,t} + p_{2,t} B_{2,t} + M_t + P_t T_t, \end{aligned}$$

where  $P_t$  is the price level,  $y_t$  is nonasset income (treated as exogenous),  $q_t$  is the price of the real asset,  $d_t$  is the dividend,  $s_{t-1}$  equals the shares of the real asset carried into period  $t$ ,  $i_{1,t-1}$  is the nominal interest rate on one-period bonds  $B_{1,t-1}$  purchased at  $t - 1$  at a price of one dollar,  $B_{2,t-1}$  are two-period bonds carried over from period  $t - 1$  and which

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23. See, for example, Walsh (1982a).



are now one-period bonds and so have a price of one dollar. The price of a two-period bond is denoted by  $p_{2,t}$ . Finally,  $M_{t-1}$  equals nominal money balances held by the agent,  $c_t$  is consumption, and  $T_t$  represents any lump-sum taxes (or transfers, if negative). In real terms, this budget constraint can be written as

$$y_t + W_t = c_t + q_t s_t + b_{1,t} + p_{2,t} b_{2,t} + m_t + T_t,$$

where

$$W_t \equiv \left[ (q_t + d_t) s_{t-1} + \left( \frac{1 + i_{1,t-1}}{1 + \pi_t} \right) b_{1,t-1} + \left( \frac{1}{1 + \pi_t} \right) (b_{2,t-1} + m_{t-1}) \right]$$

and  $1 + r_{1,t} \equiv (1 + i_{1,t-1}) / (1 + \pi_t)$ ,  $b_{i,t} = B_{i,t} / P_t$  for  $i = 1, 2$ ,  $m_t = M_t / P_t$ , and  $1 + \pi_t = P_t / P_{t-1}$ .

The household's decision problem involves choosing the sequence of consumption, money holdings, bond holdings, and real asset holdings to maximize the expected present discounted value of utility, subject to a sequence of budget constraints. The chapter appendix shows that this problem can be written in Lagrangian form as

$$\begin{aligned} \mathcal{L}_t = & E_t \sum_{i=0}^{\infty} \beta^i U(c_{t+i}, m_{t+i}) + \lambda_t W_t \\ & + E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[ \begin{array}{l} (y_{t+i} - c_{t+i} - T_{t+i}) + \Delta_{1,t+i} b_{1,t+i} \\ + \Delta_{s,t+i} s_{t+i} + \Delta_{2,t+i} b_{2,t+i} + \Delta_{m,t+i} m_{t+i} \end{array} \right], \end{aligned} \quad (11.26)$$

where

$$\begin{aligned} \Delta_{1,t} & \equiv \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (1 + r_{1,t+1}) - 1, \\ \Delta_{2,t} & \equiv \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{1}{1 + \pi_{t+1}} \right) - p_{2,t}, \\ \Delta_{s,t} & \equiv \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (q_{t+1} + d_{t+1}) - q_t, \\ \Delta_{m,t} & \equiv \beta \left( \frac{\lambda_{t+1+i}}{\lambda_{t+i}} \right) \left( \frac{1}{1 + \pi_{t+1}} \right) - 1. \end{aligned}$$

Now consider the first-order conditions for the household's choice of  $c_t$ ,  $b_{1,t}$ ,  $b_{2,t}$ ,  $s_t$ , and  $m_t$ . For consumption this takes the form  $U_c(c_t, m_t) = \lambda_t$ , and for the asset holdings they take the form

$$\Delta_{1,t} = \Delta_{2,t} = \Delta_{s,t} = 0, \quad (11.27)$$

$$U_m(c_t, m_t) + \lambda_t \Delta_{m,t} = 0.$$

That is,  $\Delta_{j,t} = 0$  for bonds and the real asset, and  $\Delta_{m,t} \neq 0$  if there is a nonpecuniary return to holding money, i.e.,  $U_m \neq 0$ . These results yield the standard results on asset pricing. For example,  $\Delta_{1,t} = 0$  implies

$$\lambda_t = U_c(c_t, m_t) = \beta E_t (1 + r_{1,t+1}) U_c(c_{t+1}, m_{t+1}), \quad (11.28)$$

which is the normal Euler equation, while  $\Delta_{2,t} = 0$  implies (using  $\Delta_{1,t} = 0$  and  $1 + r_{1,t+1} = (1 + i_{1,t}) / (1 + \pi_{t+1})$ )

$$\begin{aligned} p_{2,t} &= \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{1 + i_{1,t}}{1 + \pi_{t+1}} \right) \left( \frac{1}{1 + i_{1,t}} \right) \\ &= \left( \frac{1}{1 + i_{1,t}} \right), \end{aligned}$$

which in turn implies that a two-period bond held to maturity yields a nominal rate of return of

$$(1 + i_{1,t+1})(1 + i_{1,t}) - 1$$

as implied by the expectations hypothesis of the term structure.<sup>24</sup>

From  $\Delta_{s,t} = 0$ ,

$$q_t = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (q_{t+1} + d_{t+1}),$$

and the price of the asset is equal to the expected discounted future dividend plus price.

From  $U_m + \lambda_t \Delta_{m,t} = 0$  and  $\Delta_{1,t} = 0$ ,

$$\begin{aligned} \frac{U_m}{U_c} &= -\Delta_{m,t} = 1 - \beta \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{1}{1 + \pi_{t+1}} \right) \\ &= 1 - \left( \frac{1}{1 + i_{1,t}} \right) = \left( \frac{i_{1,t}}{1 + i_{1,t}} \right); \end{aligned}$$

the marginal rate of substitution between money and consumption is equal to  $i_{1,t} / (1 + i_{1,t})$  (see chapter 2).

Now consider the implications for central bank balance sheet policies. Let  $\bar{U}$  denote the expected present discounted value of utility, and define

$$\bar{R}_t \equiv \lambda_t (1 + r_{1,t}) W_t + E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} (y_{t+i} - c_{t+i} - T_{t+i}).$$

24. A two-period bond is purchased at time  $t$  at the price  $p_{2,t}$ . At  $t + 1$ , it becomes a one-period bond with a price of one dollar and a nominal yield at  $t + 1$  of  $1 + i_{1,t+1}$ . Held to maturity, its return is

$$\frac{1 + i_{1,t+1} - p_{2,t}}{p_{2,t}} = (1 + i_{1,t+1})(1 + i_{1,t}) - 1.$$

Then (11.26) becomes

$$\mathcal{L}_t = \bar{U} + \bar{R}_t + E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} [\Delta_{1,t+i} b_{1,t+i} + \Delta_{s,t+i} s_{t+i} + \Delta_{2,t+i} b_{2,t+i} + \Delta_{m,t+i} m_{t+i}].$$

Using the first-order conditions, this reduces to

$$\mathcal{L}_t = \bar{U}_t + \bar{R}_t - E_t \sum_{i=0}^{\infty} \beta^i U_m(c_{t+i}, m_{t+i}) m_{t+i}.$$

The stocks of the nonmonetary assets do not appear. It follows that central bank open-market operations involving  $m$  and  $b_1$  or  $m$  and  $b_2$  matter only insofar as they affect  $m$ . Similarly, having the central bank buy the asset  $s$  to increase  $m$  has the same effects as buying one-period or two-period bonds to increase  $m$ . And selling  $b_1$  to buy  $b_2$ , holding  $m$  constant, has no direct effect on a representative agent's budget. Finally, if the one-period nominal interest rate is zero for  $t$  to  $t+T$ , then  $\Delta_{m,t+i} = i_{1,t+i} / (1 + i_{1,t+i})$  is zero for  $i = 0, \dots, T$ , and variations in the money supply while the nominal rate is zero have no effect on the household's decision problem.<sup>25</sup>

If, however, the  $\Delta_{j,t}$  terms for bonds and other assets are not zero, then

$$E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} [\Delta_{1,t+i} b_{1,t+i} + \Delta_{s,t+i} s_{t+i} + \Delta_{2,t+i} b_{2,t+i} + \Delta_{m,t+i} m_{t+i}]$$

matters. If  $\Delta_{1,t+i} \neq \Delta_{2,t+i} \neq \Delta_{s,t}$ , buying the one-period bond to increase the real supply of money has a different effect than buying two-period bonds or private assets to engineer the same increase in the money supply.

The different  $\Delta_j$  can be interpreted as asset pricing wedges, deviations of the asset's price from the value implied by (11.25). Because the effectiveness of balance sheet policies rely on deviations from standard asset pricing relationships, the question is, what might account for such deviations? In the case of money, the pricing wedge is accounted for by the direct utility money yields.<sup>26</sup> What other factors might account for these pricing wedges? Suppose the household had a nonpecuniary reason for preferring long-term bonds, as in the preferred habitat model of Modigliani and Sutch (1967) or Vayanos and Vila (2009) (see chapter 10). Then, just as in an MIU model, the pricing wedges would be functions of the marginal utilities that capture nonpecuniary reasons for holding particular assets. This approach was developed by Walsh (1982a) as a means of characterizing the imperfect substitutability of different assets.

25. Consistent with the earlier analysis of interest rate policies at the ELB, variations in  $m_{t+T+k}$  for  $k > 0$  can still matter. Changes in the central bank's balance sheet can also have fiscal implications. Assume these are offset via adjustments in lump-sum taxes or transfers  $\bar{T}_t$ .

26. In a cash-in-advance model,  $\Delta_{m,t}$  would depend on the Lagrangian multiplier on the cash-in-advance constraint. See chapter 3.

Borrowing limits or restrictions on issuing assets can also lead to pricing wedges. For example, suppose private agents can hold positive levels of the one-period bonds but cannot issue such bonds. Then the first-order condition for one-period bond holdings would take the form

$$\Delta_{1,t} = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (1 + r_{1,t+1}) - 1 \leq 0, \Delta_{1,t} b_{1,t} = 0.$$

If  $b_{1,t} > 0$ , then  $\Delta_{1,t} = 0$  and the standard pricing formula holds. But at the corner solution,  $b_{1,t} = 0$  and the expected discounted pecuniary return is less than the price of the bond.

Wedges can also arise if there are transaction costs or if agents face market segmentation such that holding certain assets can only be done using intermediation services provided by financial institutions. In this case, the wedges would be associated with these transaction costs or the costs of the intermediation sector.

The following sections explore in more detail some of the modern modeling approaches to understanding balance sheet policies. One can view the models as providing economic environments in which the terms denoted here by  $\Delta_{j,t}$  can be nonzero. And, critically, it is important to understand how these pricing wedges might vary with changes in the composition of the central bank's balance sheet and whether variations in these wedges affect aggregate spending.

The presence of asset pricing wedges may account for the effects of balance sheet policies on financial markets, asset prices, and interest rates, but they may not be sufficient to guarantee such policies will affect aggregate spending and output. For example, suppose  $\Delta_{2,t} \neq 0$  but  $\Delta_{1,t} = \Delta_{s,t} = 0$ . Then there is a spread between the one-period returns on the two-period bond and on the one-period bond.<sup>27</sup> But in a basic NK model in which aggregate demand consists only of consumption, the household's Euler condition takes the standard form given by (11.28) when  $\Delta_{1,t} = 0$ . Only the one-period real interest rate matters for consumption decisions and aggregate demand. Hence, models developed to understand the potential effects of balance sheet policies deviate from the basic NK model by accounting for wedges in standard asset pricing relationships and by accounting for how aggregate spending may be affected by more than just the one-period interest rate. For example, the first models to be discussed introduce heterogeneity among households, with consumption decisions by some households affected by the short-term interest rate and the decisions of others affected by the long-term interest rate.

### 11.5.2 Market Segmentation and Transaction Costs

Andrés, López-Salido, and Nelson (2004) provided an example of a new Keynesian DSGE model that motivated asset pricing wedges as arising from transaction costs in asset markets

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27. The gross one-period return on the one-period bond is  $1 + i_{1,t}$ ; on the two-period bond it is  $1/p_{2,t} = (1 + i_{1,t})/[1 - (1 + i_{1,t})\Delta_{2,t}]$ , which equals  $1 + i_{1,t}$  if and only if  $\Delta_{2,t} = 0$ .

and from risk factors. Their model was designed to capture the imperfect asset substitutability that was central to the transmission channel of monetary policy in the view of early Keynesians such as Tobin (1969). When assets are imperfect substitutes, altering the outstanding stocks held by the public requires relative rates of return to adjust to restore asset market equilibrium.

Assume households can hold money, short-term bonds, or long-term bonds. All bonds are issued by the government. Purchases of long-term bonds are subject to a stochastic transaction cost  $\zeta_t$ . Thus, in the notation of the previous section, the first-order condition for the two-period bond would be

$$(1 + \zeta_t)p_{2,t} = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{1}{1 + \pi_{t+1}} \right),$$

or  $\Delta_{2,t} = \zeta_t p_{2,t} \neq 0$ . Consequently,

$$(1 + \zeta_t)p_{2,t} = \left( \frac{1}{1 + i_{1,t}} \right). \quad (11.29)$$

Given the one-period rate,  $p_{2,t}$  simply moves inversely with  $1 + \zeta_t$ , leaving the price including transaction costs unchanged and the yield from holding a two-period bond to maturity unaffected.<sup>28</sup> However, Andrés, López-Salido, and Nelson also assumed that increased holdings of the long-term bond is perceived as exposing the household to greater risk, and that in response to this increase in risk, households desire to hold more money relative to long-term bonds. In this case, the pricing wedge reflects both transaction costs and risk factors and so becomes  $\Delta_{2,t} = \zeta_t p_{2,t} + f(M_t/B_{2,t})$ , where the risk component is represented by the function  $f(M_t/B_{2,t})$ . They assumed  $f' < 0$ ; holding more money reduces the spread between the price of the two-period bond relative to the value given by standard frictionless pricing formula. A central bank open-market operation involving purchasing the two-period bond to increase the money supply would lower  $B_{2,t}$  held by the public and increase  $M_t$ , reducing the pricing wedge.

As the authors noted, however, unless the price of long-term bonds has an independent effect on household spending decisions, variations in the transaction costs variable  $\zeta_t$  or policy-induced changes in  $M_t/B_{2,t}$  will not influence aggregate demand. Suppose, for example, only transaction costs are present and (11.29) holds. Then an increase in  $\zeta_t$  simply lowers  $p_{2,t}$ . The one-period rate  $i_{1,t}$  and the household's standard Euler condition for the optimal intertemporal allocation of consumption are unaffected. Thus, to generate real effects, they introduced household heterogeneity by assuming not all households have

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28. The yield to maturity is

$$\frac{(1 + i_{1,t+1}) - (1 + \zeta_t)p_{2,t}}{(1 + \zeta_t)p_{2,t}} = (1 + i_{1,t+1})(1 + i_{1,t}) - 1,$$

which is independent of  $\zeta_t$ .

the same access to all financial markets. To illustrate how transaction costs can interact with market segmentation, it is useful to turn to the model developed by Chen, Cúrdia, and Ferrero (2012).

Chen, Cúrdia, and Ferrero provided a model for studying balance sheet policies in which pricing wedges arise because of market segmentation and transaction costs imposed on certain financial trades. Their model includes both short-term and long-term bonds, but households differ in terms of the assets they are allowed to trade and the costs they face in making asset trades. A fraction  $\omega_r$  of households are restricted in that they can only trade in long-term bonds. The remaining  $\omega_u = 1 - \omega_r$  fraction are unrestricted in that they can hold both long-term and short-term bonds. However, unrestricted households face a per unit transaction fee  $\zeta_t$  for trades in long-term bonds. This structure reflects the idea that many households only save through assets such as pension funds that are long-term in nature.<sup>29</sup> The model displays a form of (exogenous) market segmentation in that restricted households cannot participate in the long-term bond market.<sup>30</sup> However, absent the transaction costs, unrestricted households who operate in both the short-term and the long-term bond markets would arbitrage away any asset pricing wedges. It is the transaction costs that will account for wedges in the pricing relationships.

The short-term bond is a one-period bond that pays a nominal return  $i_t$  at  $t + 1$ . The long-term bond is a perpetuity that sells for  $P_{L,t}$  at time  $t$ . These bonds pay an exponentially decaying coupon  $\kappa^s$  at  $t + s + 1$ , where  $0 \leq \kappa^s < 1$ .<sup>31</sup> If  $j \in \{r, u\}$  indexes restricted and unrestricted household types, the preferences of a type  $j$  household are

$$E_t \sum_{s=0}^{\infty} \beta_j^s \left\{ \left( \frac{1}{1 - \sigma_j} \right) (C_{t+s}^j)^{1 - \sigma_j} - \frac{\varphi_{t+s}^j [h_{t+s}^j]^{1 + \nu}}{1 + \nu} \right\},$$

where  $C^j$  is real consumption and  $h^j$  equals hours worked. The budget constraint for type  $u$  households is

$$P_t C_t^u + B_t^u + (1 + \zeta_t) P_{L,t} B_t^{L,u} \leq (1 + i_{t-1}) B_{t-1}^u + \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,u} + W_t h_t^u + \Pi_t^u,$$

where  $B_t^u$  ( $B_t^{L,u}$ ) are holdings of short- and long-term bonds. Long-term bonds purchased at  $t - s$ ,  $B_{t-s}^{L,u}$ , yield a coupon  $\kappa^{s-1}$  at time  $t$ . The terms  $W_t h_t^u + \Pi_t^u$  represent the labor and

29. This is consistent with the evidence in Kaplan, Violante, and Weidner (2014) that many households can be described as wealthy hand-to-mouth households. These households have sizable levels of wealth, but little of this is held in the form of liquid assets that can be used to smooth temporary fluctuations in income. However, the long-term bonds held by restricted households in the model of Chen, Cúrdia, and Ferrero (2012) can be liquidated without any transaction costs, and so these households do not behave as hand-to-mouth consumers.

30. An earlier example of a model based on asset market segmentation is King and Thomas (2007).

31. See Woodford (2001a).

profit income plus transfers received by the household. Note that the price at  $t$  of a long-term bond issued at  $t - s$  is simply  $\kappa^s P_{L,t}$ . This follows because such a bond yields a stream of coupons in periods  $t + 1, \dots$  of  $\kappa^s, \kappa^{s+1}, \kappa^{s+2}, \dots$  while a long-term bond purchased at  $t$  yields a coupon stream in  $t + 1, \dots$  of  $1, \kappa, \kappa^2, \dots$ . Hence, the time  $t$  price of  $B_{t-s}^{L,u}$  is  $\kappa^s P_{L,t}$ . Defining  $i_{L,t}$  as the gross yield to maturity,

$$P_{L,t} = \frac{1}{1 + i_{L,t}} + \frac{\kappa}{(1 + i_{L,t})^2} + \frac{\kappa^2}{(1 + i_{L,t})^3} + \dots, \quad \text{or} \quad P_{L,t} = 1 / (1 + i_{L,t} - \kappa).$$

Because type  $r$  households cannot hold short-term bonds and do not face transaction costs in purchasing long-term bonds, their budget constraint is

$$P_t C_t^r + P_{L,t} B_t^{L,r} \leq \sum_{s=1}^{\infty} \kappa^{s-1} B_{t-s}^{L,r} + W_t h_t^r + \Pi_t^r.$$

The first-order conditions for the optimal consumption choice of restricted households takes the form of a familiar Euler equation involving the one-period return on holding a long-term bond. Let  $r_{L,t}$  be the real return on holding the long-term bond for one period. Because the long-term bond pays one dollar at  $t + 1$ , the real return on a long-term bond held from  $t$  to  $t + 1$  is

$$1 + r_{L,t} \equiv \frac{P_t}{P_{t+1}} \frac{P_{L,t+1} (1/P_{L,t+1} + \kappa)}{P_{L,t}} = \frac{P_t}{P_{t+1}} \frac{P_{L,t+1} (1 + i_{L,t})}{P_{L,t}}.$$

Hence, if  $\lambda_t^r$  is the marginal utility of consumption for a restricted household,

$$\lambda_t^r = \beta_r E_t \left( \frac{P_{L,t+1}}{P_{L,t}} \frac{1 + i_{L,t}}{1 + \pi_{t+1}} \right) \lambda_{t+1}^r = \beta_r E_t (1 + r_{L,t}) \lambda_{t+1}^r, \quad (11.30)$$

where  $1 + \pi_{t+1} = P_{t+1}/P_t$ .

Unrestricted households hold both long-term and short-term bonds, so for these households there are two Euler conditions. One links current and future marginal utility of consumption to the real return on the short-term bond, and one links current and future marginal utility of consumption to the real return net of transaction costs on the long-term bond. For short-term bonds,

$$\lambda_t^u = \beta_u E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) \lambda_{t+1}^u, \quad (11.31)$$

and for long-term bonds,

$$(1 + \zeta_t) \lambda_t^u = \beta_u E_t (1 + r_{L,t}) \lambda_{t+1}^u. \quad (11.32)$$

Suppose all households are unrestricted. Then the only relevant intertemporal conditions are (11.31) and (11.32), and they can be written as

$$\Delta_{S,t} \equiv \beta_u E_t \left( \frac{\lambda_{t+1}^u}{\lambda_t^u} \right) \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) - 1 = 0,$$

for the short-term bond and

$$\Delta_{L,t} \equiv \beta_u E_t \left( \frac{\lambda_{t+1}^u}{\lambda_t^u} \right) (1 + r_{L,t}) - 1 = \zeta_t,$$

for the long-term bond. In terms of the pricing wedges discussed earlier, the pricing wedge on the long-term bond is due to the transaction costs  $\zeta_t$  incurred in purchasing this asset. If  $\zeta_t$  is exogenous, then fluctuations in  $\zeta_t$  would cause the real return on the long-term bond to fluctuate but would have no other real effects if the central bank held the short-term rate fixed.<sup>32</sup> To see this more clearly, suppose transaction costs are small, and linearize (11.31) and (11.32) around a zero inflation steady state to obtain

$$i_t - \rho + E_t \left( \hat{\lambda}_{t+1}^u - \hat{\lambda}_t^u - \pi_{t+1} \right) = E_t r_{L,t} - \hat{\zeta}_t - \rho + E_t \left( \hat{\lambda}_{t+1}^u - \hat{\lambda}_t^u \right),$$

where  $\rho$  is the steady-state real return,  $\hat{\zeta}_t \equiv \zeta_t - \zeta$ , and  $\zeta$  is the steady-state value of  $\zeta_t$ . This equation implies that

$$E_t r_{L,t} = (i_t - E_t \pi_{t+1}) + \hat{\zeta}_t,$$

so that the spread between the expected one-period returns on the long- and short-term bonds is simply equal to the transaction cost.

Now return to the basic model with restricted and unrestricted households. To see why changes in  $\hat{\zeta}_t$  have real effects on the model's equilibrium, suppose transaction costs rise and  $r_{L,t}$  rises by the full increase in  $\hat{\zeta}_t$ . This would leave unrestricted households that hold both short-term and long-term bonds unaffected. However, the rise in  $r_{L,t}$  has a real effect on the consumption decisions of the restricted households, which hold only long-term bonds. From (11.30), the rise in the long-term rate leads restricted households to adjust by reducing current consumption relative to future consumption. The fall in current spending by these households leads, in a sticky-price model, to a decline in current output.

To analyze central bank balance sheet policies, Chen, Cúrdia, and Ferrero (2012) assumed  $\zeta_t$  is not exogenous but depends on the relative supplies of long-term and short-term bonds held by the public:

$$\zeta_t = \zeta \left( \frac{P_{L,t} B_t^L}{B_t} \right) > 0, \quad \zeta' > 0.$$

A shift in the composition of bond holdings by unrestricted households toward long-term bonds increases the transaction costs associated with trading in these assets. This rise in  $\zeta_t$  increases the spread between the returns on long-term bonds and short-term bonds and is contractionary, as restricted households reduce their consumption spending. Thus, for a given short-term rate  $i_t$ , the central bank can engineer a short-run economic expansion by

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32. This is because, as is common in DSGE models, the return on the long-term bond does not appear elsewhere in the model of Chen, Cúrdia, and Ferrero (2012).



reducing  $P_{L,t}B_t^L/B_t$ , that is, by selling short-term bonds from its balance sheet and purchasing long-term bonds, as this increases (decreases) the quantity of short-term (long-term) bonds held by the private sector. This type of portfolio composition adjustment corresponds to the Federal Reserve program known as QE2 and the maturity extension program (MEP) undertaken between September 2011 and October 2014. The QE2 policy involved purchasing long-term Treasury securities by increasing reserves, while MEP funded the purchases of long-term securities by selling short-term Treasuries from the Fed's portfolio.

In addition to the financial market segmentation in their model, Chen, Cúrdia, and Ferrero (2012) incorporated sticky prices, habit persistence in consumption, and investment and capital adjustment costs to help the model better match empirical data. They estimated their model using data from 1987 to 2009. They used the model to simulate the effects of a policy that corresponds to the Federal Reserve's purchases of long-term Treasuries financed by selling short-term Treasuries when the economy is at the ELB. Hence, this policy constituted a change in the composition of the Fed's balance sheet rather than an expansion in its size. Impulse responses from their estimated model show an expansionary effect of this policy, operating through a fall in the long-term interest rate (Chen et al. 2012, fig. 3, p. F309). They also simulated the effects of this policy with and without a commitment to maintaining the short-term interest rate at zero for an extended period of time. In the absence of such an interest rate commitment, the effects of the balance sheet policy are small (Chen et al., fig. 4, p. F311).

### 11.5.3 Costly Intermediation

In the models of Andrés, López-Salido, and Nelson (2004) and Chen, Cúrdia, and Ferrero (2012), the source of the transaction costs is not fully specified. In the Cúrdia and Woodford (2011) model, these costs arise from the real costs associated with financial intermediation. The model has two types of households: borrowers and savers. Let  $\lambda_t^\tau$  denote the marginal utility of consumption of a type  $\tau$  household, where  $\tau = b$  for a borrower and  $\tau = s$  for a saver. The key distinction between household types is that

$$\lambda^b > \lambda^s \quad \text{for all } c.$$

Thus, when evaluated at the same level of consumption, a type  $b$  always has a higher marginal utility of consumption than a type  $s$  and consequently is more impatient to consume. A type  $b$  will want to borrow to increase current consumption; a type  $s$  will want to save to increase future consumption. The household's type is not a fixed characteristic but follows a Markov process. For simplicity, assume a type  $\tau$  at time  $t$  remains a type  $\tau$  with probability one-half and changes type with probability one-half.<sup>33</sup>

Market segmentation is introduced by assuming a type  $s$  household cannot lend directly to a type  $b$  household. Instead, savers supply funds to a financial intermediary (a bank)

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33. Cúrdia and Woodford (2011) considered a more general transition process for household type.

and these banks issue loans to borrowers. In addition to bank deposits, type  $s$  households may hold government bonds; type  $b$  households take out loans from banks. Cúrdia and Woodford assumed bonds are perfect substitutes for bank deposits, so both yield the same return in equilibrium.

Both household types choose their consumption optimally, but as their preferences differ, there will be two Euler equations to characterize optimal consumption paths. Let  $\lambda_t^\tau$  denote the marginal utility of consumption for a type  $\tau$  household and  $\lambda_t^{-\tau}$  the marginal utility for the other type. Given the assumed transition process for household type, the future marginal utility of consumption for a type  $\tau$  is  $\lambda_{t+1}^\tau$  with probability one-half and  $\lambda_{t+1}^{-\tau}$  with probability one-half. The Euler equation for a type  $\tau$  takes the form

$$\lambda_t^\tau = \beta E_t \left( \frac{1 + i_t^\tau}{1 + \pi_{t+1}} \right) \frac{1}{2} (\lambda_{t+1}^\tau + \lambda_{t+1}^{-\tau}),$$

where  $i_t^\tau$  is the nominal interest rate faced by a type  $\tau$ . For a type  $b$ ,  $i_t^b$  is the rate on bank loans; for a type  $s$ ,  $i_t^d$  is the rate on bank deposits, which is also the rate on government bonds. The Euler equations then imply

$$\lambda_t^b - \lambda_t^s = \beta E_t \left( \frac{i_t^b - i_t^d}{1 + \pi_{t+1}} \right) \frac{1}{2} (\lambda_{t+1}^b + \lambda_{t+1}^s). \quad (11.33)$$

The spread between the borrowing and saving interest rates reflects a gap between the marginal utilities of consumption of the two household types.

The model has two interest rates,  $i_t^b$  and  $i_t^d$ . The credit spread  $\omega_t$  is defined as

$$\omega_t \equiv i_t^b - i_t^d \geq 0.$$

This spread will depend on the role of financial intermediaries (banks). They take in deposits and make one-period loans. They also hold reserves  $M_t$  with the central bank, and these reserves pay a nominal return of  $i_t^m$ . Intermediaries operate in a competitive environment, and they take interest rates as given. The credit spread is positive for two reasons in the Cúrdia-Woodford model. First, real resources must be used in originating loans. The credit spread must be positive to cover these costs. Second, some borrowers do not repay their loans, so the spread must also cover the losses on these loans.

The balance sheet of the bank is

$$d_t = m_t + L_t + \chi_t(L_t), \quad (11.34)$$

where liabilities  $d$  equal deposits,  $m = M/P$  are real reserve holdings,  $L$  are loans, and  $\chi_t(L_t)$  equals the volume of bad loans extended. Bad loans are a function of the total volume of loans, and the function  $\chi(L)$  is assumed to be increasing and convex and potentially subject to shifts. Banks maximize the payout to shareholders, defined as earnings on good loans (bad loans return zero) and on reserves net of the cost of deposits and operating expenses,  $T^{\text{FI}}(L, m)$ . Cúrdia and Woodford assumed costs are increasing in  $L_t$  but

decreasing in reserve holdings  $m_t$ , and operating expenses are taken to be convex in both arguments. In addition, they assumed that for any  $L_t$  there exists an  $\bar{m}_t(L_t)$ , defined as a satiation level of reserves, such that  $T_{t,m}^{\text{FI}}(L, m) \equiv \partial T^{\text{FI}}(L, m)/\partial m = 0$  for all  $m \geq \bar{m}_t(L_t)$ . Hence, the bank pays out to its shareholders

$$\pi_t^{\text{FI}} = (1 + i_t^b) L_t + (1 + i_t^m) m_t - T_t^{\text{FI}}(L_t, m_t) - (1 + i_t^d) d_t,$$

which it attempts to maximize subject to (11.34). Using the balance sheet constraint,

$$\pi_t^{\text{FI}} = (i_t^b - i_t^d) L_t + (i_t^m - i_t^d) m_t - T_t^{\text{FI}}(L_t, m_t) - (1 + i_t^d) \chi_t(L_t).$$

The first-order conditions for the bank's optimal choice of  $L_t$  and  $m_t$ , taking interest rates as given, are

$$(1 + i_t^d) \chi_{t,L}(L_t) + T_{t,L}^{\text{FI}}(L_t, m_t) = i_t^b - i_t^d = \omega_t, \quad (11.35)$$

$$T_{t,m}^{\text{FI}}(L_t, m_t) = i_t^m - i_t^d \leq 0, \quad (11.36)$$

where  $\chi_{t,L}$  is the partial derivative of  $\chi$  with respect to  $L$ , and  $T_{t,x}^{\text{FI}}$  is the partial derivative of  $T^{\text{FI}}$  with respect to  $x \in \{L, m\}$ .<sup>34</sup> These first-order conditions have straightforward interpretations. From (11.35), the intermediary's optimal lending is at the point where the marginal cost of expanding lending is equal to the marginal gain. The marginal cost consists of the increase in bad loans plus the increase in operating costs that occur when  $L_t$  is increased. The marginal gain is the spread  $\omega_t$  between the return on loans and the cost of the deposits used to fund lending. Similarly, reserves are held to the point where the marginal cost savings (recall  $T_m^{\text{FI}} \leq 0$ ) equals the marginal gain, where the latter is the spread between the interest rate the central bank pays on reserves and the rate banks pay on deposits.

Define the spread between the deposit rate and the rate paid on reserves as  $\delta_t^m \equiv i_t^d - i_t^m$ . The opportunity cost of holding reserves is  $\delta_t^m$ . The demand for reserves  $m_t^d$  implied by (11.36) is a function of  $L_t$  and  $\delta_t^m$  and is denoted by  $m_t^d(L_t, \delta_t^m)$ , where  $m_t^d(L_t, 0)$  is defined as equal to the satiation level of reserves  $\bar{m}(L_t)$ .<sup>35</sup> The demand for reserves and the spread  $\delta^m$  satisfy joint inequalities given by

$$m_t \geq m_t^d(L_t, \delta_t^m),$$

$$\delta_t^m \geq 0,$$

34. Cúrdia and Woodford (2011) set up the intermediary's problem differently, so they obtained slightly different expressions for these first-order conditions. The economic interpretations are the same, though. See problem 8 at the end of this chapter.

35. Recall from chapter 2 that the Friedman rule called for setting the nominal interest rate to zero to ensure the opportunity cost of holding money was zero. When interest is paid on money (reserves in the present model), this opportunity cost no longer requires that nominal interest rates equal zero. Instead,  $\delta_t^m = 0$  can be achieved at any positive level of  $i_t^d$  as long as  $i_t^m = i_t^d$ .

with at least one holding with equality. If the opportunity cost of holding reserves measured by  $\delta_t^m$  is positive, equilibrium requires that the demand for reserves equal the reserve supply provided by the central bank:  $m_t = m_t^d(L_t, \delta_t^m)$ . If  $\delta_t^m = 0$ , then  $m_t > m_t^d(L_t, 0) = \bar{m}_t(L_t)$  is consistent with equilibrium.

Equilibrium in the loan market requires the demand for loans by borrowers equal the supply of loans. This supply consists of lending by banks plus any lending to the private sector by the central bank, denoted by  $L_t^{\text{cb}}$ :

$$b_t = L_t + L_t^{\text{cb}}.$$

Incorporating direct lending by the central bank to the nonfinancial sector (i.e., household borrowers) allows the model to be used to investigate the impact of policies, sometimes called credit-easing policies, in which the central bank extends credit directly to the private sector.

The central bank's balance sheet consists of reserves, which are liabilities of the central bank, and assets consisting of loans and central bank holdings of government debt  $b_t^{\text{cb}}$ :

$$m_t = L_t^{\text{cb}} + b_t^{\text{cb}}.$$

Cúrdia and Woodford assumed the central bank incurs costs  $T_t^{\text{cb}}(L_t^{\text{cb}})$  in lending to the private sector. The central bank pays interest  $i_t^m$  on reserves and receives  $i_t^d$  on its holdings of government debt.<sup>36</sup>

The central bank has three policy instruments: (1) the interest rate paid on reserves  $i_t^m$ , (2) the nominal quantity of reserves  $M_t$ , and (3) the composition of the asset side of the central bank's balance sheet between  $L_t^{\text{cb}}$  and  $b_t^{\text{cb}}$ . Cúrdia and Woodford embedded this financial structure into a model with sticky prices. The nominal rigidity allows the central bank to affect real reserves by controlling the nominal supply of reserves. Thus, by affecting  $m$ , the central bank can control  $\delta^m$  when  $\delta^m > 0$ . By adjusting  $i_t^m$ , it can control the level of  $i_t^d$  for a given  $\delta_t^m$ .

The real side of Cúrdia and Woodford's model parallels a fairly standard new Keynesian model; as a result, welfare depends on the usual distortions arising from imperfect competition and relative price dispersion. However, two new distortions are present in the model. First, a social planner not constrained to use financial intermediaries would ensure the marginal utility of consumption is equalized across the two types of households. As (11.33) shows,  $\Omega_t \equiv \lambda_t^b - \lambda_t^s$  will be increasing in the credit spread  $\omega_t$ ; the social planner

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36. Lending to the private sector imposes risks on the central bank. It might have to take losses on its balance sheet. Similarly, if the central bank holds long-term government bonds, it will take losses when interest rates rise. Benigno and Nistico (2015) developed a model in which unconventional open-market operations have consequences for inflation and output because of income losses on the central bank balance sheet. Hall and Reis (2013) evaluated the solvency of the Federal Reserve and the ECB under various scenarios for interest rates.

would ensure  $\omega_t = 0$ .<sup>37</sup> Second, welfare also depends on the resources absorbed in the financial sector.

The optimal monetary policy in this framework implements the Friedman rule: supply reserves up to the satiation level  $\bar{m}$ . This ensures the opportunity cost of holding reserves is zero;  $\delta^m = 0$ , or  $i_t^m = i_t^d$ . Once  $m_t = \bar{m}$ , and conditional on the paths of  $i_t^d$ ,  $i_t^m$ , and  $L_t^{\text{cb}}$ , there is generally no value in expanding the central bank's balance sheet beyond  $\bar{m}$  by engaging in open-market purchases of government debt. An exception occurs if it is desirable to increase central bank lending to households. From the central bank's balance sheet,  $L_t^{\text{cb}} = m_t - b_t^{\text{cb}}$ . Suppose the central bank has reduced its holdings of government debt to zero and  $m_t = \bar{m}$ . Then it would be necessary to expand the size of the balance sheet above  $\bar{m}$  to increase  $L_t^{\text{cb}}$ .

Policies to alter the composition of the central bank's balance sheet would involve, in the case of a credit-easing policy, a sale of government debt to finance loans to the household sector. If central bank lending simply reduced bank lending dollar for dollar, the impact on welfare would depend on whether the fall in spreads as the costs of the banking sector fall with the decline in  $L_t$  are less than offset by any costs to the central bank associated with the rise in  $L_t^{\text{cb}}$ . If the costs of central bank lending are sufficiently high, it is never optimal to have  $L_t^{\text{cb}} > 0$ . However, if the marginal cost of the central bank providing credit is less than the marginal cost of private lenders, it is optimal for the central bank to engage in lending. If a financial crisis is interpreted as a shock that disrupts the ability of the private sector to intermediate credit, thereby raising the cost of providing credit through financial intermediaries, central bank credit-easing policies may be called for.

#### 11.5.4 Moral Hazard in Banking

Gertler and Karadi (2011; 2013) developed models to analyze balance sheet policies in which financial frictions arise because of moral hazard problems of the type studied by Kiyotaki and Moore (1997; 2012), and Gertler and Kiyotaki (2010) (see chapter 10). In Gertler and Karadi's (2011) work, the economy consists of five agents: households, banks, nonfinancial firms, a government, and the central bank. As in the model of Cúrdia and Woodford (2011), households hold (real) deposits and one-period (real) government bonds that are viewed as perfect substitutes, so both pay a gross real return of  $1 + r_t$ . Banks take in deposits from households and lend to nonfinancial firms and the government. Households are precluded from lending directly to nonfinancial firms. This market segmentation means that households cannot arbitrage any spread between the interest rate on deposits and the rate on loans to nonfinancial firms.

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37. The term  $\Omega_t$  also affects the inflation adjustment equation through its effect on real marginal costs faced by firms, as there is an inefficient allocation of hours across household types when  $\Omega_t \neq 0$ .

A bank uses funds raised from households and its own net worth (equity, or bank capital) to invest in claims on nonfinancial firms and long-term government bonds. A bank's balance sheet is

$$n_t + d_t = Q_t s_t + q_t b_t,$$

where  $n$  is the bank's equity,  $d$  represents deposit liabilities,  $Q$  is the price of nonfinancial claims,  $s$  is the number of such claims the bank holds,  $q$  is the price of long-term government bonds, and  $b$  is the quantity of bonds the bank holds. Assume the bank retains all earnings.<sup>38</sup> In this case, it enters period  $t$  with equity

$$n_t = R_{k,t} Q_{t-1} s_{t-1} + R_{b,t} q_{t-1} b_{t-1} - R_t d_{t-1},$$

where  $R_k$  is the gross return on shares,  $R_b$  is the gross interest rate on long-term government bonds, and  $R_t$  is the gross interest rate paid on deposits. If the bank's problem is to maximize  $E_t n_{t+1}$  subject to the balance sheet constraint,  $d_t$ ,  $s_t$ , and  $b_t$  are chosen to maximize

$$E_t n_{t+1} = E_t (R_{k,t+1} - R_{t+1}) Q_t s_t + E_t (R_{b,t+1} - R_{t+1}) q_t b_t + E_t R_{t+1} n_t. \quad (11.37)$$

In equilibrium, arbitrage ensures  $E_t (R_{k,t+1} - R_{t+1}) = E_t (R_{b,t+1} - R_{t+1}) = 0$ . Otherwise—if, for example,  $E_t (R_{k,t+1} - R_{t+1}) > 0$ —each bank would have an incentive to expand its deposit liabilities to purchase more claims because their expected return would exceed the marginal cost of deposits, and if  $E_t (R_{k,t+1} - R_{t+1}) < 0$ , no bank would want to hold claims and the market would not clear. A similar argument holds for the expected return on bonds. Interest rate spreads would be zero.

To motivate spreads between the rates of return on nonfinancial claims, government long-term bonds, and deposits, Gertler and Karadi (2013) introduced a moral hazard problem, building on the work of Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). Specifically, assume the bank owners can divert funds that would otherwise be available to repay their liabilities (deposits). Assume that a fraction  $\theta$  of the bank's claims on nonfinancial firms can be diverted, and a fraction  $\theta \Delta < \theta$  of its long-term bond portfolio can be diverted. Requiring that  $0 \leq \Delta < 1$  assumes it is harder to divert funds from bank's portfolio of government bonds than from its holdings of claims on firms. The bank owners will divert funds if the amount they can divert,  $\theta (Q_t s_t + \Delta q_t b_t)$ , exceeds the continuation value of the bank, namely, the present discounted value of the profits from remaining in business. Let this continuation value be denoted by  $V_t$ . Then the bank faces an incentive constraint of the form

$$V_t \geq \theta Q_t s_t + \Delta \theta q_t b_t. \quad (11.38)$$

If this constraint is not satisfied, no depositor will put money into the bank.

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38. Gertler and Karadi (2011) assumed banks exit with a constant probability per period, and they discussed conditions to ensure the bank retains all earnings until it exits.

In the Gertler-Karadi model, the bank maximizes the expected discounted present value of the bank subject to (11.37) and the incentive constraint (11.38). However, the key implications of this moral hazard problem can be illustrated in a simplified static version of the model. Suppose the bank wants to maximize  $n_{t+1}$ , subject to (11.37) and  $n_{t+1} \geq \theta Q_t s_t + \Delta \theta q_t b_t$ . The decision problem of the bank can be written as

$$\max_{s_t, b_t} n_{t+1} + \lambda (n_{t+1} - \theta Q_t s_t - \Delta \theta q_t b_t),$$

where  $\lambda_t$  is the Lagrangian multiplier on the incentive constraint and  $n_{t+1}$  is given by (11.37). Using (11.37), the first-order conditions for  $s_t$  and  $b_t$  are

$$E_t (R_{k,t+1} - R_{t+1}) = \left( \frac{\lambda}{1 + \lambda} \right) \theta,$$

$$E_t (R_{b,t+1} - R_{t+1}) = \left( \frac{\lambda}{1 + \lambda} \right) \theta \Delta.$$

In the absence of a moral hazard problem ( $\theta = 0$ ), or if the incentive constraint is not binding ( $\lambda = 0$ ), the spreads are zero, and the situation is the same as obtained with frictionless financial markets. However, if  $\theta > 0$  and the constraint binds, so that  $\lambda > 0$ , the return on the bank's assets exceeds the return on deposits. Banks would like to borrow more (raise more deposits) to invest in nonfinancial claims, but they are limited in their ability to borrow by the incentive constraint. Note that the assumption of market segmentation is important here. Households cannot purchase long-term government bonds or claims on nonfinancial firms. If they could, then (absent some other friction that limits arbitrage) households would never hold deposits if, for example,  $E_t (R_{k,t+1} - R_{t+1}) > 0$ .

With  $0 \leq \Delta < 1$ , the expected long-term bond rate exceeds the rate on deposits but is less than or equal to the return on nonfinancial assets:

$$E_t (R_{k,t+1} - R_{b,t+1}) = \left( \frac{\lambda}{1 + \lambda} \right) \theta (1 - \Delta) > 0.$$

Because it is easier to divert from the bank's holdings of  $s$ , attempts by the bank to expand investments in nonfinancial claims to take advantage of potential arbitrage opportunities when  $E_t (R_{k,t+1} - R_{t+1}) > 0$  act to tighten the incentive constraint more than does expansion of the bank's bond holdings. Consequently, more of the long-term bond spread can be arbitrated away, leaving  $E_t (R_{k,t+1} - R_{b,t+1}) > 0$ .

Using (11.37) and the expressions for the interest rate spreads, the incentive constraint can be written as

$$\frac{Q_t s_t + \Delta q_t b_t}{n_t} \leq \phi_t, \tag{11.39}$$

where  $\phi_t \equiv E_t R_{t+1} / [\theta - E_t (R_{k,t+1} - R_{t+1})]$  defines the limit the incentive constraint places on the size of a bank's portfolio relative to its net worth.

Gertler and Karadi (2013) argued that  $\phi_t$  can be interpreted as a maximum leverage ratio; it is the maximum ratio of assets (adjusted by  $\Delta$ ) to net worth that the bank may hold without violating the incentive constraint. This constraint limits the bank's portfolio size to the point where its incentive to divert funds is exactly balanced by the cost of losing the value of the bank, and it acts as an endogenous capital constraint. The two assets the bank can hold do not enter with equal weights. Long-term government bonds enter with a weight  $\Delta < 1$ , reflecting the weaker constraint on arbitrage for this asset compared to loans.

The ratio  $\phi_t$  is independent of bank-specific factors, allowing (11.39) to be aggregated over all banks. Thus, the value of aggregate assets held by the banking system is constrained to be less than or equal to the multiple  $\phi_t$  of aggregate bank capital. Let  $N_t$  denote aggregate bank capital, and let  $S_{pt}$  and  $B_{pt}$  denote aggregate bank holdings of nonfinancial claims and government long-term bonds. When the constraint is binding,  $\lambda_t > 0$  and

$$\phi_t N_t = Q_t S_{pt} + \Delta q_t B_{pt}. \quad (11.40)$$

Changes in bank equity  $N_t$  will induce fluctuations in overall asset demand by banks.

Suppose the economy starts with a binding aggregate incentive constraint (11.40). Consider a crisis experiment in which there is an exogenous sharp decrease in bank capital. Because of the balance sheet constraint, the decrease in  $N_t$  generates a drop in banks' asset demand and forces a fire sale of assets to satisfy the incentive and balance sheet constraints. In Gertler and Karadi's general equilibrium version of their model, asset prices  $Q_t$  and  $q_t$  decline. This further weakens bank balance sheets, the balance sheet constraint tightens even more, and this limits arbitrage between assets inducing an increase in interest rate spreads.

Now consider the implications of this model for balance sheet policies by the central bank. Consider the possibility that the central bank can purchase assets  $S_{gt}$  or long-term government bonds  $B_{gt}$  from the banking system. The central bank's balance sheet is

$$Q_t S_{gt} + q_t B_{gt} = D_{gt},$$

where  $D_{gt}$  is the central bank's issuance of short-term debt. Purchases of  $S_{gt}$  correspond to credit-easing policies and essentially represent central bank intermediation; they are like direct loans to banks that end up as loans to firms. If balance sheet policies are to be effective, the central bank must have some advantage over banks that can overcome the factors limiting private sector provision of credit. Consequently, Gertler and Karadi (2013) assumed the central bank is able to obtain funds elastically by issuing short-term government debt, and the government is able to commit credibly to honoring its debt. This ensures there is no moral hazard or agency conflict between the central bank and the private sector of the type that limits the leverage of banks. But if this is the case, then the logic of the model suggests in an efficient equilibrium all lending should be done by the central bank. Gertler and Karadi assumed, therefore, that the central bank is less efficient than



banks at making loans. Specifically, they assumed there are efficiency costs  $\tau_s$  and  $\tau_b$  for each private loan and government bond purchased by the central bank.

The aggregate balance sheet constraint of the banking sector can now be written as

$$\phi_t N_t \geq Q_t (S_t - S_{gt}) + \Delta q_t (B_t - B_{gt}),$$

where  $B_t = B_{pt} + B_{gt}$  is the total stock of government debt. This inequality can be rewritten as

$$Q_t S_t \leq \phi_t N_t + Q_t S_{gt} + \Delta q_t (B_{gt} - B_t). \quad (11.41)$$

The left side of (11.41) can be interpreted as the total demand for securities used to finance nonfinancial firms. If the aggregate balance sheet constraint is not binding, that is, (11.41) holds with inequality, then asset purchases by the central bank are neutral; they displace private intermediation without affecting total credit or asset prices. Spreads are zero because  $\lambda_t = 0$  when the constraint does not bind. Firms obtain all the loans they need, and banks supply all the loans they want. Banks obtain the funds they need (deposits) without any problem.

Now suppose the constraint binds. Then

$$Q_t S_t = \phi_t N_t + Q_t S_{gt} + \Delta q_t (B_{gt} - B_t).$$

The total quantity of credit to nonfinancial firms is limited by a binding constraint, and any policy actions that increase the right side of this expression will relax the constraint and increase credit to nonfinancial firms. Given the total quantity of bank equity ( $N_t$ ), an increase in the central bank's holdings of private securities ( $S_{gt}$  loans) or government bonds ( $B_{gt}$ ) raises the total supply of credit to nonfinancial firms (that is, it relaxes the constraint on the banking sector and increases the demand for nonfinancial assets by banks). Such actions raise the total supply of credit by freeing up bank capital. This increased demand by banks for assets pushes up the price of assets  $Q_t$  and  $q_t$  (given that asset supplies are inelastic in the short run). Excess returns fall.

Because  $\Delta < 1$ , a given dollar amount of private sector asset purchases by the central bank is more effective in expanding credit than the same dollar amount of purchases of government bonds by the central bank. From (11.40), removing  $1/Q_t$  in shares from the bank's balance sheet frees up one dollar that can be used by the bank to purchase assets. Removing  $1/q_t$  in government bonds from the bank's balance sheet frees up  $\Delta < \$1$ . This has two implications. First, expanding the central bank's balance sheet by purchasing loans from the banking sector has a larger effect in expanding credit to the nonfinancial sector than does purchasing long-term government bonds from the banking sector. Second, the central bank can expand credit to the private sector without expanding its own balance sheet by selling long-term government bonds to the banking system and using the proceeds to purchase loans from the banking system.

The key mechanism that causes these central bank balance sheet policies to have real effects is the underlying moral hazard problem that limits the ability of private banks to expand credit, together with the assumption that the incentive constraint is affected by the composition of the assets held by the banking system. Removing loans or government bonds from the balance sheets of private banks by expanding the total size of the central bank's balance sheet could be effective in relaxing the binding incentive constraint, but if  $\Delta = 1$ , so that the moral hazard issue applies symmetrically to both the loans and government bonds held by banks, then changing the composition of the central bank's balance sheet would not affect total credit.

### 11.5.5 Resaleability Constraints

Another type of limit on financial transactions can arise if some assets are more difficult to sell than others, that is, if some assets are illiquid. DelNegro et al. (2016) built on the model of Kiyotaki and Moore (2012) to study central bank balance sheet policies when government debt is more liquid than private assets. The illiquidity of private assets arises in their model from what they characterize as a resaleability constraint.

They assumed a continuum of households, each of which consists of a continuum of members. Each period, a fraction  $\varkappa$  of household members receive investment opportunities and become entrepreneurs. Other members become workers who supply labor. At the end of each period, all household members pool resources, so that the model preserves the tractability properties of a representative household model. Entrepreneurs own the capital stock and rent it to goods-producing firms, but they require financial resources to take advantage of their investment opportunities. They issue equity, hold capital, hold equity of other entrepreneurs, and hold bonds. Financial markets, however, impose constraints on the evolution of entrepreneurs' balance sheets. Specifically, entrepreneurs face borrowing constraints in that they can issue new equity only up to a fraction  $\theta$  of their investment. This constraint can be motivated by the types of moral hazard considerations discussed in chapter 10. In addition, they face a resaleability constraint; in any period, an entrepreneur can sell only a fraction  $\phi_t$  of its privately issued equity holdings.

The balance sheet assets of a household consist of nominal government debt, capital, and equity shares in the capital held by other households. The household's liabilities consist of the equity shares of its own capital it has retained and its net worth. Thus, the balance sheet constraint for the household, in real terms, is

$$b_t + q_t N_t^o + q_t K_t = q_t N_t^I + NW_t,$$

where  $b_t$  equals real holdings of government bonds,  $q_t$  is the price of equity,  $q_t N_t^o$  equals the household's holdings of equity issued by other households,  $K_t$  is capital held by the household,  $q_t N_t^I$  is the real value of claims on its own capital held by other households,

and  $NW_t$  is net worth. Net worth is  $NW_t = q_t N_t + b_t$ , where  $N_t = K_t - N_t^I + N_t^O$  is the net equity held by the household.<sup>39</sup>

The role of financial frictions shows up in the evolution of the household's net liabilities in the form of equity issued. In period  $t$ , let  $j$  refer to a household member who is an entrepreneur. The household holds capital  $K_t$  and has mortgaged (sold equity shares)  $N_t^I$  of it to other households. Of the remaining  $K_t - N_t^I$ , a fraction  $\delta$  depreciates, leaving  $(1 - \delta)(K_t - N_t^I)$  held by the household. The resaleability constraint means that at most  $\phi_t$  of this can be sold. Finally, the financing constraint implies that a member  $j$  undertaking investment  $I_{j,t}$  can issue at most  $\theta I_{j,t} < I_{j,t}$  in new equity to finance investment of  $I_{j,t}$ . Thus, equity issued by  $j$  must satisfy

$$N_{j,t+1}^I \leq (1 - \delta) N_t^I + \phi_t (1 - \delta) (K_t - N_t^I) + \theta I_{j,t}.$$

If  $\theta = 1$ , the entrepreneur can finance the entire investment project by selling equity. It can sell new equity equal at most to a fraction  $\phi_t$  of its remaining capital stock not already mortgaged. For holdings of equity in the capital of other households, the resaleability constraint implies

$$N_{j,t+1}^O \geq (1 - \delta) N_t^O - \phi_t (1 - \delta) N_t^O.$$

Combining these two expressions,

$$N_{j,t+1} \geq (1 - \phi_t) (1 - \delta) N_t + (1 - \theta) I_{j,t}. \quad (11.42)$$

Aggregating this equation over all  $j$ ,

$$N_{t+1} \geq (1 - \phi_t) (1 - \delta) N_t + (1 - \theta) I_t. \quad (11.43)$$

In contrast to equity, household holdings of government bonds are not subject to any constraint on their sale. As these bonds can only be issued by the government, the only constraint on the household's bonds holdings is that  $B_{j,t} \geq 0$ .

The budget constraint of entrepreneur  $j$  is

$$\left[ r_t^k + q_t (1 - \delta) \right] N_t + \frac{(1 + i_{t-1})}{1 + \pi_t} b_{t-1} = C_{j,t} + p_t^I I_{j,t} + q_t (N_{j,t+1} - I_{j,t}) + b_{j,t}, \quad (11.44)$$

where  $r^k$  is the return on capital,  $i_t$  is the nominal interest rate,  $\pi_t$  is the inflation rate, and  $p_t^I$  is the cost of new capital including any capital adjustment costs.<sup>40</sup> Entrepreneurs are

39. Households also own the goods-producing firms in the economy, but the portfolio consisting of the shares in these firms is assumed to be fully diversified and nontradeable.

40. DelNegro et al. (2016) also incorporated a labor-leisure choice to allow for variable hours worked. This is important, as they calibrate their model and use their model to estimate the impact of Federal Reserve balance sheet policies during the Great Recession, but the labor supply decision is not central to understanding the implications of their model for central bank policies.

also subject to the constraint (11.42). If  $q_t > p_t^I$ , then the price of equity exceeds the price of newly produced and installed capital. The household's welfare is maximized if each entrepreneur purchases as much capital as feasible, consuming zero and holding no government bonds. This also implies (11.42) will bind, so  $N_{j,t+1} - I_{j,t} = (1 - \phi_t)(1 - \delta)N_t - \theta I_{j,t}$ . This in turn allows (11.44) to be written as

$$I_{j,t} = \frac{[r_t^k + q_t \phi_t (1 - \delta)] N_t + \frac{(1+i_t-1)}{1+\pi_t} b_{t-1}}{p_t^I - q_t \theta}.$$

The right side of this expression is independent of  $j$ , so when aggregated over all entrepreneurs in the household,

$$I_t = \int_0^{\varkappa} I_{j,t} dj = \varkappa \frac{[r_t^k + q_t \phi_t (1 - \delta)] N_t + \frac{(1+i_t-1)}{1+\pi_t} b_{t-1}}{p_t^I - q_t \theta}. \quad (11.45)$$

Because entrepreneurs do not consume or supply labor hours, the decision problem of the typical household, which consists of a fraction  $\varkappa$  of entrepreneurs and a fraction  $1 - \varkappa$  of workers, can be written as

$$\max_{C_{t+i}, H_{t+i}} E_t \sum_{i=0}^{\infty} \beta^i (1 - \varkappa) U(C_{t+i}, H_{t+i}),$$

where  $H$  denotes hours worked, subject to a budget constraint of the form

$$\begin{aligned} \left[ r_t^k + q_t (1 - \delta) \right] N_t + \frac{(1 + i_t - 1)}{1 + \pi_t} b_{t-1} + \omega_t (1 - \varkappa) H_t + D_t = (1 - \varkappa) C_t + p_t^I I_t \\ + q_t (N_{t+1} - I_t) + b_t, \end{aligned}$$

and the investment constraint given by (11.45), where  $D_t$  equals profits from firms.

Let  $\lambda_{t+i}$  and  $\varphi_{t+i}$  be the Lagrangian multipliers on the two constraints. Then the first-order conditions for  $C_t$ ,  $H_t$ ,  $I_t$ ,  $b_t$ , and  $N_{t+1}$  take the form

$$U_C(C_t, H_t) - \lambda_t = 0,$$

$$U_H(C_t, H_t) + \omega_t \lambda_t = 0,$$

$$\lambda_t (q_t - p_t^I) - \varphi_t = 0,$$

$$\lambda_t = \beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) \left[ \lambda_{t+1} + \varkappa \frac{\varphi_{t+1}}{p_{t+1}^I - q_{t+1} \theta} \right],$$

$$\lambda_t q_t = \beta E_t \left\{ \left[ r_{t+1}^k + q_{t+1} (1 - \delta) \right] \lambda_{t+1} + \varkappa \frac{[r_{t+1}^k + q_{t+1} \phi_{t+1} (1 - \delta)] \varphi_{t+1}}{p_{t+1}^I - q_{t+1} \theta} \right\}.$$

From the third of these equations,  $\varphi_t = \lambda_t (q_t - p_t^I) \geq 0$ , so the last two conditions can be written as

$$1 = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) (1 + \Delta_{b,t}),$$

$$1 = \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left[ \frac{r_{t+1}^k + q_{t+1} (1 - \delta)}{q_t} \right] (1 + \Delta_{e,t}),$$

where

$$\Delta_{b,t} \equiv \varkappa \left( \frac{q_{t+1} - p_{t+1}^I}{p_{t+1}^I - q_{t+1}\theta} \right) \geq 0,$$

$$\Delta_{e,t} \equiv \frac{[r_{t+1}^k + q_{t+1}\phi_{t+1}(1 - \delta)]}{[r_{t+1}^k + q_{t+1}(1 - \delta)]} \Delta_{b,t} < \Delta_{b,t}.$$

Standard frictionless pricing relationships for equity and bonds are obtained when  $\Delta_{b,t} = \Delta_{e,t} = 0$ . The wedges are nonzero when  $p_t^I \neq q_t$ , as otherwise  $\varphi_t = 0$  and the constraint on investment is not binding. The wedges are increasing in  $\theta$ , the parameter governing the borrowing limits. Holding bonds carries an extra premium relative to the frictionless case, and this premium is measured by  $\Delta_{b,t} = \varkappa(q_{t+1} - p_{t+1}^I)/(p_{t+1}^I - q_{t+1}\theta)$ . Increased bond holdings relax the constraint on investment by providing entrepreneurs in the household with extra liquidity because bond holdings are not subject to a resaleability constraint. This extra liquidity can finance  $\varkappa/(p_{t+1}^I - q_{t+1}\theta)$ , and relaxing this constraint is worth  $q_{t+1} - p_{t+1}^I$  to the household. Holding additional equity also relaxes the constraint on entrepreneurs. But the premium measured by  $\Delta_{b,t}$  applies less to equity because equity is less liquid ( $\phi_t < 1$ ) than government bonds. If  $\phi_t = 1$ , so that equity is as liquid as bonds, then  $\Delta_{e,t} = \Delta_{b,t}$ .

DelNegro et al. (2016) embedded this model of investment and financial frictions into an otherwise standard NK model with sticky prices and sticky wages. They treat the 2008–2009 financial crisis as caused by a shock to the liquidity of private assets (a fall in  $\phi_t$ ) and examine whether, when the short-term nominal interest rate falls to zero, central bank purchases of private assets in exchange for government bonds can mitigate the liquidity shock. They concluded that interventions such as those undertaken by the Federal Reserve were important in preventing the Great Recession from becoming another Great Depression. The mechanism through which such interventions operate is straightforward. The shock to  $\phi_t$  reduces liquidity and tightens the constraint on investment spending. Because  $\Delta_{b,t} > \Delta_{e,t}$ , bonds are more effective in relaxing the constraint on investment, so increasing the stock of government bonds in the hands of the public by removing less liquid assets from private sector portfolios can offset the effects of a shock to the liquidity of the assets issued by the private sector.

### 11.5.6 Summary on Balance Sheet Policies

This section has reviewed alternative approaches to modeling the effects of policies that either expand the balance sheet of the central bank or alter the compositions of the assets held by the central bank. The approaches differed in how they motivated the wedges between expected rates of returns on different assets and the asset pricing relationships implied by frictionless financial markets. One approach, exemplified by the models of Andrés, López-Salido, and Nelson (2004) and Chen, Cúrdia, and Ferrero (2012), limited arbitrage by assuming some households' portfolio choices were restricted; these households could hold long-term government bonds but not short-term government bonds. Other households could hold both long-term and short-term government bonds, but the spread between the rates of return on these government bonds reflected transaction costs. These costs were then assumed to depend on the central bank's balance sheet. Cúrdia and Woodford (2010; 2011) also introduced household heterogeneity, market segmentation, and transaction costs to account for limits to arbitrage and a potential role for balance sheet policies. In their models, the transaction costs arise because financial institutions serve to intermediate the flow of household saving to firms seeking to borrow, and this intermediation service absorbs real resources.

The second approach, developed by Gertler and Karadi (2011; 2013), explains pricing wedges based on moral hazard considerations that limit banks' ability to arbitrage away differences in expected rates of return. Banks face an incentive constraint that, when binding, limits their ability to expand lending. Central bank balance sheet policies have the potential to relax this constraint and expand the supply of credit.

A third example, that of DelNegro et al. (2016), emphasized differences in the liquidity of government bonds and private assets, with these differences modeled as a restriction on the fraction of a household's holdings of equity that could be sold at any time. Central bank purchases of private assets, funded by selling government bonds to the public, increase total private sector liquidity and can play a role in offsetting liquidity shocks suffered by the private sector.

### 11.6 Appendix: Derivation of the Asset Pricing Wedges

The household's decision problem involves choosing the sequence of consumption and real asset holdings to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \lambda_{t+i} \left( \begin{array}{c} U(c_{t+i}, m_{t+i}) + \\ y_{t+i} + \left[ (p_{t+i} + d_{t+i}) s_{t-1+i} + \left( \frac{1+i_{t+i}-1}{1+\pi_{t+i}} \right) b_{1,t-1+i} \right] \\ + \left( \frac{1}{1+\pi_{t+i}} \right) (b_{2,t-1+i} + m_{t-1+i}) \\ -c_{t+i} - T_{t+i} - q_{t+i} s_{t+i} - b_{1,t+i} - p_{2,t+i} b_{2,t+i} - m_{t+i} \end{array} \right) \right],$$

taking income, dividends, asset prices, and interest rates as given. Define

$$W_t \equiv \left[ (p_t + d_t) s_{t-1} + \left( \frac{1 + i_{t-1}}{1 + \pi_t} \right) b_{1,t-1} + \left( \frac{1}{1 + \pi_t} \right) (b_{2,t-1} + m_{t-1}) \right].$$

The terms involving current and future budget constraints can be written as

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} (y_{t+i} - c_{t+i} - T_{t+i}) + \lambda_t W_t \\ & + E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left( \begin{aligned} & \beta \left( \frac{\lambda_{t+i+1}}{\lambda_{t+i}} \right) \left[ (p_{t+i+1} + d_{t+i+1}) s_{t+i} + \left( \frac{1+i_{t+i}}{1+\pi_{t+i+1}} \right) b_{1,t+i} \right] \\ & + \left( \frac{1}{1+\pi_{t+i+1}} \right) (b_{2,t+i} + m_{t+i}) \\ & - q_{t+i} s_{t+i} - b_{1,t+i} - p_{2,t+i} b_{2,t+i} - m_{t+i} \end{aligned} \right). \end{aligned}$$

Notice that the second summation is equal to

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left( \begin{aligned} & \left[ \beta \left( \frac{\lambda_{t+i+1}}{\lambda_{t+i}} \right) (p_{t+i+1} + d_{t+i+1}) - q_{t+i} \right] s_{t+i} + \\ & \left[ \beta \left( \frac{\lambda_{t+i+1}}{\lambda_{t+i}} \right) \left( \frac{1+i_{t+i}}{1+\pi_{t+i+1}} \right) - 1 \right] b_{1,t+i} \\ & + \left[ \beta \left( \frac{\lambda_{t+i+1}}{\lambda_{t+i}} \right) \left( \frac{1}{1+\pi_{t+i+1}} \right) - p_{2,t+i} \right] (b_{2,t+i} + m_{t+i}) \\ & + \left[ \beta \left( \frac{\lambda_{t+i+1}}{\lambda_{t+i}} \right) \left( \frac{1}{1+\pi_{t+i+1}} \right) - 1 \right] m_{t+i} \end{aligned} \right) \\ & = E_t \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} (\Delta_{s,t+i} s_{t+i} + \Delta_{1,t+i} b_{1,t+i} + \Delta_{2,t+i} b_{2,t+i} + \Delta_{m,t+i} m_{t+i}), \end{aligned}$$

where the  $\Delta_{k,t+i}$  terms are defined in the text.

## 11.7 Problems

1. Suppose  $r_t^n = r^{\text{ELB}} < 0$  with probability  $q$  and exits with probability  $1 - q$ . What is the expected time the economy will remain at the ELB? How is the expected duration at the ELB affected by an increase in  $q$ ?
2. Derive (11.12) and (11.13). Assume  $\sigma(1 - q)(1 - \beta q) - q\kappa > 0$  and graph your solution. With Calvo price adjustment,  $\kappa$  is a decreasing function of the degree of nominal price stickiness. Suppose prices become more flexible, so that  $\kappa$  increases. For a given value of  $r^{\text{ELB}}$ , what happens to the equilibrium output gap and inflation while the ELB binds? Explain why the output gap and inflation become more negative as prices become more flexible.
3. Consider the following model with  $r_t^n = \bar{r} < 0$  and the nominal interest rate equal to zero:

$$y_t = E_t y_{t+1} + \left( \frac{1}{\sigma} \right) (E_t \pi_{t+1} + r_t^n),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^f),$$

where  $y_t$  is output and  $y_t^f$  is the economy's flexible-price output level. Suppose at  $t + 1$ ,  $i_{t+1} = r_{t+1}^n$ ,  $y_{t+1} = y_{t+1}^f$ , and  $\pi_{t+1} = 0$  with probability  $1 - q$ , and with probability  $q$ ,  $r_{t+1}^n = \bar{r}$  and  $i_{t+1} = 0$ .

- a. Solve for  $y_t$  and  $\pi_t$  as a function of  $y_t^f$ .
  - b. What is the effect on  $y_t$  and  $\pi_t$  of a one-period positive productivity shock that increases  $y_t^f$  but leaves future values  $y_{t+i}^f$ ,  $i > 0$  unchanged? Explain the intuition behind your results.
  - c. Now suppose the productivity increase is permanent. How are  $y_t$  and  $\pi_t$  affected? Are your answers different than in part (b)? If so, explain why.
4. Solve for the eigenvalues of the matrix  $Q$  in (11.19). Let  $\beta = 0.99$  and  $\sigma = 1$ . Plot both eigenvalues as a function of  $\kappa$ , letting  $\kappa$  range from 0.05 (prices very sticky) to 2.0 (prices very flexible). What happens to the largest eigenvalue? What does this imply about the effects of forward guidance? Is it more powerful when prices are sticky or when prices are flexible? Explain.
5. Suppose the model is given by

$$x_t = E_t \bar{x}_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t^n),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

and the policymaker's objective is to minimize

$$\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2).$$

In period  $t$  the economy is at the ELB, so  $i_t = 0$ . In period  $t + 1$ , the economy is out of the ELB and assume  $x_{t+i} = \pi_{t+i} = 0$  for  $i \geq 1$  is feasible. Assume that the policymaker commits to setting  $x_{t+i} = \pi_{t+i} = 0$  for  $i \geq 2$ .

- a. At time  $t$ , what policy should it commit to for period  $t + 1$  if it wants to minimize

$$\frac{1}{2} \left[ (\pi_t^2 + \lambda x_t^2) + \beta (\pi_{t+1}^2 + \lambda x_{t+1}^2) \right] ?$$

At  $t + 1$ ,  $x_{t+1} = \pi_{t+1} = 0$  is feasible. Is it optimal? (The assumption that  $x_{t+i} = \pi_{t+i} = 0$  for  $i = 2, 3, \dots$  makes this a problem that can be solved analytically.)

- b. Explain why it is optimal to promise  $i_{t+1} < r_{t+1}^n$ .
6. Suppose the setup is the same as in problem 5, but assume that the central bank commits to setting  $x_{t+i} = \pi_{t+i} = 0$  for  $i \geq 3$  rather than  $i \geq 2$ . At time  $t$ , what policy should it commit to for period  $t + 1$  and  $t + 2$  if it wants to minimize

$$\frac{1}{2} \left[ (\pi_t^2 + \lambda x_t^2) + \beta (\pi_{t+1}^2 + \lambda x_{t+1}^2) + \beta^2 (\pi_{t+2}^2 + \lambda x_{t+2}^2) \right] ?$$



At  $t + 1$ ,  $x_{t+1} = \pi_{t+1} = 0$  is feasible. Is it optimal? (The assumption that  $x_{t+i} = \pi_{t+i} = 0$  for  $i = 3, 4, \dots$  makes this a problem that can be solved analytically.)

7. Show that (11.39) is implied by (11.37) and the bank's balance sheet.
8. Suppose the balance sheet of an intermediary consists of deposit liabilities  $d_t$  and assets consisting of loans

$$d_t = m_t + L_t + \chi_t(L_t) + T_t^{\text{FI}}(L_t, m_t) + \pi_t^{\text{FI}},$$

where  $m = M/P$  is real reserve holdings,  $L$  is loans,  $\chi_t(L_t)$  equals the volume of bad loans extended, real resource costs are equal to  $T_t(L_t, m_t)$ , and  $\pi_t^{\text{FI}}$  is any payouts to shareholders. The costs of operating the intermediary are assumed to depend on the volume of loans and reserves, while bank loans are a function of the total volume of loans. Both the bad loan and the cost functions are increasing and convex in their arguments and potentially subject to shifts. Cúrdia and Woodford assumed financial intermediaries choose a level of loans  $L$  and reserves  $m$  and then secure a level of deposit liabilities such that earnings  $(1 + i^b)L + (1 + i^m)m$  are just equal to repayment on deposits  $(1 + i^d)d$ . Using this assumption to eliminate  $d_t$  from (11.34) yields, with some rearranging, the bank's problem as one of maximizing

$$\pi_t^{\text{FI}} = \frac{(i^b - i_t^d)L + (i_t^m - i_t^d)m_t}{1 + i_t^d} - \chi_t(L_t) - T_t^{\text{FI}}(L_t, m_t).$$

Derive the first-order conditions  $L$  and  $m$  for the problem of maximizing  $\pi^{\text{FI}}$ . How do they differ from (11.35) and (11.36)?

9. For the model of DelNegro et al. (2016), show how the borrowing restrictions and resaleability constraints on each household imply the aggregate relationship given in (11.43).



# 12 Monetary Policy Operating Procedures

## 12.1 Introduction

Previous chapters treated the nominal money supply, the nominal interest rate, or even inflation as the variable directly controlled by the monetary policymaker. This approach ignores the actual problems surrounding policy implementation. Central banks do not directly control the nominal money supply, inflation, or long-term interest rates likely to be most relevant for aggregate spending. Instead, narrow reserve aggregates, such as the monetary base or very short-term interest rates like the U.S. federal funds rate, are the variables over which the central bank can exercise close control. Among the issues considered in this chapter are the specific relationships between short-term interest rates, other reserve aggregates such as nonborrowed reserves or the monetary base, and the broader monetary aggregates such as  $M1$  or  $M2$ , as well as why many central banks choose to use a short-term interest rate rather than a monetary aggregate as their policy instrument.

The actual implementation of monetary policy involves a variety of rules, traditions, and practices, collectively called *operating procedures*. Operating procedures differ according to the instrument the central bank uses in its daily conduct of policy, the operating target whose control is achieved over short horizons (e.g., a short-term interest rate versus a reserve aggregate), the conditions under which the instruments and operating targets are automatically adjusted in light of economic developments, the information about policy and the types of announcements the monetary authority might make, the choice of variables for which the bank establishes targets (e.g., for money supply growth or the inflation rate), and whether these targets are formal or informal.

The objective in examining monetary policy operating procedures is to understand which instruments are under the control of the monetary authority, the factors that determine the optimal instrument choice, and how the choice of instrument affects the manner in which short-term interest rates, reserve aggregates, or the money stock might reflect policy actions and nonpolicy disturbances. After discussing the role of instruments and goals, the chapter examines the factors that determine the optimal choice of an operating procedure and the response of the market for bank reserves to various economic disturbances. Then,

a model of a channel system for setting interest rates is presented. In a channel system, the central bank pays interest on bank reserves, and this provides the policymaker with a new instrument. In contrast to a traditional model of bank reserves and interest rates as alternative instruments of monetary policy, the payment of interest on reserves allows the central bank to separate its decisions over the level of reserves from its decisions over the level of interest rates. The chapter concludes with a history of the Fed's operating procedures and a brief discussion of operating procedures in other countries.

## 12.2 From Instruments to Goals

Discussions of monetary policy implementation focus on *instruments*, *operating targets*, *intermediate targets*, and *policy goals*. Instruments are the variables that are directly controlled by the central bank. These typically include an interest rate charged on reserves borrowed from the central bank, the reserve requirement ratios that determine the level of reserves banks must hold against their deposit liabilities, and the composition of the central bank's own balance sheet (e.g., its holdings of government securities). The instruments of policy are manipulated to achieve a prespecified value of an operating target, typically some measure of bank reserves (total reserves, borrowed reserves, or nonborrowed reserves—the difference between total and borrowed reserves), or a very short-term rate of interest, usually an overnight interbank rate (the federal funds rate in the case of the United States).

Goals such as inflation or deviations of unemployment from the natural rate are the ultimate variables of interest to policymakers; instruments are the actual variables under their direct control. Intermediate target variables fall between operating targets and goals in the sequence of links that run from policy instruments to real economic activity and inflation. Because observations on some or all of the goal variables are usually obtained less frequently than are data on interest rates, exchange rates, or monetary aggregates, the behavior of these variables can often provide the central bank with information about economic developments that will affect the goal variables. For example, faster than expected money growth may signal that real output is expanding more rapidly than was previously thought. The central bank might change its operating target (e.g., raise the interbank rate or contract reserves) to keep the money growth rate on a path believed to be consistent with achieving its policy goals. In this case, money growth serves as an intermediate target variable. Under inflation-targeting policies, the inflation forecast plays the role of an intermediate target (Svensson and Woodford 2005).

Instruments, operating targets, intermediate targets, and goals have been described in a sequence running from the instruments directly controlled by the central bank to goals, the ultimate objectives of policy. Actually, policy design operates in the reverse fashion: from the goals of policy, to the values of the intermediate targets consistent with the goals, to the values of the operating targets needed to achieve the intermediate targets, and finally

to the instrument settings that yield the desired values of the operating targets (Tinbergen 1956). In earlier chapters, inflation and the money supply were sometimes treated as policy instruments, ignoring the linkages from reserve markets to interest rates to banking sector behavior to aggregate demand. Similarly, it is often useful to ignore reserve market behavior and treat an operating target variable, such as the overnight interbank interest rate or a reserve aggregate, as the policy instrument. Since these two variables can be controlled closely over short time horizons, they are often also described as *policy instruments*.

### 12.3 The Instrument Choice Problem

If the monetary policy authority can choose between employing an interest rate or a monetary aggregate as its policy instrument, which should it choose? The classic analysis of this question is due to Poole (1970). He showed how the stochastic structure of the economy—the nature and relative importance of different types of disturbances—would determine the optimal choice of instrument.

#### 12.3.1 Poole's Analysis

Suppose the central bank must set policy before observing the current disturbances to the goods and money markets, and assume that information on interest rates, but not output, is immediately available. This informational assumption reflects a situation in which the central bank can observe market interest rates essentially continuously, but data on inflation and output might be available only monthly or quarterly. In such an environment, the central bank will be unable to determine from a movement in market interest rates the exact nature of any economic disturbances. To make a simple parallel with a model of supply and demand, observing a rise in price does not indicate whether there has been a positive shock to the demand curve or a negative shock to the supply curve. Only by observing both price and quantity can these two alternatives be distinguished, because a demand shift would be associated with a rise in both price and quantity, whereas a supply shift would be associated with a rise in price and a decline in quantity. At the macroeconomic level, an increase in the interest rate could be due to expanding aggregate demand (which might call for contractionary monetary policy to stabilize output) or an exogenous shift in money demand (which might call for letting the money supply expand). With imperfect information about economic developments, it is impossible to determine the source of shocks that have caused interest rates to move.

Poole asked, in this environment, whether the central bank should try to hold market interest rates constant or should hold a monetary quantity constant while allowing interest rates to move. And he assumed that the objective of policy was to stabilize real output, so he answered this question by comparing the variance of output implied by the two alternative policies.

Poole treated the price level as fixed; to highlight his basic results, the same is done here. Since the instrument choice problem primarily relates to the decision to hold either a market rate or a monetary quantity constant over a fairly short period of time (say, the time between policy board meetings), ignoring price level effects is not unreasonable as a starting point for the analysis. Poole's result can be derived in a simple model given in log terms by

$$y_t = -\alpha i_t + u_t, \quad (12.1)$$

$$m_t = y_t - ci_t + v_t. \quad (12.2)$$

Equation (12.1) represents an aggregate demand relationship in which output is a decreasing function of the interest rate; demand also depends on an exogenous disturbance  $u_t$  with variance  $\sigma_u^2$ . Equation (12.2) gives the demand for money as a decreasing function of the interest rate and an increasing function of output. Money demand is subject to a random shock  $v_t$  with variance  $\sigma_v^2$ . Equilibrium requires that the demand for money equal the supply of money  $m_t$ . For simplicity,  $u$  and  $v$  are treated as mean zero serially and mutually uncorrelated processes. These two equations represent a simple IS-LM model of output determination, given a fixed price level.<sup>1</sup>

The final aspect of the model is a specification of the policymaker's objective, assumed to be the minimization of the variance of output deviations:

$$E[y_t]^2, \quad (12.3)$$

where all variables have been normalized so that the economy's equilibrium level of output in the absence of shocks is  $y = 0$ .

The timing is as follows. The central bank sets either  $i_t$  or  $m_t$  at the start of the period, then the stochastic shocks  $u_t$  and  $v_t$  occur, determining the values of the endogenous variables (either  $y_t$  and  $i_t$  if  $m_t$  is the policy instrument or  $y_t$  and  $m_t$  if  $i_t$  is the policy instrument).

When the money stock is the policy instrument, (12.1) and (12.2) can be solved jointly for equilibrium output:

$$y_t = \frac{\alpha m_t + cu_t - \alpha v_t}{\alpha + c}.$$

Then, setting  $m_t$  such that  $E[y_t] = 0$ ,<sup>2</sup> one obtains  $y_t = (cu_t - \alpha v_t)/(\alpha + c)$ . Given that  $u$  and  $v$  are assumed to be uncorrelated, the value of the objective function under a money supply procedure is

$$E_m [y_t]^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}. \quad (12.4)$$

1. Note that the price level has been normalized to equal 1 so that the log of the price level is zero;  $p = 0$ . The income elasticity of money demand has also been set equal to 1.

2. This just requires  $m = 0$  because of the normalizations.

Under the alternative policy,  $i_t$  is the policy instrument, and (12.1) can be solved directly for output. That is, the money market condition is no longer needed, although it will determine the level of  $m_t$  necessary to ensure money market equilibrium. By fixing the rate of interest, the central bank lets the money stock adjust endogenously to equal the level of money demand given by the interest rate and the level of income. Setting  $i_t$  such that  $E[y_t] = 0$ , output equals  $u_t$  and

$$E_i [y_t]^2 = \sigma_u^2. \quad (12.5)$$

The two alternative policy choices can be evaluated by comparing the variance of output implied by each. The interest rate operating procedure is preferred to the money supply operating procedure if and only if

$$E_i [y_t]^2 < E_m [y_t]^2,$$

and, from (12.4) and (12.5), this condition is satisfied if and only if

$$\sigma_v^2 > \left(1 + \frac{2c}{\alpha}\right) \sigma_u^2. \quad (12.6)$$

Thus, an interest rate procedure is more likely to be preferred when the variance of money demand disturbances is larger, the LM curve is steeper (the slope of the LM curve is  $1/c$ ), and the IS curve is flatter (the slope of the IS curve is  $-1/\alpha$ ). A money supply procedure is preferred if the variance of aggregate demand shocks ( $\sigma_u^2$ ) is large, the LM curve is flat, or the IS curve is steep.<sup>3</sup>

If only aggregate demand shocks are present (i.e.,  $\sigma_v^2 = 0$ ), a money rule leads to a smaller variance for output. Under a money rule, a positive IS shock leads to an increase in the interest rate. This reduces aggregate spending, thereby partially offsetting the original shock. Since the adjustment of  $i$  automatically stabilizes output, preventing this interest rate adjustment by fixing  $i$  leads to larger output fluctuations. If only money demand shocks are present, (i.e.,  $\sigma_u^2 = 0$ ), output can be stabilized perfectly under an interest rate rule. Under a money rule, money demand shocks cause the interest rate to move to maintain money market equilibrium; these interest rate movements then lead to output fluctuations. With both types of shocks occurring, the comparison of the two policy rules depends on the relative variances of  $u$  and  $v$  as well as on the slopes of the IS and the LM curves, as shown by (12.6).

This framework is quite simple and ignores many important factors. To take just one example, no central bank has direct control over the money supply. Instead, control can be exercised over a narrow monetary aggregate such as the monetary base, and variations

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3. In the context of an open economy in which the IS relationship is  $y_t = -\alpha_1 i_t + \alpha_2 s_t + u_t$ , where  $s_t$  is the exchange rate, Poole's conclusions go through without modification if the central bank's choice is expressed not in terms of  $i_t$  but in terms of the monetary conditions index  $i_t - (\alpha_2/\alpha_1) s_t$ .

in this aggregate are then associated with variations in broader measures of the money supply. To see how the basic framework can be modified to distinguish between the base as a policy instrument and the money supply, suppose the two are linked by

$$m_t = b_t + hi_t + \omega_t, \quad (12.7)$$

where  $b$  is the (log) monetary base, and the money multiplier ( $m_t - b_t$  in log terms) is assumed to be an increasing function of the rate of interest (i.e.,  $h > 0$ ). In addition,  $\omega_t$  is a random money multiplier disturbance. Equation (12.7) could arise under a fractional reserve system in which excess reserves are a decreasing function of the rate of interest.<sup>4</sup> Under an interest rate procedure, (12.7) is irrelevant for output determination, so  $E_i(y_t)^2 = \sigma_u^2$ , as before. But now, under a monetary base operating procedure,

$$y_t = \frac{(c+h)u_t - \alpha v_t + \alpha \omega_t}{\alpha + c + h},$$

$$E_b(y_t)^2 = \left( \frac{1}{\alpha + c + h} \right)^2 \left[ (c+h)^2 \sigma_u^2 + \alpha^2 (\sigma_v^2 + \sigma_\omega^2) \right].$$

The interest rate procedure is preferred over the monetary base procedure if and only if

$$\sigma_v^2 + \sigma_\omega^2 > \left[ 1 + \frac{2(c+h)}{\alpha} \right] \sigma_u^2.$$

Because  $\omega$  shocks do not affect output under an interest rate procedure, the presence of money multiplier disturbances makes a base rule less attractive and makes it more likely that an interest rate procedure will lead to a smaller output variance. This simple extension reinforces the basic message of Poole's analysis; increased financial sector volatility (money demand or money multiplier shocks in the model used here) increases the desirability of an interest rate policy procedure over a monetary aggregate procedure. If money demand is viewed as highly unstable and difficult to predict over short time horizons, greater output stability can be achieved by stabilizing interest rates, letting monetary aggregates fluctuate. If, however, the main source of short-run instability arises from aggregate spending, a policy that stabilizes a monetary aggregate will lead to greater output stability.

This analysis is based on the realistic assumption that policy is unable to identify and respond directly to underlying disturbances. Instead, policy is implemented by fixing, at least over some short time interval, the value of an operating target or policy instrument. As additional information about the economy is obtained, the appropriate level at which to fix the policy instrument changes. So the critical issue is not so much which variable is used as a policy instrument but how that instrument should be adjusted in light of new but imperfect information about economic developments.

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4. See, for example, Modigliani, Rasche, and Cooper (1970) or McCallum and Hoehn (1983).



Poole's basic model ignores such factors as inflation, expectations, and aggregate supply disturbances. These factors and many others have been incorporated into models examining the choice between operating procedures based on an interest rate and those based on a monetary aggregate (e.g., Canzoneri, Henderson, and Rogoff 1983). B. Friedman (1990) contains a useful and comprehensive survey. In addition, as Friedman stressed, the appropriate definition of the policymaker's objective function is unlikely to be simply the variance of output once inflation is included in the model. The choice of instrument is an endogenous decision of the policymaker and therefore depends on the objectives of monetary policy.

This dependence is highlighted in the analysis of Collard and Dellas (2005). They employed a new Keynesian model of the type studied in chapter 8 in which households optimally choose consumption and firms maximize profit subject to a restriction on the frequency with which they can change prices, as in the model of Calvo (1983).<sup>5</sup> Two policy rules are considered. One is a fixed growth rate for the nominal quantity of money. The second is an interest rate rule that is close to a nominal interest rate peg. The rule does allow a long-run response to inflation that slightly exceeds 1 to ensure determinacy of the rational-expectations equilibrium (see section 8.3.3). Unlike Poole's original analysis, in which an ad hoc loss function was used to evaluate policies, Collard and Dellas ranked each rule according to its effect on the welfare of the representative agent. In a calibrated version of their model, they found that the relative ranking of the rules can differ from the ones obtained in Poole's analysis. For example, a fiscal policy shock raises nominal interest rates, so the interest rate rule must allow the money supply to expand to prevent the nominal rate from rising. This represented a procyclical policy in Poole's framework, and made the interest rate rule less desirable than the money rule. However, in the new Keynesian and other neoclassical frameworks, a rise in government spending reduces consumption, so the interest rate rule turns out to be countercyclical with respect to consumption. By stabilizing consumption (which enters the welfare function), the interest rate rule could actually dominate the money rule for some values of the calibrated parameters. In response to a positive money demand shock, a money rule causes consumption and output to fall. However, this induces a negative correlation between consumption and leisure that can actually stabilize utility. Thus, depending on parameter values, a money rule may outperform an interest rate rule in the face of money demand shocks. While Collard and Dellas employed an interest rate rule that is close to a peg, Ireland (2000) evaluated a money rule and an interest rate rule estimated from post-1980 Federal Reserve behavior. He found the estimated policy rule dominates a fixed money growth rule. The general lesson to be drawn is that the objectives used to evaluate alternative policy rules and the parameter values used to calibrate the model can be critical to the results.

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5. Collard and Dellas included capital in their model and allowed firms to index prices to nominal growth.

### 12.3.2 Policy Rules and Information

The alternative policies considered in the previous section can be viewed as special cases of the following policy rule:<sup>6</sup>

$$b_t = \mu i_t. \quad (12.8)$$

According to (12.8), the monetary authority adjusts the base, its actual instrument, in response to interest rate movements. The parameter  $\mu$ , both its sign and its magnitude, determines how the base is varied by the central bank as interest rates vary. If  $\mu = 0$ , then  $b_t = 0$ , and one has the case of a monetary base operating procedure in which  $b$  is fixed (at zero by normalization) and is not adjusted in response to interest rate movements. If  $\mu = -h$ , then (12.7) implies that  $m_t = \omega_t$ , and one has the case of a money supply operating procedure in which the base is automatically adjusted to keep  $m_t$  equal to zero on average; the actual value of  $m_t$  varies as a result of the control error  $\omega_t$ . In this case,  $b_t$  is the policy instrument and  $m_t$  is the operating target. Equation (12.8) is called a *policy rule* or an *instrument rule* in that it provides a description of how the policy instrument is set.

By combining (12.7) and (12.8) with (12.1) and (12.2),

$$i_t = \frac{v_t - \omega_t + u_t}{\alpha + c + \mu + h}, \quad (12.9)$$

so that large values of  $\mu$  reduce the variance of the interest rate. As  $\mu \rightarrow \infty$ , an interest rate operating procedure is approximated in which  $i_t$  is set equal to a fixed value (zero due to normalization). By representing policy in terms of the policy rule and then characterizing policy in terms of the choice of a value for  $\mu$ , one can consider intermediate cases to the extreme alternatives considered in section 12.3.1.

Substituting (12.9) into (12.1), output is given by

$$y_t = \frac{(c + \mu + h)u_t - \alpha(v_t - \omega_t)}{\alpha + c + \mu + h}.$$

From this expression, the variance of output can be calculated:

$$\sigma_y^2 = \frac{(c + \mu + h)^2 \sigma_u^2 + \alpha^2 (\sigma_v^2 + \sigma_\omega^2)}{(\alpha + c + \mu + h)^2}.$$

Minimizing with respect to  $\mu$ , the optimal policy rule (in the sense of minimizing the variance of output) is given by

$$\mu^* = - \left[ c + h - \frac{\alpha (\sigma_v^2 + \sigma_\omega^2)}{\sigma_u^2} \right]. \quad (12.10)$$

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6. Recall that constants are normalized to be zero in equations such as (12.8). More generally, one might have a rule of the form  $b_t = b_0 + \mu(i_t - E_t i_t)$ , where  $b_0$  is a constant and  $E_t i_t$  is the expected value of  $i_t$ . Issues of price level indeterminacy can arise if the average value of  $b_t$  is not tied down (as it is in this case by  $b_0$ ); see chapter 10.

In general, neither the interest rate ( $\mu \rightarrow \infty$ ) nor the base ( $\mu = 0$ ) nor the money supply ( $\mu = -h$ ) operating procedures will be optimal. Instead, Poole (1970) demonstrated that the way policy (in the form of the setting for  $b_t$ ) should respond to interest rate movements will depend on the relative variances of the three underlying economic disturbances.

To understand the role these variances play, suppose first that  $v \equiv \omega \equiv 0$ , so that  $\sigma_v^2 = \sigma_\omega^2 = 0$ ; there are no shifts in either money demand or money supply, given the base. In this environment, the basic Poole analysis concludes that a base rule dominates an interest rate rule. Equation (12.10) shows that the central bank should reduce  $b_t$  when the interest rate rises (i.e.,  $b_t = -(c + h)i_t$ ). With interest rate movements signaling aggregate demand shifts (since  $u_t$  is the only source of disturbance), a rise in the interest rate indicates that  $u_t > 0$ . A policy designed to stabilize output should reduce  $m_t$ ; this decline in  $m_t$  can be achieved by reducing the base. Rather than “leaning against the wind” to offset the interest rate rise, the central bank should engage in a contractionary policy that pushes  $i_t$  up even further.

When  $\sigma_v^2$  and  $\sigma_\omega^2$  are positive, interest rate increases may now be the result of an increase in money demand or a decrease in money supply. Since the appropriate response to a positive money demand shock or a negative money supply shock is to increase the monetary base and offset the interest rate rise (i.e., it is appropriate to lean against the wind),  $\mu^* > -(c + h)$ ; it will become optimal to actually increase the base as  $\sigma_v^2 + \sigma_\omega^2$  becomes sufficiently large.

The value for the policy rule parameter in (12.10) can also be interpreted in terms of a signal extraction problem faced by the policy authority. Recall that the basic assumption in the Poole analysis was that the policymaker could observe and react to the interest rate, but perhaps because of information lags, the current values of output and the underlying disturbances could not be observed. Suppose instead that the shocks  $u$ ,  $v$ , and  $e$  are observed, and the central bank can respond to them. That is, suppose the policy rule could take the form  $b_t = \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t$  for some parameters  $\mu_u$ ,  $\mu_v$ , and  $\mu_\omega$ . If this policy rule is substituted into (12.1) and (12.2), one obtains

$$y_t = \frac{(c + h + \alpha\mu_u)u_t - \alpha(1 - \mu_v)v_t + \alpha(1 + \mu_\omega)\omega_t}{\alpha + c + h}.$$

In this case, which corresponds to a situation of perfect information about the basic shocks, it is clear that the variance of output can be minimized if  $\mu_u = -(c + h)/\alpha$ ,  $\mu_v = 1$  and  $\mu_\omega = -1$ .

If the policymaker cannot observe the underlying shocks, then policy needs to be set on the basis of forecasts of these disturbances. Given the linear structure of the model and the quadratic form of the objective, the optimal policy can be written  $b_t = \mu_u \hat{u}_t + \mu_v \hat{v}_t + \mu_\omega \hat{\omega}_t = -[(c + h)/\alpha] \hat{u}_t + \hat{v}_t - \hat{\omega}_t$ , where  $\hat{u}_t$ ,  $\hat{v}_t$ , and  $\hat{\omega}_t$  are the forecasts of the shocks.<sup>7</sup>

7. The linear-quadratic structure of the policy problem implies certainty equivalence holds. Under certainty equivalence, optimal policy depends only on the expected values of the disturbances.

In the Poole framework, the central bank observes the interest rate and can set policy conditional on  $i_t$ . Thus, the forecasts of shocks will depend on  $i_t$  and will take the form  $\hat{u}_t = \delta_u i_t$ ,  $\hat{v}_t = \delta_v i_t$ , and  $\hat{\omega}_t = \delta_\omega i_t$ . The policy rule can then be written as

$$b_t = - \left( \frac{c+h}{\alpha} \right) \hat{u}_t + \hat{v}_t - \hat{\omega}_t = \left( -\frac{c+h}{\alpha} \delta_u + \delta_v - \delta_\omega \right) i_t. \quad (12.11)$$

Using this policy rule to solve for the equilibrium interest rate, determining the  $\delta_i$  from the assumption that forecasts are equal to the projections of the shocks on  $i_t$ , it is straightforward to verify that the coefficient on  $i_t$  in the policy rule (12.11) is equal to the value  $\mu^*$  given in (12.10).<sup>8</sup> Thus, the optimal policy response to observed interest rate movements represents an optimal response to the central bank's forecasts of the underlying economic disturbances.

### 12.3.3 Intermediate Targets

The previous section showed how the optimal response coefficients in the policy rule could be related to the central bank's forecast of the underlying disturbances. This interpretation of the policy rule parameter is important, since it captures a very general way of thinking about policy. When the central bank faces imperfect information about the shocks to the economy, it should respond based on its best forecasts of these shocks.<sup>9</sup> In the example, the only information variable available was the interest rate, so forecasts of the underlying shocks were based on  $i$ . In more general settings, information on other variables may be available on a frequent basis, and this should also be used in forecasting the sources of economic disturbances. Examples of such information variables include, besides market interest rates, exchange rates, commodity prices, and asset prices.<sup>10</sup>

Because the central bank must respond to partial and incomplete information about the true state of the economy, monetary policy is often formulated in practice in terms of intermediate targets. Intermediate targets are variables whose behavior provides information useful in forecasting the goal variables.<sup>11</sup> Deviations in the intermediate targets from their expected paths indicate a likely deviation of a goal variable from its target and signal the need for a policy adjustment. For example, if money growth, which is observed weekly,

8. See problem 4 at the end of this chapter.

9. Brainard (1967) showed that this statement is no longer true when there is uncertainty about the model parameters in addition to the additive uncertainty considered here. Parameter uncertainty makes it optimal to adjust less than completely. See section 8.4.7.

10. As discussed in chapter 1, commodity prices eliminate the price puzzle in VAR estimates of monetary policy effects because of the informational role they appear to play.

11. See Kareken, Muench, and Wallace (1973) and B. Friedman (1975; 1977b; 1990) for early treatments of the informational role of intermediate targets. More recently, Svensson (1997a; 1999a) stressed the role of inflation forecasts as an intermediate target. Bernanke and Woodford (1997) showed, however, how multiple equilibria may arise if policy is based on private sector forecasts, which are in turn based on expectations of future policy.

is closely related to subsequent inflation, which is observed only monthly, then faster than expected money growth signals the need to tighten policy. When action is taken to keep the intermediate target variable equal to its target, the hope is that policy will be adjusted automatically to keep the goal variables close to their targets as well.<sup>12</sup>

To see the role of intermediate targets in a very simple framework, consider the following aggregate supply, aggregate demand, and money demand system, expressed in terms of the rate of inflation:

$$y_t = a(\pi_t - E_{t-1}\pi_t) + z_t, \quad (12.12)$$

$$y_t = -\alpha(i_t - E_t\pi_{t+1}) + u_t, \quad (12.13)$$

$$m_t - p_t = m_t - \pi_t - p_{t-1} = y_t - ci_t + v_t. \quad (12.14)$$

Equation (12.12) is a standard Lucas supply curve; (12.13) gives aggregate demand as a decreasing function of the expected real interest rate; and (12.14) is a simple money demand relationship. Assume that each of the three disturbances  $z$ ,  $u$ , and  $v$  follows a first-order autoregressive process:

$$z_t = \rho_z z_{t-1} + e_t,$$

$$u_t = \rho_u u_{t-1} + \varphi_t,$$

$$v_t = \rho_v v_{t-1} + \psi_t,$$

where  $-1 < \rho_i < 1$  for  $i = z, u, v$ . The innovations  $e$ ,  $\varphi$ , and  $\psi$  are assumed to be mean zero and serially and mutually uncorrelated processes. The interest rate  $i$  is taken to be the policy instrument.

Suppose that the monetary authority's objective is to minimize the expected squared deviations of the inflation rate around a target level  $\pi^*$ . Hence,  $i_t$  is chosen to minimize<sup>13</sup>

$$V = \frac{1}{2}E(\pi_t - \pi^*)^2. \quad (12.15)$$

To complete the model, one must specify the information structure. Suppose that  $i_t$  must be set before observing  $e_t$ ,  $\varphi_t$ , or  $\psi_t$  but that  $y_{t-1}$ ,  $\pi_{t-1}$ , and  $m_{t-1}$  (and therefore  $p_{t-1}$ ,  $z_{t-1}$ ,  $u_{t-1}$ , and  $v_{t-1}$ ) are known when  $i_t$  is set. The optimal setting for the policy instrument can be found by solving for the equilibrium price level in terms of the policy instrument and then evaluating the loss function given by (12.15).

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12. B. Friedman (1990) and McCallum (1990b) provided discussions of the intermediate target problem.

13. Note that for this example the loss function in output deviations is replaced with one involving only inflation stabilization objectives. As is clear from (12.12), stabilizing inflation to minimize unexpected movements in  $\pi$  is consistent with minimizing output variability if there are no supply disturbances ( $z \equiv 0$ ). If the loss function depends on output and inflation variability and there are supply shocks, the optimal policy will depend on the relative weight placed on these two objectives.

Solving the model is simplified by recognizing that  $i_t$  will always be set to ensure that the expected value of inflation equals the target value  $\pi^*$ .<sup>14</sup> Actual inflation will differ from  $\pi^*$  because policy cannot respond to offset the effects of the shocks to aggregate supply, aggregate demand, or money demand, but policy will offset any expected effects of lagged disturbances to ensure that  $E_{t-1}\pi_t = E_t\pi_{t+1} = \pi^*$ . Using this result, (12.12) can be used to eliminate  $y_t$  from (12.13) to yield

$$\pi_t = \frac{(a + \alpha)\pi^* - \alpha i_t + u_t - z_t}{a}. \quad (12.16)$$

Equation (12.16) shows that under an interest rate policy,  $\pi_t$  is independent of  $v_t$  and the parameters of the money demand function. If the policymaker had full information on  $u_t$  and  $z_t$ , the optimal policy would be to set the interest rate equal to  $i_t^* = \pi^* + (1/\alpha)(u_t - z_t)$ , since this would yield  $\pi_t = \pi^*$ . If policy must be set prior to observing the realization of the shocks at time  $t$ , the optimal policy can be obtained by taking expectations of (12.16), conditional on time  $t - 1$  information, yielding the optimal setting for  $i_t$ :

$$i_t^* = \pi^* + \left(\frac{1}{\alpha}\right)(\rho_u u_{t-1} - \rho_z z_{t-1}). \quad (12.17)$$

Substituting (12.17) into (12.16) shows that the actual inflation rate under this policy is equal to<sup>15</sup>

$$\pi_t(i_t^*) = \pi^* + \frac{\varphi_t - e_t}{a}, \quad (12.18)$$

and the value of the loss function is equal to

$$V(i_t^*) = \frac{1}{2} \left(\frac{1}{a}\right)^2 (\sigma_\varphi^2 + \sigma_e^2),$$

where  $\sigma_x^2$  denotes the variance of a random variable  $x$ .

An alternative approach to setting policy in this example would be to derive the money supply consistent with achieving the target inflation rate  $\pi^*$  and then set the interest rate to achieve this level of  $m_t$ . Using (12.14) to eliminate  $i_t$  from (12.13),

$$y_t = \left(\frac{\alpha}{\alpha + c}\right)(m_t - \pi_t - p_{t-1} - v_t) + \left(\frac{c}{\alpha + c}\right)(u_t + \alpha\pi^*).$$

14. The first-order condition for the optimal choice of  $i_t$  is

$$\frac{\partial V}{\partial i_t} = E(\pi_t - \pi^*) \frac{\partial \pi_t}{\partial i_t} = 0,$$

implying  $E\pi_t = \pi^*$ .

15. Note that under this policy,  $E_{t-1}\pi_t = \pi^*$ , as assumed.

Using the aggregate supply relationship (12.12), the equilibrium inflation rate is

$$\begin{aligned}\pi_t &= \pi^* + \frac{1}{a} \left[ \left( \frac{\alpha}{\alpha + c} \right) (m_t - \pi_t - p_{t-1} - v_t) + \left( \frac{c}{\alpha + c} \right) (u_t + \alpha \pi^*) - z_t \right] \\ &= \frac{[a(\alpha + c) + \alpha c] \pi^* + \alpha (m_t - p_{t-1} - v_t) + cu_t - (\alpha + c)z_t}{a(\alpha + c) + \alpha}.\end{aligned}$$

The value of  $m_t$  consistent with  $\pi_t = \pi^*$  is therefore

$$m_t^* = (1 - c)\pi^* + p_{t-1} - \left( \frac{c}{\alpha} \right) u_t + \left( 1 + \frac{c}{\alpha} \right) z_t + v_t.$$

If the money supply must be set before observing the time  $t$  shocks, the optimal target for  $m$  is

$$\hat{m}_t = (1 - c)\pi^* + p_{t-1} - \left( \frac{c}{\alpha} \right) \rho_u u_{t-1} + \left( 1 + \frac{c}{\alpha} \right) \rho_z z_{t-1} + \rho_v v_{t-1}. \quad (12.19)$$

As can be easily verified, the interest rate consistent with achieving the targeted money supply  $\hat{m}_t$  is just  $\hat{i}_t$ , given by (12.17). Thus, an equivalent procedure for deriving the policy that minimizes the loss function is first to calculate the value of the money supply consistent with the target for  $\pi$  and then set  $i$  equal to the value that achieves the targeted money supply.

Now suppose the policymaker can observe  $m_t$  and respond to it. Under the policy that sets  $i_t$  equal to  $\hat{i}_t$ , (12.14) implies that the actual money supply will equal  $m_t = \pi_t(\hat{i}) + p_{t-1} + y_t(\hat{i}) - c\hat{i}_t + v_t$ , which can be written<sup>16</sup> as

$$m_t(i_t^*) = \hat{m}_t - \left( \frac{1}{a} \right) e_t + \left( 1 + \frac{1}{a} \right) \varphi_t + \psi_t. \quad (12.20)$$

Observing how  $m_t$  deviates from  $\hat{m}_t$  reveals information about the shocks, and this information can be used to adjust the interest rate to keep inflation closer to target. For example, suppose aggregate demand shocks ( $\varphi$ ) are the only source of uncertainty (i.e.,  $e \equiv \psi \equiv 0$ ). A positive aggregate demand shock ( $\varphi > 0$ ) will, for a given nominal interest rate, increase output and inflation, both of which contribute to an increase in nominal money demand. Under a policy of keeping  $i$  fixed, the policymaker automatically allows reserves to increase, letting  $m$  rise in response to the increased demand for money. Thus, an increase in  $m_t$  above  $\hat{m}_t$  would signal that the nominal interest rate should be increased

16. Substitute the solution (12.18) into the aggregate supply function to yield  $y(i_t^*) = \varphi_t - e_t + z_t = \varphi_t + \rho_z z_{t-1}$ . Using this result in (12.17) and (12.18) in (12.14),

$$\begin{aligned}m_t(i_t^*) &= \pi^* + \frac{\varphi_t - e_t}{a} + p_{t-1} + \varphi_t + \rho_z z_{t-1} - c\hat{i}_t + v_t \\ &= \pi^* + \left( \frac{\varphi_t - e_t}{a} \right) + p_{t-1} + \varphi_t + \rho_z z_{t-1} - c \left[ \pi^* + \frac{\rho_u u_{t-1} - \rho_z z_{t-1}}{\alpha} \right] + v_t.\end{aligned}$$

Collecting terms and using (12.18) yields (12.20).

to offset the demand shock. Responding to the money supply to keep  $m_t$  equal to the targeted value  $\hat{m}_t$  would achieve the ultimate goal of keeping the inflation rate equal to  $\pi^*$ . This is an example of an intermediate targeting policy; the nominal money supply serves as an intermediate target, and by adjusting policy to achieve the intermediate target, policy is also better able to achieve the target for the goal variable  $\pi_t$ .

Problems arise, however, when there are several potential sources of economic disturbances. Then it can be the case that the impact on the goal variable of a disturbance would be exacerbated by attempts to keep the intermediate target variable on target. For example, a positive realization of the money demand shock  $\psi_t$  does not require a change in  $i_t$  to maintain inflation on target.<sup>17</sup> But (12.20) shows that a positive money demand shock causes  $m_t$  to rise above the target value  $m_t(\hat{i})$ . Under a policy of adjusting  $i$  to keep  $m$  close to its target, the nominal interest rate would be raised, causing  $\pi$  to deviate from  $\pi^*$ . Responding to keep  $m$  on target will not produce the appropriate policy for keeping  $\pi$  on target.

Automatically adjusting the nominal interest rate to ensure that  $m_t$  always equals its target  $\hat{m}_t$  requires that the nominal interest rate equal<sup>18</sup>

$$i_t^T = \hat{i}_t + \frac{(1+a)\varphi_t - e_t + a\psi_t}{ac + \alpha(1+a)}. \quad (12.21)$$

In this case, inflation is equal to

$$\begin{aligned} \pi_t(i_t^T) &= \pi^* + \left(\frac{1}{a}\right) [-\alpha(i_t^T - \hat{i}) + \varphi_t - e_t] \\ &= \pi^* + \frac{c\varphi_t - (\alpha + c)e_t - \alpha\psi_t}{ac + \alpha(1+a)}. \end{aligned}$$

Comparing this expression for inflation to  $\pi_t(\hat{i}_t)$  from (12.18), the value obtained when information on the money supply is not used, one can see that the impact of an aggregate demand shock,  $\varphi$ , on the price level is reduced ( $c/[ac + \alpha(1+a)] < 1/a$ ); because a positive  $\varphi$  shock tends to raise money demand, the interest rate must be increased to offset the effects on the money supply to keep  $m$  on target. This interest rate increase partially offsets the impact of a demand shock on inflation. The impact of an aggregate supply shock ( $e$ ) under an intermediate money-targeting policy is also decreased. However, money demand shocks,  $\psi$ , now affect inflation, which they did not do under a policy of keeping  $i$  equal to  $\hat{i}$ ; a positive  $\psi$  tends to increase  $m$  above target. If  $i$  is increased to offset this shock, inflation will fall below target.

17. Equation (12.18) shows that inflation is independent of  $v_t$ .

18. Note that this discussion does not assume that the realizations of the individual disturbances can be observed by the policymaker; as long as  $m_t$  is observed,  $i_t$  can be adjusted to ensure that  $m_t = \hat{m}_t$ , and this results in  $i$  being given by (12.21). Equation (12.21) is obtained by solving (12.12)–(12.14) for  $m_t$  as a function of  $i_t$  and the various disturbances. Setting this expression equal to  $\hat{m}_t$  yields the required value of  $i_t^T$ .



The value of the loss function under the money-targeting procedure is

$$V(i_t^T) = \frac{1}{2} \left( \frac{1}{ac + \alpha(1+a)} \right)^2 \left[ c^2 \sigma_\varphi^2 + (\alpha + c)^2 \sigma_e^2 + \alpha^2 \sigma_\psi^2 \right].$$

Comparing this to  $V(\hat{i})$ , the improvement from employing an intermediate targeting procedure in which the policy instrument is adjusted to keep the money supply on target will be decreasing in the variance of money demand shocks,  $\sigma_\psi^2$ . As long as this variance is not too large, the intermediate targeting procedure will do better than a policy of simply keeping the nominal rate equal to  $\hat{i}$ . If this variance is too large, the intermediate targeting procedure will do worse.

An intermediate targeting procedure represents a rule for adjusting the policy instrument to a specific linear combination of the new information contained in movements of the intermediate target. Using (12.20) and (12.21), the policy adjustment can be written as

$$\begin{aligned} i_t^T - \hat{i} &= \left[ \frac{a}{ac + \alpha(1+a)} \right] (m_t(\hat{i}) - \hat{m}) \\ &= \mu^T [m_t(\hat{i}) - \hat{m}]. \end{aligned}$$

In other words, if the money supply realized under the initial policy setting,  $m_t(\hat{i})$ , deviates from its expected level,  $\hat{m}$ , the policy instrument is adjusted. Since the money supply will deviate from target because of  $\varphi$  and  $e$  shocks, which do call for a policy adjustment, as well as  $\psi$  shocks, which do not call for any change in policy, an *optimal* adjustment to the new information in money supply movements would depend on the relative likelihood that movements in  $m$  are caused by the various possible shocks. An intermediate targeting rule, by adjusting to deviations of money from target in a manner that does not take into account whether fluctuations in  $m$  are more likely to be due to  $\varphi$  or  $e$  or  $\psi$  shocks, represents an inefficient use of the information in  $m$ .

To derive the optimal policy response to fluctuations in the nominal money supply, let

$$\begin{aligned} i_t - \hat{i} &= \mu (m_t - \hat{m}) \\ &= \mu x_t, \end{aligned} \tag{12.22}$$

where  $x_t = (1 + a^{-1})\varphi_t - a^{-1}e_t + \psi_t$  is the new information obtained from observing  $m_t$ .<sup>19</sup> Under an intermediate targeting rule, the monetary authority would adjust its policy

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19. The expression for  $x_t$  is obtained by solving (12.12)–(12.14) for  $m_t$  as a function of the interest rate, yielding

$$m_t = \pi_t + p_{t-1} + y_t - ci_t + v_t = \left[ \pi^* + \frac{y_t - z_t}{a} \right] + p_{t-1} + y_t - ci_t + v_t, \quad \text{or}$$

$$\begin{aligned} m_t &= \left[ \pi^* + \frac{-\alpha i_t + \alpha \pi^* + u_t - z_t}{a} \right] + p_{t-1} + [-\alpha i_t + \alpha \pi^* + u_t] - ci_t + v_t \\ &= (1 + \alpha(1 + a^{-1}))\pi^* + p_{t-1} - (c + \alpha(1 + a^{-1}))i_t - a^{-1}z_t + (1 + a^{-1})u_t + v_t, \end{aligned}$$

so that, conditional on  $i_t$ ,

$$m_t - E_{t-1}m_t = -a^{-1}e_t + (1 + a^{-1})\varphi_t + \psi_t \equiv x_t.$$

instrument to minimize deviations of the intermediate target from the value consistent with achieving the ultimate policy target, in this case an inflation rate of  $\pi^*$ . But under a policy that optimally uses the information in the intermediate target variable,  $\mu$  will be chosen to minimize  $E(\pi_t - \pi^*)^2$ , not  $E(m_t - \hat{m})^2$ . Using (12.22) in (12.16), one finds that the value of  $\mu$  that minimizes the loss function is

$$\mu^* = \frac{1}{\alpha} \left[ \frac{a(1+a)\sigma_\varphi^2 + a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2} \right].$$

This is a messy expression, but some intuition for it can be gained by recognizing that if the policymaker could observe the underlying shocks, (12.16) implies the optimal policy would set the nominal interest rate  $i$  equal to  $\hat{i} + (1/\alpha)(\varphi_t - e_t)$ . The policymaker cannot observe  $\varphi_t$  or  $e_t$ , but information that can be used to estimate them is available from observing the deviation of money from its target. As shown, observing  $m_t$  provides information on the linear combination of the underlying shocks given by  $x_t$ . Letting  $E^x[\ ]$  denote expectations conditional on  $x$ , the policy instrument should be adjusted according to

$$i(x_t) = \hat{i} + \frac{1}{\alpha} (E^x \varphi_t - E^x e_t). \quad (12.23)$$

Evaluating these expectations gives

$$E^x \varphi_t = \left[ \frac{a(1+a)\sigma_\varphi^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2} \right] x_t,$$

$$E^x e_t = \left[ \frac{-a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2} \right] x_t.$$

Substituting these expressions into (12.23) yields

$$i(x_t) = \hat{i} + \left( \frac{1}{\alpha} \right) \left[ \frac{a(1+a)\sigma_\varphi^2 + a\sigma_e^2}{(1+a)^2\sigma_\varphi^2 + \sigma_e^2 + a^2\sigma_\psi^2} \right] x_t$$

$$= \hat{i} + \mu^* x_t.$$

Under this policy, the information in the intermediate target is used optimally. As a result, the loss function is reduced relative to a policy that adjusts  $i$  to keep the money supply always equal to its target:

$$V^* \leq V(i^T),$$

where  $V^*$  is the loss function under the policy that adjusts  $i$  according to  $\mu^* x_t$ .

As long as money demand shocks are not too large, an intermediate targeting procedure does better than following a policy rule that fails to respond at all to new information. The intermediate targeting rule does worse, however, than a rule that optimally responds to the

new information. This point was first made by Kareken, Muench, and Wallace (1973) and B. Friedman (1975).

Despite the general inefficiency of intermediate targeting procedures, central banks often implement policy as if they were following an intermediate targeting procedure. During the 1970s, there was strong support in the United States for using money growth as an intermediate target. Support faded in the 1980s, when money demand became significantly more difficult to predict.<sup>20</sup> The Bundesbank (prior to being superseded by the European Central Bank) and the Swiss National Bank continued to formulate policy in terms of money growth rates that can be interpreted as intermediate targets, and money formed one of the pillars in the two-pillar strategy of the European Central Bank.<sup>21</sup> Other central banks seem to use the nominal exchange rate as an intermediate target. Recently, many central banks have shifted to using inflation itself as an intermediate target.

Intermediate targets provide a simple framework for responding automatically to economic disturbances. The model of this section can be used to evaluate desirable properties that characterize good intermediate targets. The critical condition is that  $\sigma_\psi^2$  be small. Since  $\psi_t$  represents the innovation or shock to the money demand equation, intermediate monetary targeting works best if money demand is relatively predictable. Often this has not been the case. The unpredictability of money demand is an important reason that most central banks moved away from using monetary targeting during the 1980s. The shock  $\psi$  can also be interpreted as arising from control errors. For example, assuming that the monetary base was the policy instrument, unpredictable fluctuations in the link between the base and the monetary aggregate being targeted (corresponding to the  $\omega$  disturbance in (12.7)) would reduce the value of an intermediate targeting procedure. Controllability is therefore a desirable property of an intermediate target.

Lags in the relationship between the policy instrument, the intermediate target, and the final goal variable represent an additional important consideration. The presence of lags introduces no new fundamental issues; as the simple framework here shows, targeting an intermediate variable allows policy to respond to new information, either because the intermediate target variable is observed contemporaneously (as in the example) or because it helps to forecast future values of the goal variable. In either case, adjusting policy to achieve the intermediate target forces policy to respond to new information in a manner that is generally suboptimal. But this inefficiency will be smaller if the intermediate target is relatively easily controllable (i.e.,  $\sigma_\psi^2$  is small) yet is highly correlated with the variable of ultimate interest (i.e.,  $\sigma_\varphi^2$  and  $\sigma_e^2$  are large), so that a deviation of the intermediate variable from its target provides a clear signal that the goal variable has deviated from its target. For central banks that target inflation, the inflation forecast serves as an intermediate target.

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20. B. Friedman and Kuttner (1996) examined the behavior of the Fed during the era of monetary targeting.

21. Laubach and Posen (1997) argued that the targets were used to signal policy intentions rather than serving as strict intermediate targets. See Beck and Wieland (2007) on the role of money in the ECB's strategy.

An efficient forecast is based on all available information and should be highly correlated with the variable of ultimate interest (future inflation). Svensson and Woodford (2005) discussed the implementation of optimal policies through the use of inflation forecasts.

### 12.3.4 Real Effects of Operating Procedures

The traditional analysis of operating procedures focuses on volatility; the operating procedure adopted by the central bank affects the way disturbances influence the variability of output, prices, real interest rates, and monetary aggregates. The average values of these variables, however, are treated as independent of the choice of operating procedure. Canzoneri and Dellas (1998) showed that the choice of procedure can have a sizable effect on the average level of the real rate of interest by affecting the variability of aggregate consumption.

The standard Euler condition relates the current marginal utility of consumption to the expected real return and the future marginal utility of consumption:

$$u_c(c_t) = \beta R_{ft} E_t u_c(c_{t+1}),$$

where  $\beta$  is the discount factor,  $R_{ft}$  is the gross risk-free real rate of return, and  $u_c(c_t)$  is the marginal utility of consumption at time  $t$ . The right side of this expression can be written as

$$\beta R_{ft} E_t u_c(c_{t+1}) \approx \beta R_{ft} u_c(E_t c_{t+1}) + \frac{1}{2} \beta R_{ft} u_{ccc}(E_t c_{t+1}) \text{Var}_t(c_{t+1}),$$

where  $u_{ccc}$  is the third derivative of the utility function and  $\text{Var}_t(c_{t+1})$  is the conditional variance of  $c_{t+1}$ . If the variance of consumption differs under alternative monetary policy operating procedures, then either the marginal utility of consumption must adjust (i.e., consumption will change) or the risk-free real return must change. Because the expected real interest rate can be expressed as the sum of the risk-free rate and a risk premium, average real interest rates will be affected if the central bank's operating procedure affects  $R_{ft}$  or the risk premium.

Canzoneri and Dellas developed a general equilibrium model with nominal wage rigidity and simulated the model under alternative operating procedures (interest rate targeting, money targeting, and nominal income targeting). They found that real interest rates, on average, are highest under a nominal interest rate targeting procedure. To understand why, suppose the economy is subject to money demand shocks. Under a procedure that fixes the nominal money supply, such shocks induce a positive correlation between consumption (output) and inflation. This generates a negative risk premium (when consumption is lower than expected, the ex post real return is high because inflation is lower than expected). A nominal interest rate procedure accommodates money demand shocks and so results in a higher average risk premium. By calibrating their model and conducting simulations, Canzoneri and Dellas concluded that the choice of operating procedure can have a significant effect on average real interest rates.

## 12.4 Operating Procedures and Policy Measures

Understanding a central bank's operating procedures is important for two reasons. First, it is important in empirical work to distinguish between endogenous responses to developments in the economy and exogenous shifts in policy. Whether movements in a monetary aggregate or a short-term interest rate are predominantly endogenous responses to disturbances unrelated to policy shifts or are exogenous shifts in policy will depend on the nature of the procedures used to implement policy. Thus, some understanding of operating procedures is required for empirical investigations of the impact of monetary policy.

Second, operating procedures, by affecting the automatic adjustment of interest rates and monetary aggregates to economic disturbances, can have implications for the macroeconomic equilibrium. For example, operating procedures that lead the monetary authority to smooth interest rate movements can introduce a unit root into the price level,<sup>22</sup> and the analysis in chapter 8 illustrated how the response of interest rates to inflation was important in ensuring a unique stationary equilibrium.

Analyses of operating procedures are based on the market for bank reserves. In the United States, this is the federal funds market. While the focus in this section is on the United States and the behavior of the Federal Reserve, similar issues arise in the analysis of monetary policy in other countries, although institutional details can vary considerably. Discussions of operating procedures in major OECD countries can be found in Batten et al. (1990), Bernanke and Mishkin (1992), Morton and Wood (1993), Kasman (1993), Borio (1997), Bank for International Settlements (2007), and B. Friedman and Kuttner (2010).

### 12.4.1 Money Multipliers

Theoretical models of monetary economies often provide little guidance to how the quantity of money appearing in the theory should be related to empirical measures of the money supply. If  $m$  is viewed as the quantity of the means of payment used in the conduct of exchange, then cash, demand deposits, and other checkable deposits should be included in the empirical correspondence.<sup>23</sup> If  $m$  is viewed as a variable set by the policy authority, then an aggregate such as the monetary base, which represents the liabilities of the central bank and so can be directly controlled, would be more appropriate. The *monetary base* is equal to the sum of the reserve holdings of the banking sector and the currency held by

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22. See Goodfriend (1987) and Van Hoose (1989).

23. Whether these different components of money should simply be added together, as they are in monetary aggregates such as  $M1$  and  $M2$ , or should be weighted to reflect their differing degree of liquidity is a separate issue. Barnett (1980) argued for the use of Divisia indices of monetary aggregates. See also Spindt (1985).

the nonbank public.<sup>24</sup> These are liabilities of the central bank and can be affected by open-market operations. Most policy discussions, however, focus on broader monetary aggregates, but these are not the direct instruments of monetary policy. A traditional approach to understanding the linkages between a potential instrument such as the monetary base and the various measures of the money supply is to express broader measures of money as the product of the monetary base and a money multiplier. Changes in the money supply can then be decomposed into those resulting from changes in the base and those resulting from changes in the multiplier. The multiplier is developed using definitional relationships, combined with some simple behavioral assumptions.

Denoting total reserves by  $TR$  and currency by  $C$ , the monetary base  $MB$  is given by

$$MB = TR + C.$$

In the United States, currency represents close to 90 percent of the base. Aggregates such as the monetary base and total reserves are of interest because of their close connection to the actual instruments central banks can control and because of their relationship to broader measures of the money supply. A central bank can control the monetary base through open-market operations. By purchasing securities, the central bank can increase the supply of bank reserves and the base. Securities sales reduce the base.<sup>25</sup>

In the United States, the monetary aggregate  $M1$  is equal to currency in the hands of the public plus demand deposits and other checkable deposits. If the deposit component is denoted  $D$  and there is a reserve requirement ratio of  $rr$  against all such deposits, one can write

$$MB = RR + ER + C = (rr + ex + c)D,$$

where total reserves have been divided into required reserves ( $RR$ ) and excess reserves ( $ER$ ), and where  $ex = ER/D$  is the ratio of excess reserves to deposits that banks choose to hold and  $c = C/D$  is the currency-to-deposit ratio. Then,

$$M1 = D + C = (1 + c)D = \left( \frac{1 + c}{rr + ex + c} \right) MB. \quad (12.24)$$

Equation (12.24) is a very simple example of money multiplier analysis: a broad monetary aggregate such as  $M1$  is expressed as a multiplier, in this case  $(1 + c)/(rr + ex + c)$  times the monetary base. Changes in the monetary base translate into changes in broader measures of the money supply, given the ratios  $rr$ ,  $ex$ , and  $c$ . Of course, the ratios  $rr$ ,  $ex$ , and  $c$  need not remain constant as  $MB$  changes. The ratio  $ex$  is determined by bank decisions

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24. There are two commonly used data series on the U.S. monetary base, one produced by the Board of Governors of the Federal Reserve System and one by the Federal Reserve Bank of St. Louis. The two series treat vault cash and the adjustment for changes in reserve requirements differently.

25. In the United States, daily Fed interventions are chiefly designed to smooth temporary fluctuations and are conducted mainly through repurchase and sale-purchase agreements rather than outright purchases or sales.

and the Fed's policies on discount lending, and  $c$  is determined by the decisions of the public concerning the level of cash they wish to hold relative to deposits. The usefulness of this money multiplier framework was illustrated by M. Friedman and Schwartz (1963), who employed it to organize their study of the causes of changes in the money supply.

In terms of an analysis of the market for bank reserves and operating procedures, the most important of the ratios appearing in (12.24) is  $ex$ , the excess reserve ratio. Because reserves traditionally earned no interest, banks faced an opportunity cost in holding excess reserves.<sup>26</sup> As market interest rates rise, banks tend to hold a lower average level of excess reserves. This drop in  $ex$  works to increase  $M1$ . This implies that, holding the base constant, fluctuations in market interest rates induce movements in the money supply.

### 12.4.2 The Reserve Market

Traditional models of the reserve market generally have a very simple structure; reserve demand and reserve supply interact to determine the interbank interest rate.<sup>27</sup> In the United States, the federal funds rate is the interest rate banks in need of reserves pay to borrow reserves from banks with surplus reserves. The Federal Reserve can use open-market operations to affect the supply of reserves, and it is by intervening in the reserve market that the Fed attempts to affect the money supply, market interest rates, and ultimately economic activity and inflation.

Models of the demand for reserves model bank reserve holdings as arising from the need to meet reserve requirements and settlement payments in the interbank market. Banks balance the opportunity cost of holding reserves in excess of their needs against the cost of being forced to borrow in the face of a reserve shortfall. When payment flows are random, the optimal level of reserves will depend on the variability of shocks to the level of the bank's reserve holdings. The first model to incorporate these elements is due to Poole (1968). Examples include Furfine (2000), Furfine and Stehem (1998), and Heller and Lengwiler (2003), who allow banks to balance liquidity costs against the cost of liquidity management. Hamilton (1996) provided a model that emphasizes the microstructure of the reserve market, and Bartolini, Bertola, and Prati (2002) developed a model designed to capture the day-to-day operations of the reserve market when the central bank targets the funds rate. The way reserve market variables (various reserve aggregates and the funds rate) respond to disturbances depends on the operating procedure followed by the Fed. One objective of a model of the reserve market is to disentangle movements in reserves and the funds rate that are due to nonpolicy sources from those caused by policy actions.

Traditional models of the U.S. reserve market assumed that reserves earned zero interest and that the discount rate at which reserves could be borrowed from the Fed was below

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26. The Federal Reserve began paying interest on bank reserves in late 2008.

27. In the United States, the development of the modern reserve market dates from the mid-1960s. See Meulendyke (1998).

the funds rate. To prevent banks from exploiting the profit opportunity available when the rate paid on borrowed funds was below the rate to be obtained by lending funds, discount borrowing was rationed through nonprice mechanisms that limited a bank's ability to borrow. Such models have become less relevant as many central banks, including the Fed, pay interest on reserves and impose penalty rates on borrowing that allow central banks to eliminate nonprice mechanisms for limiting borrowing. For example, the Federal Reserve established a penalty borrowing rate in 2003<sup>28</sup> and began paying interest on reserve balances on October 6, 2008.<sup>29</sup> In addition, the use of sweep accounts had reduced the level of required reserves, and in some countries, banks are not required to hold reserves. These institutional changes led to interest in models of the reserve market that focus on banks' need to hold reserves because of the volatility of payment balances rather than from the need to meet a reserve requirement. Furfine (2000), for example, stated that banks active in the payment system might send and receive payments that total more than 30 times their reserve balance on a typical day. The expansion of the Federal Reserve's balance sheet since the 2008–2009 financial crisis has led to a huge increase in bank holdings of reserves. When combined with payment of interest on reserves, these changes have made traditional models of the reserve market obsolete. Thus, section 12.5 presents a simple model of interest rate determination that incorporates stochastic variability in bank payments, interest on reserves, a penalty rate on reserves borrowed from the central bank, and a large supply of reserves. The next section, however, begins with a more traditional model because such a model is helpful in understanding the implications of the various operating procedures discussed in section 12.6 and that the Fed has employed since 1960.

### A Traditional Model of the Reserve Market

The demand for reserves depends on the costs of reserves and on any factors that influence money demand—aggregate income, for example, as these factors affect the level of required reserves and the volume of payments banks must settle daily. In order to focus on the very short-run determination of reserve aggregates and the funds rate, factors such as aggregate income and prices are simply treated as part of the error term in the total reserve demand relationship, resulting in

$$TR^d = -ai^f + v^d, \quad (12.25)$$

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28. The Federal Reserve set the discount rate 100 basis points above the federal funds rate target beginning on January 6, 2003. This spread was maintained at this level until early 2007. (It was reduced to 75 basis points on June 30, 2005, but increased back to 100 basis points the following month.) From August 17, 2007, the spread was cut to 50 basis points and then reduced further to 25 basis points on March 18, 2008.

29. Authority for the Federal Reserve to pay interest was originally scheduled to come into effect in 2011, but accelerated authority was granted as part of the Emergency Economic Stabilization Act of 2008. This act's primary purpose was to establish the Troubled Asset Relief Program (TARP). An earlier major change in Fed operating procedures took place on October 6, 1979, when the Volcker Fed shifted to a reserve aggregates operating procedure that saw interest rates rise significantly as the Fed moved to bring inflation down. For a discussion of monetary policy implementation when interest is paid on reserves, see Goodfriend (2002).



where  $TR^d$  represents total reserve demand,  $i^f$  is the funds rate (the rate at which a bank can borrow reserves in the private market), and  $v^d$  is a demand disturbance. This disturbance reflects variations in income or other factors that produce fluctuations in deposit demand. One interpretation of (12.25) is that it represents a relationship between the innovations in total reserve demand and the funds rate after the lagged effects of all other factors have been removed. For example, Bernanke and Mihov (1998) attempted to identify policy shocks by focusing on the relationships among the innovations to reserve demand, reserve supply, and the funds rate obtained as the residuals from a VAR model of reserve market variables. They characterized alternative operating procedures in terms of the parameters linking these innovations.<sup>30</sup>

The total supply of reserves held by the banking system can be expressed as the sum of the reserves that banks have borrowed from the Federal Reserve System plus nonborrowed reserves:

$$TR_t^s = BR_t + NBR_t.$$

The Federal Reserve can control the stock of nonborrowed reserves through open-market operations: by buying or selling government securities, the Fed affects the stock of nonborrowed reserves. For example, a purchase of government debt by the Fed raises the stock of nonborrowed reserves when the Fed pays for its purchase by crediting the reserve account of the seller's bank with the amount of the purchase. Open-market sales of government debt by the Fed reduce the stock of nonborrowed reserves. So the Fed can, even over relatively short time horizons, exercise close control over the stock of nonborrowed reserves.

The stock of borrowed reserves depends on the behavior of private banks and on their decisions about borrowing from the Fed (borrowing from the discount window). Bank demand for borrowed reserves depends on the opportunity cost of borrowing from the Fed (the discount rate) and the cost of borrowing reserves in the federal funds market (the federal funds rate). An increase in the funds rate relative to the discount rate makes borrowing from the Fed more attractive and leads to an increase in bank borrowing. The elasticity of borrowing with respect to the spread between the funds rate and the discount rate depends on the Fed's management of the discount window. At one time, the Fed maintained the discount rate below the federal funds rate. This created an incentive for banks to borrow reserves at the discount rate and then lend these reserves at the higher market interest rates. To prevent banks from exploiting this arbitrage opportunity, the Fed used nonprice methods to ration bank borrowing. This nonprice rationing affected the degree to which banks turned to the discount window to borrow as the incentive to do so, the spread between the funds rate and the discount rate, widened. Banks had to weigh the benefits of borrowing

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30. Kasa and Popper (1997) employed a similar approach to study monetary policy in Japan. Leeper, Sims, and Zha (1996) developed a more general formulation of the links between reserve market variables in an identified VAR framework.

reserves in a particular week against the possible cost in terms of reduced future access to the discount window. Banks reduced their current borrowing if they expected the funds rate to be higher in the future because they preferred to preserve their future access to the discount window, timing their borrowing for periods when the funds rate was high.

This type of intertemporal substitution also occurred because required reserves in the United States were based on an average over a two-week maintenance period; except for the last day of the maintenance period, banks had an incentive to hold reserves on the days reserves were least costly. Therefore, borrowing decisions depended on the expected future funds rate as well as the current funds rate. This can be modeled by a simple demand function of the form

$$BR_t = b_1 (i_t^f - i_t^d) - b_2 E_t (i_{t+1}^f - i_{t+1}^d) + v_t^b, \quad (12.26)$$

where  $i^d$  is the discount rate (a policy variable) and  $v^b$  is a borrowing disturbance.

In 2003 the Fed changed the way it administered the discount window and set the discount rate above the federal funds rate. Banks that qualified for *primary credit* could borrow at a rate 1 percent above the funds rate; *secondary credit* was available at a rate 1.5 percent above the funds rate. By converting the discount rate into a penalty rate, the arbitrage opportunity created when the discount rate was below the funds rate was eliminated, as was the need for nonprice rationing. Because much of the empirical work on the U.S. reserve market was based on data from periods when the discount rate was kept below the funds rate, the model of this section assumes that  $i^f > i^d$ . The case of a penalty rate is discussed in section 12.5.

The simplest versions of a reserve market model often postulate a borrowing function of the form

$$BR_t = b(i_t^f - i_t^d) + v_t^b. \quad (12.27)$$

The manner in which an innovation in the funds rate affects borrowings, given by the coefficient  $b$  in (12.27), varies depending on how such a funds rate innovation affects expectations of future funds rate levels. Suppose, for example, that borrowings are actually given by (12.26) and that policy results in the funds rate following the process  $i_t^f = \rho i_{t-1}^f + \xi_t$ . Then  $E_t i_{t+1}^f = \rho i_t^f$  and from (12.26),  $BR_t = b i_t^f$ , where  $b = b_1 - \rho b_2$ .<sup>31</sup> A change in operating procedures that leads the funds rate to be more highly serially correlated (increases  $\rho$ ) will reduce the response of borrowings to the funds rate–discount rate spread.<sup>32</sup> While relationships such as (12.27) can cast light on the linkages that affect the correlations among

31. For simplicity, this ignores the discount rate  $i^d$  for the moment.

32. Goodfriend (1983) provided a formal model of borrowed reserves; see also Waller (1990). For a discussion of how alternative operating procedures affect the relationship between the funds rate and reserve aggregates, see Walsh (1982b; 1990). Attempts to estimate the borrowings function can be found in Peristiani (1991) and Pearce (1993).

reserve market variables for a given operating procedure, one should not expect the parameter values to remain constant across operating procedures.

To consider a variety of different operating procedures, assume the Fed responds contemporaneously to the various disturbances to the reserve market, so that nonborrowed reserves are given by

$$\text{NBR}_t = \phi^d v_t^d + \phi^b v_t^b + v_t^s, \quad (12.28)$$

where  $v_t^s$  is a *monetary policy shock*. Different operating procedures are characterized by alternative values of the parameters  $\phi^d$  and  $\phi^b$ .<sup>33</sup> Equilibrium in the reserve market requires that total reserve demand equal total reserve supply:

$$\text{TR}_t^d = \text{BR}_t + \text{NBR}_t. \quad (12.29)$$

If a month is the unit of observation, reserve market disturbances are likely to have no contemporaneous effect on real output or the aggregate price level.<sup>34</sup> Using this identifying restriction, Bernanke and Mihov (1998) obtained estimates of the innovations to TR, BR,  $i^f$ , and NBR from a VAR system that also includes GDP, the GDP deflator, and an index of commodity prices but in which the reserve market variables are ordered last.<sup>35</sup> Whether any of these VAR residuals can be interpreted directly as a measure of the policy shock  $v^s$  will depend on the particular operating procedure being used. For example, if  $\phi^d = \phi^b = 0$ , (12.28) implies that  $\text{NBR} = v^s$ ; this corresponds to a situation in which the Fed does not allow disturbances to total reserve demand or to borrowed reserves to affect nonborrowed reserves, so innovations to nonborrowed reserves can be interpreted directly as policy shocks. Under such an operating procedure, using nonborrowed reserve innovations (i.e., NBR) as the measure of monetary policy, as in Christiano and Eichenbaum (1992b), is correct. However, if either  $\phi^d$  or  $\phi^b$  differs from zero, NBR will reflect nonpolicy shocks as well as policy shocks.

Substituting (12.25), (12.27), and (12.28) into the equilibrium condition (12.29) and solving for the innovation in the funds rate yields

$$i_t^f = \left( \frac{b}{a+b} \right) i_t^d - \left( \frac{1}{a+b} \right) \left[ v_t^s + (1 + \phi^b) v_t^b - (1 - \phi^d) v_t^d \right]. \quad (12.30)$$

33. Note that  $\phi^d$  and  $\phi^b$  correspond to  $\phi$  in (1.9) of chapter 1, since they reflect the impact of nonpolicy-originating disturbances on the policy variable NBR.

34. Referring back to the discussion in section 1.3.4, this assumption corresponds to the use of the assumption that  $\theta = 0$  to identify VAR innovations.

35. The commodity price index is included to eliminate the price puzzle discussed in chapter 1. This creates a potential problem for Bernanke and Mihov's identification scheme, since forward-looking variables such as asset prices, interest rates, and commodity prices may respond immediately to policy shocks. See the discussion of this issue in Leeper, Sims, and Zha (1996), who distinguished between policy, banking sector, production, and information variables.

The reduced-form expressions for the innovations to borrowed and total reserves are then found to be

$$\text{BR}_t = -\left(\frac{ab}{a+b}\right) i_t^d - \left(\frac{1}{a+b}\right) [bv_t^s - (a - b\phi^b)v_t^b - b(1 - \phi^d)v_t^d], \quad (12.31)$$

$$\text{TR}_t = -\left(\frac{ab}{a+b}\right) i_t^d + \left(\frac{1}{a+b}\right) [av_t^s + a(1 + \phi^b)v_t^b + (b + a\phi^d)v_t^d]. \quad (12.32)$$

How does the Fed's operating procedure affect the interpretation of movements in nonborrowed reserves, borrowed reserves, and the federal funds rate as measures of monetary policy shocks? Under a federal funds rate operating procedure, the Fed offsets total reserve demand and borrowing demand disturbances so that they do not affect the funds rate. According to (12.30), this policy requires that  $\phi^b = -1$  and  $\phi^d = 1$ . In other words, a shock to borrowed reserves leads to an equal but opposite movement in nonborrowed reserves to keep the funds rate (and total reserves) unchanged (see 12.28), while a shock to total reserve demand leads to an equal change in reserve supply through the adjustment of nonborrowed reserves. The innovation in nonborrowed reserves is equal to  $v_t^s - v_t^b + v_t^d$  and so does not reflect solely exogenous policy shocks.

Under a nonborrowed reserve procedure,  $\phi^b = 0$  and  $\phi^d = 0$  as innovations to nonborrowed reserves reflect policy shocks. In this case, (12.30) becomes

$$i_t^f = \left(\frac{b}{a+b}\right) i_t^d - \left(\frac{1}{a+b}\right) (v_t^s + v_t^b - v_t^d), \quad (12.33)$$

so innovations in the funds rate reflect both policy changes and disturbances to reserve demand and the demand for borrowed reserves. In fact, if  $v_t^d$  arises from shocks to money demand that lead to increases in measured monetary aggregates, innovations to the funds rate can be positively correlated with innovations to broader monetary aggregates. Positive innovations in an aggregate such as *M1* would then appear to increase the funds rate, a phenomenon found in the VAR evidence reported in chapter 1.

From (12.31), a borrowed reserves policy corresponds to  $\phi^d = 1$  and  $\phi^b = a/b$ , because adjusting nonborrowed reserves in this manner insulates borrowed reserves from nonpolicy shocks. That is, nonborrowed reserves are fully adjusted to accommodate fluctuations in total reserve demand. Under a borrowed reserves procedure, innovations to the funds rate are, from (12.30),

$$i_t^f = \left(\frac{b}{a+b}\right) i_t^d - \left(\frac{1}{a+b}\right) \left[ v_t^s + \left(1 + \frac{a}{b}\right) v_t^b \right],$$

so the funds rate reflects both policy and borrowing disturbances.

Table 12.1 summarizes the values of  $\phi^d$  and  $\phi^b$  that correspond to different operating procedures.

**Table 12.1**  
Parameters under Alternative Operating Procedures

	Operating Procedure			
	Funds Rate	Nonborrowed	Borrowed	Total
$\phi^d$	1	0	1	$-\frac{b}{a}$
$\phi^b$	-1	0	$\frac{a}{b}$	-1

In general, the innovations in the observed variables can be written (ignoring discount rate innovations) as

$$\begin{bmatrix} i_t^f \\ BR_t \\ NBR_t \end{bmatrix} \equiv u_t = \begin{bmatrix} -\frac{1}{a+b} & -\frac{1+\phi^b}{a+b} & \frac{1-\phi^d}{a+b} \\ -\frac{b}{a+b} & \frac{a-b\phi^b}{a+b} & \frac{b(1-\phi^d)}{a+b} \\ 1 & \phi^b & \phi^d \end{bmatrix} \begin{bmatrix} v_t^s \\ v_t^b \\ v_t^d \end{bmatrix} \equiv Av_t. \tag{12.34}$$

By inverting the matrix  $A$ , one can solve for the underlying shocks, the vector  $v$ , in terms of the observed innovations  $u$ :  $v = A^{-1}u$ . This operation produces

$$\begin{bmatrix} v_t^s \\ v_t^b \\ v_t^d \end{bmatrix} = \begin{bmatrix} b\phi^b - a\phi^d - (\phi^d + \phi^b) & 1 - \phi^d \\ -b & 1 & 0 \\ a & 1 & 1 \end{bmatrix} \begin{bmatrix} i_t^f \\ BR_t \\ NBR_t \end{bmatrix}.$$

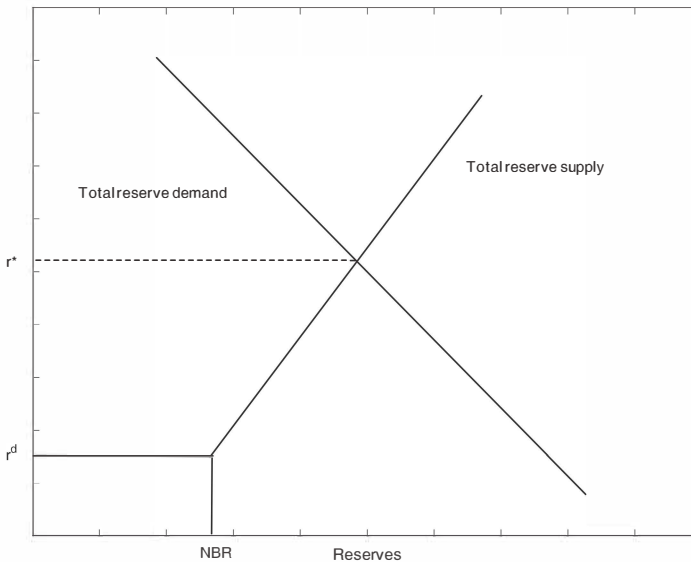
Hence,

$$v_t^s = (b\phi^b - a\phi^d)i_t^f - (\phi^d + \phi^b)BR_t + (1 - \phi^d)NBR_t, \tag{12.35}$$

so that the policy shock can be recovered as a specific linear combination of the innovations to the funds rate, borrowed reserves, and nonborrowed reserves. The policy shock cannot generally be identified with innovations in any one of the reserve market variables. Only for specific values of the parameters  $\phi^d$  and  $\phi^b$ , that is, for specific operating procedures, might the policy shock be recoverable from the innovation to just one of the reserve market variables.

**Reserve Market Responses**

This section uses the basic reserve market model to discuss how various disturbances affect reserve quantities and the funds rate under alternative operating procedures. Figure 12.1 illustrates reserve market equilibrium between total reserve demand and supply. For values of the funds rate less than the discount rate, reserve supply is vertical and equal to nonborrowed reserves. With the discount rate serving as a penalty rate, borrowed reserves fall to zero in this range, so that total reserve supply is just NBR. As the funds rate increases above the discount rate, borrowings become positive (see 12.27) and the total supply of reserves



**Figure 12.1**

Reserve market with no interest on reserves and a nonpenalty discount rate.

increases. Total reserve demand is decreasing in the funds rate according to (12.25). The funds rate is determined where supply equals demand. If the discount rate were a penalty rate, borrowing would be zero; total reserve supply would be represented by a vertical line at NBR.

Consider first a positive realization of the policy shock  $v^s$ . The effects on  $i^f$ , BR, and NBR can be found from the first column of the matrix  $A$  in (12.34). The policy shock increases nonborrowed reserves (one could think of it as initiating an open-market purchase that increases banking sector reserve assets). In figure 12.1, the reserve supply curve shifts to the right horizontally by the amount of the increase in NBR. Given the borrowed reserves and total reserve demand functions, this increase in reserve supply causes the funds rate to fall. Bank borrowing from the Fed decreases because the relative cost of borrowed reserves ( $i^d - i^f$ ) has risen, partially offsetting some of the increase in total reserve supply.<sup>36</sup> A policy shock is associated with an increase in total reserves, a fall in the funds rate, and if  $i^d < i^f$ , a fall in borrowed reserves.

Suppose there is a positive disturbance to total reserve demand,  $v^d > 0$ . This shifts total reserve demand to the right. In the absence of any policy response (i.e., if  $\phi^d = 0$ ), the funds

36. This analysis assumes that the discount rate has not changed; the Fed could, for example, change the discount rate to keep  $i^f - i^d$  constant and keep borrowed reserves unchanged. Since the total supply of reserves has increased, the funds rate must fall, so this would require a cut in the discount rate.

rate increases. This increase reduces total reserve demand (if  $a > 0$ ), offsetting to some degree the initial increase in reserve demand. If  $i^d < i^f$ , the rise in the funds rate induces an increase in reserve supply as banks increase their borrowing from the Fed. Under a funds rate operating procedure, however,  $\phi^d = 1$ ; the Fed lets nonborrowed reserves rise by the full amount of the rise in reserve demand to prevent the funds rate from rising. Both reserve demand and reserve supply shift to the right by the amount of the disturbance to reserve demand, and the new equilibrium is at an unchanged funds rate. Thus, total reserve demand shocks are completely accommodated under a funds rate procedure. If the positive reserve demand shock originated from an increase in the demand for bank deposits as a result of an economic expansion, a funds rate procedure automatically accommodates the increase in money demand and has the potential to produce procyclical movements of money and output.<sup>37</sup>

In contrast, under a total reserves operating procedure, the Fed would adjust nonborrowed reserves to prevent  $v^d$  from affecting total reserves. From (12.32), this requires that  $\phi^d = -b/a$ ; nonborrowed reserves must be reduced in response to a positive realization of  $v^d$ . It is not sufficient to just hold nonborrowed reserves constant if  $i^d < i^f$ ; the rise in the funds rate in this case will induce an endogenous rise in reserve supply as banks increase their borrowing. To offset this, nonborrowed reserves are reduced. Thus, while a funds rate procedure offsets none of the impact of a reserve demand shock on total reserves, a total reserves procedure offsets all of it.

Under a nonborrowed reserves procedure,  $\phi^d = 0$ ; hence, a positive shock to reserve demand raises the funds rate and borrowed reserves. Total reserves rise by  $-a i^f + v^d = [b/(a+b)]v^d < v^d$ . So reserves do rise (in contrast to the case under a total reserves procedure) but by less than under a funds rate procedure.

Finally, under a borrowed reserves procedure, a positive shock to total reserve demand will, by increasing the funds rate, also tend to increase bank borrowing if  $i^d < i^f$ . To hold borrowed reserves constant, the Fed must prevent the funds rate from rising (i.e., it must keep  $i^f = 0$ ; see (12.27)). This objective requires letting nonborrowed reserves rise. So in the face of shocks to total reserve demand, a funds rate operating procedure and a borrowed reserves procedure lead to the same response. As (12.33) shows, however, a borrowed reserves operating procedure is an inefficient procedure for controlling the funds rate in that it allows disturbances to the borrowings function (i.e.,  $v^b$  shocks) to affect the funds rate.

Now suppose there is a positive shock to bank borrowing;  $v^b > 0$ . The increase in borrowed reserves, by increasing total reserves, will lower the funds rate. Under a funds rate procedure, the Fed prevents this outcome by reducing nonborrowed reserves ( $\phi^b = -1$ ) to

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37. Because operating procedures have been defined in terms of the innovations to reserves and the funds rate, nothing has been said about the extent to which the funds rate might be adjusted in subsequent periods to offset movements in reserve demand induced by output or inflation.

fully neutralize the effect of  $v^b$  on the total reserve supply. The same response would occur under a total reserves operating procedure. In contrast, under a nonborrowed reserves procedure,  $\phi^b = 0$ , so the increase in borrowed reserves also increases total reserve supply, and the funds rate must decline to clear the reserve market.

Following Bernanke and Mihov (1998),  $\phi^d$  and  $\phi^b$  characterize different operating procedures. Their estimates of these parameters are used in section 12.6 when past Fed procedures are reviewed.

### 12.5 Interest on Reserves in a Channel System

Several central banks (e.g., Australia, Canada, New Zealand, Sweden, and Switzerland, all of whom, with the exception of Switzerland, are inflation targeters) employ what is known as a channel system for interest rate control. In a channel system, the central bank sets lower and upper bounds for the interest rate. The upper bound is provided by the penalty rate on reserve borrowing, since no bank would borrow in the private market at any interest rate that exceeds the rate at which reserves can be obtained from the central bank. The lower bound is provided by the interest rate paid on reserves, since no bank would lend in the private market at an interest rate that is less than the rate received on reserves. Under a channel system, the central bank's target interest rate may not provide a complete description of policy because, conditional on the target rate, the spread between the interest rate paid on reserves and the penalty borrowing rate can affect bank behavior. A channel system can also allow the central bank to alter the market interest rate without altering the level of reserves (see Woodford 2001b).

The simplest form of channel system is one in which there are no reserve requirements, the central bank pays an interest rate  $i^* - s_1$  on any reserve balances, and charges a rate  $i^* + s_2$  on any bank reserve overdrafts (i.e., it lends reserves at the rate  $i^* + s_2$  to any bank that has a negative reserve balance at the central bank). The central bank's target interest rate is  $i^*$ . Assume, as Whitesell (2006) does, that the central bank sets a symmetric window around  $i^*$  and  $s_1 = s_2 = s$ .<sup>38</sup> Assume further that loans from the central bank are perfect substitutes for funds obtained in the private market, and overnight balances with the central bank are perfect substitutes for lending in the private market. These assumptions ensure that no bank will borrow in the private market if the interest on such loans exceeds  $i^* + s$  or lend to another bank if the private market interest rate is less than  $i^* - s$ . The willingness of the central bank to pay a fixed rate on balances and to lend automatically at a fixed rate is called a standing facility. Hence, if  $i$  is the private market interbank rate,

$$i^* - s \leq i \leq i^* + s.$$

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38. Berentsen and Monnet (2008) developed a model in which money demand is motivated by the assumption that goods market transactions are anonymous. They derived the optimal spread and noted that the impact of monetary policy is characterized by both the target interest rate and the spread between borrowing and lending rates from the central bank's standing facility.



The rate  $i$  is constrained to remain within the corridor established by the spread  $s$ . Hence, this type of system is also called a corridor system.<sup>39</sup>

Consider the bank's decision problem in a channel system. Assume bank balances are subject to stochastic fluctuations. Let  $T$  be the expected value of the bank's end-of-day balances, and let actual end-of-day balances be  $T + \varepsilon$ , where  $\varepsilon$  is a mean zero random variable with continuous distribution function  $F(\cdot)$ . The model presumes that the realization of  $\varepsilon$  occurs after the interbank market closes. The representative bank chooses  $T$  to balance two costs. First, if it sets  $T$  too high, it is likely to end the day with a positive reserve balance that earns  $i^* - s$  rather than the rate  $i$  that could have been obtained by lending to another bank. Hence, the opportunity cost of holding positive end-of-day balances relative to lending them out is  $i - (i^* - s)$ . Second, if  $T$  is set too low, the bank may end the day with a negative balance and need to borrow from the central bank at the rate  $i^* + s$  rather than from another bank at the rate  $i$ . Hence, the opportunity cost of ending the day with an overdraft and borrowing from central bank is  $i^* + s - i$ . The bank will choose  $T$  to minimize the expected sum of these two costs, subject to the probability distribution of the stochastic process  $\varepsilon$ .<sup>40</sup>

The problem of a risk-neutral bank is to pick  $T$  to minimize

$$\int_{-T}^{\infty} (i - i^* + s)(T + \varepsilon) dF(\varepsilon) - \int_{-\infty}^{-T} (i^* + s - i)(T + \varepsilon) dF(\varepsilon).$$

The first term is the opportunity cost of ending the day with positive balances. This occurs whenever  $\varepsilon > -T$ . The second term is the opportunity cost of borrowing from the central bank, and this occurs whenever  $\varepsilon < -T$ . The first-order condition for the optimal choice of  $T$  is

$$(i - i^* + s) \int_{-T}^{\infty} dF(\varepsilon) - (i^* + s - i) \int_{-\infty}^{-T} dF(\varepsilon) = 0.$$

This can be expressed as  $(i - i^* + s)[1 - F(-T^*)] - (i^* + s - i)F(-T^*) = 0$ , where  $T^*$  is the optimal level of planned reserve balances, or

$$T^* = -F^{-1} \left( \frac{1}{2} + \frac{i - i^*}{2s} \right). \tag{12.36}$$

If the market rate equals the target rate,  $i = i^*$ , then  $T^* = -F^{-1}(1/2)$ , so if the distribution function  $F$  is symmetric,  $T^* = 0$ . However, if the net supply of clearing balances differs

39. Whitesell (2006) noted that between June 2003 and June 2004, the Fed's target for the federal funds rate was 1 percent, and it charged a 2 percent penalty rate on discount window borrowing. Since the Fed did not pay interest on reserves at the time, the Fed was essentially using a symmetric channel system with  $i^* = 1$  percent and  $s = 1$  percent.

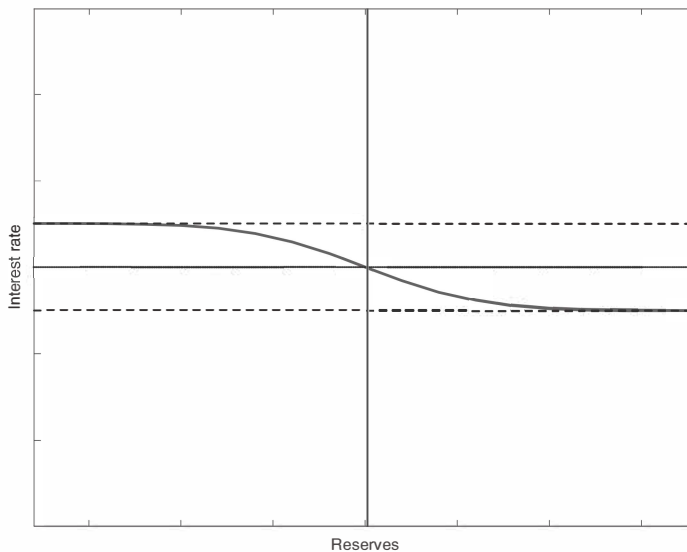
40. Reserve management problems are essentially equivalent to inventory management problems. When sales and/or production are stochastic, the optimal level of inventories balances the costs of stocking out versus the cost of carrying unsold goods. Ashcraft, McAndrews, and Skeie (2011) reported evidence of a precautionary demand for reserves on the part of banks during the global financial crisis.

from zero, the market rate will differ from  $i^*$ . For example, if the net supply is  $\bar{T}$ , then market equilibrium requires that  $T^* = \bar{T}$  and

$$i = i^* - s[1 - 2F(-\bar{T})], \quad (12.37)$$

so that  $i^* - s \leq i \leq i^* + s$ , showing that the private market rate  $i$  remains in the corridor defined by the target rate and the spread. In the case of New Zealand, for example, net settlement cash is quite small and  $s$  is equal to 25 basis points. Thus, with  $\bar{T}$  small but positive,  $F(-\bar{T})$  is slightly less than  $1/2$ , and  $i$  will be slightly below  $i^*$ .

Figure 12.2 illustrates the demand for reserves given by (12.36) when  $\varepsilon$  is normally distributed, the target interest rate is 3 percent, and  $s$  is equal to 50 basis points. The supply of reserves when  $\bar{T} = 0$  is given by the solid line at 2.5 percent up to the net balance of zero and then it jumps up to the penalty rate of 3.5 percent. Equilibrium occurs where the demand curve cuts the supply curve, and the interest rate is equal to the target rate. The figure illustrates the case of a zero net settlement balance, but the same argument would apply for any positive level; the supply curve would simply shift to the right. If the central bank were unable to control reserve supply exactly due to random variation, then the overnight rate might end up slightly above or below the actual target  $i^*$  (see Woodford 2001b on both points).

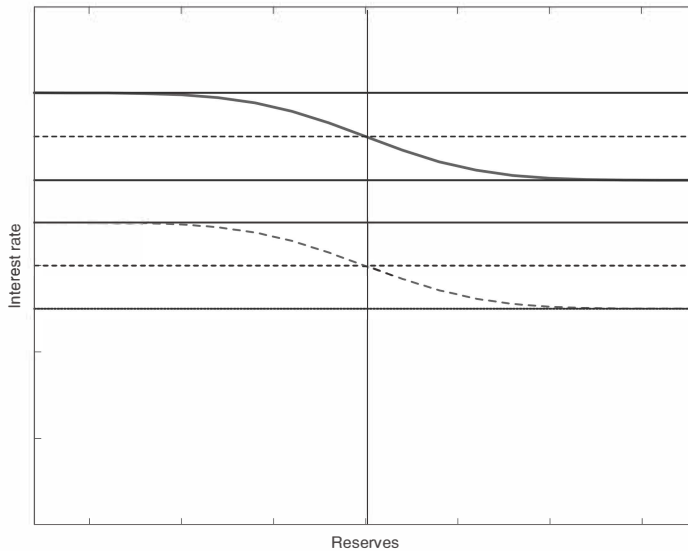


**Figure 12.2**

Reserve market equilibrium under a channel system with a target rate of 3 percent, a symmetric spread of  $\pm 50$  basis points, and zero net supply of settlement balances. Demand curve for reserves is downward-sloping within the corridor defined by  $i^* - s$  and  $i^* + s$ .

An interesting implication of (12.37) is that the central bank can affect the equilibrium market rate without altering the supply of settlement balances. It can do so by simply announcing a change in its target rate  $i^*$  and the rates paid on reserves and charged on borrowing without altering  $\bar{T}$ .<sup>41</sup> This is illustrated in figure 12.3. The ability of the central bank in a channel system to affect the level of interest rates without engaging in open-market operations to affect the supply of reserves is in marked contrast to the traditional model of the reserve market.

The model of a channel system assumes that borrowing from the private market and borrowing from the central bank are perfect substitutes, so  $i$  can never rise above  $i^* + s$ . Similarly, lending to another bank and leaving deposits at the central bank are assumed to be perfect substitutes, so  $i$  can never fall below  $i^* - s$ . In practice, these various options are not equivalent. For example, in the U.S. interbank market, loans are generally unsecured but central bank (Fed) borrowings are collateralized, so the two are not perfect substitutes. In this case, the cost of borrowing from the central bank would be  $i^* + s$  plus the cost of collateral (see Berentsen and Monnet 2008 and Berentsen, Marchesiani, and Waller 2014). Since private lending is unsecured, it is riskier than holding a riskless account balance at



**Figure 12.3**

Increasing the market rate without changing reserve supply by shifting up the rate paid on reserves and the discount rate.

41. Guthrie and Wright (2000) characterized the Reserve Bank of New Zealand as employing “open mouth” operations rather than open-market operations to affect market rates.

the central bank. So the opportunity cost of having overnight balances with the central bank is  $(i - r) - (i^* - s)$ , where  $r$  is a risk premium. In addition, not all banks may have access to the central bank's standing facilities. For example, according to Whitesell (2006), only 60 percent of U.S. depository institutions have completed the paper work necessary to borrow at the discount window. The Bank of Canada and the Bank of England limit access to certain institutions, and the European Central Bank and the Bank of England pay interest on reserves only if banks shift funds out of reserve accounts and into special deposit accounts each day.

Channel systems are becoming increasingly common. When reserve holdings by banks are small, as a result either of innovations that reduce the level of required reserves or because reserve requirements have been eliminated, channel systems provide an operating procedure for controlling the overnight bank rate. Open-market operations are still necessary to offset random fluctuations in the stock of reserves, but these operations are not used to control interest rate levels, as they are under traditional models of operating procedures.

When the quantity of reserves is large, as it is now in the United States, a channel system becomes in effect a floor system, with the effective interest rate equal to the rate paid on reserves. Variations in the quantity of reserves when the quantity is already very large would not affect the level of interest rates, while changes in the interest rate paid on reserves would alter the general level of rates without any need for the central bank to change the level of reserves. Kashyap and Stein (2012) argued that the interest rate on reserves and the quantity of reserves provide the central bank with two separate policy instruments with which it can achieve two separate objectives. The central bank can use the interest rate on reserves to affect the general level of interest rates, allowing it to pursue its objectives related to general macroeconomic inflation and output gap stability, while the quantity of reserves can be used to address distortions in debt issuance by financial intermediaries. Cochrane (2014) argued that the rate paid on reserves can be used to eliminate the monetary distortion that M. Friedman (1969) emphasized and, with this friction eliminated, the price level is determined by the fiscal theory (see chapter 4) rather than conventional monetary theories of the price level.

The discussion so far has been limited to a partial equilibrium analysis of the market for reserves. Ireland (2014) embedded a model of interest on reserves into a general equilibrium new Keynesian model. The demand for monetary services by households is based on a shopping-time model (see chapter 3) in which monetary services are produced by currency and bank deposits. Banks use reserves and labor to create deposits. Prices are sticky, monetary policy uses a variant of a Taylor rule to determine the short-term interest rate, and the spread between the short-term interest rate, and the rate paid on reserves is a stochastic process. Ireland calibrated his model and investigated the effects of paying interest on reserves on the economy's steady state and on its dynamic responses to shocks. He found that paying interest on reserves had large effects on steady-state reserve holdings but little effect on output or inflation. This was also true of the dynamic responses to

productivity and monetary policy shocks; the responses of reserve variables were affected by the payment of interest on reserves, but those of output and inflation were not.

One final aspect of the payment of interest on reserves is important to note. As discussed earlier, efficiency in monetary economies generally requires that the Friedman rule be satisfied. This rule calls for the opportunity cost of money to be zero. If reserves pay interest, then the opportunity cost of holding reserves can be set to zero if the rate paid on reserves equals the nominal market interest rate. Thus, paying interest on reserves is a means of eliminating the classical Friedman distortion without requiring the market interest rate to be reduced to zero.

## 12.6 A Brief History of Fed Operating Procedures

In the United States, the operating procedures employed by the Federal Reserve have changed over time. Consequently, the manner in which the reserve market has responded to disturbances has varied, and the appropriate measure of policy shocks has also changed.

Federal Reserve operating procedures have been discussed by various authors,<sup>42</sup> and major studies of operating procedures have been undertaken by the Federal Reserve (Federal Reserve System 1981; Goodfriend and Small 1993). Fed operating procedures have varied over the past 40 years. There have been periods corresponding to nonborrowed reserves, borrowed reserves, and funds rate operating procedures, although in no case did the Fed's behavior reflect pure examples of any one type.<sup>43</sup>

### 12.6.1 1972–1979

The first period dates from the end of the Bretton Woods exchange rate system in the early 1970s to October 6, 1979. The Fed is usually described as having followed a federal funds rate operating procedure during this period. Under such a policy, the Fed allowed nonborrowed reserves to adjust automatically to stabilize the funds rate within a narrow band around its target level. Thus, a shock to total reserve demand that in the absence of a policy response would have led to an increase in both the funds rate and borrowed reserves was offset by open-market purchases that expanded nonborrowed reserves sufficiently to prevent the funds rate from rising (i.e.,  $\phi^d = 1$ ). As a result, expansions in reserve demand were fully accommodated by increases in reserve supply.<sup>44</sup>

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42. Examples include Walsh (1990), Goodfriend (1991; 1993), Strongin (1995), and Meulendyke (1998) and the references they cite.

43. From 1975 to 1993 the Fed announced targets for various monetary aggregates, and these played a role as intermediate targets during some periods; see B. Friedman and Kuttner (1996).

44. While the discussion here focuses on reserve market adjustments, changes in the funds rate target then lead to changes in market interest rates. For evidence, see Cook and Hahn (1989), Rudebusch (1995a), or Roley and Sellon (1996). International evidence on the response of market interest rates to changes in the short-run interest rate used to implement policy can be found in Buttiglione, Del Giovane, and Tristani (1998).

A funds rate operating procedure only implies that shocks to the funds rate are offset initially; the targeted funds rate could, in principle, respond strongly beginning in period  $t + 1$ . However, the funds rate operating procedure came under intense criticism during the 1970s because of the Fed's tendency to stabilize interest rates for longer periods of time. Such interest rate–smoothing behavior can have important implications for price level behavior, as shown by Goodfriend (1987). Because a rise in the price level increases the nominal demand for bank deposits as private agents attempt to maintain their real money holdings, periods of inflation lead to increases in the nominal demand for bank reserves. If the central bank holds nonborrowed reserves fixed, the rising demand for reserves pushes up interest rates, thereby moderating the rise in money demand and real economic activity. If the central bank instead attempts to prevent interest rates from rising, it must allow the reserve supply to expand to accommodate the rising demand for reserves. Thus, interest rate–stabilizing policies can automatically accommodate increases in the price level, contributing to ongoing inflation. Under some circumstances, an interest rate policy can even render the price level indeterminate; an arbitrary change in the price level produces a proportionate change in nominal money demand, which the central bank automatically accommodates to keep interest rates from changing. Since market interest rates incorporate a premium for expected inflation, an increase in expected inflation would, under a policy of stabilizing market interest rates, also be automatically accommodated.

Recall from the reserve market model that under a funds rate procedure, nonborrowed reserves are automatically adjusted to offset the impact on the funds rate of shocks to total reserve demand and to borrowed reserves. In terms of the model parameters, this adjustment requires that  $\phi^d = 1$  and  $\phi^b = -1$ . Bernanke and Mihov (1998), using both monthly and biweekly data, reported that these restrictions are not rejected for the period 1972:11 to 1979:09. Thus, innovations in the funds rate provide an appropriate measure of monetary policy during this period.

### 12.6.2 1979–1982

In October 1979, as part of a policy shift to lower inflation, the Fed moved to a non-borrowed reserves operating procedure. An operating procedure that focused on a reserve quantity was viewed as more consistent with reducing money growth rates to bring down inflation.

The Fed had, in fact, begun announcing target growth rates for several monetary aggregates in 1975. Under the Humphrey-Hawkins Act, the Fed was required to establish monetary targets and report these to Congress.<sup>45</sup> Because growth rate target ranges were set

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45. The targets for  $M1$  for the period 1975–1986 and for  $M2$  and  $M3$  for the period 1975–1991 are reported by Bernanke and Mishkin (1992, table 1). Preliminary targets for the following calendar year were set each July and confirmed in January. Discussions of the targets can be found in the various issues of the Federal Reserve's *Monetary Policy Report to Congress*. The Fed stopped setting growth rate targets for  $M1$  after 1986 because of the apparent breakdown in the relationship between  $M1$  and nominal income.

for several measures of the money supply (there were targets for  $M1$ ,  $M2$ ,  $M3$ , and debt), the extent to which these targets actually influenced policy was never clear. The move to a nonborrowed reserves operating procedure was thought by many economists to provide a closer link between the policy instrument (nonborrowed reserves) and the intermediate target of policy (the monetary growth targets). B. Friedman and Kuttner (1996) provide an evaluation of the actual effects of these targets on the conduct of policy.

Under a nonborrowed reserves procedure, an increase in expected inflation would no longer automatically lead to an accommodative increase in bank reserves. Instead, interest rates would be allowed to rise, reducing nominal asset demand and restraining money growth. Similarly, if money growth rose above the Fed's target growth rate, reserve demand would rise, pushing up the funds rate. The resulting rise in the funds rate would tend to reduce money demand automatically.

Whether the Fed actually followed a nonborrowed reserves procedure after October 1979 has often been questioned. After the switch in policy procedures, the funds rate was clearly both higher and more volatile. Many commentators felt that the policy shift in late 1979 was designed to allow the Fed to increase interest rates substantially while reducing the political pressures on the Fed to prevent rates from rising. Under the former funds rate procedure, changes in short-term interest rates were (correctly) perceived as reflecting Fed decisions. By adopting a nonborrowed reserves operating procedure and focusing more on achieving its targeted growth rates for the money supply, the Fed could argue that the high interest rates were due to market forces, not Fed policy. Cook (1989) estimated, however, that fully two-thirds of all funds rate changes during this period were the result of "judgmental" Fed actions; only one-third represented automatic responses to nonpolicy disturbances.

The 1979–1982 period was characterized by increased attention by the Fed to its monetary targets. In principle, nonborrowed reserves were adjusted to achieve a targeted growth rate for the money stock. If the money stock was growing faster than desired, the nonborrowed reserves target would be adjusted downward to place upward pressure on the funds rate. This in turn would reduce money demand and tend to bring the money stock back on target. As a result, market interest rates responded sharply to each week's new information on the money supply. If the money supply exceeded the market's expectation, market interest rates rose in anticipation of future policy tightening (see Roley and Walsh 1985 and the references listed there).

The actual practice under the nonborrowed reserves procedure was complicated by several factors. First, the Fed established and announced targets for several different definitions of the money stock.<sup>46</sup> This policy reduced the transparency of the procedure because

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46. The Fed established target cones for each aggregate. For example, the target cone for  $M1$  set in January 1980 was 4.0 percent to 6.5 percent from a base of the actual level of  $M1$  in the fourth quarter of 1979. The use of actual levels as the base for new target cones resulted in base drift; past target misses were automatically incorporated into the new base. See Broadus and Goodfriend (1984). For a discussion of the optimal degree of base drift, see Walsh (1986).

often one monetary aggregate might be above its target while another would be below, making the appropriate adjustment to the nonborrowed reserves path unclear. Second, under the system of lagged reserve accounting then in effect, the level of reserves a bank was required to hold during week  $t$  was based on its average deposit liabilities during week  $t - 2$ . With reserve demand essentially predetermined each week, variations in the funds rate had little contemporaneous effect on reserve demand. Changes in reserve supply required large swings in the funds rate to equilibrate the reserve market. A rise in interest rates had no immediate effect on the banking sector's reserve demand, leading to a delay in the impact of a policy tightening on money growth. This system was criticized by McCallum and Hoehn (1983) as reducing the ability of the Fed to control the growth rate of the monetary aggregates.<sup>47</sup>

Referring back to the earlier reserve market model, with  $\phi^d = \phi^b = 0$  under a nonborrowed reserves operating procedure, (12.30) implies that

$$i_{\text{NBR}}^f = \left( \frac{b}{a+b} \right) i^d - \left( \frac{1}{a+b} \right) [v^s + v^b - v^d], \quad (12.38)$$

so that, ignoring discount rate changes, the variance of funds rate innovations rises from  $\sigma_s^2/(a+b)^2$  under a pure funds rate operating procedure to  $[\sigma_s^2 + \sigma_d^2 + \sigma_b^2]/(a+b)^2$  under a pure nonborrowed reserves operating procedure, where  $\sigma_i^2$  is the variance of  $v^i$  for  $i = s, d, b$ . The variance of funds rate innovations is decreasing in  $a$ , and with lagged reserve accounting,  $a = 0$ , further increasing the variance of the funds rate. Changes in reserve supply would require large swings in the funds rate to equilibrate the reserve market.

In practice, it was argued, the Fed actually set its nonborrowed reserves target so as to achieve the level of the funds rate it desired. That is, the Fed started with a desired path for the money stock; since equilibrium required that money demand equal money supply, it used an estimated money demand function to determine the level of the funds rate consistent with the targeted level of money demand. Then, based on total reserve demand (predetermined under lagged reserve accounting) and an estimated borrowed reserves function, it determined the level of nonborrowed reserves required to achieve the desired funds rate. A nonborrowed reserves operating procedure designed to achieve a desired funds rate is simply an inefficient funds rate procedure. However, by shifting the focus of policy away from a concern for stabilizing interest rates, the 1979 policy shift did reflect a substantive policy shift consistent with reducing the rate of inflation.

Using biweekly data for the period October 1979 to October 1982, Bernanke and Mihov (1998) reported estimates of  $\phi^d$  and  $\phi^b$ ; neither estimate is statistically significantly different from zero. These estimates are consistent, then, with the actual use of a nonborrowed reserves operating procedure during this period.

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47. Lagged reserve accounting was replaced by contemporaneous reserve accounting in 1984. See Hamilton (1996) for a detailed discussion of the reserve accounting system.



Key to a nonborrowed reserves operating procedure is the need to predict the relationship between changes in nonborrowed reserves and the resulting impact on broader monetary aggregates, inflation, and real economic activity. During the late 1970s and early 1980s, there seemed to be a fairly stable relationship between monetary aggregates such as  $M1$  and nominal income. This relationship could be used to work backward from a desired path of nominal income growth to a growth path for  $M1$  to a growth path for nonborrowed reserves. Unfortunately, this relationship appeared to break down in the early and mid-1980s (see, e.g., B. Friedman and Kuttner 1996). In the absence of a reliable link between reserve measures and nominal income, the Fed eventually moved away from a nonborrowed reserves operating procedure.

### 12.6.3 1982–1988

After 1982 the Fed generally followed a borrowed reserves operating procedure. As noted earlier, such a procedure is, in practice, similar to a funds rate operating procedure, at least in the face of reserve demand shocks. The basic Poole analysis implied that an operating procedure oriented toward interest rates tends to dominate one oriented toward monetary aggregates as the variance of money demand shocks rises relative to aggregate demand shocks. B. Friedman and Kuttner (1996) provided a plot of the ratio of the variance of money demand shocks to the variance of aggregate demand shocks based on an estimated VAR. The plot shows this ratio reaching a minimum during 1981 and then steadily increasing. The shift back to an interest rate operating procedure after 1982 is consistent with the recommendations of Poole's model.

From the earlier discussion, a borrowed reserves operating procedure implies values of 1 and  $a/b$  for  $\phi^d$  and  $\phi^b$ . Bernanke and Mihov (1998) obtained point estimates for  $\phi^d$  and  $\phi^b$  for February 1984 to October 1988 that are more consistent with a funds rate procedure ( $\phi^d = 1$ ;  $\phi^b = -1$ ) than with a borrowed reserves procedure. However, for biweekly data during the post-1988 period, Bernanke and Mihov found estimates consistent with a borrowed reserves procedure with  $a = 0$ . This last parameter restriction agrees with the characterization of policy provided by Strongin (1995).

Cosimano and Sheehan (1994) estimated a biweekly reserve market model using data from 1984 to 1990. Their results are consistent with a borrowed reserves procedure over this period, not with a funds rate procedure, although they noted that actual policy under this procedure was similar to what would occur under a funds rate procedure. The evidence also suggests that after the October 1987 stock market crash, the Fed moved toward a more direct funds rate procedure.

### 12.6.4 1988–2008

From the late 1980s until the global financial crisis, the Fed targeted the funds rate directly ( $\phi^d = 1$ ;  $\phi^b = -1$ ). Open-market operations were conducted once each day, so the actual funds rate fluctuated slightly around the target rate daily. Taylor (1993a) was one of the

earliest to model Fed interest rate–setting behavior in terms of a policy rule. He showed that a simple rule that made the funds rate a function of inflation and the output gap did a good job in tracking the actual behavior of the funds rate. Taylor rules have become a standard way of representing Fed policies and those of other central banks. A huge literature has estimated such rules for different time periods and for different central banks. Early examples include Clarida, Gali, and Gertler (1999) and Orphanides (2001); more recent contributions are discussed in chapter 8. Taylor rules, usually augmented to include past interest rates, are commonly used to represent policy in empirical DSGE models (e.g., Smets and Wouters 2003; 2007; Christiano, Eichenbaum, and Evans 2005). Interest rate rules in new Keynesian models are discussed in chapter 8 and more references to the literature are provided.

One of the biggest changes in operating procedures since 1990 is the increase in the transparency with which the Fed and other central banks conduct monetary policy.<sup>48</sup> Since 1994, the Federal Open Market Committee has announced its policy decisions at the time they are made. These announced changes in the target rate receive prominent coverage in the press, and the FOMC’s press releases convey its assessment of the economy to the public and give some signal of possible future changes in policy. The Fed began providing more information on FOMC members’ projections of future growth and inflation. The medium-term projections for inflation have been interpreted as giving the implicit inflation targets of the FOMC members. While these statements contributed to policy transparency, the Fed, unlike many other central banks, did not formally translate its “long-run goal of price stability” into an explicit target for the rate of inflation until January 2012, when it announced that its target for inflation is 2 percent.

### 12.6.5 2009–2016

The financial market crisis that began during the second half of 2007 led to major changes in the Federal Reserve’s operating procedures. During 2008 the Fed cut the funds rate target, and the effective funds rate was essentially at zero by the end of 2008. In such an environment, policy clearly can no longer be represented in terms of a simple rule for setting the policy interest rate. To deal with the financial crisis and the sharp decline in economic activity, the Fed developed new policy tools. For example, new auction facilities were introduced to expand the range of institutions able to borrow from the Fed, and the assets that qualified as collateral were greatly expanded. However, even as the funds rate fell to zero, rates on corporate debt rose, reflecting increases in risk premiums, and the Fed moved directly to reduce risk spreads. It employed statements about the future path of interest rates (forward guidance) and actions that expanded the Fed’s balance sheet and altered

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48. Eijffinger and Geraats (2006) and Dincer and Eichengreen (2007) constructed indexes of central bank transparency. The latter index is updated in Dincer and Eichengreen (2014). Geraats (2002; 2009) provided surveys of research on monetary policy transparency. Recent work on transparency is cited in Walsh (2007).

its composition. Cecchetti (2009) and Reis (2009) discussed some of these new policies; these policies were the focus of chapter 11. Empirical evidence on the effectiveness of balance sheet policies was discussed in chapter 1.

In December 2008, the FOMC cut the federal funds rate target range to (0 percent, 0.25 percent). It remained at this level until December 2015, when the Fed raised the range to (0.25 percent, 0.50 percent). During this seven-year period, in which the traditional policy interest rate was unchanged and at a level viewed as its effective lower bound, the Federal Reserve attempted to use forward guidance about the future path of interest rates and balance sheet policies that affected both the size and the composition of its balance sheet to stimulate economic activity (see chapter 11).

As of October 2008, the Federal Reserve has had the authority to pay interest on required and excess reserves. For several years prior to December 2015, the Federal Reserve was paying 0.25 percent on required and excess reserves.<sup>49</sup> During this same period, the Federal Reserve was charging 0.75 percent on borrowed reserves. Thus, the channel for interest rates initially had a lower bound of 25 basis points and an upper bound of 75 basis points. These rates were raised to 50 and 125 basis points, respectively, in December 2015. At the same time, the level of reserves held by the banking system had expanded tremendously, reflecting the huge expansion in the size of the Federal Reserve's balance sheet. Excess reserves, for example, rose from under \$2 billion in January 2008 to over \$2,200 billion by the end of 2015.

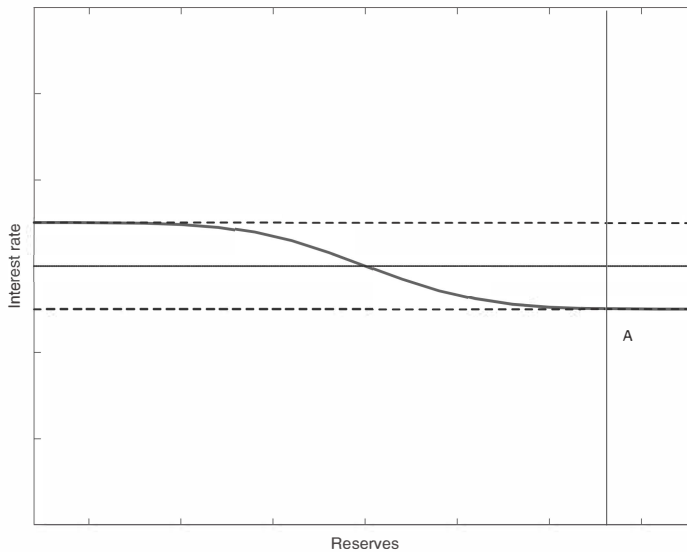
The huge supply of reserves means the Fed was (and continues to be, at present) operating a floor system; the quantity of reserves has been so large that the equilibrium is at the floor of the corridor defined by the rate paid on reserves and the rate charged for discount window borrowing.<sup>50</sup> This is illustrated in figure 12.4, which can be compared to figure 12.2. Equilibrium is at point A, where the vertical reserve supply curve intersects the floor of the corridor given by the rate paid on reserves.

In fact, the rate paid on reserves has not served as a floor for market interest rates; the federal funds rate and the 3-month Treasury bill rate have both been below the floor during the period of extraordinarily low U.S. interest rates. As argued by Bech and Klee (2011), this situation can arise because some government agencies that hold reserves cannot legally be paid interest on their reserves. These nonbanks are willing to lend in the interbank market at a rate below the rate paid on reserves. Banks that can receive interest on their reserve balances have an incentive to borrow from these agencies, earning a risk-free return equal to the rate paid on reserves minus the rate paid to borrow the reserves. By bidding for reserves, banks should drive up the federal funds rate until it equals the rate paid on

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49. As of 2016, the Federal Reserve paid the same rate of interest on required reserves and on excess reserves, although it has the authority to set different rates on these two components of total reserves. The growth of reserve balances since 2008 means that excess reserves constitute the overwhelming bulk of total reserves.

50. For a discussion of why banks are holding such high levels of reserves, see Keister and McAndrews (2009). For a model of policy with interest on reserves, see Ireland (2014).



**Figure 12.4**

Channel system with a large quantity of reserves. Equilibrium is at point A, where the interbank rate is equal to the interest rate paid on reserves.

reserves so that the two interest rates are equal. The fact that the rates have differed is a sign that there is some limit to arbitrage.

Because the interest rate on excess reserves has not provided a floor for key rates such as the 3-month Treasury bill rate, there is concern that control over the rate paid on reserves may not provide the tight control over the general level of interest rates that the model of a channel system suggests. The Federal Reserve has therefore experimented with new policy tools designed to ensure it is able to exercise broad influence over interest rates. Martin et al. (2013) and Ihrig, Meade, and Gretchen (2015) discussed the tools the Fed has used to raise interest rates and to manage reserves from the near-zero interest rate levels seen since December 2008. Once such tool is overnight reverse repurchase operations. Before the financial crisis, the Fed would use repurchase agreements, or repos, to manage temporary changes in the level of reserves. To increase reserves, the Fed would buy a security from a bank or primary security dealer with the seller agreeing to repurchase the security the next day. This had the effect of increasing reserves held by the private sector when the security was sold to the Fed and reversing that increase in reserves the next day when the Fed sold the security back. A reverse repo involves the Fed selling securities to the private sector, thereby reducing reserves, and repurchasing the security the next day. The Fed is, in effect, borrowing overnight from the private sector, putting up securities from its portfolio as collateral. The Fed has experimented with setting an interest rate

at which it will borrow and the aggregate amount. Perhaps more important, the number of counterparties allowed to participate in these operations has been increased. This may reduce the arbitrage limitations that led market rates to fall below the rate paid on reserves. Nonbank institutions that cannot receive interest on reserves can now lend directly to the Fed at the rate set on reverse repos.

Three additional options are available should the Fed wish to reduce the supply of reserves for longer periods of time. The first is a term reverse repo agreement. These involve the repurchasing part of the agreement occurring at a later date than the next day. Second, the Fed can offer term deposits to banks. Much like a CD offered by a bank to a household, term deposits at the Fed involve a bank depositing reserves with the Fed for a fixed term, during which those balances are no longer part of the reserve supply. Finally, the Fed could use traditional open-market operations to shrink its balance sheet. That is, by selling securities from its portfolio without the commitment to repurchase that is part of a repo agreement, the Fed can permanently reduce reserve balances. See Ihrig, Meade, and Gretchen (2015) for a detailed discussion of all these instruments and how each works to affect market interest rates.

## 12.7 Other Countries

The preceding discussion focused on the United States. If measuring monetary policy requires an understanding of operating procedures, then the appropriate measure of policy in the United States will not necessarily be appropriate for other countries. Operating procedures generally depend on the specific institutional structure of a country's financial sector, and the means used to implement monetary policy have varied over time in most countries as financial markets have evolved as the result of either deregulation or financial innovations. The way major central banks operate has been affected dramatically by the global financial crisis, the ensuing recessions, and the debt crises in Europe. The instruments of policy have been expanded, and new ways of injecting liquidity into markets have been developed.

For the period before the global financial crisis, Borio (1997) surveyed policy implementation in the industrial economies. Detailed discussions of the operating procedures in France, Germany, Japan, the United Kingdom, and the United States can be found in Batten et al. (1990). Bernanke and Mishkin (1992) provided case studies of monetary policy strategies in the United States, the United Kingdom, Canada, Germany, Switzerland, and Japan. These countries, plus France, are discussed in Kasman (1993) and Morton and Wood (1993). The behavior of the Bundesbank was examined by Clarida and Gertler (1997). Cargill, Hutchison, and Ito (1997) provided a discussion of Japan. Goodhart and Viñals (1994) discussed policy behavior in a number of European and Antipodian countries. A more recent survey of central bank operating procedures, but still dated before the

2008–2009 financial crisis, was provided in Bank for International Settlements (2007). See also B. Friedman and Kuttner's (2010) comparison of monetary policy interest rate–setting frameworks in the United States, Europe, and Japan.

The experiences with monetary policy operating procedures in all these countries have been broadly similar over the past 20 years. Beginning in the mid-1970s, many countries publicly established monetary targets. Germany, Canada, and Switzerland began announcing monetary targets in 1975, the United Kingdom in 1976, and France in 1977. The weight placed on these targets, however, varied greatly over time. In general, the financial innovations that occurred in the 1980s, together with significant deregulation of financial markets that took place after 1985, reduced reliance on monetary targets. This finding is consistent with the implications of Poole's model, which suggested that increased financial market instability that makes money demand less predictable would lessen the advantages of any operating procedure oriented toward monetary aggregates.

Morton and Wood (1993) argued that a common theme among the six industrial countries they examined has been the move to more flexible interest rate policies. Rather than rely on officially established interest rates, often combined with direct credit controls, central banks have moved toward more market-oriented interest rate policies.<sup>51</sup> These involve control over a reserve aggregate (such as nonborrowed reserves in the United States) through which the central bank influences liquidity in the money market. This provides the central bank with control over a short-term money market rate that balances reserve supply and demand. Typically, central banks do not intervene in the market continuously; instead they estimate reserve demand and then add or subtract bank reserves to achieve the targeted interbank interest rate. Because these operations are based on reserve projections and because actual reserve demand may differ from projections, the actual value of the interest rate can differ from the central bank's target. However, by intervening daily, the central bank can normally keep target deviations quite small.

Finally, it is worth emphasizing that the choice of operating procedure is, in principle, distinct from the choice of ultimate goals and objectives of monetary policy. For example, a policy under which price stability is the sole objective of monetary policy could be implemented through either an interest rate procedure or a reserve aggregate procedure. A policy that incorporates output stabilization or exchange rate considerations can similarly be implemented through different procedures. The choice of operating procedure is significant, however, for interpreting the short-term response of financial markets to economic disturbances. And inefficient procedures can introduce unnecessary volatility into financial markets.

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51. Similarly, Kasman (1993) noted that innovation and liberalization in financial markets have made the institutional settings in which policy is conducted increasingly similar among the industrial countries.

## 12.8 Summary

This chapter has focused on the implementation of monetary policy. It began with the classic instrument choice problem due to Poole (1970): Should the central bank focus on controlling a monetary aggregate or a nominal interest rate? It was shown how the optimal adjustment of the policy instrument involves a signal extraction problem when the central bank is responding to partial and incomplete information. The choice of operating procedure for implementing policy has consequences for whether monetary shocks are best measured by the interest rate in the reserve market or a measure of a market aggregate. In traditional models of the reserve market, the central bank could control the quantity of reserves and let the interest rate adjust to clear the market, or it could target the interest rate and let the quantity of reserves adjust to clear the market. When the central bank pays interest on reserves, it can employ a channel system in which the quantity of reserves and the target level of interest rates are decoupled. When the quantity of reserves is large, the interest rate on reserves becomes the key policy rate. The chapter also reviewed the various operating procedures employed by the Federal Reserve since 1972.

## 12.9 Problems

1. Suppose the basic Poole model (12.1 and 12.2) is modified by allowing the disturbances to be serially correlated. Specifically, assume that the disturbance in (12.1) is given by  $u_t = \rho_u u_{t-1} + \varphi_t$ , while the disturbance in (12.2) is given by  $v_t = \rho_v v_{t-1} + \psi_t$ , where  $\varphi$  and  $\psi$  are white noise processes (assume all shocks can be observed with a one-period lag). Assume the central bank's loss function is  $E(y_t)^2$ .
  - a. Under a money supply operating procedure, derive the value of  $m_t$  that minimizes  $E(y_t)^2$ .
  - b. Under an interest rate operating procedure, derive the value of  $i_t$  that minimizes  $E(y_t)^2$ .
  - c. Explain why your answers in parts (a) and (b) depend on  $\rho_u$  and  $\rho_v$ .
  - d. Does the choice between a money supply procedure and an interest rate procedure depend on the  $\rho_i$ ? Explain.
  - e. Suppose the central bank sets its instrument for two periods (for example,  $m_t = m_{t+1} = m^*$ ) to minimize  $E(y_t)^2 + \beta E(y_{t+1})^2$ , where  $0 < \beta < 1$ . How is the instrument choice problem affected by the  $\rho_i$ ?
2. Suppose (12.1) is replaced by a forward-looking IS curve of the form

$$y_t = E_t y_{t+1} - \alpha i_t + u_t.$$

The LM curve is given by (12.2). Assume  $u_t$  and  $v_t$  are given by  $u_t = \rho_u u_{t-1} + \varphi_t$  and  $v_t = \rho_v v_{t-1} + \psi_t$ , where  $\varphi$  and  $\psi$  are white noise processes (assume all shocks can be observed with a one-period lag). Assume the central bank's loss function is  $E(y_t)^2$ .

- a. Under a money supply operating procedure, derive the value of  $m_t$  that minimizes  $E(y_t)^2$ .
  - b. Under an interest rate operating procedure, derive the value of  $i_t$  that minimizes  $E(y_t)^2$ .
  - c. Explain why your answers in parts (a) and (b) depend on  $\rho_u$  and  $\rho_v$ .
  - d. Does the choice between a money supply procedure and an interest rate procedure depend on the  $\rho_i$ ? Explain.
3. Suppose the utility of the representative household depends on consumption, leisure, and real money balances, as in the MIU models of chapter 2. In the context of a new Keynesian model of the sort developed in chapter 8 (optimizing agents, sticky prices), discuss how consumption, leisure, and real money balances would respond to a positive money demand shock under the following policies:
    - a. The central bank stabilizes the nominal supply of money.
    - b. The central bank stabilizes the nominal rate of interest (subject to satisfying the Taylor principle for determinacy).
    - c. Given your results in parts (a) and (b), is there a clear ranking of the two policies in terms of their implications for welfare?
  4. Solve for the  $\delta_i$  in (12.11), and show that the optimal rule for the monetary base is the same as that implied by the value of  $\mu^*$  given in (12.10).
  5. Suppose the money demand relationship is given by  $m = -c_1 i + c_2 y + v$ . Show how the choice of an interest rate versus a money supply operating procedure depends on  $c_2$ . Explain.
  6. Prices and aggregate supply shocks can be added to Poole's analysis by using the following model:

$$y_t = y_n + a(\pi_t - E_{t-1}\pi_t) + e_t,$$

$$y_t = y_n - \alpha(i_t - E_t\pi_{t+1}) + u_t,$$

$$m_t - p_t = c_0 - c_1 i_t + y_t + v_t.$$

Assume that the central bank's objective is to minimize  $E[\lambda(y - y_n)^2 + \pi^2]$  and that disturbances are mean zero white noise processes. Both the private sector in setting  $E_{t-1}\pi_t$  and the monetary authority in setting its policy instrument must act prior to observing the current values of the disturbances.



- a. Calculate the expected loss function if  $i_t$  is used as the policy instrument. (*Hint:* Given the objective function, the instrument will always be set to ensure that expected inflation is equal to zero.)
  - b. Calculate the expected loss function if  $m_t$  is used as the policy instrument.
  - c. How does the instrument choice comparison depend on
    - i. the relative variances of the aggregate supply, aggregate demand, and money demand disturbances?
    - ii. the weight on stabilizing output fluctuations  $\lambda$ ?
7. Using the intermediate target model of section 12.3.3 and the loss function (12.15), rank the policies that set  $i_t$  equal to  $\hat{i}_t$ ,  $i_t^T$ , and  $\hat{i}_t + \mu^* x_t$ .
  8. Show that if the nominal interest rate is set according to (12.17), the expected value of the nominal money supply is equal to  $\hat{m}$  given in (12.19).
  9. Suppose the central bank is concerned with minimizing the expected value of a loss function of the form

$$L = E(\text{TR})^2 + \chi E(i^f)^2,$$

which depends on the variances of innovations to total reserves and the funds rate ( $\chi$  is a positive parameter). Using the reserve market model of this chapter, find the values of  $\phi^d$  and  $\phi^b$  that minimize this loss function. Are there conditions under which a pure nonborrowed reserves or a pure borrowed reserves operating procedure would be optimal?

10. Suppose there is a positive shock to reserve demand. Assuming  $i^d > i^f$ , construct a table showing how the funds rate, total reserves, nonborrowed reserves, and borrowed reserves respond under (1) a funds rate operating procedure, (2) a borrowed reserves operating procedure, (3) a nonborrowed reserves operating procedure, and (4) a total reserves operating procedure.
11. Suppose there is a positive shock to reserve demand. Assume the discount rate is a penalty rate, so  $i^d > i^f$ . Construct a table showing how the funds rate and nonborrowed reserves respond under (1) a funds rate operating procedure, and (2) a nonborrowed reserves operating procedure.
12. Assume  $i^d = i^f + s$  in (12.27), where  $s$  is a penalty for discount window borrowing. How does this modification change (12.27)? How are (12.30)–(12.32) affected? How does the Fed's operating procedure affect the interpretation of movements in nonborrowed reserves, borrowed reserves, and the federal funds rate as measures of monetary policy shocks?

13. Suppose the central bank operates a channel system of the sort analyzed in section 12.5, with  $s = 0.50$  (i.e., 50 basis points). Assume end-of-day bank settlement balances are  $T + \varepsilon$ , where  $\varepsilon$  is normally distributed with mean zero and variance  $\sigma^2$ . The supply of settlement balances is fixed at  $\bar{T}$  and the target interest rate is  $i^*$ .
- Explain how the equilibrium market interest rate is affected by an increase in  $\sigma^2$ .
  - Explain how the equilibrium market interest rate is affected by an increase in  $s$ .
  - Explain why  $i^*$  is not a complete description of monetary policy under a channel system unless  $s$  is also known.

## References

- Abel, A. B. 1985. "Dynamic Behavior of Capital Accumulation in a Cash-in-Advance Model." *Journal of Monetary Economics* 16(1): 55–71.
- Abreu, D. 1988. "On the Theory of Infinitely Repeated Games with Discounting." *Econometrica* 56: 383–396.
- Adam, K., and R. Billi. 2006. "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates." *Journal of Money, Credit, and Banking* 38(7): 1877–1905.
- . 2007. "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates." *Journal of Monetary Economics* 54(3): 728–752.
- Adao, B., I. Correia, and P. Teles. 2003. "Gaps and Triangles." *Review of Economic Studies* 70(4): 699–713.
- . 2004. "The Monetary Transmission Mechanism: Is It Relevant for Policy?" *Journal of the European Economic Association* 2: 310–319.
- Adolfson, M. 2007. "Monetary Policy with Incomplete Exchange Rate Pass-Through." *Journal of International Money and Finance* 26(3): 468–494.
- Adolfson, M., S. Laséen, J. Lindé, and M. Villani. 2007. "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through." *Journal of Monetary Economics* 72(2): 481–511.
- . 2008. "Evaluating an Estimated New Keynesian Small Open Economy Model." *Journal of Economic Dynamics and Control* 32(8): 2690–2721.
- Aiyagari, S. R., and M. Gertler. 1985. "The Backing of Government Bonds and Monetarism." *Journal of Monetary Economics* 16(1): 19–44.
- Aizenman, J., and J. A. Frankel. 1986. "Targeting Rules for Monetary Policy." *Economic Letters* 21: 183–187.
- Akerlof, G. A. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* 84(3): 488–500.
- Akerlof, G. A., and J. L. Yellen. 1985. "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?" *American Economic Review* 75(4): 708–721.
- al-Nowaihi, A., and P. Levine. 1994. "Can Reputation Resolve the Monetary Policy Credibility Problem?" *Journal of Monetary Economics* 33(2): 355–380.
- . 1996. "Independent but Accountable: Walsh Contracts and the Credibility Problem." CEPR Discussion Paper 1387.
- Albanesi, S., V. V. Chari, and L. J. Christiano. 2003. "Expectation Traps and Monetary Policy." *Review of Economic Studies* 70: 715–741.
- Alchian, A. A. 1977. "Why Money?" *Journal of Money, Credit, and Banking* 9(1): 133–140.
- Alesina, A. 1987. "Macroeconomic Policy in a Two-Party System as a Repeated Game." *Quarterly Journal of Economics* 102(3): 651–678.
- . 1989. "Macroeconomics and Politics." In *NBER Macroeconomics Annual 1988*, 11–55. Cambridge, MA: MIT Press.
- Alesina, A., and R. Gatti. 1995. "Independent Central Banks: Low Inflation at No Cost?" *American Economic Review* 85(3): 196–200.

- Alesina, A., and N. Roubini. 1992. "Political Cycles in OECD Countries." *Review of Economics and Statistics* 59(4): 663–688.
- Alesina, A., N. Roubini, G. D. Cohen. 1997. *Political Cycles and the Macroeconomy*. MIT Press, Cambridge, MA.
- Alesina, A., and J. Sachs. 1988. "Political Parties and the Business Cycle in the United States: 1948–1984." *Journal of Money, Credit, and Banking* 20(1): 63–82.
- Alesina, A., and L. Summers. 1993. "Central Bank Independence and Macroeconomic Performance." *Journal of Money, Credit, and Banking* 25(2): 157–162.
- Altig, D., L. J. Christiano, M. Eichenbaum, and J. Lindé. 2011. "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." *Review of Economic Dynamics* 14(2): 225–247.
- Altissimo, F., M. Ehrmann, and F. Smets. 2006. "Inflation Persistence and Price-Setting Behaviour in the Euro Area." European Central Bank Occasional Paper 46.
- Alvarez, F., A. Atkeson, and P. J. Kehoe. 2002. "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets." *Journal of Political Economy* 110(1): 73–112.
- Alvarez, F., and M. Gonzalez-Rozada. 2011. "From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models." University of Chicago Becker Friedman Institute Working Paper.
- Alvarez, F., R. E. Lucas Jr., and W. Weber. 2001. "Interest Rates and Inflation." *American Economic Review* 91(2): 219–225.
- Alvarez, L. J., E. Dhyne, M. M. Hoeberichts, C. Kwapil, H. L. Bihan, P. Lunnemann, F. Martins, R. Sabbatini, H. Stahl, P. Vermeulen, and J. Vilmunen. 2005. "Sticky Prices in the Euro Area: A Summary of New Micro Evidence." European Central Bank Working Paper 563.
- Amato, J. D., and S. Gerlach. 2002. "Inflation Targeting in Emerging Market and Transition Economies." *European Economic Review* 46(4–5): 781–790.
- Ammer, J., and R. T. Freeman. 1995. "Inflation Targeting in the 1990s: The Experiences of New Zealand, Canada and the United Kingdom." *Journal of Economics and Business* 47(2): 165–192.
- Andersen, L., and J. Jordon. 1968. "Monetary and Fiscal Actions: A Test of their Relative Importance in Economic Stabilization." Federal Reserve Bank of St. Louis *Review* 50(Nov.): 11–24.
- Andersen, T. M. 1989. "Credibility of Policy Announcements: The Output and Inflation Costs of Disinflationary Policies." *European Economic Review* 33(1): 13–30.
- Andrés, J., R. Domenech, and J. Ferri. 2006. "Price Rigidity and the Volatility of Vacancies and Unemployment." International Economics Institute, University of Valencia, Working Paper 06-01.
- Andrés, J., J. D. López-Salido, and E. Nelson. 2004. "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium." *Journal of Money, Credit, and Banking* 36(4): 665–690.
- Ang, A., and M. Piazzesi. 2003. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics* 50: 745–787.
- Angeletos, G.-M. 2004. "Comment on Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach." In *NBER Macroeconomics Annual 2003*, 350–361. Cambridge, MA: MIT Press.
- Angelini, P. 1994a. "More on the Behavior of the Interest Rates and the Founding of the Fed." *Journal of Monetary Economics* 34(3): 537–553.
- . 1994b. "Testing for Structural Breaks: Trade-off between Power and Spurious Effects." *Journal of Monetary Economics* 34(3): 561–566.
- Angeloni, I., M. Aucremanne, M. Ehrmann, J. Galí, A. Levin, and F. Smets. 2006. "New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modelling." *Journal of the European Economic Association*, 4(2-3): 562–574.
- Aoki, K. 2002. "Optimal Monetary Policy Response to Relative Price Changes." *Journal of Monetary Economics* 48: 55–80.
- Aruoba, S. B., and F. Schorfheide. 2013. "Macroeconomic Dynamics Near the ZLB: A Tale of Two Equilibria." Federal Reserve Bank of Philadelphia Working Paper 13–29.
- Asako, K. 1983. "The Utility Function and the Superneutrality of Money on the Transition Path." *Econometrica* 51(5): 1593–1596.

- Ascari, G. 2000. "Optimizing Agents, Staggered Wages and Persistence in the Real Effects of Money Shocks." *The Economic Journal* 110: 664–686.
- . 2004. "Staggered Prices and Trend Inflation: Some Nuisances." *Review of Economic Dynamics* 7: 642–667.
- Ascari, G., and T. Ropele. 2007. "Optimal Monetary Policy Under Low Trend Inflation." *Journal of Monetary Economics* 54(8): 2568–2583.
- Ascari, G., and A. M. Sbordone. 2014. "The Macroeconomics of Trend Inflation." *Journal of Economic Literature* 52(3): 679–739.
- Ashcraft, A., J. McAndrews, and D. Skeie. 2011. "Precautionary Reserves and the Interbank Market." *Journal of Money, Credit, and Banking* 43(7): 311–348.
- Athey, S., A. Atkeson, and P. J. Kehoe. 2005. "The Optimal Degree of Discretion in Monetary Policy." *Econometrica* 73(5): 1431–1475.
- Atkinson, A. B., and J. E. Stiglitz. 1972. "The Structure of Indirect Taxation and Economic Efficiency." *Journal of Public Economics* 1(1): 97–119.
- Attfield, C., D. Demery, and N. W. Duck. 1991. *Rational Expectations in Macroeconomics*. 2nd ed. Oxford: Blackwell.
- Auernheimer, L. 1974. "The Honest Government's Guide to the Revenue from the Creation of Money." *Journal of Political Economy* 82(3): 598–606.
- Backus, D. K., and J. Driffill. 1985. "Inflation and Reputation." *American Economic Review* 75(3): 530–538.
- Bae, Y., and R. M. De Jong. 2007. "Money Demand Function Estimation by Nonlinear Cointegration." *Journal of Applied Econometrics* 22(4): 767–793.
- Bailey, M. J. 1956. "The Welfare Costs of Inflationary Finance." *Journal of Political Economy* 64(2): 93–110.
- Balduzzi, P., G. Bertola, S. Foresi, and L. Klapper. 1998. "Interest Rate Targeting and the Dynamics of Short-Term Rates." *Journal of Money, Credit, and Banking* 30(1): 26–50.
- Ball, L. 1991. "The Genesis of Inflation and the Costs of Disinflation." *Journal of Money, Credit, and Banking* 23(3): 439–452.
- . 1993. "How Costly Is Disinflation? The Historical Evidence." Federal Reserve Bank of Philadelphia Business Review (Nov./Dec.): 17–28.
- . 1994. "Credible Disinflation with Staggered Price-Setting." *American Economic Review* 84(1): 282–289.
- . 1995. "Time Consistent Inflation Policy and Persistent Changes in Inflation." *Journal of Monetary Economics* 36(2): 329–350.
- . 2001. "Another Look at Long-Run Money Demand." *Journal of Monetary Economics* 47(1): 31–44.
- Ball, L., N. G. Mankiw, and R. Reis. 2005. "Monetary Policy for Inattentive Economies." *Journal of Monetary Economics* 52(4): 703–725.
- Ball, L., and D. Romer. 1991. "Sticky Prices as Coordination Failure." *American Economic Review* 81(3): 539–552.
- Bank for International Settlements. 2007. "Monetary Policy Frameworks and Central Bank Market Operations."
- Barlevy, G. 2007. "Economic Theory and Asset Bubbles." Federal Reserve Bank of Chicago *Economic Perspectives* 31(3): 44–59.
- Barnett, W. A. 1980. "Economic Monetary Aggregates: An Application of Index Number and Aggregation Theory." *Journal of Econometrics* 14(1): 11–48.
- Barnett, W. A., J. Liu, R. S. Mattson, and J. van den Noort. 2013. "The New CFS Divisia Monetary Aggregates: Design, Construction, and Data Sources." *Open Economy Review* 24: 101–124.
- Barr, D. G., and J. Y. Campbell. 1997. "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices." *Journal of Monetary Economics* 39(3): 361–383.
- Barro, R. J. 1974. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82(6): 1095–1118.
- . 1976. "Rational Expectations and the Role of Monetary Policy." *Journal of Monetary Economics* 2(1): 1–32.
- . 1977. "Unanticipated Money Growth and Unemployment in the United States." *American Economic Review* 67(1): 101–115.

- . 1978. “Unanticipated Money, Output, and the Price Level in the United States.” *Journal of Political Economy* 86(4): 549–580.
- . 1979a. “On the Determination of the Public Debt.” *Journal of Political Economy* 87(5): 940–971.
- . 1979b. “Unanticipated Money Growth and Unemployment in the United States: Reply.” *American Economic Review* 69(5): 1004–1009.
- . 1981. *Money, Expectations, and Business Cycles*. New York: Academic Press.
- . 1986. “Reputation in a Model of Monetary Policy with Incomplete Information.” *Journal of Monetary Economics* 17(1): 3–20.
- . 1995. “Inflation and Economic Growth.” Bank of England *Quarterly Bulletin* (May): 39–52.
- . 1996. “Inflation and Growth.” Federal Reserve Bank of St. Louis *Review* 78(3): 153–169.
- Barro, R. J., and D. B. Gordon. 1983a. “A Positive Theory of Monetary Policy in a Natural-Rate Model.” *Journal of Political Economy* 91(4): 589–610.
- . 1983b. “Rules, Discretion and Reputation in a Model of Monetary Policy.” *Journal of Monetary Economics* 12(1): 101–121.
- Barro, R. J., and R. G. King. 1984. “Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles.” *Quarterly Journal of Economics* 99(4): 817–839.
- Barro, R. J., and M. Rush. 1980. “Unanticipated Money and Economic Activity.” In *Rational Expectations and Economic Policy*, ed. S. Fischer, 23–48. Chicago: University of Chicago Press.
- Barth, M.J.I., and V. A. Ramey. 2002. “The Cost Channel of Monetary Transmission.” In *NBER Macroeconomic Annual 2001*, 199–239. Cambridge, MA: MIT Press.
- Bartolini, L., G. Bertola, and A. Prati. 2002. “Day-to-Day Monetary Policy and the Volatility of the Federal Funds Rate.” *Journal of Money, Credit, and Banking* 34(1): 136–159.
- Bassetto, M. 2002. “A Game-Theoretic View of the Fiscal Theory of the Price Level.” *Econometrica* 70(6): 2167–2195.
- Batini, N., and D. Laxton. 2007. “Under What Conditions Can Inflation Targeting Be Adopted?” In *Monetary Policy under Inflation Targeting*, ed. F. S. Mishkin and K. Schmidt-Hebbel, 467–506. Santiago: Banco Central de Chile.
- Batini, N., and A. Yates. 2003. “Hybrid Inflation and Price Level Targeting.” *Journal of Money, Credit, and Banking* 35(3): 283–300.
- Batten, D. S., M. P. Blackwell, I. S. Kim, S. E. Nocera, and Y. Ozeki. 1990. “The Conduct of Monetary Policy in the Major Industrial Countries: Instruments and Operating Procedures.” International Monetary Fund Occasional Paper 70.
- Bauer, M., and G. Rudebusch. 2013. “The Signaling Channel for Federal Reserve Bond Purchases.” Federal Reserve Bank of San Francisco *Proceedings* (April).
- Baumeister, C., and L. Benati. 2013. “Unconventional Monetary Policy and the Great Recession—Estimating the Impact of a Compression in the Yield Spread at the Zero Lower Bound.” *International Journal of Central Banking* 9(2): 165–212.
- Baumol, W. 1952. “The Transactions Demand for Cash.” *Quarterly Journal of Economics* 67(4): 545–556.
- Bean, C. 1983. “Targeting Nominal Income: An Appraisal.” *Economic Journal* 93: 806–819.
- Beaudry, P., and M. B. Devereux. 1995. “Monopolistic Competition, Price Setting, and the Effects of Real and Monetary Shocks.” Working Paper.
- Bech, M. L., and E. Klee. 2011. “The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market.” *Journal of Monetary Economics* 58(5): 415–431.
- Beck, G. W., and V. Wieland. 2007. “Money in Monetary Policy Design: A Formal Characterization of European Central Bank-Style Cross-Checking.” *Journal of the European Economic Association* 5(2-3): 524–533.
- Beetsma, R., and H. Jensen. 1998. “Inflation Targets and Contracts with Uncertain Central Banker Preferences.” *Journal of Money, Credit, and Banking* 30(3): 384–403.
- Belongia, M. T., and P. N. Ireland. 2016. “Money and Output: Friedman and Schwartz Revisited.” *Journal of Money, Credit, and Banking* 48(6): 1223–1266.
- Benassy, J.-P. 1995. “Money and Wage Contracts in an Optimizing Model of the Business Cycle.” *Journal of Monetary Economics* 35(2): 303–315.

- Benhabib, J., S. Schmitt-Grohé, and M. Uribe 2001a. "Monetary Policy and Multiple Equilibria." *American Economic Review* 91(1): 167–186.
- . 2001b. "The Perils of Taylor Rules." *Journal of Economic Theory* 96: 40–69.
- . 2002. "Avoiding Liquidity Traps." *Journal of Political Economy* 110(3): 535–563.
- Benigno, G., and P. Benigno. 2003. "Price Stability in Open Economies." *Review of Economic Studies* 70(4): 743–764.
- . 2008. "Implementing International Monetary Cooperation through Inflation Targeting." *Macroeconomic Dynamics* 12(S1): 45–59.
- Benigno, P. 2004. "Optimal Monetary Policy in a Currency Area." *Journal of International Economics* 63(2): 293–320.
- Benigno, P., and S. Nistico. 2015. "Non-Neutrality of Open-Market Operations." Working Paper.
- Benigno, P., and M. Woodford. 2004. "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach." In *NBER Macroeconomics Annual 2003*, 271–333. Cambridge, MA: MIT Press.
- . 2005. "Inflation Stabilization and Welfare: The Case of a Distorted Steady state." *Journal of the European Economics Association* 3: 1185–1236.
- Berentsen, A., A. Marchesiani, and C. J. Waller. 2014. "Floor Systems for Implementing Monetary Policy: Some Unpleasant Fiscal Arithmetic." *Review of Economic Dynamics* 17(3): 523–542.
- Berentsen, A., G. Menzies, and R. Wright. 2011. "Inflation and Unemployment in the Long Run." *American Economic Review* 101(1): 371–398.
- Berentsen, A., and C. Monnet. 2008. "Monetary Policy in a Channel System." *Journal of Monetary Economics* 55(6): 1067–1080.
- Bernanke, B. S. 1986. "Alternative Explanations of the Money-Income Correlation." *Carnegie-Rochester Conference Series on Public Policy* 25: 49–99.
- . 1993. "Credit in the Macroeconomy." Federal Reserve Bank of New York *Quarterly Review* 18(1): 50–70.
- . 2005. "What Have We Learned Since October 1979?" Federal Reserve Bank of St. Louis *Review*, 87(Mar.): 277–292.
- Bernanke, B. S., and A. S. Blinder. 1992. "The Federal Funds Rate and the Channels of Monetary Transmission." *American Economic Review* 82(4): 901–921.
- Bernanke, B. S., and M. Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review* 79(1): 14–31.
- Bernanke, B. S., M. Gertler, and S. Gilchrist. 1996. "The Financial Accelerator and the Flight to Quality." *Review of Economics and Statistics* 78(1): 1–15.
- . 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, ed. J. B. Taylor and M. Woodford, vol. 1, 1341–1393. Amsterdam: Elsevier.
- Bernanke, B. S., and K. N. Kuttner. 2005. "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *Journal of Finance* 60(3): 1221–1257.
- Bernanke, B. S., T. Laubach, F. S. Mishkin, and A. Posen (1998): *Inflation Targeting: Lessons from the International Experience*. Princeton, NJ: Princeton University Press.
- Bernanke, B. S., and I. Mihov. 1998. "Measuring Monetary Policy." *Quarterly Journal of Economics* 113(3): 869–902.
- Bernanke, B. S., and F. S. Mishkin. 1992. "Central Bank Behavior and the Strategy of Monetary Policy: Observations from Six Industrialized Countries." In *NBER Macroeconomics Annual 1992*, 183–228. Cambridge, MA: MIT Press.
- . 1997. "Inflation Targeting: A New Framework for Monetary Policy?" *Journal of Economic Perspectives* 11: 97–116.
- Bernanke, B. S., and V. R. Reinhart. 2004. "Conducting Monetary Policy at Very Low Short-Term Interest Rates." *American Economic Review* 94(2): 85–90.
- Bernanke, B. S., V. R. Reinhart, and B. P. Sack. 2004. "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment." *Brookings Papers on Economic Activity* 35(2): 1–100.

- Bernanke, B. S., and M. Woodford. 1997. "Inflation Forecasts and Monetary Policy." *Journal of Money, Credit, and Banking* 29(4): 653–684.
- Betts, C., and M. Devereux. 2000. "Exchange Rate Dynamics in a Model with Pricing-to-Market." *Journal of Monetary Economics* 50(1): 215–244.
- Bewley, T. 1983. "A Difficulty with the Optimum Quantity of Money." *Econometrica* 51(5): 1485–1504.
- Billi, R. M. 2015. "A Note on Nominal GDP Targeting and the Zero Lower Bound." Riksbank Working Paper 270.
- Bils, M., and P. J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112(5): 947–985.
- Blake, A. P. 2002. "A 'Timeless Perspective' on Optimality in Forward-Looking Rational Expectations Models." In *Proceedings of the Royal Economic Society Annual Conference*.
- Blake, A. P., and T. Kirsanova. 2012. "Discretionary Policy and Multiple Equilibria in LQ RE Models." *Review of Economic Studies* 79(June): 1309–1339.
- Blanchard, O. J. 1989. "A Traditional Interpretation of Economic Fluctuations." *American Economic Review* 79(5): 1146–1164.
- . 1990. "Why Does Money Affect Output? A Survey." In *Handbook of Monetary Economics*, ed. B. M. Friedman and F. H. Hahn, vol. 2, 779–835. Amsterdam: Elsevier.
- Blanchard, O. J., and S. Fischer. 1989. *Lectures on Macroeconomics*. Cambridge, MA: MIT Press.
- Blanchard, O. J., and J. Galí. 2007. "Real Wage Rigidities and the New Keynesian Model." *Journal of Money, Credit, and Banking* 39(1): 35–65.
- . 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics* 2(2): 1–30.
- Blanchard, O. J., and C. M. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48(5): 1305–1311.
- Blanchard, O. J., and N. Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77(4): 647–666.
- Blanchard, O. J., and D. Quah. 1989. "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review* 79(4): 655–673.
- Blanchard, O. J., and M. W. Watson. 1986. "Are Business Cycles All Alike?" In *The American Business Cycle: Continuity and Change*, ed. R. J. Gordon, pp. 123–165. Chicago: University of Chicago Press.
- Blinder, A. S. 1995. "Central Banking in Theory and Practice: Lecture II: Credibility, Discretion, and Independence." Marshall Lecture, Cambridge University.
- Blinder, A. S., and J. B. Rudd. 2013. "The Supply-Shock Explanation of the Great Stagflation Revisited." In *The Great Inflation: The Rebirth of Modern Central Banking*, ed. M. Bordo, and A. Orphanides, 119–175. Chicago: University of Chicago Press.
- Bodenstein, M., J. Hebden, and R. Nunes. 2012. "Imperfect Credibility and the Zero Lower Bound." *Journal of Monetary Economics* 59(2): 135–149.
- Bohn, H. 1991a. "Budget Balance through Revenue or Spending Adjustments? Some Historical Evidence for the United States." *Journal of Monetary Economics* 27(3): 333–359.
- . 1991b. "On Cash-in-Advance Models of Money Demand and Asset Pricing." *Journal of Money, Credit, and Banking* 23(2): 224–242.
- . 1991c. "Time Inconsistency of Monetary Policy in the Open Economy." *Journal of Monetary Economics* 30(3–4): 249–266.
- . 1992. "Budget Deficits and Government Accounting." *Carnegie-Rochester Conference Series on Public Policy* 37: 1–884.
- . 1999. "Comment: A Frictionless View of U.S. Inflation." In *NBER Macroeconomics Annual 1998*, 384–389. Cambridge, MA: MIT Press.
- . 2007. "Are Stationarity and Cointegration Restrictions Really Necessary for the Intertemporal Budget Constraint?" *Journal of Monetary Economics* 54: 1837–1847.
- Boianovsky, M. 2002. "Simonsen and the Early History of the Cash-in-Advance Approach." *European Journal of the History of Economic Thought* 9(1): 57–71.



- Boivin, J., and M. P. Giannoni. 2006. "Has Monetary Policy Become More Effective?" *Review of Economics and Statistics* 88(3): 445–462.
- Boivin, J., M. Kiley, and F. Mishkin. 2010. "How Has the Monetary Transmission Mechanism Evolved over Time?" In *Handbook of Monetary Economics*, ed. B. M. Friedman and M. Woodford, vol. 3, 369–422. Amsterdam: Elsevier.
- Borio, C. 1997. "The Implementation of Monetary Policy in Industrial Countries: A Survey." Bank for International Settlements Economic Paper 47.
- Borio, C., and P. Disyatat. 2010. "Unconventional Monetary Policies: An Appraisal." *The Manchester School*, 78(S1): 1–134.
- Boschen, J. F., and L. O. Mills. 1991. "The Effects of Countercyclical Monetary Policy on Money and Interest Rates: An Evaluation of Evidence from FOMC Documents." Federal Reserve Bank of Philadelphia Working Paper 91-20.
- . 1995a. "Tests of Long-run Neutrality Using Permanent Monetary and Real Shocks." *Journal of Monetary Economics* 35(1): 25–44.
- . 1995b. "The Relation between Narrative and Money Market Indicators of Monetary Policy." *Economic Inquiry* 33(1): 24–44.
- Brainard, W. 1967. "Uncertainty and the Effectiveness of Policy." *American Economic Review* 57(2): 411–425.
- Braun, R. A. 1991. "Comment on Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit, and Banking* 23(3): 542–546.
- Braun, R. A., L. Körber, and Y. Waki. 2012. "Some Unpleasant Properties of Log-Linearized Solutions When the Nominal Rate Is Zero." Federal Reserve Bank of Atlanta Working Paper 2012-5a.
- Brayton, F., and P. Tinsley. 1996. "A Guide to the FRB/US: A Macroeconomic Model of the United States." Federal Reserve Board, Finance and Economics Discussion Series, 1942–1996.
- Briault, C., A. Haldane, and M. King. 1996. "Independence and Accountability." London: Bank of England.
- Broadbuss, A., and M. Goodfriend. 1984. "Base Drift and the Longer Run Growth of M1: Evidence from a Decade of Monetary Targeting." Federal Reserve Bank of Richmond *Economic Review* 70(6): 3–14.
- Brock, W. A. 1974. "Money and Growth: The Case of Long Run Perfect Foresight." *Journal of Economic Dynamics and Control* 15(3): 750–777.
- . 1975. "A Simple Perfect Foresight Monetary Model." *Journal of Monetary Economics* 1(2): 133–150.
- . 1990. "Overlapping Generations Models with Money and Transaction Costs." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 1, 263–295. Amsterdam: Elsevier.
- Brunner, K., A. Meltzer, and A. Cukierman. 1980. "Stagflation, Persistent Unemployment, and the Permanence of Economic Shocks." *Journal of Monetary Economics* 6: 467–492.
- Brunnermeier, M., and Y. Sannikov. 2016. "The I Theory of Money." Working Paper.
- Bruno, M., and S. Fischer. 1990. "Seigniorage, Operating Rules, and the High Inflation Trap." *Quarterly Journal of Economics* 105(2): 353–374.
- Buiter, W. 2002. "The Fiscal Theory of the Price Level: A Critique." *Economic Journal* 112(481): 459–480.
- Bullard, J. 1999. "Testing Long-Run Monetary Neutrality Propositions: Lessons from the Recent Research." Federal Reserve Bank of St. Louis *Review* 81(6): 57–77.
- Bullard, J., and J. W. Keating. 1995. "The Long-run Relationship between Inflation and Output in Postwar Economies." *Journal of Monetary Economics* 36(3): 477–496.
- Bullard, J., and K. Mitra. 2002. "Learning about Monetary Policy Rules." *Journal of Monetary Economics* 49(6): 1105–1129.
- Buttiglione, L., P. Del Giovane, and O. Tristani. 1998. "Monetary Policy Actions and the Term Structure of Interest Rates: A Cross-Country Analysis." In *Monetary Policy and Interest Rates*, ed. I. Angeloni and R. Rovelli. New York: St. Martins Press.
- Caballero, R. J., and E. Engel. 2007. "Price Stickiness in Ss Models: New Interpretations of Old Results." *Journal of Monetary Economics* 54(1): 100–121.
- Cagan, P. 1956. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money*, ed. M. Friedman, 25–117. Chicago: University of Chicago Press.

- Calvo, G. A. 1978. "On the Time Consistency of Optimal Policy in a Monetary Economy." *Econometrica* 46(6): 1411–1428.
- . 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12(3): 983–998.
- Calvo, G. A., and P. E. Guidotti. 1993. "On the Flexibility of Monetary Policy: The Case of the Optimal Inflation Tax." *Review of Economic Studies* 60(3): 667–687.
- Calvo, G. A., and L. Leiderman. 1992. "Optimal Inflation Tax Under Precommitment: Theory and Evidence." *American Economic Review* 82(1): 179–194.
- Campbell, J. R., C. L. Evans, J. D. M. Fisher, and A. Justiniano. 2012. "Macroeconomic Effects of Federal Reserve Forward Guidance." *Brookings Papers on Economic Activity* 44(1): 1–80.
- Campbell, J. Y. 1994. "Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model." *Journal of Monetary Economics* 33(3): 463–506.
- Campbell, J. Y., and N. G. Mankiw. 1989. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." In *NBER Macroeconomics Annual 1989*, 185–246. Cambridge, MA: MIT Press.
- . 1991. "The Response of Consumption to Income: A Cross-Country Investigation." *European Economic Review* 35(4): 723–756.
- Campbell, J. Y., and R. J. Shiller. 1991. "Yield Spreads and Interest Rate Movements: A Bird's Eye View." *Review of Economic Studies* 58(3): 495–514.
- Candel-Sanchez, F., and J. C. Campoy-Minarroy. 2004. "Is the Walsh Contract Really Optimal?" *Public Choice* 120(1-2): 29–39.
- Canova, F., and L. Gambetti. 2009. "Structural Change in the U.S. Economy: Is there a Role for Monetary Policy?" *Journal of Economic Dynamics and Control* 33: 477–490.
- Canzoneri, M. B. 1985. "Monetary Policy Games and the Role of Private Information." *American Economic Review* 75: 1056–1070.
- Canzoneri, M. B., R. E. Cumby, and B. T. Diba. 2001. "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review* 91(5): 1221–1238.
- . 2007. "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models." *Journal of Monetary Economics* 54(7): 1863–1881.
- . 2011. "The Interaction between Monetary and Fiscal Policy." In *Handbook Of Monetary Economics*, ed. B. M. Friedman, and M. Woodford, vol. 3B, 935–999. Amsterdam: Elsevier.
- Canzoneri, M. B., and H. Dellas. 1998. "Real Interest Rates and Central Bank Operating Procedures." *Journal of Monetary Economics* 42(3): 471–494.
- Canzoneri, M. B., and D. Henderson. 1989. *Noncooperative Monetary Policies in Interdependent Economies*. Cambridge, MA: MIT Press.
- Canzoneri, M. B., D. Henderson, and K. Rogoff. 1983. "The Information Content of the Interest Rate and Optimal Monetary Policy." *Quarterly Journal of Economics* 98(4): 545–566.
- Canzoneri, M. B., C. Nolan, and A. Yates. 1997. "Mechanisms for Achieving Monetary Stability: Inflation Targeting versus the ERM." *Journal of Money, Credit, and Banking* 29(1): 46–60.
- Caplin, A., and D. Spulber. 1987. "Menu Costs and the Neutrality of Money." *Quarterly Journal of Economics* 102: 703–726.
- Carare, A., and M. R. Stone. 2006. "Inflation Targeting Regimes." *European Economic Review* 50: 1297–1315.
- Cargill, T. F., M. M. Hutchison, and T. Ito. 1997. *The Political Economy of Japanese Monetary Policy*. Cambridge, MA: MIT Press.
- Carlson, K. M. 1978. "Does the St. Louis Equation Now Believe in Fiscal Policy?" *Federal Reserve Bank of St. Louis Review* 60(2): 13–19.
- Carlstrom, C. T., and T. S. Fuerst. 1995. "Interest Rate Rules vs. Money Growth Rules: A Welfare Comparison in a Cash-in-Advance Economy." *Journal of Monetary Economics* 36(2): 247–267.
- . 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review* 87(5): 893–910.
- . 2000. "The Fiscal Theory of the Price Level." *Federal Reserve Bank of Cleveland Economic Review* 36(1): 22–32.

- . 2001. “Timing and Real Indeterminacy in Monetary Models.” *Journal of Monetary Economics* 47(2): 285–298.
- . 2009. “Central Bank Independence and Inflation: a Note.” *Economic Inquiry* 47(1): 182–186.
- Carlstrom, C. T., T. S. Fuerst, and M. Paustian. 2012a. “How Inflationary Is an Extended Period of Low Interest Rates?” Federal Reserve Bank of Cleveland Working Paper 1202.
- . 2012b. “Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg.” Bank of England Working Paper 459.
- . 2014. “Targeting Long Rates in a Model with Segmented Markets.” Federal Reserve Bank of Cleveland Working Paper 14–19.
- . 2016. “Optimal Contracts, Aggregate Risk, and the Financial Accelerator.” *American Economic Journal: Macroeconomics* 8(1): 119–147.
- Cateau, G., O. Kryvtsov, M. Malik Shukayev, and A. Ueberfeld. 2009. “Adopting Price-Level Targeting under Imperfect Credibility in ToTEM.” Bank of Canada Working Paper 2009-17.
- Cecchetti, S. G. 2009. “Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis.” *Journal of Economic Perspectives* 23(1): 51–75.
- Champ, B., S. Freeman, and J. Haslag. 2016. *Modeling Monetary Economics*. 4th ed. Cambridge: Cambridge University Press.
- Chari, V. V., L. J. Christiano, and M. Eichenbaum. 1995. “Inside Money, Outside Money and Short-Term Interest Rates.” *Journal of Money, Credit, and Banking* 27(4): 1354–1386.
- Chari, V. V., L. J. Christiano, and P. J. Kehoe. 1991. “Optimal Fiscal and Monetary Policy: Some Recent Results.” *Journal of Money, Credit, and Banking* 23(3): 519–539.
- . 1996. “Optimality of the Friedman Rule in Economies with Distorting Taxes.” *Journal of Monetary Economics* 37(2): 203–223.
- Chari, V. V., and P. J. Kehoe. 1990. “Sustainable Plans.” *Journal of Political Economy* 98(4): 783–802.
- . 1999. “Optimal Fiscal and Monetary Policy.” In *Handbook of Macroeconomics*, ed. J. B. Taylor, and M. Woodford, vol. 1C, 1671–1745. Amsterdam: Elsevier.
- Chari, V. V., P. J. Kehoe, and E. R. McGrattan. 2000. “Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?” *Econometrica* 68(5): 1151–1179.
- . 2009. “New Keynesian Models: Not Yet Useful for Policy Analysis.” *American Economic Journal: Macroeconomics* 1(1): 242–266.
- Chen, H., V. Cúrdia, and A. Ferrero. 2012. “The Macroeconomic Effects of Large-Scale Asset Purchase Programmes.” *Economic Journal*, 122(Sept.): F289–F315.
- Chiang, A. C. 1992. *Elements of Dynamic Optimization*. New York: McGraw-Hill.
- Chortareas, G. E., and S. M. Miller. 2003. “Monetary Policy Delegation, Contract Costs, and Contract Targets.” *Bulletin of Economic Research* 55(1): 101–112.
- . 2007. “The Walsh Contracts for Central Bankers Prove Optimal After All.” *Public Choice* 131(1): 243–247.
- Christensen, J., and G. Rudebusch. 2012. “The Response of Interest Rates to US and UK Quantitative Easing.” *Economic Journal* 122: 385–414.
- Christiano, L. J. 1991. “Modelling the Liquidity Effect of a Money Shock.” Federal Reserve Bank of Minneapolis *Quarterly Review* 15(1): 3–34.
- Christiano, L. J., and M. Eichenbaum. 1992a. “Current Real Business Cycle Theories and Aggregate Labor-Market Fluctuations.” *American Economic Review* 82(3): 430–450.
- . 1992b. “Liquidity Effects and the Monetary Transmission Mechanism.” *American Economic Review* 82(2): 346–353.
- . 1995. “Liquidity Effects, Monetary Policy, and the Business Cycle.” *Journal of Money, Credit, and Banking* 27(4): 1113–1136.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans. 1996. “The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds.” *Review of Economics and Statistics* 78(1): 16–34.

- . 1997. “Sticky Prices and Limited Participation Models: A Comparison.” *European Economic Review* 41(6): 1201–1249.
- . 1999. “Monetary Policy Shocks: What Have We Learned and to What End?” In *Handbook of Macroeconomics*, ed. J. B. Taylor, and M. Woodford, vol. 1A, 65–148. Amsterdam: Elsevier.
- . 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy* 113(1): 1–45.
- Christiano, L. J., M. Eichenbaum, and S. Rebelo. 2011. “When is the Government Spending Multiplier Large?” *Journal of Political Economy* 119(1): 78–121.
- Christiano, L. J., M. Eichenbaum, and M. Trabandt. 2013. “Unemployment and Business Cycles.” NBER Working Paper 19265.
- . 2014. “Understanding the Great Recession.” NBER Working Paper 20040.
- Christiano, L. J., and T. J. Fitzgerald. 2000. “Understanding the Fiscal Theory of the Price Level.” Federal Reserve Bank of Cleveland *Economic Review* 36(2): 1–38.
- Christiano, L. J., R. Motto, and M. Rostagno. 2010. “Financial Factors in Economic Fluctuations.” European Central Bank Working Paper. 1192.
- Christiano, L. J., and T. Zha. 2010. “A Simplified Two-Period Version of the Gertler-Kiyotaki Model.” [http://faculty.wcas.northwestern.edu/~lchrist/research/GK\\_comment/model.pdf](http://faculty.wcas.northwestern.edu/~lchrist/research/GK_comment/model.pdf).
- Chung, H., J.-P. Laforte, D. Reifschneider, and J. C. Williams. 2012. “Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?” *Journal of Money, Credit, and Banking* 44: 47–82.
- Clarida, R., and M. Gertler. 1997. “How Does the Bundesbank Conduct Monetary Policy.” In *Reducing Inflation*, ed. C. Romer, and D. Romer, 363–406. Chicago: University of Chicago Press.
- Clarida, R. H., J. Galí, and M. Gertler. 1998. “Monetary Policy Rules in Practice: Some International Evidence.” *European Economic Review* 42: 1033–1067.
- . 1999. “The Science of Monetary Policy: A New Keynesian Perspective.” *Journal of Economic Literature* 37(4): 1661–1707.
- . 2000. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *Quarterly Journal of Economics* 115(1): 147–180.
- . 2001. “Optimal Monetary Policy in Closed versus Open Economies: An Integrated Approach.” *American Economic Review* 91(2): 248–252.
- . 2002. “A Simple Framework for International Monetary Policy Analysis.” *Journal of Monetary Economics* 49(5): 879–904.
- Clower, R. W. 1967. “A Reconsideration of the Microfoundations of Monetary Theory.” *Western Economic Journal* 6(1): 1–9.
- Cochrane, J. H. 1998. “What Do the VARs Mean? Measuring the Output Effects of Monetary Policy.” *Journal of Monetary Economics* 41(2): 277–300.
- . 1999. “A Frictionless View of U.S. Inflation.” in *NBER Macroeconomics Annual 1998*, 323–384. Cambridge, MA: MIT Press.
- . 2001. “Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level.” *Econometrica* 69(1): 69–116.
- . 2011a. “Determinacy and Identification with Taylor Rules.” *Journal of Political Economy* 119(3): 565–615.
- . 2011b. “Understanding Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic.” *European Economic Review* 55(1): 2–30.
- . 2013. “The New-Keynesian Liquidity Trap.” NBER Working Paper No. 19476.
- . 2014. “Monetary Policy with Interest on Reserves.” *Journal of Economic Dynamics and Control* 49: 74–108.
- Cochrane, J. H., and M. Piazzesi. 2002. “The Fed and Interest Rates: A High Frequency Identification.” *American Economic Review* 92(1): 90–95.
- Cogley, T., and J. M. Nason. 1995. “Output Dynamics in Real Business Cycle Models.” *American Economic Review* 85(3): 492–511.

- Cogley, T., and A. M. Sbordone. 2008. "Trend Inflation and Inflation Persistence in the New Keynesian Phillips Curve." *American Economic Review* 98(5): 2101–2126.
- Coibion, O. 2010. "Testing the Sticky Information Phillips Curve." *Review of Economics and Statistics* 92(1): 87–101.
- . 2012. "Are the Effects of Monetary Policy Shocks Big or Small?" *American Economic Journal: Macroeconomics* 4(2): 1–32.
- Coibion, O., and Y. Gorodnichenko. 2011. "Monetary Policy, Trend Inflation and the Great Moderation: An Alternative Interpretation." *American Economic Review* 101(1): 341–370.
- . 2015. "Information Rigidity and the Expectations Formation Process : A Simple Framework and New Facts." *American Economic Review* 105(8): 2644–2678.
- Coibion, O., Y. Gorodnichenko, and J. Wieland. 2012. "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?" *Review of Economic Studies* 79(4): 1371–1406.
- Cole, H. L., and L. E. Ohanian. 2002. "Shrinking Money: The Demand for Money and the Non-neutrality of Money." *Journal of Monetary Economics* 49(4): 653–686.
- Coleman, W.J.I. 1996. "Money and Output: A Test of Reverse Causation." *American Economic Review* 86(1): 90–111.
- Collard, F., and H. Dellas. 2005. "Poole in the New Keynesian Model." *European Economic Review* 49(4): 887–907.
- Cook, T. 1989. "Determinants of the Federal Funds Rate: 1979-1982." Federal Reserve Bank of Richmond *Economic Review* 75(1): 3–19.
- Cook, T., and T. Hahn. 1989. "The Effect of Interest Rate Changes in the Federal Funds Rate Target on Market Interest Rates in the 1970s." *Journal of Monetary Economics* 24(3): 331–351.
- Cooley, T. F., ed. 1995. *Frontiers of Business Cycle Research*, Princeton, NJ: Princeton University Press.
- Cooley, T. F., and G. D. Hansen. 1989. "The Inflation Tax in a Real Business Cycle Model." *American Economic Review* 79(4): 733–748.
- . 1991. "The Welfare Costs of Moderate Inflation." *Journal of Money, Credit, and Banking*, 23(3 pt 2): 483–503.
- . 1995. "Money and the Business Cycle." In *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, 175–216. Princeton, NJ: Princeton University Press.
- Cooley, T. F., and E. Prescott. 1995. "Economic Growth and Business Cycles." In *Frontiers of Business Cycle Research*, ed. T. F. Cooley, 1–38. Princeton, NJ: Princeton University Press.
- Cooley, T. F., and V. Quadrini. 1999. "A Neoclassical Model of the Phillips Curve Relation." *Journal of Monetary Economics* 44(2): 165–193.
- Correia, I., E. Farhi, J. P. Nicolini, and P. Teles. 2013. "Unconventional Fiscal Policy at the Zero Bound." *American Economic Review* 103(4): 1172–1211.
- Correia, I., and P. Teles. 1996. "Is the Friedman Rule Optimal When Money Is an Intermediate Good?" *Journal of Monetary Economics* 38(2): 223–244.
- . 1999. "The Optimal Inflation Tax." *Review of Economic Dynamics* 2: 325–346.
- Corsetti, G., L. Dedola, and S. Leduc. 2010. "Optimal Monetary Policy in Open Economies." In *Handbook of Monetary Economics*, ed. B. M. Friedman and M. Woodford, vol. 3, 861–933. Amsterdam: Elsevier.
- Corsetti, G., and P. Presenti. 2001. "Welfare and Macroeconomic Interdependence." *Quarterly Journal of Economics* 116(2): 421–445.
- . 2002. *International Dimensions of Optimal Monetary Policy*. University of Rome III and Federal Reserve Bank of New York.
- . 2005. "The International Dimension of Optimal Monetary Policy." *Journal of Monetary Economics* 52(2): 281–305.
- Cosimano, T. F., and R. G. Sheehan. 1994. "The Federal Reserve Operating Procedure, 1984–1990: An Empirical Analysis." *Journal of Macroeconomics* 16(4): 573–588.
- Costain, J., and A. Nakov. 2011. "Price Adjustment in a General Model of State-Dependent Pricing Model." *Journal of Money, Credit, and Banking*, 43(2-3), 385–406.

- . 2014. “Logit Price Dynamics.” European Central Bank Working Paper 1693.
- . 2015. “Precautionary Price Stickiness.” *Journal of Economic Dynamics and Control* 58: 218–234.
- Costain, J., and M. Reiter. 2008. “Business Cycles, Unemployment Insurance, and the Calibration of Matching Models.” *Journal of Economic Dynamics and Control* 32(4), 1120–1155.
- Cover, J. P. 1992. “Asymmetric Effects of Positive and Negative Money Supply Shocks.” *Quarterly Journal of Economics* 107(4): 1261–1282.
- Cox, J., J. Ingersoll, and S. Ross. 1985. “A Theory of the Term Structure of Interest Rates.” *Econometrica* 53(2): 385–408.
- Craig, B., and G. Rocheteau. 2008. “Inflation and Welfare : A Search Approach.” *Journal of Money, Banking, and Credit* 40(1): 89–119.
- Croushore, D. 1993. “Money in the Utility Function: Functional Equivalence to a Shopping-Time Model.” *Journal of Macroeconomics* 15(1): 175–182.
- Cubitt, R. P. 1992. “Monetary Policy Games and Private Sector Precommitment.” *Oxford Economic Papers* 44(3): 513–530.
- Cukierman, A. 1992. *Central bank Strategy, Credibility, and Independence: Theory and Evidence*. Cambridge, MA: MIT Press.
- . 2002. “Are Contemporary Central Banks Transparent about Economic Models and Objectives and What Difference Does it Make?” *Federal Reserve Bank of St. Louis Review* 84(4): 15–35.
- Cukierman, A., and S. Gerlach. 2003. “The Inflation Bias Revisited: Theory and Some International Evidence.” *The Manchester School* 71(5): 541–565.
- Cukierman, A., and F. Lippi. 2001. “Labour Markets and Monetary Union: A Strategic Analysis.” *Economic Journal* 111(473): 541–565.
- Cukierman, A., and N. Liviatan. 1991. “Optimal Accommodation by Strong Policymakers under Incomplete Information.” *Journal of Monetary Economics* 27(1): 99–127.
- Cukierman, A., and A. Meltzer. 1986. “A Theory of Ambiguity, Credibility and Inflation under Discretion and Asymmetric Information.” *Econometrica* 54(5): 1099–1128.
- Cúrdia, V., and M. Woodford. 2010. “Credit Spreads and Monetary Policy.” *Journal of Money, Credit, and Banking*, 42(S1), 3–35.
- . 2011. “The Central-Bank Balance Sheet as an Instrument of Monetary Policy.” *Journal of Monetary Economics* 58(1): 54–79.
- Currie, D., and P. Levine. 1991. “The International Coordination of Monetary Policy: A Survey.” In *Surveys in Monetary Economics*, ed. C. Green and D. Llewellyn, vol. 1, 379–417. Oxford: Blackwell.
- Dahlquist, M., and L. E. O. Svensson. 1996. “Estimating the Term Structure of Interest Rates for Monetary Policy Analysis.” *Scandinavian Journal of Economics* 98(2): 163–183.
- Dai, Q., and K. Singleton. 2000. “Specification Analysis of Affine Term Structure Models.” *Journal of Finance* 55: 1943–1978.
- D’Amico, S., W. English, D. López-Salido, and E. Nelson 2012. “The Federal Reserve’s Large Scale Asset Purchase Programs: Rationale and Effects.” *Economic Journal*. 122(564): F415–F446.
- Damjanovic, T., V. Damjanovic, and C. Nolan. 2008. “Unconditional Optimal Monetary Policy.” *Journal of Monetary Economics* 55: 491–500.
- Daniel, B. 2001. “The Fiscal Theory of the Price Level in an Open Economy.” *Journal of Monetary Economics* 48(2): 293–308.
- . 2007. “The Fiscal Theory of the Price Level and Initial Government Debt.” *Review of Economic Dynamics* 10(2): 193–206.
- de Carvalho, C. V. 2006. “Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks.” *B.E. Journal of Macroeconomics: Frontiers* 6(3): 1–58.
- De Fiore, F., and O. Tristani. 2013. “Optimal Monetary Policy in a Model of the Credit Channel.” *Economic Journal* 123(571): 906–931.
- De Gregorio, J. 1993. “Inflation, Taxation, and Long-Run Growth.” *Journal of Monetary Economics* 31(3): 271–298.

- De Prano, M., and T. Mayer. 1965. "Tests of the Relative Importance of Autonomous Expenditures and Money." *American Economic Review* 55(4): 729–752.
- Debortoli, D., J. Kim, J. Linde, and R. Nunes. 2015. "Designing a Simple Loss Function for the Fed: Does the Dual Mandate Make Sense?" *Working Paper*.
- DeJong, D. N., and C. Dave. 2011. *Structural Macroeconometrics*. 2nd ed. Princeton, NJ: Princeton University Press.
- DelNegro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki. 2016. "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities." NBER Working Paper No. 22259.
- DelNegro, M., M. Giannoni, and C. Patterson. 2012. "The Forward Guidance Puzzle." Federal Reserve Bank of New York Staff Reports 574.
- Demirel, U. D. 2009. "Optimal Monetary Policy in a Financially Fragile Economy." *B.E. Journal of Macroeconomics* 9(1): 1–37.
- Demopoulos, G. D., G. M. Katsimbris, and S. M. Miller. 1987. "Monetary Policy and Central-Bank Financing of Government Budget Deficits." *European Economic Review* 31(5): 1023–1050.
- den Haan, W. J., G. Ramey, and J. Watson. 2000. "Job Destruction and Propagation of Shocks." *American Economic Review* 90(3): 482–498.
- Dennis, R. 2006. "The Frequency of Price Adjustment and New Keynesian Business Cycle Dynamics." Federal Reserve Bank of San Francisco Working Paper 2006-22.
- Dennis, R., and T. Kirsanova. 2013. "Expectations Traps and Coordination Failures with Discretionary Policy-making." University of Glasgow Business School Working Paper 2013-02.
- Devereux, M. B., and J. Yetman. 2014. "Capital Controls, Global Liquidity Traps, and the International Policy Trilemma." *Scandinavian Journal of Economics* 116(1): 158–189.
- Dhyne, E., L. J. Alvarez, H. L. Bihan, G. Veronese, D. Dias, J. Hoffmann, N. Jonker, P. Lunnemann, F. Rumler, and J. Vilmunen. 2006. "Price Setting in the Euro Area and the United States: Some Facts From Individual Consumer Price Data." *Journal of Economic Perspectives* 20(2): 171–192.
- Diamond, P. A. 1984. "Money in Search Equilibrium." *Econometrica* 52(1): 1–20.
- Diamond, P. A., and J. A. Mirrlees. 1971. "Optimal Taxation and Public Production I: Production and Efficiency; II: Tax Rules." *American Economic Review* 61(3): 87–278.
- Dib, A., C. Mendicino, and Y. Zhang. 2013. "Price Level Targeting in a Small Open Economy with Financial Frictions: Welfare Analysis." *Economic Modelling* 30: 941–953.
- Diba, B. T., and H. I. Grossman. 1988a. "Explosive Rational Bubbles in Stock Prices." *American Economic Review* 78(3): 520–530.
- . 1988b. "Rational Inflationary Bubbles." *Journal of Monetary Economics* 21(1): 35–46.
- Diebold, F. X., M. Piazzesi, and G. D. Rudebusch. 2005. "Modeling Bond Yields in Finance and Macroeconomics." *American Economic Review* 95(2): 415–420.
- Dincer, N. N., and B. Eichengreen. 2007. "Central Bank Transparency: Where, Why, and with What Effects?" NBER Working Paper 13003.
- . 2014. "Central Bank Transparency and Independence: Updates and New Measures." *International Journal of Central Banking* 10(1): 189–253.
- Dittmar, R., W. T. Gavin, and F. E. Kydland. 1999. "The Inflation-Output Variability Trade-off and Price Level Targeting." Federal Reserve Bank of St. Louis *Review* 81(1): 23–31.
- Dixit, A. K. 1990. *Optimization in Economic Theory*. 2nd ed. Oxford: Oxford University Press.
- . 2000. "A Repeated Game Model of Monetary Union." *Economic Journal* 110(466): 759–780.
- Dixit, A., and L. Lambertini. 2001. "Monetary-Fiscal Policy Interactions and Commitment versus Discretion in a Monetary Union." *European Economic Review* 45(4-6), 977–987.
- . 2003. "Symbiosis of Monetary and Fiscal Policies in a Monetary Union." *Journal of International Economics* 60(2): 235–247.
- Dixit, A. K., and J. E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67(3): 297–308.
- Dotsey, M., and P. Ireland. 1995. "Liquidity Effects and Transaction Technologies." *Journal of Money, Credit, and Banking*, 27(4), 1441–1457.

- . 1996. “The Welfare Cost of Inflation in General Equilibrium.” *Journal of Monetary Economics* 37(1): 29–47.
- Dotsey, M., and R. G. King. 2005. “Implications of State-Dependent Pricing for Dynamic Macroeconomic Models.” *Journal of Monetary Economics* 52(1): 213–242.
- . 2006. “Pricing, Production and Persistence.” *Journal of the European Economic Association* 4(5): 893–928.
- Dotsey, M., R. G. King, and A. L. Wolman. 1999. “State Contingent Pricing and the General Equilibrium Dynamics of Money and Output.” *Quarterly Journal of Economics* 114(2): 655–690.
- . 2006. “Inflation and Real Activity with Firm-Level Productivity Shocks: A Quantitative Framework.” Federal Reserve Bank of Richmond.
- . 2013. “Inflation and Real Activity with Firm-Level Productivity Shocks.” FRB Philadelphia Working Paper 13–35.
- Drazen, A. 1979. “The Optimal Rate of Inflation Revisited.” *Journal of Monetary Economics* 5(2): 231–248.
- Drazen, A. 2000. *The Political Economy of Macroeconomics*. Princeton, NJ: Princeton University Press.
- Drazen, A., and P. R. Masson. 1994. “Credibility of Policies versus Credibility of Policymakers.” *Quarterly Journal of Economics* 109(3): 735–754.
- Driffill, J. 1988. “Macroeconomic Policy Games with Incomplete Information: A Survey.” *European Economic Review* 32(2-3): 513–541.
- Driffill J., G. E. Mizon, and A. Ulph. 1990. “Costs of Inflation.” In *The Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 2, 1012–1066. Amsterdam: Elsevier.
- Eggertsson, G.B.G. 2011. “What Fiscal Policy Is Effective at Zero Interest Rates?” In *NBER Macroeconomics Annual 2010*, 59–112. Cambridge, MA: MIT Press.
- Eggertsson, G.B.G., and M. Woodford. 2003. “The Zero Bound on Interest Rates and Optimal Monetary Policy.” *Brookings Papers on Economic Activity* 34(1): 139–235.
- Ehrmann, M., and F. Smets. 2003. “Uncertain Potential Output: Implications for Monetary Policy.” *Review of Economic Dynamics* 27(9): 1611–1638.
- Eichenbaum, M. 1992. “Comments: ‘Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy’ by Christopher Sims.” *European Economic Review* 36(5): 1001–1011.
- Eichenbaum, M., and J.D.M. Fisher. 2007. “Estimating the Frequency of Price Reoptimization in Calvo-Style Models.” *Journal of Monetary Economics* 54(7): 2032–2047.
- Eichenbaum, M., N. Jaimovich, and S. Rebelo. 2011. “Reference Prices and Nominal Rigidities.” *American Economic Review* 101(1): 234–262.
- Eichenbaum, M., and K. J. Singleton. 1986. “Do Equilibrium Real Business Cycle Theories Explain Postwar U.S. Business Cycles?” In *NBER Macroeconomics Annual 1986*, 91–146. Cambridge, MA: MIT Press.
- Eijffinger, S., and J. de Haan. 1996. “The Political Economy of Central-Bank Independence”. Special Papers in International Economics 19, Princeton University, Princeton, NJ.
- Eijffinger, S., and P. Geraats. 2006. “How Transparent Are Central Banks?” *European Journal of Political Economy* 22: 1–22.
- Eijffinger, S., M. Hoeberichts, and E. Schaling. 1995. “Optimal Conservativeness in the Rogoff (1985) Model: A Graphical and Closed-Form Solution.” CentER Discussion Paper 95121, Tilburg University, Netherlands.
- Eijffinger, S., and E. Schaling. 1995. “Optimal Commitment in an Open Economy: Credibility vs. Flexibility.” Bank of England Working Paper 41.
- Engel, C. 2002. “The Responsiveness of Consumer Prices to Exchange Rates and the Implications for Exchange-Rate Policy: A Survey of a Few Recent New Open-Economy Macro Models.” *The Manchester School*, 70(S): 1–15.
- . 2011. “Currency Misalignments and Optimal Monetary Policy: A Reexamination.” *American Economic Review*, 101(Oct.): 2796–2822.
- Engle, R., and C. Granger. 1987. “Cointegration and Error Correction: Representation, Estimation, and Testing.” *Econometrica* 55(2): 251–276.
- Erceg, C., D. Henderson, and A. T. Levin. 2000. “Optimal Monetary Policy with Staggered Wage and Price Contracts.” *Journal of Monetary Economics* 46(2): 281–313.



- Ercceg, C. J., and A. T. Levin. 2003. "Imperfect Credibility and Inflation Persistence." *Journal of Monetary Economics* 50(4): 915–944.
- Estrella, A., and J. C. Fuhrer. 2002. "Dynamic Inconsistencies: Counterfactual Implications of a Class of Rational Expectations Models." *American Economic Review* 92(4): 1013–1028.
- Evans, C., and D. Marshall. 1998. "Monetary Policy and the Term Structure of Nominal Interest Rates: Evidence and Theory." *Journal of Monetary Economics* 49: 53–111.
- Evans, G. W. 1991. "Pitfalls in Testing for Explosive Bubbles in Asset Prices." *American Economic Review* 81(4): 922–930.
- Evans, G. W., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton, NJ: Princeton University Press.
- . 2009. "Expectations, Learning and Monetary Policy: An Overview of Recent Research." In *Monetary Policy under Uncertainty and Learning*, ed. K. Schmidt-Hebbel, and C. E. Walsh. Santiago: Banco Central de Chile.
- Faia, E. 2008. "Optimal Monetary Policy Rules with Labor Market Frictions." *Journal of Economic Dynamics and Control* 32(5): 1600–1621.
- . 2009. "Ramsey Monetary Policy with Labor Market Frictions." *Journal of Monetary Economics* 56(4): 570–581.
- Faig, M. 1988. "Characterization of the Optimal Tax on Money When it Functions as a Medium of Exchange." *Journal of Monetary Economics* 22(1): 137–148.
- Fair, R. C. 1984. *Specification, Estimation, and Analysis of Macroeconometric Models*. Cambridge, MA: Harvard University Press.
- Faust, J. 1996. "Whom Can You Trust? Theoretical Support for the Founders' Views." *Journal of Monetary Economics* 37(2): 267–283.
- Faust, J., and J. Irons. 1999. "Money, Politics and the Postwar Business Cycles." *Journal of Monetary Economics* 44(1): 61–89.
- Faust, J., and E. Leeper. 1997. "When Do Long-Run Identifying Restrictions Give Reliable Results?" *Journal of Business Economics and Statistics* 15(3): 345–354.
- Federal Reserve System 1981. "New Monetary Control Procedures." Discussion paper.
- Feenstra, R. C. 1986. "Functional Equivalence between Liquidity Costs and the Utility of Money." *Journal of Monetary Economics* 17(2): 271–291.
- Feldstein, M. 1978. "The Welfare Costs of Capital Income Taxation." *Journal of Political Economy*, 86(2), 529–551.
- . 1979. "The Welfare Costs of Permanent Inflation and Optimal Short-Run Economic Policy." *Journal of Political Economy* 87(4): 749–768.
- . 1998. "The Costs and Benefits of Going from Low Inflation to Price Stability." In *Monetary Policy and Inflation*, ed. C. Romer, and D. Romer, 123–156. Chicago: University of Chicago Press.
- Feldstein, M., J. Green, and E. Sheshinski. 1978. "Inflation and Taxes in a Growing Economy with Debt and Equity Finance." *Journal of Political Economy*, 86(2), S53–S70.
- Fischer, M. E., and J. J. Seater. 1993. "Long-Run Neutrality and Superneutrality in an ARIMA Framework." *American Economic Review* 83(3): 402–415.
- Fischer, S. 1972. "Keynes-Wicksell and Neoclassical Models of Money and Growth." *American Economic Review* 62(5): 880–890.
- . 1974. "Money and the Production Function." *Economic Inquiry* 12(4): 517–533.
- . 1977. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy* 85(1): 191–206.
- . 1979. "Capital Accumulation on the Transition Path in a Monetary Optimizing Model." *Econometrica* 47(6): 1433–1439.
- Fishe, R., and M. Wohar. 1990. "The Adjustment of Expectations to a Change in Regime: Comment." *American Economic Review* 80(4): 968–976.
- Flood, R., and P. Isard. 1988. "Monetary Policy Strategies." NBER Working Paper 2770.

- Frankel, J., and M. Chinn. 1995. "The Stabilizing Properties of a Nominal GNP Rule." *Journal of Money, Credit, and Banking* 27(2): 318–334.
- Fratianni, M., J. von Hagen, and C. Waller. 1997. "Central Banking as a Principal Agent Problem." *Economic Inquiry* 35(2): 378–393.
- Friedman, B. M. 1975. "Targets, Instruments and Indicators of Monetary Policy." *Journal of Monetary Economics* 1(4): 443–473.
- . 1977a. "Even the St. Louis Model Now Believes in Fiscal Policy." *Journal of Money, Credit, and Banking* 9(2): 365–367.
- . 1977b. "The Inefficiency of Short-Run Monetary Targets for Monetary Policy." *Brookings Papers on Economic Activity* 8(2): 293–335.
- . 1990. "Targets and Instruments of Monetary Policy." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 2, 1183–1230. Amsterdam: Elsevier.
- Friedman, B. M., and K. N. Kuttner. 1992. "Money, Income, Prices and Interest Rates." *American Economic Review* 82(3): 472–492.
- . 1996. "A Price Target for U.S. Monetary Policy? Lessons from the Experience with Money Growth Targets." *Brookings Papers on Economic Activity* 27(1): 77–125.
- . 2010. "Implementation of Monetary Policy. How Do Central Banks Set Interest Rates?" In *Handbook of Monetary Economics*, ed. B. M. Friedman, and M. Woodford, vol. 3, 1345–1438. Amsterdam: Elsevier.
- Friedman, M. 1968. "The Role of Monetary Policy." *American Economic Review* 58(1): 1–17.
- . 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine.
- . 1977. "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* 85(3): 451–472.
- Friedman, M., and D. Meiselman. 1963. "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States: 1897–1958." In *Commission on Money and Credit: Stabilization Policies*, 165–268. Englewood Cliffs, NJ: Prentice Hall.
- Friedman, M., and A. Schwartz. 1963. *A Monetary History of the United States, 1867–1960*. Princeton, NJ: Princeton University Press.
- Fudenberg, D., and E. Maskin. 1986. "Folk Theorems for Repeated Games with Discounting or with Incomplete Information." *Econometrica* 54(3): 533–554.
- Fuerst, T. S. 1992. "Liquidity, Loanable Funds, and Real Activity." *Journal of Monetary Economics* 29(1): 3–24.
- Fuhrer, J. C. 1996. "Monetary Policy Shifts and Long-Term Interest Rates." *Quarterly Journal of Economics* 111(4): 1183–1209.
- . 1997. "The (Un)Importance of Forward-Looking Behavior in Price Specifications." *Journal of Money, Credit, and Banking* 29(3): 338–350.
- . 2011. "Inflation Persistence." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and M. Woodford, vol. 3, 423–486. Amsterdam: Elsevier.
- Furfine, C. H. 2000. "Interbank Payments and the Daily Federal Funds Rate." *Journal of Monetary Economics* 46(2): 535–553.
- Furfine, C. H., and J. Stehem. 1998. "Analyzing Alternative Intraday Credit Policies in Real-Time Gross Settlement Systems." *Journal of Money, Credit, and Banking* 30(4): 832–848.
- Gagnon, J., M. Raskin, J. Remache, and B. Sack. 2011. "The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases." *International Journal of Central Banking* 7(1): 3–43.
- Gaiotti, E., and A. Secchi. 2004. "Is there a Cost Channel of Monetary Policy Transmission? An Investigation into the Pricing Behavior of 2,000 Firms." Bank of Italy Working Paper 525.
- Gali, J. 1992. "How Well Does the IS-LM Model Fit the Postwar U.S. Data?" *Quarterly Journal of Economics* 107(2): 709–738.
- . 2003. "New Perspectives on Monetary Policy, Inflation, and the Business Cycle." In *Advances in Economics and Econometrics*, ed. M. Dewatripont, L. Hansen, and S. J. Turnovsky, 151–197. Cambridge: Cambridge University Press.
- . 2008. "The New Keynesian Approach to Monetary Policy Analysis: Lessons and New Directions." Working Paper.

- . 2011. “The Return of the Wage Phillips Curve.” *Journal of the European Economic Association* 9(3): 436–461.
- . 2015. *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. 2nd ed. Princeton, NJ: Princeton University Press.
- Galí, J., and M. Gertler. 1999. “Inflation Dynamics: A Structural Econometric Analysis.” *Journal of Monetary Economics* 44(2): 195–222.
- . 2007. “Macroeconomic Modeling for Monetary Policy Evaluation.” *Journal of Economic Perspectives* 21(4): 25–46.
- Galí, J., M. Gertler, and J. D. López-Salido. 2001. “European Inflation Dynamics.” *European Economic Review* 45(7): 1237–1270.
- . 2005. “Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve.” *Journal of Monetary Economics* 52(6): 1107–1118.
- . 2007. “Markups, Gaps, and the Welfare Costs of Business Fluctuations.” *Review of Economics and Statistics* 89(1): 44–59.
- Galí, J., and T. Monacelli. 2005. “Monetary Policy and Exchange Rate Volatility in a Small Open Economy.” *Review of Economic Studies* 72: 707–734.
- . 2008. “Optimal Monetary and Fiscal Policy in a Currency Union.” *Journal of International Economics* 76(1): 116–132.
- Galí, J., F. Smets, and R. Wouters. 2012. “Unemployment in an Estimated New Keynesian Model.” In *NBER Macroeconomics Annual 2011*, 329–360. Cambridge, MA: MIT Press.
- Gallmeyer, M., F. Hollifield, and S. E. Zin. 2005. “Taylor Rules, McCallum Rules, and the Term Structure of Interest Rates.” *Journal of Monetary Economics* 52: 921–950.
- García de Paso, J. I. 1993. “Monetary Policy with Private Information: A Role for Monetary Targets.” Working Paper 9315, Instituto Complutense de Analisis Economico.
- . 1994. “A Model of Appointing Governors to the Central Bank.” Instituto Complutense de Analisis Economico Working Paper 9416.
- Garfinkel, M., and S. Oh. 1993. “Strategic Discipline in Monetary Policy with Private Information: Optimal Targeting Horizons.” *American Economic Review* 83(1): 99–117.
- Gavin, W. T., B. D. Keen, A. W. Richter, and N. A. Throckmorton. 2015. “The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics.” *Journal of Economic Dynamics and Control* 55 14–38.
- Geraats, P. M. 2002. “Central Bank Transparency.” *Economic Journal* 112: 532–565.
- . 2009. “Trends in Monetary Policy Transparency.” *International Finance* 12(2): 235–268.
- Gertler, M. 1988. “Financial Structure and Aggregate Activity: An Overview.” *Journal of Money, Credit, and Banking* 20(3): 559–588.
- Gertler, M., S. Gilchrist, and F. M. Natalucci. 2007. “External Constraints on Monetary Policy and the Financial Accelerator.” *Journal of Money, Credit, and Banking* 39(2-3), 295–330.
- Gertler, M., and P. Karadi. 2011. “A Model of Unconventional Monetary Policy.” *Journal of Monetary Economics* 58(1): 17–34.
- . 2013. “QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool.” *International Journal of Central Banking* 9(S1): 5–53.
- Gertler, M., and N. Kiyotaki. 2010. “Financial Intermediation and Credit Policy in Business Cycle Analysis.” In *Handbook of Monetary Economics*, ed. B. M. Friedman and M. Woodford, vol. 3, 547–599. Amsterdam: Elsevier.
- Gertler, M., and J. Leahy. 2008. “A Phillips Curve with an Ss Foundation.” *Journal of Political Economy* 116(3): 533–572.
- Gertler, M., and A. Trigari. 2009. “Unemployment Fluctuations with Staggered Nash Wage Bargaining.” *Journal of Political Economy* 117(1): 38–86.
- Geweke, J. 1986. “The Superneutrality of Money in the United States: An Interpretation of the Evidence.” *Econometrica* 54(1): 1–22.
- Giavazzi, F., and M. Pagano. 1988. “The Advantage of Tying One’s Hands: EMS Discipline and Central Bank Credibility.” *European Economic Review* 32: 1055–1082.

- Gilchrist, S., and E. Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review* 102(4): 1692–1720.
- . 2013. "The Impact of the Federal Reserve's Large-Scale Asset Purchase Programs on Corporate Credit Risk." *Journal of Money, Credit, and Banking* 45(8S): 29–57.
- Gillman, M. 1995. "Comparing Partial and General Equilibrium Estimates of the Welfare Costs of Inflation." *Contemporary Economic Policy* 13(4): 60–71.
- Goldfeld, S. M. 1976. "The Case of the Missing Money." *Brookings Papers on Economic Activity* 7(3): 683–740.
- Goldfeld, S. M., and D. E. Sichel. 1990. "The Demand for Money." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 1, 299–356. Amsterdam: Elsevier.
- Golosov, M., and R. E. Lucas Jr. 2007. "Menu Costs and the Phillips Curve." *Journal of Political Economy* 115(2): 171–199.
- Gomme, P. 1993. "Money and Growth Revisited: Measuring the Costs of Inflation in an Endogenous Growth Model." *Journal of Monetary Economics* 32(1): 51–77.
- Goodfriend, M. 1983. "Discount Window Borrowing, Monetary Policy, and the Post-October 6, 1979 Federal Reserve Operating Procedure." *Journal of Monetary Economics* 12(3): 343–356.
- . 1987. "Interest-Rate Smoothing and Price Level Trend-Stationarity." *Journal of Monetary Economics* 19(3): 335–348.
- . 1991. "Interest Rate Policy and the Conduct of Monetary Policy." *Carnegie-Rochester Conference Series on Public Policy* 34: 7–30.
- . 1993. "Interest Rate Policy and the Inflation Scare Problem, 1979–1992." Federal Reserve Bank of Richmond *Economic Quarterly* 79(1): 1–24.
- . 2000. "Overcoming the Zero Bound on Interest Rate Policy." *Journal of Money, Credit, and Banking* 32(4): 1007–1035.
- . 2002. "Interest on Reserves and Monetary Policy." Federal Reserve Bank of New York *Economic Policy Review* 8(1).
- Goodfriend, M., and R. G. King. 1997. "The New Neoclassical Synthesis and the Role of Monetary Policy." In *NBER Macroeconomics Annual 1997*, 231–283. Cambridge, MA: MIT Press.
- . 2001. "The Case for Price Stability." NBER Working Paper 8423.
- Goodfriend, M., and D. Small, eds. 1993. *Operating Procedures and the Conduct of Monetary Policy: Conference Proceedings*. Federal Reserve System.
- Goodhart, C.A.E., and J. Vinals. 1994. "Strategy and Tactics of Monetary Policy: Examples from Europe and the Antipodes." In *Goals, Guidelines and Constraints Facing Monetary Policymakers*, ed. J. C. Fuhrer, 139–187. Federal Reserve Bank of Boston.
- Gordon, D. B., and E. M. Leeper. 1994. "The Dynamic Impacts of Monetary Policy: An Exercise in Tentative Identification." *Journal of Political Economy* 102(6): 1228–1247.
- Gordon, R. J. 1982. "Why Stopping Inflation May Be Costly: Evidence from Fourteen Historical Episodes." In *Inflation: Causes and Effects*, ed. R. E. Hall, 11–40. Chicago: University of Chicago Press.
- Gordon, R. J., and S. King. 1982. "The Output Costs of Disinflation in Traditional and Vector Autoregressive Models." *Brookings Papers on Economic Activity* 13(1): 205–244.
- Grandmont, J., and Y. Younes. 1972. "On the Role of Money and the Existence of a Monetary Equilibrium." *Review of Economic Studies* 39: 355–372.
- Gray, J. A. 1978. "On Indexation and Contract Length." *Journal of Political Economy* 86(1): 1–18.
- Greenwood, R., S. Hanson, and D. Vayanos. 2015. "Forward Guidance in the Yield Curve: Short Rates versus Bond Supply." NBER Working Paper 21750.
- Grier, K. B., and H. E. Neiman. 1987. "Deficits, Politics and Money Growth." *Economic Inquiry* 25(2): 201–214.
- Grossman, S., and L. Weiss. 1983. "A Transactions-Based Model of the Monetary Transmission Mechanism." *American Economic Review* 73(5): 871–880.
- Guidotti, P. E., and C. A. Vegh. 1993. "The Optimal Inflation Tax When Money Reduces Transactions Costs: A Reconsideration." *Journal of Monetary Economics* 31(2): 189–205.
- Gürkaynak, R. S., B. P. Sack, and E. T. Swanson. 2005. "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements." *International Journal of Central Banking* 1(1): 55–93.

- Guthrie, G., and J. Wright. 2000. "Open Mouth Operations." *Journal of Monetary Economics* 46(2): 489–516.
- Haefke, C., M. Sonntag, and T. van Rens. 2013. "Wage Rigidity and Job Creation." *Journal of Monetary Economics* 60(8): 887–899.
- Hagedorn, M., and I. Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98(4): 1692–1706.
- Hahn, F. H. 1965. "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy." In *The Theory of Interest Rates*, ed. F. H. Hahn, and F. P. R. Brechling, 126–135. London: Macmillan.
- Haldane, A. G., ed. 1995. *Targeting Inflation: A Conference of Central Banks on the Use of Inflation Targets*. Bank of England.
- Haldane, A. G. 1997. "Designing Inflation Targets." In *Monetary Policy and Inflation Targeting*, ed. P. Lowe, 76–112. Reserve Bank of Australia.
- Hall, R. E. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95(1): 50–65.
- Hall, R. E., and N. G. Mankiw. 1994. "Nominal Income Targeting." In *Monetary Policy*, ed. N. G. Mankiw, 71–92. Chicago: University of Chicago Press.
- Hall, R. E., and P. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review* 98(4): 1653–1674.
- Hall, R. E., and R. Reis. 2013. "Maintaining Central-Bank Solvency under New-Style Central Banking." Working Paper.
- Hamada, K. 1976. "A Strategic Analysis of Monetary Interdependence." *Journal of Political Economy* 84(4): 677–700.
- Hamilton, J. D. 1994. *Time Series Analysis*. Princeton, NJ: Princeton University Press.
- . 1996. "The Daily Market for Federal Funds." *Journal of Political Economy* 104(1): 26–56.
- Hamilton, J. D., and J. C. Wu. 2012a. "Identification and Estimation of Gaussian Affine Term Structure Models." *Journal of Econometrics* 168(2): 315–331.
- . 2012b. "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment." *Journal of Money, Credit, and Banking* 44(1): 3–46.
- Hansen, G. D., and E. C. Prescott. 1994. "Recursive Methods for Computing Equilibria of Business Cycle Models." In *Frontiers of Business Cycle Research*, ed. T. F. Cooley, 39–64. Princeton, NJ: Princeton University Press.
- Hansen, M. S. 2004. "The 'Price Puzzle' Reconsidered." *Journal of Monetary Economics* 51(7): 1385–1413.
- Hartley, P. 1988. "The Liquidity Services of Money." *International Economic Review* 29(1): 1–24.
- Havrilesky, T., and J. Gildea. 1991. "The Policy Preferences of FOMC Members as Revealed by Dissenting Votes." *Journal of Money, Credit, and Banking* 23(1): 130–138.
- . 1992. "Reliable and Unreliable Partisan Appointees to the Board of Governors." *Public Choice* 73: 397–417.
- . 1995. "The Biases of Federal Reserve Bank Presidents." *Economic Inquiry* 33(2): 274–284.
- Heer, B., and A. Maussner. 2010. "Inflation and Output Dynamics in a Model with Labor Market Search and Capital Accumulation." *Review of Economic Dynamics* 13(3): 654–686.
- Heller, D., and Y. Lengwiler. 2003. "Payment Obligations, Reserve Requirements, and the Demand for Central Bank Balances." *Journal of Monetary Economics* 50(2): 419–432.
- Herrendorf, B. 1997. "Time Consistent Collection of Optimal Seigniorage: A Unifying Framework." *Journal of Economic Surveys* 11(1): 1–46.
- . 1998. "Inflation Targeting as a Way of Precommitment." *Oxford Economic Papers* 50(3): 431–448.
- . 1999. "Transparency, Reputation, and Credibility under Floating and Pegged Exchange Rates." *Journal of Monetary Economics* 49(1): 31–50.
- Herrendorf, B., and B. Lockwood. 1997. "Rogoff's 'Conservative' Central Banker Restored." *Journal of Money, Credit, and Banking* 29(4), 476–495.
- Herrendorf, B., and M. J. M. Neumann. 2003. "The Political Economy of Inflation, Labor Market Distortions, and Central Bank Independence." *Economic Journal* 113(484): 43–64.

- Hobijn, B., F. Ravenna, and A. Tambalotti. 2006. "Menu Costs at Work: Restaurant Prices and the Introduction of the Euro." *Quarterly Journal of Economics* 121(3): 1103–1131.
- Hoffman, D. L., and R. H. Rasche. 1991. "Long-Run Income and Interest Elasticities of Money Demand in the United States." *Review of Economics and Statistics* 73: 665–674.
- Hoffman, D. L., R. H. Rasche, and M. A. Tieslau. 1995. "The Stability of Long-Run Money Demand in Five Industrial Countries." *Journal of Monetary Economics* 35(2): 317–339.
- Holman, J. A. 1998. "GMM Estimation of a Money-in-the-Utility-Function Model: The Implications of Functional Forms." *Journal of Money, Credit, and Banking* 30(4): 679–698.
- Hoover, K. D. 1995. "Resolving the Liquidity Effect: Commentary." Federal Reserve Bank of St. Louis *Review* 77(3): 26–32.
- Hoover, K. D., and S. J. Perez. 1994. "Post Hoc Ergo Propter Once More: An Evaluation of 'Does Monetary Policy Matter?' in the Spirit of James Tobin." *Journal of Monetary Economics* 34(1): 89–99.
- Hornstein, A., P. Krusell, and G. L. Violante. 2005. "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism." Federal Reserve Bank of Richmond *Economic Quarterly* 91(3): 19–51.
- Hornstein, A., and A. L. Wolman. 2005. "Trend Inflation, Firm-Specific Capital, and Sticky Prices." Federal Reserve Bank of Richmond *Economic Quarterly* 91(4): 57–83.
- Hosios, A. J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies* 57(2): 279–298.
- Huang, K.X.D., and Z. Liu. 2002. "Staggered Price-Setting, Staggered Wage-Setting, and Business Cycle Persistence." *Journal of Monetary Economics* 49(2): 405–433.
- Hutchison, M. M., and C. E. Walsh. 1992. "Empirical Evidence on the Insulation Properties of Fixed and Flexible Exchange Rates: The Japanese Experience." *Journal of Monetary Economics* 32(3-4): 241–263.
- Ihrig, J. E., E. E. Meade, and C. Gretchen. 2015. "Rewriting Monetary Policy 101: What's the Fed's Preferred Post-Crisis Approach to Raising Interest Rates?" *Journal of Economic Perspectives* 29(4): 177–198.
- Imrohorolu, A., and E. C. Prescott. 1991. "Seigniorage as a Tax: A Quantitative Evaluation." *Journal of Money, Credit, and Banking* 23(3), 462–475.
- Ireland, P. N. 1996. "The Role of Countercyclical Monetary Policy." *Journal of Political Economy* 104(4): 704–723.
- . 1997a. "A Small, Structural, Quarterly Model for Monetary Policy Evaluation." *Carnegie-Rochester Conference Series on Public Policy* 47: 83–108.
- . 1997b. "Sustainable Monetary Policies." *Journal of Economic Dynamics and Control* 22(1): 87–108.
- . 1999. "Does the Time-Inconsistency Problem Explain the Behavior of Inflation in the United States?" *Journal of Monetary Economics* 44: 279–292.
- . 2000. "Interest Rates, Inflation, and Federal Reserve Policy since 1980." *Journal of Money, Banking, and Credit* 32(3): 417–434.
- . 2001. "Sticky-Price Models of the Business Cycle: Specification and Stability." *Journal of Monetary Economics* 47(1): 3–18.
- . 2004. "Money's Role in the Monetary Business Cycle." *Journal of Money, Credit, and Banking* 36(6): 969–83.
- . 2005. "The Liquidity Trap, the Real Balance Effect, and the Friedman Rule." *International Economic Review* 46(4): 1271–1301.
- . 2009. "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review* 99(3): 1040–1052.
- . 2014. "The Macroeconomic Effects of Interest on Reserves." *Macroeconomic Dynamics* 18: 1271–1312.
- Jaffee, D., and T. Russell. 1976. "Imperfect Information, Uncertainty, and Credit Rationing." *Quarterly Journal of Economics* 90(4): 651–666.
- Jaffee, D., and J. E. Stiglitz. 1990. "Credit Rationing." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 2, 837–888. Amsterdam: Elsevier.
- Jensen, C., and B. T. McCallum. 2002. "The Non-optimality of Proposed Monetary Policy Rules under Timeless-Perspective Commitment." *Economics Letters* 77(2): 163–168.

- . 2010. “Optimal Continuation versus the Timeless Perspective in Monetary Policy.” *Journal of Money, Credit, and Banking* 42(6): 1093–1107.
- Jensen, H. 1997. “Credibility of Optimal Monetary Delegation.” *American Economic Review* 87(5): 911–920.
- . 2002. “Targeting Nominal Income Growth or Inflation?” *American Economic Review* 92(4): 928–956.
- Joines, D. H. 1985. “Deficits and Money Growth in the United States, 1872–1983.” *Journal of Monetary Economics* 16(3): 329–351.
- Jones, R. A. 1976. “The Origin and Development of Media of Exchange.” *Journal of Political Economy* 84(4): 757–775.
- Jonsson, G. 1995. “Institutions and Incentives in Monetary and Fiscal Policy.” Monograph Series 28, Institute for International Economic Studies, Stockholm University.
- . 1997. “Monetary Politics and Unemployment Persistence.” *Journal of Monetary Economics* 39: 303–325.
- Jovanovic, B. 1982. “Inflation and Welfare in the Steady-State.” *Journal of Political Economy* 90(3): 561–577.
- Joyce, M., A. Lasasoa, I. Stevens, and M. Tong. 2011. “The Financial Market Impact of Quantitative Easing in the United Kingdom.” *International Journal of Central Banking* 7(3): 113–161.
- Judd, J. P., and G. D. Rudebusch. 1997. “A Tale of Three Chairmen.” Federal Reserve Bank of San Francisco. Discussion paper.
- Judd, J. P., and J. Scadding. 1982. “The Search for a Stable Money Demand Function.” *Journal of Economic Literature* 20(3): 993–1023.
- Judd, J. P., and B. Trehan. 1989. “Unemployment Rate Dynamics: Aggregate Demand and Supply Interactions.” Federal Reserve Bank of San Francisco *Economic Review*, (Fall): 20–37.
- Jung, T., Y. Teranishi, and T. Watanabe. 2005. “Optimal Monetary Policy at the Zero-Interest-Rate Bound.” *Journal of Money, Credit, and Banking* 37(5): 813–835.
- Justiniano, A., G. E. Primiceri, and A. Tambalotti. 2013. “Is There a Trade-off between Inflation and Output Stabilization?” *American Economic Journal: Macroeconomics* 5(2): 1–31.
- Kapetanios, G., and H. Mumtaz. 2012. “Assessing the Economy-wide Effects of Quantitative Easing.” *Economic Journal* 122: 316–347.
- Kaplan, G., B. Moll, and G. L. Violante. 2016. “Monetary Policy According to HANK.” NBER Working Paper 21897.
- Kaplan, G., G. L. Violante, and J. Weidner. 2014. “The Wealthy Hand-to-Mouth.” *Brookings Papers on Economic Activity* 48(1): 77–153.
- Karabarbounis, L. 2014. “The Labor Wedge: MRS vs. MPN.” *Review of Economic Dynamics* 17(2): 206–223.
- Kareken J., T. Muench, and N. Wallace. 1973. “Optimal Open Market Strategies: The Use of Information Variables.” *American Economic Review* 63(1): 156–172.
- Kasa, K., and H. Popper. 1997. “Monetary Policy in Japan: A Structural VAR Analysis.” *Journal of the Japanese and International Economies* 11(3): 275–295.
- Kashyap, A. K., and J. C. Stein. 2012. “The Optimal Conduct of Monetary Policy with Interest on Reserves.” *American Economic Journal: Macroeconomics* 4(1): 266–82.
- Kasman, B. 1993. “A Comparison of Monetary Policy Operating Procedures in Six Industrial Countries.” In *Operating Procedures and the Conduct of Monetary Policy: Conference Proceedings*, ed. M. Goodfriend, and D. Small. Federal Reserve System.
- Kehoe, P., and V. Midrigan. 2015. “Prices are Sticky After All.” *Journal of Monetary Economics* 75: 35–53.
- Keister, T., and J. McAndrews. 2009. “Why are Banks Holding so Many Excess Reserves?” Federal Reserve Bank of New York Staff Report 380.
- Kerr, W., and R. G. King. 1996. “Limits on Interest Rate Rules in the IS Model.” Federal Reserve Bank of Richmond *Economic Quarterly* 82(2): 47–75.
- Khan, A., R. G. King, and A. L. Wolman. 2003. “Optimal Monetary Policy.” *Review of Economic Studies* 70(4): 825–860.
- Khan, H., and Z. Zu. 2006. “Estimates of the Sticky-Information Phillips Curve for the United States.” *Journal of Money, Credit, and Banking* 38(1): 195–207.

- Khoury, S. 1990. "The Federal Reserve Reaction Function." In *The Political Economy of American Monetary Policy*, ed. T. Mayer, 27–49. Cambridge: Cambridge University Press.
- Kiley, M. T. 2002a. "Partial Adjustment and Staggered Price Setting." *Journal of Money, Credit, and Banking* 34(2): 283–298.
- . 2002b. "The Lead of Output over Inflation in Sticky Price Models." *Economics Bulletin* 50(5): 1–7.
- . 2007a. "A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting." *Journal of Money, Credit, and Banking*, 39(S1), 101–125.
- . 2007b. "Is Moderate-to-High Inflation Inherently Unstable?" *International Journal of Central Banking* 3(2): 173–201.
- . 2014. "The Aggregate Demand Effects of Short- and Long-Term Interest Rates." *International Journal of Central Banking* 10(4): 69–104.
- . 2016. "Policy Paradoxes in the New Keynesian Model." *Review of Economic Dynamics* 21: 1–15.
- Kilponen, J., and K. Leitemo. 2011. "Transmission Lags and Optimal Monetary Policy." *Journal of Economic Dynamics and Control* 35(4): 565–578.
- Kimbrough, K. P. 1986a. "Inflation, Employment, and Welfare in the Presence of Transaction Costs." *Journal of Money, Credit, and Banking* 28(2): 127–140.
- . 1986b. "The Optimum Quantity of Money Rule in the Theory of Public Finance." *Journal of Monetary Economics* 18(3): 277–284.
- King, R. G., and C. I. Plosser. 1984. "Money, Credit and Prices in a Real Business Cycle." *American Economic Review* 74(3): 363–380.
- . 1985. "Money, Deficits, and Inflation." *Carnegie-Rochester Conference Series on Public Policy* 22: 147–196.
- King, R. G., C. I. Plosser, and S. Rebelo. 1988. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics* 21(2-3), 195–232.
- King, R. G., and J. K. Thomas. 2007. "Breaking the New Keynesian Dichotomy: Asset Market Segmentation and the Monetary Transmission Mechanism." Working Paper.
- King, R. G., and M. W. Watson. 1996. "Money, Prices, Interest Rates and the Business Cycle." *Review of Economics and Statistics* 78(1): 35–53.
- . 1997. "Testing Long-Run Neutrality." Federal Reserve Bank of Richmond *Economic Quarterly* 83(3): 69–101.
- . 1998. "The Solution of Singular Linear Difference Systems under Rational Expectations." *International Economic Review* 39(4): 1015–1026.
- Kirsanova, T., C. Leith, and S. Wren-Lewis. 2006. "Should Central Banks Target Consumer Prices or the Exchange Rate?" *Economic Journal* 116(512): F208–F231.
- Kiyotaki, N., and J. Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105(2): 211–248.
- . 2012. "Liquidity, Business Cycles, and Monetary Policy." NBER Working Paper 17934.
- Kiyotaki, N., and R. Wright. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy* 97(4): 927–954.
- . 1993. "A Search-Theoretic Approach to Monetary Economics." *American Economic Review* 83(1): 63–77.
- Klein, M., and M.J.M. Neumann. 1990. "Seigniorage: What Is It and Who Gets It?" *Weltwirtschaftliches Archiv* 126(2): 205–221.
- Klenow, P. J., and O. Kryvtsov. 2008. "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123(3): 863–904.
- Klenow, P., and J. Willis. 2007. "Sticky Information and Sticky Prices." *Journal of Monetary Economics* 54: 79–99.
- Knell, M., and H. Stix. 2005. "The Income Elasticity of Money Demand: A Meta-Analysis of Empirical Results." *Journal of Economic Surveys* 19(3): 513–533.
- Kocherlakota, N. R. 1998. "Money Is Memory." *Journal of Economic Theory* 81(2): 232–251.



- . 2000. “Creating Business Cycles Through Credit Constraints.” *Federal Reserve Bank of Minneapolis Quarterly Review* 24(3): 2–10.
- Kocherlakota, N. R., and C. Phelen. 1999. “Explaining the Fiscal Theory of the Price Level.” *Federal Reserve Bank of Minneapolis Quarterly Review* 23(4), 14–23.
- Kollman, R. 2001. “The Exchange Rate in a Dynamic-Optimizing Business Cycle Model with Nominal Rigidities.” *Journal of Monetary Economics* 55: 243–262.
- Kool, C., and D. Thornton. 2012. “How Effective Is Central Bank Forward Guidance?” *Federal Reserve Bank of St. Louis Working Paper* 2012-063A.
- Kormendi, R. C., and P. G. Meguire. 1984. “Cross-Regime Evidence of Macroeconomic Rationality.” *Journal of Political Economy* 92(5): 875–908.
- Krause, M. U., D. J. López-Salido, and T. A. Lubik. 2008. “Do Search Frictions Matter for Inflation Dynamics?” *European Economic Review* 52(8): 1464–1479.
- Kreps, D., and R. Wilson. 1982. “Reputation and Imperfect Information.” *Journal of Economic Theory* 27(2): 253–279.
- Krishnamurthy, A., and A. Vissing-Jorgensen. 2011. “The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy.” *Brookings Papers on Economic Activity* 43(2): 215–287.
- . 2013. “The Ins and Outs of LSAPs.” In *Proceedings, Economic Policy Symposium*. Federal Reserve Bank of Kansas City.
- Kryvtsov, O., M. Shukayev, and A. Ueberfeldt. 2008. “Adopting Price-Level Targeting under Imperfect Credibility.” *Bank of Canada Working Paper* 2008-3.
- Kudlyak, M. 2014. “The Cyclicalities of the User Cost of Labor.” *Journal of Monetary Economics* 68: 53–67.
- Kurozumi, T. 2008. “Optimal Sustainable Monetary Policy.” *Journal of Monetary Economics* 55(7): 1277–1289.
- Kurozumi, T., and W. Van Zandweghe. 2010. “Labor Market Search, the Taylor Principle, and Indeterminacy.” *Journal of Monetary Economics* 7(Oct.): 851–858.
- Kuttner, K. N. 2001. “Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market.” *Journal of Monetary Economics* 47(3): 523–544.
- Kydland, F. E., and E. C. Prescott. 1977. “Rules Rather than Discretion: The Inconsistency of Optimal Plans.” *Journal of Political Economy* 85(3): 473–492.
- . 1982. “Time to Build and Aggregate Fluctuations.” *Econometrica* 50(6): 1345–1370.
- Lacker, J. M. 1988. “Inside Money and Real Output.” *Economics Letters* 28(1): 9–14.
- Lago Alves, S. A. 2012. “Trend Inflation and the Unemployment Volatility Puzzle.” *Banco Central do Brazil Working Paper* 277.
- . 2014. “Lack of Divine Coincidence in New Keynesian Models.” *Journal of Monetary Economics* 67(C): 33–46.
- Lagos, R., and G. Rocheteau. 2005. “Inflation, Output, and Welfare.” *International Economic Review* 46(2): 495–522.
- Lagos, R., and R. Wright. 2005. “A Unified Framework for Monetary Theory and Policy Analysis.” *Journal of Political Economy* 113(3): 463–484.
- Laidler, D.E.W. 1985. *The Demand for Money: Theories, Evidence, and Problems*. 3rd ed. New York: Harper and Row.
- Lane, P. 2001. “The New Open Economy Macroeconomics: A Survey.” *Journal of Monetary Economics* 54(2): 235–266.
- Lansing, K. J. 2002. “Real-Time Estimation of Trend Output and the Illusion of Interest Rate Smoothing.” *Federal Reserve Bank of San Francisco Economic Review* (Jan.): 17–34.
- Lansing, K. J., and B. Trehan. 2003. “Forward-Looking Behavior and Optimal Discretionary Monetary Policy.” *Economics Letters* 81(2): 249–256.
- Laubach, T., and A. S. Posen. 1997. “Disciplined Discretion: The German and Swiss Monetary Targeting Frameworks in Operation.” *Federal Reserve Bank of New York Research Paper* 9707.
- Leeper, E. M. 1991. “Equilibria under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies.” *Journal of Monetary Economics* 27(1): 129–147.

- . 1993. “Has the Romers’ Narrative Approach Identified Monetary Policy Shocks?” Federal Reserve Bank of Atlanta Working Paper 93-1.
- . 1997. “Narrative and VAR Approaches to Monetary Policy: Common Identification Problems.” *Journal of Monetary Economics* 40(3): 641–657.
- Leeper, E. M., and D. B. Gordon. 1992. “In Search of the Liquidity Effect.” *Journal of Monetary Economics* 29(3): 341–369.
- Leeper, E. M., C. A. Sims, and T. Zha. 1996. “What Does Monetary Policy Do?” *Brookings Papers on Economic Activity* 27(2): 1–63.
- Leiderman, L., and L.E.O. Svensson, eds. 1995. *Inflation Targets*. London: Center for Economic Policy Research.
- LeRoy, S. 1984a. “Nominal Prices and Interest Rates in General Equilibrium: Endowment Shocks.” *Journal of Business* 57(2): 197–213.
- . 1984b. “Nominal Prices and Interest Rates in General Equilibrium: Money Shocks.” *Journal of Business* 57(2): 177–195.
- Levin, A. T., J. D. López-Salido, E. Nelson, and T. Yun. 2010. “Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound.” *International Journal of Central Banking* 6(1): 143–189.
- Levin, A. T., A. Onatski, J. C. Williams, and N. M. Williams. 2006. “Monetary Policy Under Uncertainty in Micro-Founded Macroeconomic Models.” In *NBER Macroeconomic Annual 2005*, 229–312. Cambridge, MA: MIT Press.
- Levin, A. T., and J. B. Taylor. 2013. “Falling Behind the Curve: A Positive Analysis of Stop-Start Monetary Policies and the Great Inflation.” In *The Great Inflation: The Rebirth of Modern Central Banking*, ed. A. Orphanides, and M. D. Bordo, 217–244. Chicago: University of Chicago Press.
- Levin, A. T., V. Wieland, and J. C. Williams. 1999. “Robustness of Simple Monetary Policy Rules under Model Uncertainty.” In *Monetary Policy Rules*, ed. J. B. Taylor. Chicago: Chicago University Press.
- Levin, A. T., and T. Yun. 2007. “Reconsidering the Natural Rate Hypothesis in a New Keynesian Framework.” *Journal of Monetary Economics* 54(5): 1344–1365.
- Li, C., and M. Wei. 2013. “Term Structure Modelling with Supply Factors and the Federal Reserve’s Large Scale Asset Purchase Programs.” *International Journal of Central Banking* 9(1): 3–39.
- Linde, J. 2005. “Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach.” *Journal of Monetary Economics* 52(6): 1135–1149.
- Lippi, F. 1997. *Central Bank Independence and Credibility: Essays on the Delegation Arrangements for Monetary Policy*, Tinbergen Institute Research Series No. 146. Erasmus University, Rotterdam.
- Ljungquist, L., and T. Sargent. 2000. *Recursive Macroeconomics*. Cambridge, MA: MIT Press.
- Llosa, L.-G., and V. Tuesta. 2009. “Learning about Monetary Policy Rules When the Cost-Channel Matters.” *Journal of Economic Dynamics and Control* 33(11): 1880–1896.
- Lockwood, B. 1997. “State-Contingent Inflation Contracts and Unemployment Persistence.” *Journal of Money, Credit, and Banking* 29(3): 286–299.
- Lockwood, B., and A. Philippopoulos. 1994. “Insider Power, Employment Dynamics, and Multiple Inflation Equilibria.” *Economica* 61(241): 59–77.
- Lohmann, S. 1992. “Optimal Commitment in Monetary Policy: Credibility versus Flexibility.” *American Economic Review* 82(1): 273–286.
- Lowe, P., ed. 1997. *Monetary Policy and Inflation Targeting*. Reserve Bank of Australia.
- Lubik, T. A., and F. Schorfheide. 2004. “Testing for Indeterminacy: An Application to U.S. Monetary Policy.” *American Economic Review* 94(1): 190–217.
- . 2005. “A Bayesian Look at New Open Economy Macroeconomics.” In *NBER Macroeconomic Annual 2005*, 313–366. Cambridge, MA: MIT Press.
- Lucas, R. E. 1972. “Expectations and the Neutrality of Money.” *Journal of Economic Theory* 4(2): 103–124.
- . 1973. “Some International Evidence on Output-Inflation Trade-offs.” *American Economic Review* 63(3): 326–334.
- . 1976. “Econometric Policy Evaluation: A Critique.” *Carnegie-Rochester Conference Series on Public Policy* 1: 19–46.

- . 1980a. "Equilibrium in a Pure Currency Economy." In *Models of Monetary Economies*, ed. J. H. Karaken, and N. Wallace, 131–145. Federal Reserve Bank of Minneapolis.
- . 1980b. "Two Illustrations of the Quantity Theory of Money." *American Economic Review* 70(5): 1005–1014.
- . 1982. "Interest Rates and Currency Prices in a Two-Country World." *Journal of Monetary Economics* 10(3): 335–359.
- . 1988. "Money Demand in the United States: A Quantitative Review." *Carnegie-Rochester Conference Series on Public Policy* 29: 137–168.
- . 1990. "Liquidity and Interest Rates." *Journal of Economic Theory* 50(2): 237–264.
- . 1996. "Nobel Lecture: Monetary Neutrality." *Journal of Political Economy* 104(4): 661–682.
- . 2000. "The Welfare Costs of Inflation." *Econometrica* 68(2): 247–274.
- Lucas, R. E., and N. L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics* 12(1): 55–93.
- . 1987. "Money and Interest in a Cash-in-Advance Economy." *Econometrica* 55(3): 491–514.
- . 1989. *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard University Press.
- Maddala, G. S. 1992. *Introduction to Econometrics*. 2nd ed. New York: Macmillan.
- Mankiw, N. G. 1985. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *Quarterly Journal of Economics* 101(2): 529–537.
- . 1987. "The Optimal Collection of Seigniorage: Theory and Evidence." *Journal of Monetary Economics* 20(2): 327–341.
- Mankiw, N. G., and J. A. Miron. 1986. "The Changing Behavior of the Term Structure of Interest Rates." *Quarterly Journal of Economics* 101(2): 211–228.
- Mankiw, N. G., J. A. Miron, and D. N. Weil. 1987. "The Adjustment of Expectations to a Change in Regime: A Study of the Founding of the Federal Reserve." *American Economic Review* 77(3): 358–374.
- . 1994. "The Founding of the Fed and the Behavior of Interest Rates: What Can be Learned from Small Samples?" *Journal of Monetary Economics* 34(3): 555–559.
- Mankiw, N. G., and R. Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117(4): 1295–1328.
- . 2006a. "Pervasive Stickiness." *American Economic Review* 96(2): 164–169.
- . 2006b. "Sticky Information in General Equilibrium." *Journal of the European Economic Association* 2(2-3): 603–613.
- Mankiw, N. G., and L. H. Summers. 1986. "Money Demand and the Effects of Fiscal Policy." *Journal of Money, Credit, and Banking* 18(4): 415–429.
- Martin, A., J. McAndrews, A. Palida, and D. Skeie. 2013. "Federal Reserve Tools for Managing Rates and Reserves." Federal Reserve Bank of New York Staff Report 624.
- Maskin, E., and J. Tirole. 1988. "Models of Dynamic Oligopoly I: Overview and Quality Competition with Large Fixed Costs." *Econometrica* 56(3): 549–569.
- Mattey, J., and R. Meese. 1986. "Empirical Assessment of Present Value Relationships." *Econometric Reviews* 5(2): 171–233.
- McCallum, B. T. 1983a. "On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective." *Journal of Monetary Economics* 11(2): 139–168.
- . 1983b. "The Role of Overlapping Generations Models in Monetary Economics." *Carnegie-Rochester Conference Series on Public Policy* 18: 9–44.
- . 1984a. "A Linearized Version of Lucas's Neutrality Model." *Canadian Journal of Economics* 17(1): 138–145.
- . 1984b. "On Low-frequency Estimates of Long-run Relationships in Macroeconomics." *Journal of Monetary Economics* 14(1): 3–14.
- . 1986. "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine." *Journal of Monetary Economics* 17(1): 135–160.

- . 1988. “Robustness Properties of a Rule for Monetary Policy.” *Carnegie-Rochester Conference Series on Public Policy* 29: 173–204.
- . 1989. *Monetary Economics: Theory and Policy*. New York: Macmillan.
- . 1990a. “Inflation: Theory and Evidence.” In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 2, 963–1012. Amsterdam: Elsevier.
- . 1990b. “Targets, Instruments, and Indicators of Monetary Policy.” In *Monetary Policy for a Changing Financial Environment*, ed. W. S. Haraf, and P. Cagan, 44–70. Washington, D.C. AEI Press.
- . 1995. “Two Fallacies Concerning Central Bank Independence.” *American Economic Review* 85(2): 207–211.
- . 1997a. “Critical Issues Concerning Central Bank Independence.” *Journal of Monetary Economics* 39(1): 99–112.
- . 1997b. “Inflation Targeting in Canada, New Zealand, Sweden, the United Kingdom, and in General.” In *Towards More Effective Monetary Policy*, ed. I. Kuroda, 211–241. London: Macmillan.
- . 2000. “Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates.” *Journal of Money, Credit, and Banking*, 32(4 pt. 2): 870–904.
- . 2001. “Indeterminacy, Bubbles, and the Fiscal Theory of Price Level Determination.” *Journal of Monetary Economics* 47(1): 19–30.
- . 2005. “Monetary Policy and the Term Structure of Interest Rates.” *Federal Reserve Bank of Richmond Economic Quarterly*, 91(4): 1–21.
- McCallum, B. T., and M. S. Goodfriend. 1987. “Demand for Money: Theoretical Studies.” In *The New Palgrave Dictionary of Economics*, ed. P. Newman, M. Milgate, and J. Eatwell, 775–781. Houndmills, UK: Palgrave Macmillan.
- McCallum, B. T., and J. G. Hoehn. 1983. “Instrument Choice for Money Stock Control with Contemporaneous and Lagged Reserve Accounting: A Note.” *Journal of Money, Credit, and Banking* 15(1): 96–101.
- McCallum, B. T., and E. Nelson. 1999. “An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis.” *Journal of Money, Credit, and Banking* 31(3 pt. 1): 296–316.
- . 2000. “Monetary Policy for an Open Economy: An Alternative Framework with Optimizing Agents and Sticky Prices.” *Oxford Review of Economic Policy* 16: 74–91.
- . 2004. “Timeless Perspective vs. Discretionary Monetary Policy in Forward-Looking Models.” *Federal Reserve Bank of St. Louis Review* 86(Mar.): 43–56.
- . 2005. “Fiscal and Monetary Theories of the Price Level: The Irreconcilable Differences.” *Oxford Review of Economic Policy* 21: 565–583.
- McCandless Jr., G. T., and W. E. Weber. 1995. “Some Monetary Facts.” *Federal Reserve Bank of Minneapolis Quarterly Review* 19(3): 2–11.
- McGough, B., G. D. Rudebusch, and J. C. Williams. 2005. “Using a Long-Term Interest Rate as the Monetary Policy Instrument.” *Journal of Monetary Economics* 52(5): 855–879.
- McKay, A., E. Nakamura, and J. Steinsson. 2016. “The Power of Forward Guidance Revisited.” *American Economic Review*, forthcoming.
- Mertens, K., and M. O. Ravn. 2014. “Fiscal Policy in an Expectations-Driven Liquidity Trap.” *Review of Economic Studies* 81(4): 1637–1667.
- Metzler, L. 1951. “Wealth, Saving, and the Rate of Interest.” *Journal of Political Economy* 59(2): 92–116.
- Meulendyke, A.-M. 1998. *U.S. Monetary Policy and Financial Markets*. Federal Reserve Bank of New York.
- Miao, J. 2014. *Economic Dynamics in Discrete Time*. Cambridge, MA: MIT Press.
- Miles, D. K., and J. Schanz. 2014. “The Relevance or Otherwise of the Central Bank’s Balance Sheet.” *Journal of Monetary Economics* 92: S103–S116.
- Minford, P. 1995. “Time-Inconsistency, Democracy, and Optimal Contingent Rules.” *Oxford Economic Papers* 47(2): 195–210.
- Mino, K., and S. Tsutsui. 1990. “Reputational Constraint and Signalling Effects in a Monetary Policy Game.” *Oxford Economic Papers* 42(3): 603–619.
- Mishkin, F. S. 1982. “Does Anticipated Policy Matter? An Econometric Investigation.” *Journal of Political Economy* 90(1): 22–51.

- . 1992. “Is the Fisher Effect for Real? A Reexamination of the Relationship between Inflation and Interest Rates.” *Journal of Monetary Economics* 30(2): 195–215.
- Mishkin, F. S., and A. S. Posen. 1997. “Inflation Targeting: Lessons from Four Countries.” Federal Reserve Bank of New York *Economic Policy Review* 3(3): 9–110.
- Mishkin, F. S., and K. Schmidt-Hebbel. 2002. “A Decade of Inflation Targeting in the World: What Do We Know and What Do We Need to Know?” In *Inflation Targeting: Design, Performance, Challenges*, ed. N. Loayza, and R. Soto, 171–219. Santiago: Banco Central de Chile.
- . 2007. “Does Inflation Targeting Make a Difference?” In *Monetary Policy under Inflation Targeting*, ed. F. S. Mishkin, and K. Schmidt-Hebbel, 291–372. Santiago: Banco Central de Chile.
- Mixon Jr., F. G., and M. T. Gibson. 2002. “The Timing of Partisan and Nonpartisan Appointments to the Central Bank: Some New Evidence.” *Journal of Money, Credit, and Banking* 34(2): 361–375.
- Modigliani, F. 1963. “The Monetary Mechanism and Its Interaction with Real Phenomena.” *The Review of Economics and Statistics* 45(1): 79–107.
- Modigliani, F., and A. Ando. 1976. “Impacts of Fiscal Actions on Aggregate Income and the Monetarist Controversy: Theory and Evidence.” In *Monetarism*, ed. J. L. Stein, 17–42. Amsterdam: Elsevier.
- Modigliani, F., R. Rasche, and J. P. Cooper. 1970. “Central Bank Policy, the Money Supply, and the Short-Term Rate of Interest.” *Journal of Money, Credit, and Banking* 2(2): 166–218.
- Modigliani, F., and R. Sutch. 1967. “Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience.” *Journal of Political Economy* 75(4): 569–589.
- Moen, E. 1997. “Competitive Search Equilibrium.” *Journal of Political Economy* 105(2): 385–411.
- Monacelli, T. 2005. “Monetary Policy in a Low Pass-through Environment.” *Journal of Money, Credit, and Banking* 37(6): 1047–1066.
- . 2008. “Optimal Monetary Policy with Collateralized Household Debt and Borrowing Constraints.” In *Asset Prices and Monetary Policy*, ed. J. Y. Campbell, 103–141. Chicago: University of Chicago Press.
- Monnet, C., and W. Weber. 2001. “Money and Interest Rates.” Federal Reserve Bank of Minneapolis *Quarterly Review* 25(4): 2–13.
- Mortensen, D. T., and C. A. Pissarides. 1994. “Job Creation and Job Destruction in the Theory of Unemployment.” *Review of Economic Studies* 61(3): 397–415.
- Morton, J., and P. Wood. 1993. “Interest Rate Operating Procedures of Foreign Central Banks.” In *Operating Procedures and the Conduct of Monetary Policy: Conference Proceedings*. Federal Reserve System.
- Mulligan, C. B., and X. Sala-i-Martin. 1997. “The Optimum Quantity of Money: Theory and Evidence.” *Journal of Money, Credit, and Banking*, 24(4), 687–715.
- . 2000. “Extensive Margins and the Demand for Money at Low Interest Rates.” *Journal of Political Economy* 108(5): 961–991.
- Muscattelli, A. 1998. “Optimal Inflation Contracts and Inflation Targets with Uncertain Central Bank Preferences: Accountability through Independence.” *Economic Journal* 108: 529–542.
- . 1999. “Delegation versus Optimal Inflation Contracts: Do We Really Need Conservative Central Bankers?” *Economica* 66: 241–254.
- Nakamura, E., and J. Steinsson. 2008. “Five Facts About Prices: A Reevaluation of Menu Cost Models.” *Quarterly Journal of Economics* 123: 1415–1464.
- Nakamura, E., J. Steinsson, P. Sun, and D. Villar. 2016. “The Elusive Costs of Inflation: Price Dispersion during the U. S. Great Inflation.” NBER Working Paper No. 22505.
- Nakov, A. 2008. “Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate.” *International Journal of Central Banking* 4(2): 73–127.
- Neiss, K., and E. Nelson. 2003. “The Real-Interest-Rate Gap as an Inflation Indicator.” *Macroeconomic Dynamics* 7(02): 239–262.
- . 2005. “Inflation Dynamics, Marginal Cost, and the Output Gap: Evidence from Three Countries.” *Journal of Money, Credit, and Banking* 37(6): 1019–1045.
- Nelson, E. 1998. “Sluggish Inflation and Optimizing Models of the Business Cycle.” *Journal of Monetary Economics* 42(2): 303–322.

- Nessen, M., and D. Vestin. 2005. "Average Inflation Targeting." *Journal of Money, Credit, and Banking* 37(5): 837–863.
- Niehans, J. 1978. *The Theory of Money*. Baltimore: Johns Hopkins University Press.
- Niepel, D. 2004. "The Fiscal Myth of the Price Level." *Quarterly Journal of Economics* 119(1): 277–300.
- Nolan, C., and E. Schaling. 1996. "Monetary Policy Uncertainty and Central Bank Accountability." Bank of England Working Papers 54.
- Nosal, E., and G. Rocheteau. 2011. *Money, Payments, and Liquidity*. Cambridge, MA: MIT Press.
- Obstfeld, M., and K. Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91(4): 675–687.
- . 1986. "Ruling Out Divergent Speculative Bubbles." *Journal of Monetary Economics* 17(3): 349–362.
- . 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103(3): 624–660.
- . 1996. *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.
- . 2000. "New Directions in Stochastic Open Economy Models." *Journal of International Economics* 48: 117–153.
- . 2002. "The Global Implications of Self-Oriented National Monetary Rules." *Quarterly Journal of Economics* 117(2): 503–535.
- O’Flaherty, B. 1990. "The Care and Handling of Monetary Authorities." *Economics and Politics* 2(1): 25–44.
- Oh, S. 1989. "A Theory of a Generally Accepted Medium of Exchange and Barter." *Journal of Monetary Economics* 23(1): 101–119.
- Ohanian, L., and A. Stockman. 1995. "Resolving the Liquidity Effect." Federal Reserve Bank of St. Louis *Review* 77(3): 3–25.
- Orphanides, A. 2000. "The Quest for Prosperity without Inflation." European Central Bank Working paper.
- . 2001. "Monetary Policy Rules Based on Real-time Data." *American Economic Review* 91(4): 964–985.
- Orphanides, A., and J. C. Williams. 2002. "Robust Monetary Policy Rules with Unknown Natural Rates." *Brookings Papers on Economic Activity* 33(2): 63–146.
- Papell, D., T. Molodtsova, and A. Nikolsko-Rzhevskyy. 2008. "Taylor Rules with Real-Time Data: A Tale of Two Countries and One Exchange Rate." *Journal of Monetary Economics* 55: S63–S79.
- Patinkin, D. 1965. *Money, Interest, and Prices: An Integration of Monetary and Value Theory*. 2nd ed. New York: Harper and Row.
- Pearce, D. 1993. "Discount Window Borrowing and Federal Reserve Operating Regimes." *Economic Inquiry* 31(4): 564–579.
- Perez, S. J. 2001. "Looking Back at Forward-Looking Monetary Policy." *Journal of Economics and Business* 53(5): 509–521.
- Peristiani, S. 1991. "The Model Structure of Discount Window Borrowing." *Journal of Money, Credit, and Banking* 23(1): 13–34.
- Persson, T., and G. Tabellini. 1990. *Macroeconomic Policy, Credibility and Politics*. Chur, Switzerland: Harwood Academic Publishers.
- . 1993. "Designing Institutions for Monetary Stability." *Carnegie-Rochester Conference Series on Public Policy* 39: 53–84.
- . eds. 1994. *Monetary and Fiscal Policy: Credibility*. Cambridge, MA: MIT Press.
- . 1999. "Political Economics and Macroeconomic Policy." In *Handbook of Macroeconomics*, ed. J. B. Taylor, and M. Woodford, vol. 1C, 1397–1482. Amsterdam: Elsevier.
- Petrosky-Nadeau, N., and L. Zhang. 2013a. "Solving the DMP Model Accurately." NBER Working Paper 19208.
- . 2013b. "Unemployment Crises." NBER Working Paper 19207.
- Phelps, E. S. 1973. "Inflation in the Theory of Public Finance." *Swedish Journal of Economics* 75(1): 67–82.
- Piazzesi, M. 2010. "Affine Term Structure Models." In *Handbook of Financial Econometrics: Tools and Techniques*, 691–766, ed. Y. Ait-Sahalia, and L. P. Hansen. Amsterdam: Elsevier.
- Pissarides, C. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77(5): 1339–1369.

- Poole, W. 1968. "Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy." *Journal of Finance* 23: 769–791.
- . 1970. "Optimal Choice of Monetary Policy Instrument in a Simple Stochastic Macro Model." *Quarterly Journal of Economics* 84(2): 197–216.
- Poterba, J. M., and J. J. Rotemberg. 1990. "Inflation and Taxation with Optimizing Governments." *Journal of Money, Credit, and Banking* 22(1): 1–18.
- Primiceri, G. 2006. "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U.S. Postwar Stabilization Policy." *Quarterly Journal of Economics* 121(3): 869–901.
- Ramey, V. A. 2016. "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics*, vol. 2A, ed. J. B. Taylor and H. Uhlig. Amsterdam: Elsevier.
- Ramsey, F. P. (1928). "A Mathematical Theory of Saving." *Economic Journal* 38(152): 543–559.
- Rasche, R. H. 1973. "A Comparative Static Analysis of Some Monetarist Propositions." Federal Reserve Bank of St. Louis Review 55: 15–23.
- Ravenna, F. 2000. "The Impact of Inflation Targeting in Canada: A Structural Analysis." Working Paper.
- Ravenna, F., and J. Seppälä. 2006. "Monetary Policy and Rejections of the Expectations Hypothesis." Bank of Finland Research Discussion Papers 25/2006.
- . 2007. "Monetary Policy, Expected Inflation and Inflation Risk Premia." Bank of England Research Discussion Papers 18/2007.
- Ravenna, F., and C. E. Walsh. 2006. "Optimal Monetary Policy with the Cost Channel." *Journal of Monetary Economics* 53(2): 199–216.
- . 2008. "Vacancies, Unemployment, and the Phillips Curve." *European Economic Review* 52(8): 1494–1521.
- . 2011. "Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: a Linear-Quadratic Framework." *American Economic Journal: Macroeconomics* 3(2): 130–162.
- . 2012a. "Monetary Policy and Labor Market Frictions: A Tax Interpretation." *Journal of Monetary Economics* 59(2): 180–195.
- . 2012b. "Screening and Labor Market Flows in a Model with Heterogeneous Workers." *Journal of Money, Credit, and Banking* 44(2): 31–71.
- Rebelo, S., and D. Xie. 1999. "On the Optimality of Interest Rate Smoothing." *Journal of Monetary Economics* 43(2): 263–282.
- Reichenstein, W. 1987. "The Impact of Money on Short-Term Interest Rates." *Economic Inquiry* 25(1): 67–82.
- Reis, R. 2006a. "Inattentive Consumers." *Journal of Monetary Economics* 53(8): 1731–1800.
- . 2006b. "Inattentive Producers." *Review of Economic Studies* 73(3): 793–821.
- . 2007. "The Analytics of Monetary Non-neutrality in the Sidrauski Model." *Economics Letters* 94(1): 129–135.
- . 2009. "Interpreting the Unconventional US Monetary Policy of 2007–09." *Brookings Papers on Economic Activity* 40(2): 119–182.
- Reynard, S. 2004. "Financial Market Participation and the Apparent Instability of Money Demand." *Journal of Monetary Economics* 51: 1297–1317.
- Rigobon, R., and B. Sack. 2004. "The Impact of Monetary Policy on Stock Prices." *Journal of Monetary Economics* 51: 1553–1575.
- Ritter, J. 1995. "The Transition from Barter to Fiat Money." *American Economic Review* 85(1): 134–149.
- Roberts, J. M. 1995. "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit, and Banking* 27(4): 975–984.
- . 1997. "Is Inflation Sticky?" *Journal of Monetary Economics* 39(2): 173–196.
- Rocheteau, G., and R. Wright. 2005. "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium." *Econometrica* 73(1): 175–202.
- Rockoff, H. 2015. "War and Inflation in the United States from the Revolution to the First Iraq War." NBER Working Paper 21221.

- Roger, S. 2010. "Inflation Targeting Turns 20." *Finance & Development* (Mar.): 46–49.
- Rogerson, R., and R. Shimer. 2011. *Search in Macroeconomic Models of the Labor Market*. Amsterdam: Elsevier.
- Rogoff, K. 1985. "The Optimal Degree of Commitment to an Intermediate Monetary Target." *Quarterly Journal of Economics* 100(4): 1169–1189.
- . 1989. "Reputation, Coordination, and Monetary Policy." In *Modern Business Cycle Theory*, ed. R. Barro, 236–264. Cambridge, MA: Harvard University Press.
- Roley, V. V., and G. H. J. Sellon. 1996. "The Response of the Term Structure of Interest Rates to Federal Funds Rate Target Changes." Discussion paper.
- Roley, V. V., and C. E. Walsh. 1985. "Monetary Policy Regimes, Expected Inflation, and the Response of Interest Rates to Money Announcements." *Quarterly Journal of Economics* 100(5): 1011–1039.
- Rolnick, A. J., and W. E. Weber. 1997. "Money, Inflation, and Output under Fiat and Commodity Standards." *Journal of Political Economy* 105(6): 1308–1321.
- Romer, C. D., and D. H. Romer. 1989. "Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz." In *NBER Macroeconomics Annual 1989*, 121–170. Cambridge, MA: MIT Press.
- . 2002. "The Evolution of Economic Understanding and Postwar Stabilization Policy." In *Proceedings, Economic Policy Symposium*, 11–78. Federal Reserve Bank of Kansas.
- . 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review* 94: 1055–1084.
- Romer, D. H. 1986. "A Simple General Equilibrium Version of the Baumol-Tobin Model." *Quarterly Journal of Economics* 101(4): 663–685.
- . 1993. "Openness and Inflation: Theory and Evidence." *Quarterly Journal of Economics* 108(4): 869–903.
- . 1998. "A New Assessment of Openness and Inflation: Reply." *Quarterly Journal of Economics* 113(2): 649–652.
- . 2012. *Advanced Macroeconomics*. 4th ed. New York: McGraw-Hill.
- Rose, A. K. 2014. "Recent Monetary Regimes of Small Economies." *Journal of International Money and Finance* 49: 5–27.
- Rotemberg, J. J. 1982. "Sticky Prices in the United States." *Journal of Political Economy* 80: 1187–1211.
- . 1984. "A Monetary Equilibrium Model with Transaction Costs." *Journal of Political Economy* 92(1): 40–58.
- . 1988. "New Keynesian Microfoundations." In *NBER Macroeconomics Annual 1987*, 69–104. Cambridge, MA: MIT Press.
- . 2007. "The Persistence of Inflation Versus That of Real Marginal Cost in the New Keynesian Model." *Journal of Monetary Economics* 39(1): 237–239.
- Rotemberg, J. J., and M. Woodford. 1995. "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets." In *Frontiers of Business Cycle Research*, ed. T. F. Cooley, 243–293. Princeton, NJ: Princeton University Press.
- . 1997. "An Optimization-Based Framework for the Evaluation of Monetary Policy." In *NBER Macroeconomics Annual 1997*, 297–361. Cambridge, MA: MIT Press.
- Rudd, J., and K. Whelan. 2005. "Does Labor's Share Drive Inflation?" *Journal of Money, Credit, and Banking* 37(2): 297–312.
- Rudebusch, G. D. 1995a. "Erratum." *Journal of Monetary Economics* 36: 679.
- . 1995b. "Federal Reserve Interest Rate Targeting, Rational Expectations, and the Term Structure." *Journal of Monetary Economics* 35(2): 245–274.
- . 1998. "Do Measures of Monetary Policy in a VAR Make Sense?" *International Economic Review* 39(4): 907–931.
- . 2002a. "Assessing Nominal Income Rules for Monetary Policy with Model and Data Uncertainty." *Economic Journal* 112(479): 402–432.
- . 2002b. "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia." *Journal of Monetary Economics* 49(6): 1161–1187.



- . 2006. “Monetary Policy Inertia: A Fact or Fiction.” *International Journal of Central Banking* 2: 135–865.
- Rudebusch, G. D., B. P. Sack, and E. T. Swanson. 2007. “Macroeconomic Implications of Changes in the Term Premium.” Federal Reserve Bank of St. Louis *Review* 89(4): 241–269.
- Rudebusch, G. D., and L.E.O. Svensson. 1999. “Policy Rules for Inflation Targeting.” In *Monetary Policy Rules*, ed. J. B. Taylor, 203–262. Chicago: University of Chicago Press.
- Rudebusch, G. D., and T. Wu. 2007. “Accounting for a Shift in Term Structure Behavior with No-Arbitrage and Macro-Finance Models.” *Journal of Money, Credit, and Banking* 39(2-3): 395–422.
- . 2008. “A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy.” *Economic Journal* 118: 906–926.
- Ruge-Murcia, F. J. 2003. “Does the Barro-Gordon Model Explain the Behavior of U.S. Inflation?” *Journal of Monetary Economics* 50(6): 1375–1390.
- Rupert, P., M. Schindler, and R. Wright. 2001. “Generalized Search-Theoretic Models of Monetary Exchange.” *Journal of Monetary Economics* 48(3): 605–622.
- Sack, B. 2000. “Does the Fed Act Gradually? A VAR Analysis.” *Journal of Monetary Economics* 46(1): 229–256.
- Sala, L., U. Söderstrom, and A. Trigari. 2008. “Monetary Policy under Uncertainty in an Estimated Model with Labor Market Frictions.” *Journal of Monetary Economics* 55(5): 983–1006.
- . 2010. “The Output Gap, the Labor Wedge, and the Dynamic Behavior of Hours.” IGIER Working Paper 365. Bocconi University, Milan.
- . 2012. “Structural and Cyclical Forces in the Labor Market during the Great Recession: Cross-Country Evidence.” In *Proceedings of the NBER International Seminar on Macroeconomics*, 345–404.
- Salyer, K. D. 1991. “The Timing of Markets and Monetary Transfers in Cash-in-Advance Economics.” *Economic Inquiry* 29: 762–773.
- Samuelson, P. A. 1958. “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money.” *Journal of Political Economy* 66(6): 467–482.
- Sargent, T. J. 1976. “The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics.” *Journal of Political Economy* 84(3): 631–640.
- . 1982. “Beyond Supply and Demand Curves in Macroeconomics.” *American Economic Review* 72(2): 382–389.
- . 1986. “The Ends of Four Big Inflation.” In *Rational Expectations and Inflation*, 40–109. New York: Harper and Row.
- . 1987. *Dynamic Macroeconomic Theory*. Cambridge, MA: Harvard University Press.
- . 1999. *The Conquest of American Inflation*. Princeton, NJ: Princeton University Press.
- Sargent, T. J., and P. Surico. 2011. “Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals.” *American Economic Review* 101(1): 109–128.
- Sargent, T. J., and N. Wallace. 1975. “‘Rational’ Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule.” *Journal of Political Economy* 83(2): 241–254.
- . 1981. “Some Unpleasant Monetarist Arithmetic.” Federal Reserve Bank of Minneapolis *Quarterly Review* 5(3): 1–17.
- Sbordone, A. M. 2002a. “An Optimizing Model of U.S. Wage and Price Dynamics.” Federal Reserve Bank of San Francisco *Proceedings*, (Mar.).
- . 2002b. “Prices and Unit Labor Costs: A New Test of Price Stickiness.” *Journal of Monetary Economics* 49(2): 265–292.
- Schelde-Andersen, P. 1992. “OECD Country Experiences with Disinflation.” In *Inflation, Disinflation and Monetary Policy*, ed. A. Blundell-Wignall, 104–173. Reserve Bank of Australia.
- Schellekens, P. 2002. “Caution and Conservatism in the Making of Monetary Policy.” *Journal of Money, Credit, and Banking* 34(1): 160–177.
- Schmitt-Grohe, S., and M. Uribe. 2000. “Price Level Determinacy and Monetary Policy under a Balanced-Budget Requirement.” *Journal of Monetary Economics* 45(1): 211–246.
- Sheffrin, S. M. 1995. “Identifying Monetary and Credit Shocks.” In *Monetarism and the Methodology of Economics*, ed. K. D. Hoover, and S. M. Sheffrin, 151–163. Aldershot, UK: Edward Elgar.

- Shi, S. 1995. "Money and Prices: A Model of Search and Bargaining." *Journal of Economic Theory* 67(2): 467–496.
- . 1997. "A Divisible Search Model of Fiat Money." *Econometrica* 65(1): 75–102.
- . 1999. "Search, Inflation and Capital Accumulation." *Journal of Monetary Economics* 44(1): 81–103.
- . 2005. "Nominal Bonds and Interest Rates." *International Economic Review* 46(2): 579–612.
- Shiller, R. J. 1981. "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71(3): 421–436.
- . 1990. "The Term Structure of Interest Rates." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and F. Hahn, vol. 1, 626–722. Amsterdam: Elsevier.
- Shimer, R. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95(1): 25–49.
- Sidrauski, M. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 57(2): 534–544.
- Sims, C. A. 1972. "Money, Income and Causality." *American Economic Review* 62(4): 540–542.
- . 1980. "Comparison of Interwar and Postwar Business Cycles." *American Economic Review* 70(2): 250–257.
- . 1988. "Identifying Policy Effects." In *Empirical Macroeconomics for Interdependent Economies*, ed. R. C. Bryant, D. W. Henderson, G. Holtham, P. Hooper, and S. A. Symansky, 305–321. Washington, DC: Brookings Institution.
- . 1992. "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy." *European Economic Review* 36(5): 975–1000.
- . 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4: 381–399.
- . 1998a. "Comment on Glenn Rudebusch's 'Do Measures of Monetary Policy in a VAR Make Sense?'" *International Economic Review* 39(4): 933–941.
- . 1998b. "The Role of Interest Rate Policy in the Generation and Propagation of Business Cycles: What Has Changed Since the 30s?" In *Proceedings of the Annual Research Conference*, 121–167. Federal Reserve Bank of Boston.
- . 2003. "Implications of Rational Inattention." *Journal of Monetary Economics* 50(3): 665–690.
- . 2013. "Paper Money." *American Economic Review* 103(2): 563–584.
- Small, D. H. 1979. "Unanticipated Money Growth and Unemployment in the United States: Comment." *American Economic Review* 69(5): 996–1003.
- Smets, F., and R. Wouters. 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1(5): 1123–1175.
- . 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97(3): 586–606.
- Smith, B. 1983. "Limited Information, Credit Rationing, and Optimal Government Lending Policy." *American Economic Review* 73(3): 305–318.
- Smith, J. M., and J. B. Taylor. 2009. "The Term Structure of Policy Rules." *Journal of Monetary Economics* 56(7): 907–917.
- Söderlind, P., and L.E.O. Svensson. 1997. "New Techniques to Extract Market Expectations from Financial Instruments." *Journal of Monetary Economics* 40(2): 383–429.
- Söderström, U. 2002. "Monetary Policy with Uncertain Parameters." *Scandinavian Journal of Economics* 104(1): 125–145.
- Soller, E. V., and C. J. Waller. 2000. "A Search Theoretic Model of Legal and Illegal Currency." *Journal of Monetary Economics* 45(1): 155–184.
- Solow, R. 1956. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics* 70(1): 65–94.
- Spindt, P. 1985. "Money Is What Money Does: Monetary Aggregation and the Equations of Exchange." *Journal of Political Economy* 93(1): 175–204.

- Stein, J. 1969. "Neoclassical and Keynes-Wicksell Monetary Growth Models." *Journal of Money, Credit, and Banking* 1(2): 153–171.
- Steinsson, J. 2003. "Optimal Monetary Policy in an Economy with Inflation Persistence." *Journal of Monetary Economics* 50(7): 1425–1456.
- Stiglitz, J. E., and A. Weiss. 1981. "Credit Rationing in Markets with Imperfect Information." *American Economic Review* 71(3): 393–410.
- Stock, J. H., and M. W. Watson. 1989. "Interpreting the Evidence on Money-Income Causality." *Journal of Econometrics* 40(1): 161–181.
- . 1993. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica* 61: 783–820.
- Stockman, A. 1981. "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8(3): 387–393.
- Stokey, N. 2002. "'Rules versus Discretion' after Twenty-Five Years." In *NBER Macroeconomics Annual 2002*, 9–45. Cambridge, MA: MIT Press.
- Strongin, S. 1995. "The Identification of Monetary Policy Disturbances: Explaining the Liquidity Puzzle." *Journal of Monetary Economics* 35(3): 463–497.
- Summers, L. H. 1981. "Optimal Inflation Policy." *Journal of Monetary Economics* 7(2): 175–194.
- Svensson, L.E.O. 1985. "Money and Asset Prices in a Cash-in-Advance Economy." *Journal of Political Economy* 93(5): 919–944.
- . 1997a. "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review* 41(6): 1111–1146.
- . 1997b. "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts." *American Economic Review* 87(1): 98–114.
- . 1999a. "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics* 43: 607–654.
- . 1999b. "Inflation Targeting: Some Extensions." *Scandinavian Journal of Economics* 101(3): 337–361.
- . 1999c. "Price Level Targeting vs. Inflation Targeting." *Journal of Money, Credit, and Banking* 31: 277–295.
- . 2001. "The Foolproof Way of Escaping from a Liquidity Trap: Is It Really, and Can It Help Japan?" Frank D. Graham Memorial Lecture, Princeton University, Princeton, NJ.
- Svensson, L.E.O., and N. Williams. 2008. "Optimal Monetary Policy Under Uncertainty: A Markov Jump-Linear-Quadratic Approach." *Federal Reserve Bank of St. Louis Review* 90(4): 275–293.
- Svensson, L.E.O., and M. Woodford. 2003. "Indicator Variables for Optimal Policy." *Journal of Monetary Economics* 50(3): 691–720.
- . 2004. "Indicator Variables for Optimal Policy under Asymmetric Information." *Journal of Economic Dynamics and Control* 28(4): 661–690.
- . 2005. "Implementing Optimal Policy through Inflation-Forecast Targeting." In *The Inflation-Targeting Debate*, ed. B. S. Bernanke and M. Woodford, 19–83. Chicago: University of Chicago Press.
- Swanson, E. T. 2011. "Let's Twist Again: A High-Frequency Event-Study Analysis of Operation Twist and Its Implications for QE2." *Brookings Papers on Economic Activity* 42(1): 151–207.
- . 2015. "Measuring the Effects of Unconventional Monetary Policy on Asset Prices." NBER Working Paper 21816.
- Swanson, E. T., and J. C. Williams. 2013. "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates." Federal Reserve Bank of San Francisco Working Paper 2012-02.
- Tabellini, G. 1988. "Centralized Wage Setting and Monetary Policy in a Reputational Equilibrium." *Journal of Money, Credit, and Banking* 20(1): 102–118.
- Taylor, J. B. 1979. "Staggered Wage Setting in a Macro Model." *American Economic Review* 69(2): 108–113.
- . 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88(1): 1–24.
- . 1983. "Comments on 'Rules, Discretion and Reputation in a Model of Monetary Policy' by R. J. Barro and D. B. Gordon." *Journal of Monetary Economics* 12(1): 123–125.

- . 1985. “What Would Nominal GNP Targeting Do to the Business Cycle?” *Carnegie-Rochester Conference Series on Public Policy* 22: 61–84.
- . 1993a. “Discretion versus Policy Rules in Practice.” *Carnegie Rochester Conference Series on Public Policy* 39(1): 195–214.
- . 1993b. *Macroeconomic Policy in a World Economy*. New York: W. W. Norton.
- . 1996. “How Should Monetary Policy Respond to Shocks While Maintaining Long-Run Price Stability?” In *Proceedings, Economic Policy Symposium*, 181–195. Federal Reserve Bank of Kansas City.
- . 2016. “The Staying Power of Staggered Wage and Setting Models in Macroeconomics.” In *Handbook of Macroeconomics*, vol. 2A, ed. J. B. Taylor and H. Uhlig. Amsterdam: Elsevier.
- Teles, P., and R. Zhou. 2005. “A Stable Money Demand: Looking for the Right Monetary Aggregate.” *Federal Reserve Bank of Chicago Economic Perspectives* 29(1): 50–63.
- Temple, J. B. 2002. “Openness, Inflation, and the Phillips Curve: A Puzzle.” *Journal of Money, Credit, and Banking* 34(2): 450–468.
- Terra, C. T. 1998. “Openness and Inflation: A New Assessment.” *Quarterly Journal of Economics* 113(2): 641–648.
- Thomas, C. 2008. “Search and Matching Frictions and Optimal Monetary Policy.” *Journal of Monetary Economics* 55: 936–956.
- . 2011. “Search Frictions, Real Rigidities, and Inflation Dynamics.” *Journal of Money, Credit, and Banking* 43(6): 1131–1164.
- Tinbergen, J. 1956. *Economic Policy: Principles and Design*. Amsterdam: Elsevier.
- Tobin, J. 1956. “The Interest Elasticity of the Transactions Demand for Cash.” *Review of Economics and Statistics* 38(3): 241–247.
- . 1965. “Money and Economic Growth.” *Econometrica* 33(4): 671–684.
- . 1969. “A General Equilibrium Approach to Monetary Theory.” *Journal of Money, Credit, and Banking* 1(1): 15–29.
- . 1970. “Money and Income: Post Hoc Ergo Proctor Hoc?” *Quarterly Journal of Economics* 84(2): 301–317.
- Tootell, G. 1991. “Are District Presidents More Conservative Than Board Governors?” *Federal Reserve Bank of Boston New England Economic Review* (Sept.-Oct.): 3–12.
- Trabandt, M. 2003. “Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework.” Humbolt University, Berlin.
- Trehan, B., and C. E. Walsh. 1988. “Common Trends, the Government’s Budget Balance, and Revenue Smoothing.” *Journal of Economic Dynamics and Control* 12(2-3): 425–444.
- . 1990. “Seigniorage and Tax Smoothing in the United States, 1914–1986.” *Journal of Monetary Economics* 25(1): 97–112.
- . 1991. “Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Federal Budget and Current Account Deficits.” *Journal of Money, Credit, and Banking* 23(2): 206–223.
- Trejos, A., and R. Wright. 1993. “Search, Bargaining, Money and Prices: Recent Results and Policy Implications.” *Journal of Money, Credit, and Banking* 25(3): 558–576.
- . 1995. “Search, Bargaining, Money, and Prices.” *Journal of Political Economy* 103(1): 118–141.
- Trigari, A. 2009. “Equilibrium Unemployment, Job Flows, and Inflation Dynamics.” *Journal of Money, Credit, and Banking* 41(1): 1–33.
- Uhlig, H. 1999. “A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily.” In *Computational Methods for the Study of Dynamic Economies*, ed. R. Marimon and A. Scott, 30–61. Oxford: Oxford University Press.
- . 2005. “What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure.” *Journal of Monetary Economics* 52(2): 381–419.
- Van Hoose, D. 1989. “Monetary Targeting and Price Level Nontrend Stationarity.” *Journal of Money, Credit, and Banking* 21(2): 232–239.
- Vayanos, D., and J. Vila. 2009. “A Preferred-Habitat Model of the Term Structure of Interest Rates.” NBER Working Paper No. 15487.

- Vestin, D. 2006. "Price-Level versus Inflation Targeting." *Journal of Monetary Economics* 53(7): 1361–1376.
- Vickers, J. 1986. "Signalling in a Model of Monetary Policy with Incomplete Information." *Oxford Economic Papers* 38(3): 443–455.
- Wallace, N. 1981. "A Modigliani-Miller Theorem for Open-Market Operations." *American Economic Review* 71(3): 267–274.
- Waller, C. J. 1989. "Monetary Policy Games and Central Bank Politics." *Journal of Money, Credit, and Banking* 21(4): 422–431.
- . 1990. "Administering the Window: A Game Theoretic Model of Discount Window Borrowing." *Journal of Monetary Economics* 25(2): 273–287.
- . 1992. "A Bargaining Model of Partisan Appointments to the Central Bank." *Journal of Monetary Economics* 29(3): 411–428.
- . 1995. "Performance Contracts for Central Bankers." *Federal Reserve Bank of St. Louis Review* 77(5): 3–14.
- Waller, C. J., and C. E. Walsh. 1996. "Central Bank Independence, Economic Behavior and Optimal Term Lengths." *American Economic Review* 86(5): 1139–1153.
- Walsh, C. E. 1982a. "Asset Substitutability and Monetary Policy: An Alternative Characterization." *Journal of Monetary Economics* 9: 59–71.
- . 1982b. "The Effects of Alternative Operating Procedures on Economic and Financial Relationships." In *Proceedings, Economic Policy Symposium*, 133–163. Federal Reserve Bank of Kansas City.
- . 1984. "Optimal Taxation by the Monetary Authority." NBER Working Paper 1375.
- . 1986. "In Defense of Base Drift." *American Economic Review* 76(4): 692–700.
- . 1987. "Monetary Targeting and Inflation: 1976–1984." *Federal Reserve Bank of San Francisco Economic Review* (1): 5–15.
- . 1990. "Issues in the Choice of Monetary Policy Operating Procedures." In *Monetary Policy for a Changing Financial Environment*, ed. W. S. Haraf, and P. Cagan, 8–37. Washington, DC: AEI Press.
- . 1993. "What Caused the 1990-1991 Recession?" *Federal Reserve Bank of San Francisco Economic Review* (2): 33–48.
- . 1995a. "Is New Zealand's Reserve Bank Act of 1989 an Optimal Central Bank Contract?" *Journal of Money, Credit, and Banking* 27(4): 1179–1191.
- . 1995b. "Optimal Contracts for Central Bankers." *American Economic Review* 85(1): 150–167.
- . 1999. "Announcements, Inflation Targeting and Central Bank Incentives." *Economica* 66(262): 255–269.
- . 2000. "Market Discipline and Monetary Policy." *Oxford Economic Papers* 52: 249–271.
- . 2002. "When Should Central Bankers Be Fired?" *Economics of Governance* 3(1): 1–21.
- . 2003a. "Labor Market Search and Monetary Shocks." In *Dynamic Macroeconomic Analysis: Theory and Policy in General Equilibrium*, ed. S. Altug, J. Chadha, and C. Nolan, 451–486. Cambridge: Cambridge University Press.
- . 2003b. "Speed Limit Policies: The Output Gap and Optimal Monetary Policy." *American Economic Review* 93(1): 265–278.
- . 2005a. "Endogenous Objectives and the Evaluation of Targeting Rules for Monetary Policy." *Journal of Monetary Economics* 52(5): 889–911.
- . 2005b. "Labor Market Search, Sticky Prices, and Interest Rate Policies." *Review of Economic Dynamics* 8(4): 829–849.
- . 2007. "Optimal Economic Transparency." *International Journal of Central Banking* 3(1): 5–36.
- . 2009. "Inflation Targeting: What Have We Learned?." *International Finance* 12(2): 195–233.
- . 2011. "The Future of Inflation Targeting." *Economic Record* 87: 23–36.
- . 2014. "Monetary Policy Transmission Channels and Policy Instruments." Working Paper, University of California, Santa Cruz.
- . 2015. "Goals and Rules in Central Bank Design." *International Journal of Central Banking*, 11(Sept.): 295–352.

- Wang, P., and C. K. Yip. 1992. "Alternative Approaches to Money and Growth." *Journal of Money, Credit, and Banking* 24(4): 553–562.
- Werning, I. 2011. "Managing a Liquidity Trap: Monetary and Fiscal Policy." NBER Working Paper 17344.
- West, K. D. 1986. "Targeting Nominal Income: A Note." *Economic Journal* 96: 1077–1083.
- . 1987. "A Specification Test for Speculative Bubbles." *Quarterly Journal of Economics* 102(3): 553–580.
- . 1988. "Dividend Innovation and Stock Price Volatility." *Econometrica* 56(1): 37–61.
- Whitesell, W. 2006. "Interest Rate Corridors and Reserves." *Journal of Monetary Economics* 53(6): 1177–1195.
- Wickens, M. 2008. *Macroeconomic Theory: A Dynamic General Equilibrium Approach*. Princeton, NJ: Princeton University Press.
- Wieland, J. 2014. "Are Negative Supply Shocks Expansionary at the Zero Lower Bound." University of California, San Diego.
- Wieland, V. 2000a. "Learning By Doing and the Value of Optimal Experimentation." *Journal of Economic Dynamics and Control* 24: 501–534.
- . 2000b. "Monetary Policy, Parameter Uncertainty and Optimal Learning." *Journal of Monetary Economics* 46(1): 199–228.
- Williams, J. C. 2009. "Heeding Daedalus: Optimal Inflation and the Zero Lower Bound." *Brookings Papers on Economic Activity* 40(2): 1–49.
- Williamson, S. D. 1986. "Costly Monitoring, Financial Intermediation and Equilibrium Credit Rationing." *Journal of Monetary Economics* 18(2): 159–179.
- . 1987a. "Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing." *Quarterly Journal of Economics* 102(1): 135–145.
- . 1987b. "Financial Intermediation, Business Failures, and Real Business Cycles." *Journal of Political Economy* 95(6): 1196–1216.
- . 2004. "Limited Participation, Private Money, and Credit in a Spatial Model of Money." *Economic Theory* 24: 857–875.
- . 2005. "Limited Participation and the Neutrality of Money." Federal Reserve Bank of Richmond *Economic Quarterly* 91(2): 1–20.
- Williamson, S. D., and R. Wright. 2011. "New Monetarist Economics: Models." In *Handbook of Monetary Economics*, ed. B. M. Friedman, and M. Woodford, vol. 3A, 25–96. Amsterdam: Elsevier.
- Woodford, M. 1995. "Price-level Determinacy without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy* 43(1): 1–46.
- . 1998. "Doing without Money: Controlling Inflation in a Post-Monetary World." *Review of Economic Dynamics* 1: 173–219.
- . 1999. "Optimal Monetary Policy Inertia." *The Manchester School* 67: 1–35.
- . 2000. "Pitfalls of Forward-Looking Monetary Policy." *American Economic Review* 90(2): 100–104.
- . 2001a. "Fiscal Requirements for Price Stability?" *Journal of Money, Credit, and Banking* 33(3): 669–728.
- . 2001b. "Monetary Policy in the Information Economy." In *Proceedings, Economic Policy Symposium*. Federal Reserve Bank of Kansas City.
- . 2003a. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.
- . 2003b. "Optimal Interest-Rate Smoothing." *Review of Economic Studies* 70: 861–886.
- Woolley, J. 1995. *Nixon, Burns, 1972, and Independence in Practice*. Santa Barbara: University of California.
- Wren-Lewis, S., and C. Leith. 2006. "The Optimal Monetary Policy Response to Exchange Rate Misalignments." Centre for Dynamic Macroeconomic Analysis Conference Papers CDMC 06/05.
- Wu, J. C., and F. D. Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit, and Banking* 48(2-3): 253–291.
- Yun, T. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37(2): 345–370.
- Zha, T. 1997. "Identifying Monetary Policy: A Primer." Federal Reserve Bank of Atlanta *Economic Review* 82(2): 26–43.

## Name Index

- Abel, A. B., 112  
Abreu, D., 240  
Adam, K., 517, 521–522  
Adao, B., 345  
Adolfson, M., 307–308, 438, 448  
Aiyagari, S. R., 148–149  
Aizenman, J., 262  
Akerlof, G. A., 285, 481  
Albanesi, S., 234, 270  
Alesina, A., 251, 257–258  
al-Nowaihi, A., 238–239, 241, 256  
Alvarez, F., 207, 212–214, 300  
Amato, J. D., 358  
Ammer, J., 358  
Andersen, L., 12  
Andersen, T., 241  
Ando, A., 12  
Andrés, J., 38, 377, 534, 538–539, 543, 556  
Ang, A., 467  
Angelini, P., 476  
Aoki, K., 442, 447  
Ascari, G., 292, 308, 334–335  
Athey, S., 256  
Atkeson, A., 207, 212–214, 256  
Atkinson, A. B., 174, 181  
Attfield, C. L. F., 218, 505  
Auernheimer, L., 226  
  
Backus, D. K., 241–243  
Bae, Y., 59–60  
Bailey, M. J., 61–62  
Balduzzi, P., 473, 476  
Ball, L., 30, 59, 64, 201, 241–243, 269, 284–286  
Barlevy, G., 162  
Barnett, W. A., 13  
Barr, D. G., 465  
Barro, R. J., 7, 14, 67, 171, 198–200, 222–223, 225, 227–228, 234, 236–238, 241–243  
Barth, M. J., 22  
Bassetto, M., 167  
Batini, N., 360  
  
Batten, D. S., 579, 603  
Bauer, M., 37  
Baumeister, C., 39  
Baumol, W., 42, 97  
Bean, C., 264  
Beaudry, P., 286  
Bech, M. L., 601  
Beetsma, R., 276  
Belongia, M. T., 13  
Benassy, J.-P., 282  
Benati, L., 39  
Benhabib, J., 52, 335, 513–514  
Benigno, G., 341, 344, 397, 414–415, 419, 422, 424  
Benigno, P., 397, 414–415, 419, 422, 424, 442, 444, 446  
Berentsen, A., 2, 7, 67, 593  
Bermanke, B. S., 14, 19, 21–22, 24, 28–29, 31, 37, 40, 358, 474, 479, 489–491, 495–496, 501, 504, 509, 579, 583, 585, 590, 596, 598–599, 603  
Betts, C., 397  
Billi, R. M., 360, 517, 521–522  
Bils, M., 302  
Blake, A. P., 348  
Blanchard, O. J., 19, 48, 91, 286, 332, 340, 370, 376  
Blinder, A. S., 14, 19, 22, 265, 268  
Bohn, H., 147, 162, 169  
Boivin, J., 9, 17, 20, 22  
Borio, C. E. V., 579, 603  
Boschen, J. F., 7, 29  
Brainard, W., 363–365  
Braun, R. A., 180, 186  
Briault, C., 241  
Brock, W. A., 42, 52, 102  
Brunner, K., 205  
Bruno, M., 157, 159  
Buiter, W., 144, 162  
Bullard, J., 7, 333–334  
Buttiglione, L., 466

- Cagan, P., 154–155, 157–159  
 Calvo, G. A., 154–155, 169, 172–174, 223, 226,  
 292, 320, 323, 567  
 Campbell, J. R., 28, 33  
 Campbell, J. Y., 74, 463, 465  
 Canover, F., 22  
 Canzoneri, M. B., 162, 168, 239–241, 253–254, 256,  
 422, 424, 458–459, 567, 578  
 Cargill, T. F., 603  
 Carlson, K. M., 12  
 Carlstrom, C. T., 38, 44, 94, 162, 251, 459–460, 462,  
 501–504, 525  
 Carvalho, C. V. de, 312  
 Cateau, G., 360  
 Cecchetti, S. G., 601  
 Chari, V. V., 21, 59, 62, 174, 178–179, 233–234,  
 240, 270, 286, 289, 292, 306, 316, 369  
 Chen, H., 38–39, 540–543, 556  
 Chinn, M., 264  
 Chortareas, G. E., 254–255  
 Christensen, J., 37  
 Christiano, L. J., 9, 17, 21, 23, 26, 59, 62, 162, 174,  
 178–179, 207–208, 214–215, 233–234, 270, 292,  
 307, 310, 335, 337, 366, 368, 505, 585, 600  
 Clarida, R., 334, 341, 353, 361–362, 397–398,  
 402–403, 412–415, 419–423, 428, 434, 447–448,  
 600, 603  
 Clower, R. W., 42, 97, 103  
 Cochrane, J. H., 25, 144–145, 162, 169, 335, 362,  
 475, 528, 594  
 Cogley, T., 308, 319  
 Coibion, O., 29, 204, 313, 334, 362  
 Coleman, W. J., III, 11  
 Collard, F., 567  
 Cook, T., 28, 597  
 Cooley, T. F., 41, 69, 78–79, 112–113, 207  
 Correia, I., 62, 174, 180, 182–183, 186, 345  
 Corsetti, G., 397, 414, 421–424, 438, 440  
 Cosimano, T. F., 599  
 Costain, J., 301  
 Cover, J. P., 14  
 Cox, J. C., 463  
 Craig, B., 127  
 Croushore, D., 42  
 Cukierman, A., 205, 224–225, 241, 243, 247, 251,  
 265, 270, 275  
 Cumby, R. E., 162, 168  
 Cúrdia, V., 38–39, 505, 540–544, 546–547, 556, 559  
 Currie, D., 423  
  
 Dahlquist, M., 465  
 Dai, Q., 467  
 D'Amico, S. G., 33, 36–37  
 Damjanovic, T., 348  
 Damjanovic, V., 348  
 Daniel, B., 162, 167  
 Dave, C., 91, 505  
 Dedola, L., 397, 414, 421, 423–424, 440  
  
 De Fiore, F., 505  
 De Gregorio, J., 66  
 de Haan, J., 251  
 DeJong, D. N., 59–60, 91, 505  
 Del Giovane, P., 466  
 Dellas, H., 567, 578  
 DelNegro, M. G., 39, 525, 552, 555–556, 559  
 Demery, D., 218, 505  
 Demirel, U. D., 504  
 Demopoulos, G. D., 147  
 Dennis, R., 309  
 De Prano, M., 12  
 Devereux, M. B., 286, 397, 424  
 Diamond, P. A., 120, 174, 182  
 Dib, A. C., 360  
 Diba, B. T., 162, 168  
 Diebold, F. X., 467  
 Dittmar, R., 360  
 Dixit, A. K., 256, 258, 320, 343  
 Domenech, R., 377  
 Dotsey, M., 66, 207, 214, 296, 298, 301, 303, 337  
 Drazen, A., 241  
 Driffill, J., 63, 241–243  
 Duck, N. W., 218, 505  
  
 Eggertsson, G. B. G., 517, 519, 521–522  
 Eichenbaum, M., 9, 14, 17, 20–21, 23, 26, 59,  
 207–208, 214–215, 292, 304, 307, 309–310, 312,  
 335, 337, 366, 368, 585, 600  
 Eijffinger, S., 251  
 Engel, C., 397, 440  
 Erceg, C. J., 206, 345, 366–368  
 Estrella, A., 306  
 Evans, C., 9, 17, 23, 26, 59, 214–215, 307, 310, 335,  
 337, 366, 368, 473, 600  
 Evans, G. W., 162, 205–206, 365  
  
 Faia, E., 375  
 Faig, M., 174, 182  
 Fair, R. C., 26  
 Faust, J., 19, 257–258  
 Feenstra, R. C., 102  
 Feldstein, M., 65, 69, 186, 188  
 Ferrero, A., 38–39, 540–543, 556  
 Ferri, J., 377  
 Fischer, M. E., 6  
 Fischer, S., 48, 54, 68–69, 96, 157, 159, 277  
 Fische, R., 476  
 Fisher, I., 47  
 Fisher, J. D. M., 309–310, 312  
 Fitzgerald, T. J., 162  
 Frankel, J. A., 262, 264  
 Freeman, R. T., 358  
 Friedman, B. M., 10, 12, 14, 567, 577, 579, 581, 597,  
 599, 604  
 Friedman, M., 9–13, 61–62, 83, 180, 194–195, 233,  
 259, 345, 581, 594



- Fudenberg, D., 235  
 Fuerst, T. S., 38, 44, 94, 162, 207–208, 251,  
 459–460, 462, 501–504, 525  
 Fuhrer, J. C., 26, 306, 308, 473, 476  
 Furfine, C. H., 581–582
- Gagnon, J. M., 33, 36–37  
 Gaiotti, E., 22  
 Galí, J., 19, 304–305, 307, 312, 319, 330, 334,  
 340–342, 353, 361–362, 365, 368–370, 376, 381,  
 397–398, 402–403, 412–415, 419–424, 428,  
 434–435, 437, 442, 447–448, 600  
 Gallmeyer, M. F., 475  
 Gambetti, L., 22  
 Garcia de Paso, J. I., 241  
 Garfinkel, M., 251  
 Gatti, R., 258  
 Gavin, W. T., 360  
 Gerlach, S., 270, 358  
 Gertler, M., 148–149, 298–300, 304–305, 307, 312,  
 330, 334, 341, 353, 361–362, 365, 368–370, 377,  
 397–398, 402–403, 412–415, 419–423, 428, 434,  
 447–448, 478, 489–496, 501, 504–505, 508,  
 547–548, 550, 556, 600, 603  
 Giannoni, M., 22, 525  
 Giavazzi, F., 259  
 Gilchrist, S., 36–37, 495, 504–505  
 Gillman, M., 65  
 Goldfeld, S. M., 59, 66  
 Golosov, M., 295, 298, 301  
 Gomme, P., 66  
 Gonzales-Rozada, M., 300  
 Goodfriend, M., 42, 319, 368, 466, 534, 595–596  
 Goodhart, C. A. E., 603  
 Gordon, D. B., 19, 21, 222–223, 225, 227–228, 234,  
 236–238  
 Gorodnichenko, Y., 204, 334, 362  
 Grandmount, J., 103  
 Gray, J. A., 316  
 Greenwood, R. S., 36  
 Gretchen, C., 602–603  
 Grier, K. B., 146  
 Grossman, H. I., 162  
 Guidotti, P. E., 173–174  
 Gürkaynak, R. S., 28
- Hahn, T., 28  
 Haldane, A. G., 241  
 Hall, R. E., 264, 377  
 Hamada, K., 422  
 Hamilton, J. D., 21, 36–37, 467, 473, 581  
 Hansen, G. D., 21, 78, 112–113  
 Hanson, S., 36  
 Hartley, P., 112  
 Heller, D., 581  
 Henderson, D., 345, 366–368, 422, 424, 458–459,  
 567  
 Herrendorf, B., 170, 240–241, 254
- Hobijn, B., 303  
 Hoehn, J. G., 598  
 Hoffman, D. L., 59–60  
 Hollifield, B., 475  
 Holman, J. A., 59  
 Honkapohja, S., 205–206, 365  
 Hoover, K. D., 29  
 Hornstein, A., 334  
 Hosios, A. J., 375–376  
 Huang, K. X. D., 368  
 Hutchison, M. M., 19, 603
- Ihrig, J. E., 602–603  
 Imrohorolu, A., 66  
 Ingersoll, J. E., 463  
 Ireland, P. N., 13, 26, 59–60, 66, 79, 207, 214,  
 240–241, 267, 285, 338, 462, 515, 567, 594  
 Irons, J., 257  
 Ito, T., 603
- Jaffee, D., 478, 480  
 Jaimovich, N., 304  
 Jensen, C., 348, 360  
 Jensen, H., 255, 276, 348, 351, 358, 360–361  
 Joines, D. H., 146  
 Jones, R. A., 120  
 Jonsson, G., 254–255, 269, 274  
 Jordon, J., 12  
 Joyce, M. A., 33  
 Judd, J. P., 19, 59, 362  
 Jung, T., 522, 525  
 Justiniano, A., 308, 369
- Kahn, C. M., 91, 332  
 Karadi, P., 547–548, 550, 556  
 Kareken, J. H., 577  
 Kashyap, A. K., 594  
 Kasman, B., 579, 603  
 Katsimbris, G. M., 147  
 Keating, J. W., 7  
 Kehoe, P. J., 59, 62, 174, 178–179, 207, 212–214,  
 240, 256, 286, 289, 292, 304, 306, 316, 369  
 Kerr, W., 332  
 Khan, A., 345  
 Khan, H., 204  
 Kiley, M. T., 17, 20, 22, 294, 312, 334, 525, 528  
 Kimbrough, K. P., 174, 182  
 King, M., 241  
 King, R. G., 7, 9–11, 75, 91, 147, 207, 214, 286, 296,  
 298, 301, 303, 319, 332, 337, 345, 368  
 Kirsanova, T. C., 414, 422  
 Kiyatoki, N., 42, 120, 286, 492–500, 508,  
 547–548, 552  
 Klee, E., 601  
 Klenow, P. J., 204, 302–303, 313  
 Knell, M., 59  
 Kocherlakota, N. R., 162

- Kollman, R., 397  
 Kormendi, R. C., 7  
 Krause, M. U., 370  
 Kreps, D., 241  
 Krishnamurthy, A., 33, 36  
 Kryvtsov, O., 302–303, 360  
 Kurozumi, T., 335, 375  
 Kuttner, K. N., 9–10, 14, 28, 579, 597, 599, 604  
 Kydland, F. E., 221, 223, 233, 360
- Lago Alves, S. A., 370, 377  
 Lagos, R., 42, 120, 122–123, 125, 127, 135–136  
 Laidler, D. E. W., 59  
 Lambertini, L., 256, 343  
 Lane, P., 397  
 Lansing, K. J., 361, 363  
 Laseén, S., 307  
 Leahy, J., 298–300  
 Leduc, S., 397, 414, 421, 423–424, 440  
 Leeper, E. M., 9, 17, 19, 21, 23, 25, 29, 144–145, 151, 162  
 Leiderman, L., 154–155, 170, 172–173, 358  
 Leith, C., 414, 422, 442  
 Lengwiler, Y., 581  
 Levin, A. T., 26, 206, 268, 300, 307, 345, 366–368, 525  
 Levine, P., 238–239, 241, 256, 423  
 Li, C., 36  
 Linde, J., 307, 395  
 Lippi, F., 258  
 Liu, Z., 368  
 Liviatan, N., 241, 243, 247, 265, 275  
 Llosa, G., 335  
 Lockwood, B., 254–255, 269  
 Lohmann, S., 250–251  
 López-Salido, D., 38, 307, 330, 368–370, 534, 538–539, 556  
 Lowe, P., 358  
 Lubik, T. A., 26, 334–335, 362, 370, 448  
 Lucas, R. E., Jr., 2, 8, 16, 26, 59–60, 63, 65, 103–104, 106, 112–113, 123, 159, 184, 195, 198–200, 207–208, 277, 295, 298, 301
- Mankiw, N. G., 169, 172–174, 187, 191, 201, 203–204, 264, 285, 313, 476  
 Marchesiani, A., 593  
 Marshall, D., 473  
 Martin, A., 602  
 Maskin, E., 235, 243  
 Masson, P. R., 241  
 Matthey, J., 162  
 Mayer, T., 12  
 McCallum, B. T., 6, 42, 68, 103–104, 162, 218, 264–265, 279, 319, 338, 341, 347–348, 351, 358, 360, 451, 457–459, 465, 473–475, 506, 515, 598  
 McCandless, G. T., Jr., 1, 6–7  
 McGough, B., 475  
 McGrattan, E. R., 59, 286, 289, 292, 306, 316, 369
- McKay, A. E., 525–526, 528  
 Meade, E. E., 602–603  
 Meese, R. A., 162  
 Meguire, P. G., 7  
 Meiselman, D., 12  
 Meltzer, A. H., 205, 241  
 Mendicino, C., 360  
 Menzio, G., 2, 7, 67  
 Metzler, L., 148  
 Miao, J., 91, 505  
 Midrigan, V., 304  
 Mihov, I., 19, 21–22, 24, 29, 583, 585, 590, 596, 598–599  
 Miller, S. M., 147, 254–255  
 Mills, L. O., 7, 29  
 Minford, P., 258  
 Mino, K., 241  
 Miron, J. A., 476  
 Mirrlees, J. A., 174, 182  
 Mishkin, F. S., 14, 17, 20, 22, 199, 358, 467, 579, 603  
 Mitra, K., 333–334  
 Mizon, G. E., 63  
 Modigliani, F., 12, 280, 469, 537  
 Moen, E., 127  
 Monacelli, T., 397, 424, 434–435, 437–438, 440, 442, 447, 451, 505  
 Monnet, C., 7, 593  
 Moore, G. R., 26  
 Moore, J., 496–500, 548, 552  
 Mortensen, D. T., 366, 370–371  
 Morton, J., 579, 603–604  
 Motto, R., 505  
 Muench, T., 577  
 Mulligan, C. B., 61–62, 174, 178–179, 186  
 Muscatelli, A., 254
- Nakamura, E., 302–304, 312, 525–526, 528  
 Nakov, A., 301, 517, 521–522  
 Nason, J. M., 319  
 Natalucci, F., 505  
 Neiman, H. E., 146  
 Neiss, K. S., 305  
 Nelson, E., 38, 162, 279, 305–306, 319, 338, 341, 347, 351, 451, 534, 538–539, 543, 556  
 Nessen, M., 360  
 Niehans, J., 97  
 Niepelt, D., 167  
 Nolan, C., 241, 254, 348  
 Nosal, E., 120
- Obstfeld, M., 52–53, 397, 422  
 O’Flaherty, B., 258  
 Oh, S., 120, 251  
 Orphanides, A., 334–335, 362, 600
- Pagano, M., 259  
 Patinkin, D., 148, 280, 338

- Patterson, C., 525  
 Paustian, M., 38, 504, 525  
 Perez, S. J., 29, 362  
 Persson, T., 254–255  
 Phelen, C., 162  
 Phelps, E. S., 62, 169, 194  
 Philippopoulos, A., 269  
 Piazzesi, M., 467–468, 475  
 Pissarides, C. A., 366, 370–371, 377  
 Plosser, C. I., 10, 75, 147  
 Poole, W., 462, 563–567, 581, 605  
 Poterba, J. M., 169, 172  
 Prati, A., 581  
 Prescott, E. C., 66, 78–79, 221, 223, 233  
 Presenti, P., 397, 414, 422, 438  
 Primiceri, G., 22, 308, 369  
  
 Quadrini, V., 207  
 Quah, D., 19  
  
 Ramey, V. A., 9, 17, 22  
 Ramsey, F. P., 41, 175  
 Rasche, R. H., 59–60  
 Ravenna, F., 22, 303, 335, 370, 375–377, 465, 475  
 Rebelo, S., 75, 304, 460–461  
 Reichenstein, W., 21  
 Reinhart, V. R., 31, 37, 40, 509  
 Reis, R., 54–55, 69, 201, 203–204, 313, 601  
 Reynard, S., 60  
 Rigobon, R., 28  
 Ritter, J. A., 120, 128–129  
 Roberts, J. M., 305, 309  
 Rocheteau, G., 120, 127, 136  
 Rockoff, H., 174  
 Rogers, S., 358  
 Rogoff, K., 52–53, 241, 248–249, 251, 254, 397, 422, 458–459, 567  
 Roley, V. V., 28, 597  
 Romer, C. D., 19, 23, 29–30, 268  
 Romer, D. H., 19, 23, 29–30, 266–268, 285–286  
 Ropele, T., 308, 334  
 Rose, A. K., 358  
 Ross, S. A., 463  
 Rostagno, M., 505  
 Rotemberg, J. J., 26, 169, 172, 284, 286, 301, 317, 319, 342  
 Roubini, N., 257  
 Rudd, J. B., 265, 268, 305–306  
 Rudebusch, G. D., 24, 26, 36–37, 306–307, 358, 362–363, 465, 467, 469, 473, 475–476  
 Ruge-Murcia, F. J., 270  
 Rupert, P. M., 120  
 Rush, M., 14  
 Russell, T., 478, 480  
  
 Sachs, J., 257  
 Sack, B., 28, 36–37, 40, 362  
 Sala, L., 369–370  
  
 Sala-i-Martin, X., 61–62, 174, 178–179, 186  
 Salyer, K. D., 94, 107  
 Samuelson, P., 42  
 Sargent, T. J., 2, 4, 15–16, 30, 144–146, 148–149, 151, 157, 159–160, 188, 199–200, 271, 331, 365  
 Sbordone, A. M., 305, 308, 310, 330, 335, 368  
 Scadding, J., 59  
 Schaling, E., 241  
 Schelde-Andersen, P., 30  
 Schellekens, P., 254  
 Schindler, M., 120  
 Schmidt-Hebbel, K., 358  
 Schmitt-Grohé, S., 52, 335, 513–514  
 Schorfheide, F., 27, 334–335, 362, 448  
 Schwartz, A., 9–10, 13, 581  
 Seater, J. J., 6  
 Secchi, A., 22  
 Seppälä, J., 465, 475  
 Sheehan, R. G., 599  
 Sheffrin, S. M., 24  
 Shi, S., 120, 122–123, 128  
 Shiller, R. J., 162, 463  
 Shimer, R., 376–377  
 Shukayev, M. M., 360  
 Sichel, D. E., 59  
 Sidrauski, M., 42, 48, 66  
 Sims, C. A., 9, 13, 17, 19–20, 23–25, 29, 42, 144–145, 162, 168, 189, 193  
 Singleton, K. J., 14, 467  
 Small, D., 14, 595  
 Smets, F., 26, 307, 366, 369, 600  
 Smith, J. M., 473, 476–478, 507  
 Söderlind, P., 465  
 Söderström, U., 364, 369–370  
 Soller, E. V., 127  
 Solow, R., 41  
 Stehem, J., 581  
 Stein, J. C., 69, 594  
 Steinsson, J., 302–303, 312, 365, 525–526, 528  
 Stiglitz, J. E., 174, 181, 320, 478–480, 483  
 Stix, H., 59  
 Stock, J. H., 14, 59  
 Stockman, A., 112  
 Stokey, N., 112–113, 184  
 Strongin, S., 21, 599–600  
 Summers, L. H., 69, 251  
 Surico, P., 2, 4  
 Sutch, R., 469, 537  
 Svensson, L. E. O., 24, 26, 104, 107, 110, 113, 248, 254–256, 262, 269, 274, 307, 341, 348, 352, 354, 358–361, 365, 395, 465, 515, 562, 578  
 Swanson, E. T., 28, 36–38  
  
 Tabellini, G., 241, 254–255  
 Tambalotti, A., 304, 308, 369  
 Taylor, J. B., 7, 26, 264–265, 268, 278, 283, 289, 291, 334, 358, 361, 473, 476–478, 507, 599

- Teles, P., 59, 62, 174, 180, 182–183, 186, 345  
 Temple, J. B., 267  
 Teranishi, Y., 522, 525  
 Thomas, C., 370, 375  
 Tieslau, M. A., 59–60  
 Tinbergen, J., 563  
 Tirole, J., 243  
 Tobin, J., 10, 42, 69, 97, 533, 539  
 Trabant, M., 313, 335  
 Trehan, B., 19, 147, 169, 172, 361  
 Trejos, A., 120, 122, 135  
 Trigari, A., 369–370, 377  
 Tristani, O., 466, 505  
 Tsutsui, S., 241  
 Tuesta, V., 335
- Ueberfeldt, A., 360  
 Uhlig, H., 19, 74, 85  
 Ulph, A., 63  
 Uribe, M., 52, 335, 513–514
- Van Zandweghe, W., 335, 375  
 Vayanos, D., 36, 469–473, 537  
 Vestin, D., 354, 360  
 Vickers, J., 241  
 Vila, J., 469–473, 537  
 Viñals, J., 603  
 Vissing-Jorgensen, A., 33, 36
- Wallace, N., 146, 148, 151, 188, 199–200,  
 533–534, 577  
 Waller, C. J., 127, 257, 593  
 Walsh, C. E., 19, 22, 24, 28, 36, 147, 169, 172, 241,  
 251, 254–255, 257–258, 335, 341, 351, 358,  
 360–361, 365–366, 370–371, 375–377, 537, 597  
 Wang, P., 80, 99–100, 103  
 Watanabe, T., 522, 525  
 Watson, M. W., 7, 9, 14, 19, 59, 91, 207, 214, 286  
 Weber, W. E., 1, 6–7, 207  
 Wei, M., 36  
 Weil, D. N., 476  
 Weiss, A., 479–480, 483
- Werning, I., 522  
 West, K. D., 162, 264  
 Whelan, K., 305–306  
 Whitesell, V., 590, 594  
 Wickens, M., 91, 505  
 Wieland, J., 521  
 Wieland, V., 365  
 Williams, J., 37, 365, 395, 475  
 Williamson, S. D., 120, 207, 484–485, 488, 496  
 Willis, J., 204, 313  
 Wilson, R., 241  
 Wohar, M., 476  
 Wolman, A. L., 296, 301, 303, 334, 345  
 Wood, P., 579, 603–604  
 Woodford, M., 26, 144–145, 162, 189, 286, 307, 319,  
 337–338, 341–342, 344, 347–348, 352, 358–362,  
 365, 381, 460, 474, 505, 517, 519, 522, 544,  
 546–547, 556, 559, 562, 578, 590  
 Woolley, J., 24  
 Wouters, R., 26, 307, 366, 369, 600  
 Wren-Lewis, S., 414, 422, 442  
 Wright, R., 2, 7, 42, 67, 120, 122–123, 125, 127,  
 135–136  
 Wu, J. C., 32, 36–39, 467, 473  
 Wu, T., 467, 469
- Xia, F. D., 32, 39, 467  
 Xie, D., 460–461
- Yates, A., 254, 360  
 Yellen, J. L., 285  
 Yetman, J., 424  
 Yip, C. K., 80, 99–100, 103  
 Younes, Y., 103  
 Yun, T., 26, 300, 319
- Zakrajšek, E., 36–37  
 Zha, T., 9, 17, 23, 25, 29  
 Zhang, Y., 360  
 Zhou, R., 59  
 Zin, S. E., 475  
 Zu, Z., 204

## Subject Index

- AD-AS frameworks, 331–332
- Adverse selection, 479–484, 488, 505
- Affine term structure models, 455, 467–469, 476–478
- Agency costs, 478–479, 488–491, 496, 501–505
- Aggregate demand–aggregate supply (AD-AS) models, 331–332
- Aggregate supply–demand (AS-IS-LM) models, 320
- Aggregate supply relationship, 225, 233
- Arbitrageurs, 470–473
- Argentina, 173
- Asset prices
  - asset pricing wedges, 537–538, 556–557
  - and balance sheet policies, 533–534
  - and quantitative easing (QE), 34–38
- Atomistic agents, 238
- Auditing costs, 490–491
- Austria, 30
- Average inflation targeting, 360
  
- Balance sheet policies, 40, 515, 532–556
  - and asset prices, 533–534
  - asset pricing wedges, 537–538, 556–557
  - empirical evidence, 35–38
  - intermediation costs, 543–547
  - and liquidity effects, 552, 556
  - market segmentation, 540–543, 549
  - MIU model, 534–537
  - moral hazard, 547–552
  - resaleability constraints, 552–555
  - transaction costs, 538–543, 556
- Bank for International Settlements, 579, 604
- Banking. *See also* Central banks; Reserve market and federal funds rate, 601–602
  - interbank market, 492–495, 593–594
  - and money supply, 10, 13
  - moral hazard, 547–552
- Bank of Canada, 594
- Bank of England, 594
- Barro-Gordon model, 225, 233, 235, 238, 240, 254, 266, 270, 353
- Barter exchange, 120
  
- Bayesian estimation, 26–27
- Bias, inflation. *See* Inflation bias
- Bilateral exchange rates, 428
- Bonds
  - and inflation, 465
  - and interest rates, 36–37
  - market segmentation, 540–543
  - and money, 512–513
  - preferred habitat model, 469–470
  - search models, 128
- Borrowers. *See* Credit market models
- Brazil, 173
- Budget accounting, 138–144
- Budget identity, 137–143. *See also* Government
- Bundesbank, 577, 603
- Business cycles, 41
  - capital stock, 319–320
  - and money supply, 9–10
  - and policy shocks, 11–12, 23
  - political, 257–258
  
- Cagan’s model, 157–159
- Calvo model, 292–295, 299–300, 302–304, 310
- Canada, 60, 603–604
- Capital
  - firm-specific, 310–311
  - fluctuations in, 279
  - new Keynesian models, 319–320, 337
  - and output variations, 310
- Capital-labor ratio, 68–69
- Cash goods, 234
- Cash-in-advance (CIA) models, 98, 103–121, 129
  - calibration and simulations, 117–120
  - cash and credit goods, 112, 182
  - certainty case, 104–113
  - consumption, 104, 109–111
  - dynamics, 116–117
  - inflation, 113, 137
  - interest rates, 459–462
  - investment goods, 112
  - labor supply, 116, 181

- Cash-in-advance (CIA) models (cont.)  
 linear approximation, 130–132  
 liquidity effects, 109, 207  
 marginal utility of wealth, 107–108, 111–112  
 and MIU models, 109–111, 116–118  
 monetary shocks, 118  
 nominal interest rates, 106, 117  
 optimal inflation rate, 111–113  
 optimal taxation, 180–182  
 search theory, 120–129  
 steady state, 110–111, 116, 130  
 stochastic, 113–120  
 timing, 104  
 unanticipated shocks, 118  
 welfare costs of inflation, 111–114
- Central banks. *See also* Inflation bias; Operating procedures  
 asset prices, 34  
 balance sheet policies, 35–38, 40, 515, 532–556  
 budget identity, 138–139  
 channel system, 590–595  
 commitment/discretion regimes, 266, 346–354, 517–519, 522  
 credibility of, 239–240  
 currency union, 446–447  
 debt accommodation, 147  
 discretionary model, 223–234  
 estimation, 365  
 exchange rates, 406–407  
 forward guidance, 360, 524–531  
 incentives, 222, 232–233, 251–256  
 independence, 224, 248–251, 256  
 and inflation, 333–335, 353–354  
 institutional structure, 256–257  
 instruments, 562  
 and interest rates, 455, 509, 563, 593–594, 604  
 intermediate targets, 577  
 as lender of last resort (LOLR), 34  
 lending, 547  
 and monetary base, 580  
 and monetary union, 258–259  
 objectives, 342–344  
 open economy models, 406–407, 414–415, 424  
 policy rules, 221, 232–233  
 preferences, 223–224, 234–235, 247–251, 270  
 reputation, 222, 235–247  
 short-term interest rates, 32–33  
 targeting regimes, 358–363  
 targeting rules, 234–235, 259–264, 270  
 temptation/enforcement, 237  
 time inconsistency, 222, 233  
 Treasury transfers (RCB), 142  
 types, 235, 240–247, 269–270  
 uncertainty, 363–365  
 weight conservatism, 248  
 Channel system model, 590–595  
 Chisel-proof credibility, 239
- Classical dichotomy, 280, 410  
 Cointegration, 147  
 Commitment policy, 232, 240, 252–255, 259  
 and effective lower bound, 531  
 and forward guidance, 522, 524  
 in new Keynesian model, 347–351, 356, 360  
 in open economy, 416, 418  
 and targeting rules, 260, 264  
 Competition, imperfect, 233–234, 285–288  
 Constant elasticity of substitution (CES), 57  
 Consumption  
 CIA models, 104, 109–111  
 international consumption risk sharing, 401–402, 428–429  
 and leisure, 137  
 and market segmentation, 213–214  
 MIU models, 47, 50–51, 54, 68  
 new Keynesian models, 321–323, 336, 339–340  
 open economy models, 401–402  
 two-country models, 421–422  
 Corporate credit risk, 37  
 Corridor system, 591  
 Cost channel, 22, 335  
 Cost shocks, 340–341  
 Credibility, chisel-proof, 239  
 Credit channels, 478–479, 495–496  
 Credit-easing policies, 509, 547. *See also* Balance sheet policies  
 Credit goods, 234  
 Credit market models, 478–505  
 adverse selection, 480–482  
 agency costs, 488–491, 501–505  
 asymmetric information, 491  
 credit rationing, 478–479  
 farmers/gatherers, 496–500  
 financial accelerator, 504–505  
 general equilibrium models, 496–504  
 incomplete collateralization, 491  
 intermediary-to-intermediary credit flows, 491–495  
 monitoring costs, 484–488  
 moral hazard, 482–484  
 productivity shocks, 500–501, 503  
 and sticky prices, 504–505  
 Credit rationing, 478–479  
 Credit risk, 37  
 Credit view, 478  
 Currency unions, 442–447
- Debt, government, 148–150, 534  
 Debt accommodation, 147  
 Deficits  
 equilibrium seigniorage, 153–157  
 and hyperinflation, 158–159  
 and inflation, 145–147  
 and money supply, 146–147  
 Ricardian/non-Ricardian policies, 148–151, 168–169

- Deflation
  - explosive, 513
  - and nominal interest rates, 62
  - and price level, 360
- Delphic effect, 33
- Demand shock, negative, 531
- Denmark, 509
- Discount rate, 581–584, 587–588
- Discretionary policy, 221–234. *See also* Inflation
  - bias
  - equilibrium inflation, 158, 227–234, 240
  - inflation, 223–234
  - new Keynesian models, 349–353, 358–361
  - objectives, 223–225
  - and policy rules, 221, 232–233
  - sustainable plans, 240
  - time-consistent/time-inconsistent, 222, 233, 240, 265, 268
- Disinflation, 30–31
- Divine coincidence, 340
- DMP model, 370, 376–377
- Domestic markup shock, 417
- Double coincident of wants, 120
- Dynamic stochastic general equilibrium (DSGE)
  - models, 26–27, 38–39, 41. *See also* New Keynesian models
- Edgeworth complements, 74
- Effective lower bound (ELB), 31–33, 509–532.
  - See also* Zero lower bound (ZLB)
  - analytics, 518–522
  - equilibria, 516–518
  - forward guidance, 524–531
  - liquidity traps, 512–515
  - optimal commitment, 522–524
- Elasticity of money demand, 57–61
- Empirical evidence, 1–31
  - balance sheet policies, 35–38
  - business cycles, 9–12, 23
  - disinflation case studies, 30–31
  - fiscal theory of the price level, 168–169
  - Granger causality, 13–14
  - inflation bias, 265–267
  - interest elasticity of money demand, 59–61
  - long-run relationships, 1–2, 6–7
  - monetary shocks, 193
  - and monetary theory, 16
  - money demand, 59–61
  - narrative policy measures, 29
  - observationally equivalent equations, 15
  - and operating procedures, 579
  - price adjustment, 303
  - short-run relationships, 2–3, 9
  - structural econometric models, 25–27, 40
  - term structure, 473
  - VAR approach, 1, 17–25, 27
- Equilibrium
  - local uniqueness of, 332–335
  - Nash, 236, 242–243, 424
  - sequential, 241–242
  - small open economy model, 430–434
  - steady-state, 48–55, 73–74, 83–85, 110–111, 116, 130
  - two-country model, 404–405, 411, 448–449
  - types of, 244–247
- Equilibrium credit rationing, 479
- Equilibrium inflation, 158, 227–234, 240
- Equilibrium price level, 162–164
- Equilibrium seigniorage, 153–157
- European Central Bank, 258, 509, 577, 594
- Excess reserve ratio, 581
- Exchange rates
  - bilateral, 428
  - exchange rate peg, 437
  - fixed/flexible, 240
  - and nominal interest rates, 405–409
  - open economy, 433–434
  - real exchange rate, 429
  - targeting, 259
- Expectational traps, 234
- Expectations theory, 463–465, 511
- Factor-augmented VAR (FAVAR) approach, 19–20, 32
- Federal funds rate, 14, 475–476, 509, 581. *See also*
  - Effective lower bound (ELB)
  - and borrowed/nonborrowed reserves, 587–590, 595–598
  - and discount rate, 583–584
  - and GDP, 11
  - and inflation, 4–6
  - and monetary policy, 22–34
  - and price puzzle, 21
  - and shadow rate, 32
  - targeting, 31, 597–598, 601
  - Taylor rule, 361
- Federal Open Market Committee (FOMC), 28–29, 600
- Federal Reserve, 475, 581. *See also* Federal funds
  - rate; Reserve market
  - announcements, 28
  - balance sheet, 532, 582, 601
  - borrowed/nonborrowed reserves, 583, 585–588, 595–599
  - discount rate, 581–584, 587–588
  - and financial market crisis of 2007, 600
  - lagged interest rates, 362–363
  - maturity extension program (MEP), 35, 543
  - operating procedures, 595–603
  - policy rules, 600
  - QE2 program, 543
  - quantitative easing (QE) policies, 33–36
  - reverse repurchase operations, 602–603
  - structure, 258
  - target setting, 475–476
- Feedback rules, 15–16, 25
- Fiat money, 129

- Final goods-producing firms, 398, 402–403  
 Financial accelerator, 504–505  
 Financial markets. *See* Credit market models  
 Fiscal dominance, 145–147  
 Fiscal policy  
   definitions of, 142–143  
   and monetary policy, 142–145, 151  
   and money supply, 162–164  
   Ricardian/non-Ricardian, 145  
   and zero nominal interest rate, 520–521  
 Fiscal theory of the price level, 53, 162–169  
   empirical evidence, 168–169  
   equilibrium price level, 162–164  
 Fisher equation, 7–8, 130  
 Fisher hypothesis, 466–467  
 Fisher relationship, 47, 49  
 Flexible-price models, 207, 288  
 Flexible targeting rules, 260–262  
 Forward guidance, 360, 524–531  
 France, 603–604  
 Free primary good, money as, 186  
 Friedman rule, 83, 127, 137, 174–186, 345, 595
- General equilibrium models. *See* Cash-in-advance (CIA) models; Dynamic stochastic general equilibrium (DSGE) models; Money-in-the-utility function (MIU) model; New Keynesian models; Price adjustment models
- Germany, 30, 60, 603–604
- Government  
   budget identity, 137–143  
   and central bank independence, 250–256  
   debt, 148–150, 534  
   fiscal theory of the price level, 162–169  
   hyperinflation, 159–162  
   intertemporal budget constraint, 143–148, 151–153, 163, 167  
   optimal taxation, 170–188  
   Ricardian/non-Ricardian policies, 148–151, 166–169  
   seigniorage, 129, 140–142, 145–146, 153–157, 170–174  
   types, 241–247
- Granger causality, 13–14  
 Great Depression, 4  
 Great Moderation, 3–4, 7  
 Great Recession of 2008–2009, 370, 555  
 Gross domestic product (GDP)  
   and federal funds rate, 11  
   and inflation, 6–7  
   and interest rates, 4–5  
   and monetary policy, 22  
   and money supply, 4–10  
   and primary surplus, 169
- Growth. *See* Output growth
- High-powered money, 139–140, 142  
 Hodrick-Prescott (HP) filter, 3
- Hosios condition, 127  
 Humphrey-Hawkins Act, 596  
 Hungary, 30, 157  
 Hybrid price level inflation targeting, 360  
 Hyperinflation, 30  
   as bubble, 160–162  
   Cagan's model, 157–159  
   explanations of, 160  
   MIU model, 513  
   rational, 159–162
- Imperfect competition, 233–234, 285–288  
 Imperfect information models, 194–200, 215–219, 277  
 Imperfect pass-through, 437–440  
 Implementability condition, 184–185
- Income  
   elasticity, 59  
   and labor-leisure choice, 70  
   nominal, 12–13, 599  
   nominal income targeting, 264, 361
- Incomplete collateralization, 491  
 Indexation, 307–308, 312  
 Indexed bonds, 465  
 Individual rationality constraint, 253  
 Inefficiency gap, 368  
 Inflation. *See also* Hyperinflation; Inflation bias;  
   Inflation rate; Seigniorage  
   and anticipated policy changes, 31  
   bank/government types, 241–247  
   and bonds, 465  
   CIA models, 113, 137  
   closed economies, 415  
   and deficits, 145–147  
   deflation, 62, 360, 513  
   and discretionary policy, 223–234  
   disinflation, 30–31  
   equilibrium, 158, 227–234, 240  
   and federal funds rate, 4–6  
   forecasting, 359–360, 577–578  
   and GDP, 6–7  
   lagged, 354–357  
   marginal cost of, 305–306, 311, 328–330  
   and market segmentation, 212–214  
   and MIU model, 137, 513  
   and money growth, 1–7, 40, 67, 78, 120, 159–160, 200  
   and money supply, 1–2, 30–31  
   and nominal interest rates, 7, 63, 465–467  
   open economy models, 416–419  
   and output gap, 310  
   and output growth, 1–2, 6–7, 40, 199–200, 224  
   and political business cycle, 257  
   and prices, 304  
   regression of, 2  
   shocks, 340  
   targeting, 358–360



- and tax rates, 172, 188
  - and term structure, 465–467
  - unanticipated, 226
  - and unemployment, 7, 194, 267–268
  - welfare cost of, 61–66, 111–114
  - Inflation bias, 222–223, 227, 234–271
    - contract models, 251–256, 262
    - empirical evidence, 265–267
    - institutional structure, 256–259
    - and open economies, 266–267
    - persistence, 255, 269, 306–309, 354–358
    - preference models, 234, 247–251
    - rationale, 232
    - relocation, 255–256
    - repeated games, 234–240
    - reputational models, 222, 235–247
    - stabilization bias, 350, 353–354
    - targeting rules, 234–235, 251, 259–264
    - weight conservatism, 248, 262
  - Inflation rate, 223. *See also* Optimal inflation rate
    - and central bank independence, 256
    - and deficit, 145–147
    - and seigniorage revenue, 153–157
    - and sticky prices, 284
    - and supply shocks, 269
    - target, 260
    - and Taylor principle, 333–334
  - Inflation tax, 137–138, 146, 153, 158, 215, 313
    - and CIA model, 112–113, 116, 118
    - and Lucas model, 195
    - and MIU model, 62–63
    - and optimal inflation, 169, 172–175
    - and Ramsey problem, 178, 181–182
  - Informational frictions, 194–206
    - imperfect information models, 194–200, 215–219
    - learning model, 204–206
    - Lucas model, 195–200, 215–219, 277
    - sticky information, 200–204, 312–313
  - Inside money, 10–11
  - Instruments, 562–579
    - choice of, 563–568
    - instrument rules, 358, 361–364, 568
    - intermediate targets, 562, 570–578
    - policy, 563
    - policy rules, 568–570
  - Interbank interest rate, 581
  - Interbank market, 492–493, 581, 593–594
  - Interest elasticity of money demand, 57–61
  - Interest rate gap, 337
  - Interest rate peg, 456–458, 460–462
  - Interest rates, 456–467. *See also* Nominal interest rates; Term structure
    - and balance sheet policies, 35–36
    - and bond supply, 36–37
    - and central banks, 455, 509, 563, 593–594, 604
    - channel system, 590–595
    - CIA model, 459–462
    - expectations theory, 463–465
    - and GDP, 4–5
    - general equilibrium models, 459–462
    - liquidity traps, 78
    - long-term/short-term, 32–33, 462–467
    - MIU model, 51–53, 55, 63
    - and monetary policy, 11–12, 20, 473
    - and money supply, 14
    - and output, 14, 21–22
    - as policy instrument, 566–568
    - and prices, 78, 456–459
    - real interest rate, 336–337, 578
    - shadow rate, 32, 39
    - smoothing, 254, 363, 474
  - Intermediate goods-producing firms, 398, 402–403
  - Intermediation costs, 543–547
  - International risk sharing, 401–402, 428–429
  - Intertemporal budget constraint, 143–148, 151–153, 163, 167
  - Intertemporal nominal adjustment models, 282–285
  - Intertemporal optimality, 171
  - Investment-saving (IS) curve, 331–332
  - Islands model, 195–200, 215–219, 277
  - Israel, 173
  - Italy, 147, 361
- Japan, 60, 509, 603
- Kim-Wright model, 37
- Labor-leisure choice
  - CIA models, 111–112
  - MIU models, 70
- Labor market frictions
  - currency unions, 446
  - search and matching, 375
  - sticky wages and prices, 366–369, 442
  - unemployment, 369–377
- Labor supply
  - CIA models, 116, 181
  - MIU models, 116
  - and monetary shocks, 199
  - price-adjustment models, 292
  - and real wages, 194, 211–212
  - shopping-time models, 100–101
- Lagged inflation, 354–357
- Large-scale asset purchases (LSAP), 33. *See also* Quantitative easing (QE)
- Learning model, 204–206
- Leisure
  - CIA models, 98, 137
  - MIU models, 68
  - shopping-time models, 98–100, 102
- Lender of last resort (LOLR) function, 34
- Lending. *See* Credit market models
- Level factor, 468. *See also* Term structure

- Limited-participation models, 208–215
- Liquidity effects, 206–208. *See also* Portfolio rigidities  
 and balance sheet policies, 552, 556  
 CIA models, 109, 207  
 interest rates, 78  
 limited-participation models, 208–215  
 liquidity-effect model, 214
- Liquidity services, 107–109
- Liquidity traps, 512–515
- Loans. *See* Credit market models
- Local currency pricing (LCP), 440–441
- Long-run relationships, 1–2, 6–7
- Long-term yields, 37
- Loss function, 225, 229, 342–343
- Lucas model, 195–200, 215–219, 277
- Lucas supply function, 225, 277, 281
- Macrofinance, 467–473
- Marginal cost of inflation, 228, 234–235, 238, 254, 261
- Marginal utility of consumption  
 and balance sheet policies, 533, 541–546  
 and effective lower bound, 510–511  
 and MIU model, 46, 50–51, 54, 68, 74, 80, 87–88  
 and new Keynesian model, 338–339  
 and open economy, 400, 446
- Marginal utility of leisure, 68, 74, 99, 100, 196, 216, 460
- Markets. *See* Credit market models
- Market segmentation, 212–214, 540–543, 549
- Maturity extension program (MEP), 35, 543
- Mechanism design theory, 256
- Menu costs, 285–286, 298, 301–304, 307
- Mixed-strategy equilibrium, 244, 247
- Models, economic  
 AD-AS, 331–332  
 affine term structure models, 455, 467–469, 476–478  
 AS-IS-LM, 320  
 Barro-Gordon model, 225, 233, 235, 238, 240, 254, 266, 270, 353  
 Calvo model, 292–295, 299–300, 302–304, 310  
 channel system model, 590–595  
 CIA, 98, 103–121, 129  
 dynamic stochastic general equilibrium (DSGE) models, 26–27, 38–39, 41  
 flexible-price, 207, 288  
 imperfect information, 194–200, 215–219, 277  
 intertemporal nominal adjustment, 282–285  
 islands model, 195–200, 215–219, 277  
 learning model, 204–206  
 limited-participation, 208–215  
 Lucas model, 195–200, 215–219, 277  
 MIU, 41–93, 120–121, 152, 182  
 monopolistic competition, 278, 285–288  
 new Keynesian, 26, 319–388  
 open economy, 397–449  
 overlapping generations, 148, 195, 258, 515  
 preferred habitat model, 469–473  
 price adjustment, 282–301  
 principal agent, 251  
 rational-expectations, 26, 159, 204–205  
 repeated game model, 234–240  
 representative agent, 167, 496  
 reputational, 241  
 Rogoff's model, 249–251, 261–262  
 search and matching, 370, 375–377  
 search models, 120–129  
 shopping-time, 61, 98–103, 129  
 staggered nominal adjustment, 283–284, 289–295  
 state-dependent pricing (SDP), 295–301, 303  
 structural econometric models, 25–27, 40  
 time-dependent pricing (TDP), 288–296  
 two-country model, 398–424  
 two-party model, 257–258  
 vector autoregressions (VARs), 1, 17–25, 27  
 vector error correction model (VECM), 147
- Monetary base, 139, 565–566, 579–580
- Monetary dominance, 144–145
- Monetary policy. *See also* Commitment policy;  
 Discretionary policy; Operating procedures  
 and business cycles, 11–12, 23  
 cost channel of, 22, 335  
 and effective lower bound, 515–531  
 European, 603–604  
 and fiscal policy, 142–145, 151  
 and funds rate, 22, 29  
 and interest rates, 11–12, 20, 473  
 narrative measures of, 29  
 and nominal income, 12–13  
 and output, 8–14  
 policy rules, 15, 25–26  
 and term structure, 473–478  
 and theory, 16
- Monetary shocks, 118–120  
 CIA models, 118  
 empirical evidence, 193  
 and labor supply, 199  
 MIU model, 70, 80–81  
 new Keynesian model, 337  
 and open economy, 435  
 and output growth, 193, 200  
 price adjustment models, 298
- Monetary targeting, 577
- Monetary targets, 596–597, 604
- Money, interest on, 66–67
- Money demand  
 empirical evidence, 59–61  
 interest elasticity of, 57–61  
 log-log specification, 60, 66  
 modeling, 42, 97  
 and nominal interest rate, 60–61  
 and price level, 162–163

- and utility, 44
- and welfare costs of inflation, 65–66
- Money growth
  - anticipated, 77
  - and deficits, 146–147
  - Federal Reserve targets, 28, 596–597
  - and inflation, 1–7, 40, 67, 78, 120, 159–160, 200
  - and nominal interest rates, 7–8, 206–207
  - and output, 7, 9
  - as policy indicator, 570–571, 577
  - shocks, 70, 80–81, 118–120
- Money-in-the-utility function (MIU) model, 41–93, 120–121, 152, 182. *See also* Solving MIU model
  - balance sheet policies, 534–537
  - baseline parameter values, 79
  - calibration, 78–80
  - and CIA models, 109–111, 116–118
  - classical dichotomy, 280
  - consumption in, 47, 50–51, 54, 68
  - decision problem, 70–73
  - end-of-period holdings, 45
  - Fisher relationship, 47, 49
  - imperfect information model, 195
  - and inflation, 137, 513
  - interest elasticity of money demand, 57–61
  - interest rates, 51–53, 55, 63
  - labor supply, 116
  - limitations of, 61
  - linear approximation, 74–78, 85–90
  - marginal utility of money, 129
  - monetary shocks, 70, 80–81
  - multiple equilibria, 55–57
  - negative nominal rates, 511–512
  - neutrality of money, 49
  - and new Keynesian models, 319–327
  - nominal rigidities, 278–282
  - and price level, 55–57
  - representative households, 44–48, 70–73
  - and shopping-time models, 98–99
  - simulations, 69–70, 80–82
  - steady-state equilibrium, 48–55, 73–74, 83–85
  - sticky-wage model, 314–315
  - superneutrality/nonsuperneutrality of money, 50, 66–69
  - time-varying money stock, 53–55
  - and transaction cost approach, 103
  - wage rigidity, 278–282
  - welfare cost of inflation, 61–66
- Money multipliers, 579–581
- Money supply
  - and business cycles, 9–10
  - and deficits, 146–147
  - and fiscal policy, 162–164
  - and GDP, 4–10
  - high-powered money, 139–140, 142
  - and imperfect information, 199
  - and inflation, 1–2, 30–31
  - and interest rates, 14
  - as intermediate target, 574
  - measures of, 13
  - and monetary base, 565–566
  - nominal, 19
  - and nominal interest rate, 14, 459
  - and output, 7, 10–11, 15, 20–23
  - and price level, 145, 148, 151–153, 162
  - variations in, 144
- Money zero maturity (MZM), 59
- Monitoring costs, 484–488
- Monopolistic competition models, 278, 285–288
- Moral hazard, 482–484, 547–552
- Multiplicative uncertainty, 363–364
- Nash bargaining, 374
- Nash equilibrium, 236, 242–243, 424
- Neoclassical growth model, 41
- Neutrality of money, 49, 194
- New Keynesian models, 26, 319–388. *See also* Open economy models
  - capital stock, 319–320, 337
  - commitment/discretion regimes, 346–354
  - consumption, 321–323, 336, 339–340
  - discretionary policy, 349–353, 358–361
  - economic shocks, 339–341
  - effective lower bound (ELB), 516–517, 525
  - endogenous persistence, 354–357
  - firms, 323–325
  - households, 321–323
  - inflation targeting, 358–360
  - instrument rules, 358, 361–364
  - level/slope factors, 468
  - linearized IS curve, 331–332
  - linearized Phillips curve, 327–330
  - local uniqueness of equilibrium, 332–335
  - market clearing, 325–327
  - and MIU model, 319–327
  - monetary transmission mechanism, 336–338
  - nominal interest rates, 320, 325–326, 333, 366–369
  - nominal rigidities, 345
  - open economy models, 413–414, 434
  - policy objectives and trade-offs, 342–345
  - price stability, 345
  - real interest rate, 336–337
  - stabilization bias, 350, 353–354
  - sticky wages and prices, 325–326, 366–375
  - targeting regimes, 358–363
  - targeting rules, 348
  - Taylor principle/Taylor rule, 332–335, 358, 361
  - two-equation model, 331–332
  - uncertainty in, 363–365
  - unemployment, 369–377
- New Keynesian Phillips curve (NKPC), 304–313, 378–381
  - derivation of, 328, 378–380
  - linearized, 327–330
  - marginal cost, 305–306, 311, 328–330

- New Keynesian Phillips curve (NKPC) (cont.)
  - nominal price rigidity, 309–312
  - persistence, 306–309
  - and SIPC, 312–313
  - utility approximation, 381–388
- New Zealand, 256, 258, 358, 592
- No-arbitrage models. *See* Affine term structure models
- Nominal income, 12–13, 599
- Nominal income targeting, 264, 361
- Nominal interest rates. *See also* Effective lower bound (ELB); Interest rates
  - CIA models, 106, 117
  - and deflation, 62
  - and equilibrium price level, 163
  - and exchange rates, 405–409
  - and government budget constraint, 151–153
  - and inflation, 7, 63, 465–467
  - interest rate peg, 456–458, 460–462
  - liquidity effects, 78, 512–515
  - and market segmentation, 214
  - MIU model, 511–512
  - and money demand, 60–61
  - and money growth, 7–8, 206–207
  - and money supply, 14, 459
  - negative, 510–511
  - new Keynesian models, 320, 325–326, 333, 366–369
  - nominal anchor, 458
  - open economy, 406–407
  - optimal inflation rate, 83, 129
  - search models, 126–129
  - targeting, 578
  - zero, 515
- Nominal rigidities
  - dynamic stochastic general equilibrium (DSGE) models, 319
  - imperfect competition, 285–288
  - imperfect pass-through, 437–440
  - intertemporal nominal adjustment models, 282–285
  - menu costs, 285–286, 303
  - MIU one-period model, 278–282
  - monetary/productivity shocks, 282, 298–300
  - monopolistic competition model, 286–288
  - new Keynesian models, 345
  - new Keynesian Phillips curve (NKPC), 304–312
  - open economy models, 437–442
  - productivity shocks, 298–300
  - staggered nominal adjustment model, 283–284, 289–295
  - state-dependent pricing (SDP) models, 295–301, 303
  - time-dependent pricing (TDP) models, 288–296
  - wages, 368–369
- Nonindexed tax systems, 186–188
- Nonmonetary model, 41
- Nonprice rationing, 456, 583–584
- Non-Ricardian regime, 145, 149, 167–169
- Nonsuperneutrality of money, 67–69
- Nontraded goods, 442
- No Ponzi condition, 143
- Odyssean effect, 33
- Open economy models, 397–449
  - central banks, 406–407, 414–415, 424
  - and closed-economy NK model, 413–414, 434
  - consumption in, 401–402
  - exchange rates, 433–434
  - imperfect pass-through, 437–440
  - inflation, 416–419
  - international risk sharing, 401–402, 428–429
  - local currency pricing, 440–441
  - monopolistic competition, 414
  - nominal rigidities, 437–442
  - policy coordination, 422–424
  - price stabilization, 414–415
  - productivity shocks, 436
  - small open economy model, 424–437
  - sticky prices, 442
  - two-country models, 398–424
- Operating procedures, 561–605. *See also* Reserve market
  - channel system, 590–595
  - and empirical evidence, 579
  - federal funds rate, 587–590
  - Federal Reserve, 595–603
  - instrument rules, 358, 361–364, 568
  - intermediate targets, 562, 570–578
  - money multipliers, 579–581
  - operating targets, 562
  - policy instruments, 562–563
  - policy rules, 567–570
  - Poole's analysis, 563–568, 599
  - real effects of, 578
- Opportunity cost
  - CIA models, 106–107, 129
  - and market segmentation, 212
  - MIU models, 47–48, 61, 66–67
- Optimal inflation rate, 61–62
  - CIA models, 111–113
  - Friedman rule, 83, 137, 174–186, 345
  - nominal interest rates, 83, 129
- Optimal quantity of money, 62, 67
- Optimal taxation, 169–188. *See also* Optimal inflation rate
  - CIA model, 180–182
  - money as intermediate input, 182–186
  - nonindexed tax systems, 186–188
  - optimal seigniorage, 170, 172–174
  - partial equilibrium model, 170–173
  - Ramsey problem, 175–180
- Output gap, 310, 337
- Output growth
  - and inflation, 1–2, 6–7, 40, 199–200, 224
  - and interest rates, 14, 21–22

- and monetary policy, 8–14
- and monetary shocks, 193, 200
- and money growth, 7, 9
- and money supply, 10–11, 15, 20–23
- neoclassical model, 41
- and productivity shocks, 118–120
- real, 13–14
- Outside money, 11
- Overlapping generations model, 148, 195, 258, 515
- Overnight interbank interest rate, 562–563
  
- Persistence, inflation, 255, 269, 306–309, 354–358
- Phillips curves, 268. *See also* New Keynesian Phillips curve (NKPC)
- Pigou effect, 338
- Poland, 30
- Policy coordination, 422–424. *See also* Monetary policy
- Policy instruments. *See* Instruments
- Policy irrelevance hypothesis, 200
- Policy Targets Agreement (PTA), 256
- Poole's analysis, 563–568, 599, 604
- Pooling equilibrium, 244–247, 269
- Portfolio rigidities
  - limited-participation models, 208–215
  - market segmentation, 212–214
- Preferred habitat model, 469–473
- Price adjustment models, 282–301. *See also* New Keynesian Phillips curve (NKPC)
  - costs of adjustment, 285–286, 296, 313
  - empirical evidence, 303
  - firm-specific shocks, 298
  - frequency of adjustment, 300–301, 312
  - frictions in timing, 301
  - labor supply, 292
  - monetary shocks, 298
  - productivity shocks, 298
  - quadratic costs model, 284–285
  - selection effect, 295, 301
  - speed of adjustment, 312
  - staggered nominal adjustment models, 283–284, 289–295
  - state-dependent pricing (SDP), 295–301, 303
  - time-dependent pricing (TDP), 288–296
- Price level
  - equilibrium, 162–164
  - fiscal theory of, 53, 162–169
    - and government debt, 151
    - indeterminacy, 457–458
    - and interest rates, 456–459
  - MIU model, 55–57
  - and money demand, 162–163
  - and money supply, 145, 148, 151–153, 162
  - targeting, 360–361
  - and taxation, 186
- Price markup, 368
- Price puzzle, 21–23
  
- Prices. *See also* Fiscal theory of the price level; Sticky prices
  - and imperfect pass-through, 437–440
  - and inflation, 304
  - and interest rates, 78
  - microeconomic data, 302–304
  - and misperceptions, 199
  - and money supply, 2
  - price changes, 301–303, 313
  - price dispersion, 343–344, 442
  - reference/regular, 304
  - and sticky information, 200–204
  - and wages, 194–195, 278, 283
- Price shocks, 340
- Price stability
  - new Keynesian models, 345
  - open economy models, 414–415
- Principal agent models, 251
- Producer currency pricing (PCP), 440
- Productivity shocks, 70
  - and credit markets, 500–501, 503
  - and money growth, 118–120
  - nominal rigidity, 298–300
  - open economy models, 436
  - and price adjustment, 298
- Public finance, 137–192. *See also* Government deficits, 145–147
  - equilibrium seigniorage, 153–157
  - fiscal theory of the price level, 162–169
  - government budget identity, 137–143
  - hyperinflation, 159–162
  - optimal taxation, 169–188
  - Ricardian/non-Ricardian fiscal policies, 148–151, 166
- Purchasing power parity (PPP), 405–406
  
- Quantitative easing (QE), 33–40. *See also* Balance sheet policies
  - asset prices and yields, 35–38
  - and macroeconomy, 38–40
- Quantity theory of money, 2, 163
  
- Ramsey problem, 175–180, 183–185
- Rational-expectations models, 26, 159, 204–205
- Rational hyperinflation, 159–162
- RCB, 142
- Real balance effect, 515
- Real exchange rate, 429
- Real interest rate, 336–337, 578
- Real marginal cost, 305–306, 311, 328, 412
- Real output, 13–14
- Real wages, 194, 211–212, 280, 366
- Repeated game model, 234–240
- Representative agent models, 167, 496
- Reputational models, 241
- Reserve market, 581–590. *See also* Credit market models; Federal Reserve
  - borrowed/nonborrowed reserves, 583, 585–588
  - channel system, 590–595

- Reserve market (cont.)  
 discount rate, 581–584, 587–588  
 interest on reserves, 595  
 traditional model of, 582–587
- Revenue. *See* Seigniorage; Taxation
- Reverse causation argument, 10–11
- Ricardian regime, 145, 148–151, 166–169
- Risk premiums, 475
- Risk sharing, international, 401–402, 428–429
- Rogoff model, 249–251, 261–262
- Search and matching models, 370, 375–377
- Search models, 120–129  
 bonds, 128  
 centralized and decentralized markets, 122–126  
 day and night markets, 122  
 marginal utility of money, 129  
 nominal interest rates, 126–129  
 trading, 120–121  
 welfare costs of inflation, 127–129
- Seigniorage, 129  
 and budget constraint, 145–146  
 equilibrium, 153–157  
 and inflation rate, 155–156  
 measures of, 142  
 optimal tax approach, 170, 172–174  
 sources of, 140–141
- Separability, 410
- Separating equilibrium, 244–247
- Sequential equilibrium, 241–242
- Shadow interest rate, 32, 39
- Shopping-time models, 61, 98–103, 129  
 Friedman's rule, 182–183  
 labor supply, 100–101  
 and MIU models, 98–99
- Short-run relationships, 1–3, 9
- Sidrauski model. *See* Money-in-the-utility function (MIU) model
- Signaling channel, 37
- Slope factor, 468. *See also* Term structure
- Small open economy model, 424–437  
 and closed-economy NK model, 434  
 equilibrium conditions, 430–434  
 firms, 429–430  
 households, 424–428  
 international risk sharing, 428–429  
 local currency pricing, 440–441  
 monetary policy specification, 433–437  
 world output, 432
- Solving MIU model, 83–93  
 capital accumulation, 86  
 Euler condition, 88  
 Fisher equation, 90  
 goods market clearing, 86  
 labor hours, 86–87  
 linear approximation, 85–90  
 linear rational-expectations models, 91–93  
 marginal product. *See* real return condition, 89  
 marginal utility of consumption, 87–88  
 Matlab code, 93  
 money holdings, 89  
 production function, 86  
 real money growth, 89–90  
 Speed limit policy, 361  
 Stabilization bias, 350, 353–354  
 Stackelberg leader, 424  
 Staggered nominal adjustment models, 283–284, 289–295  
 State-dependent pricing (SDP) models, 295–301, 303  
 Steady-state equilibrium  
 CIA models, 110–111, 116, 130  
 MIU models, 48–55, 73–74, 83–85  
 Sticky information, 200–204, 312–313  
 Sticky information Phillips curve (SIPC), 202–203, 312–313  
 Sticky prices, 312–313. *See also* New Keynesian models; New Keynesian Phillips curve (NKPC); Nominal rigidities; Price adjustment models and credit markets, 504–505  
 currency union, 446  
 and inflation rate, 284  
 new Keynesian models, 325–326, 366–375, 377  
 open economy models, 442  
 and reference prices, 304  
 two-country model, 411–413  
 and wages, 278  
 Sticky wages  
 currency union, 446  
 new Keynesian models, 366–369  
 search and matching models, 376–377  
 sticky-wage MIU model, 314–315  
 St. Louis equations, 12  
 Stochastic models. *See* Dynamic stochastic general equilibrium (DSGE) models  
 Strict targeting rules, 262–264  
 Structural econometric models, 25–27, 40  
 Superneutrality of money, 50, 66–68  
 Supply shocks, 24, 242, 248–249  
 Surplus, 62–63  
 Sweden, 509  
 Swiss National Bank, 577  
 Switzerland, 509, 511, 603–604
- Targeting regimes, 358–363  
 Targeting rules, 234–235, 251, 259–264, 348  
 Taxation. *See also* Optimal taxation  
 inflation tax, 62–63, 69, 113, 137, 169, 173–174  
 and interest on money, 67  
 nonindexed, 186–188  
 and price levels, 186  
 Ricardian regime, 167  
 and unanticipated shocks, 173–174  
 Tax-smoothing model, 171–172  
 Taylor model, 283–284, 289–292, 294–295, 303  
 Taylor principle, 332–335, 362  
 Taylor rule, 333–335, 358, 361–362, 600

- Term structure, 455, 462–478
  - affine models of, 455, 467–469, 476–478
  - empirical evidence, 473
  - expectations theory of, 463–465, 511
  - and expected inflation, 465–467
  - and monetary policy, 473–478
  - preferred habitat model, 469–473
- 3-month Treasury bill rate (3MTB), 79
- Time consistency/inconsistency, 222, 233, 240, 265, 268
- Time-dependent pricing (TDP) models, 288–296
- Timeless perspective commitment policy, 347–348
- Time series estimates, 304, 309
- Tobin effect, 69
- Traditional non-Ricardian regime, 149
- Transactions, 97–136
  - CIA models, 98, 103–120
  - costs models, 97
  - and MIU models, 103
  - resource costs of, 98–103
  - search models, 120–129
  - shopping-time models, 98–102
  - and utility, 97
- Transfer function, 252–254
- Treasury, U.S.
  - bills, 34–35, 37–38
  - budget constraint, 138
- Trigger strategy, 236, 238–239
- Two-country models, 398–424
  - and closed-economy NK model, 413–414
  - consumption, 421–422
  - domestic policy, 415–419
  - equilibrium conditions, 404–405, 411, 448–449
  - exchange rates, 405–409
  - firms, 398, 402–404
  - flexible prices, 409–410
  - households, 398–400
  - international consumption risk sharing, 401–402, 428–429
  - local currency pricing, 441
  - policy coordination, 422–424
  - sticky prices, 411–413
  - welfare, 414–415, 419–422
- Two-equation model, 331–332
- Two-party model, 257–258
  
- Uncertainty, 363–365
- Uncovered interest rate parity (UIP), 405–406, 429
- Unemployment
  - DMP model, 370, 376–377
  - and inflation, 7, 194, 267–268
  - new Keynesian models, 369–377
  - persistence, 255, 269
  - and quantitative easing (QE), 39–40
- United Kingdom, 60, 603–604
- United States. *See also* Federal Reserve
  - deficits in, 146, 169
  - GDP and money supply, 9–10, 14, 78
  - Great Depression, 4
  - Great Recession of 2008–2009, 370
  - inflation in, 2–4, 157, 160, 267–268, 270
  - interest rates, 14, 469
  - monetary policy in, 2–4, 9, 11–12, 21
  - money demand, 60, 577
  - seigniorage, 172, 174
  - unemployment, 267–268
  - welfare cost of inflation, 64–65
- Utility functions
  - Cobb-Douglas, 68
  - constant elasticity of substitution (CES), 57
  - MIU models, 43, 54, 57
  - and money growth, 69
  
- Vector autoregressions (VARs), 1, 17–25, 27
  - criticisms of, 23–25
  - factor-augmented VAR (FAVAR) approach, 19–20
  - money and output, 20–23
- Vector error correction model (VECM), 147
  
- Wage markup, 368
- Wage rigidities. *See also* Nominal rigidities
  - intertemporal nominal adjustment models, 282–285
  - MIU one-period model, 278–282
  - new Keynesian models, 366–369
  - search and matching frictions, 375
  - staggered adjustment models, 294
- Wages. *See also* Sticky wages
  - nominal rigidities, 368–369
  - and prices, 194–195, 278, 283
  - real, 194, 211–212, 280, 366
- Wealth effects, 337–338, 412
- Weight conservatism, 248, 262
- Welfare costs of inflation
  - CIA models, 111–114
  - MIU models, 61–66
  - and money demand, 65–66
  - search models, 127–129
- Wicksellian real interest rate, 337
  
- Zero lower bound (ZLB), 31. *See also* Effective lower bound (ELB)
- Zero nominal interest rate, 515, 520–521

